

HW 6 Due: 7 mar 2025

1. The truth value of a logical expression is defined recursively as:

- The truth value of t is t .
- The truth value of f is f .
- The truth value of $(x_1 \wedge x_2)$ is t if both x_1 and x_2 have truth value t , it is f otherwise.
- The truth value of $(x_1 \vee x_2)$ is f if both x_1 and x_2 have truth value f , it is t otherwise.
- The truth value of $\neg(x)$ is f if x has truth value t , it is t otherwise.

Define a CFG that generates the following language over $\{t, f, \wedge, \vee, \neg, (,), =\}$:

$$L = \{w = x : w \text{ is a logical expression over } \{t, f\}, x \in \{t, f\}, \text{ and } x \text{ is the truth value of } w\}$$

Thus, “ $t = t$ ”, “ $((t \wedge f) \vee f) = f$ ”, and “ $\neg(((t \wedge f) \vee f)) = t$ ” are in L , but “ $((t \wedge f) \vee f) = t$ ” and “ $(t \wedge f) \vee f = f$ ” are not: the former because $((t \wedge f) \vee f)$ is false and not true, the latter because the expression lacks the outermost set of parentheses.

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2. Define a context-free grammar for the language $L = \{0^n 1^m 0^m 1^n : n, m \in \mathbb{N}\}$. 50
3. Define a context-free grammar for the language $L = \{uv \in \{0, 1\}^* : |u| = |v| \wedge u \neq v^R\}$. 50
4. Define a context-free grammar for the language $L = \{a^n b^m : n \leq 3m\}$. 50
5. Find a CFG for the language $L = \{a^n b^m : n, m \in \mathbb{N}, n \neq m\}$. 50