# HW 1

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**Question 1:**
Give a regular expression, simplified to the best of your abilities, for the language of all strings over $\sum = \{a, b, c\}$ where there are at least two non-overlapping occurrences of the string $\alpha\alpha$, where $\alpha$ is a given symbol in $\sum$ (i.e., the string $\alpha\alpha\alpha\alpha$ qualifies, but the string $\alpha\alpha\alpha$ does not, because the first $\alpha\alpha$ and the second $\alpha\alpha$ share a common position in the string).

To solve this problem, we first need to think about what the question is really asking. We are looking for every string of symbols $a, b$, and $c$, where there is at least one sequence of $aa$ or $bb$ or $cc$ with any number of symbols in between, followed by the same respective $aa$ or $bb$ or $cc$ sequence.

We can start by picking any amount of $a$'s, $b$'s, or $c$'s. This is represented like: $(a + b + c)^*$. Next, we need to incorporate our $\alpha\alpha\alpha\alpha$ rule. For $a$'s this can be shown as follows: $aa(a+b+c)^*aa$. However, we need this for $a$'s, $b$'s, and $c$'s. Additionally, we need another $(a + b + c)^*$ to handle any pattern or combination of symbols following the rule. Altogether this looks like the following:

**Final answer:** $(a + b + c)^*((aa(a + b + c)^*aa) + (bb(a + b + c)^*bb) + (cc(a + b + c)^*cc))(a + b + c)^*$

**Question 2:**

Simplify the following regular expressions (give an equivalent regular expression with the smallest number of symbols and operators, you can use the * and + operators in your answers).

**(a)** $a^*aa^*aa^*$

To start, we can simplify both $aa^*$ to $a^+$. This gets us $a^*a^+a^+$. This can further be simplified to $a^*aa^+$. Finally, we can remove the unnecessary $a^*$ at the front to get $aa^+$.

**Final answer:** $aa^+$

**(b)** $(a^*b^*)^*(a + b + \epsilon)$

To start, we can simply remove the $(a + b + \epsilon)$ portion of the expression. This is because the expression $(a^*b^*)^*$ already covers ending with an $a$, $b$, and $\epsilon$. So, this entire expression can be simplified down to $(a^*b^*)^*$.

**Final answer:** $(a^*b^*)^*$

**(c)** $(a^*abb) + (ba^*bb)$

To start, we can easily see that no matter what our answer will end in $bb$. On top of this, we know that our equation will always start with an $a$ or $b$. This means so far we have $(a + b)bb$. On top of this, we need to account for multiple $a$'s in the middle or start of the expression respectively. This can be handled by writing the following: $(b + a)a^*bb$.

**Final answer:** $(b + a)a^*bb$

**(d)** $a^*(\emptyset b + bb)$

To start, we know that any symbol times an empty set results in an empty set. This is treated as nothing in respect to the expression. For example, $\emptyset b = \emptyset$. This results in $a^*bb$ as we can disregard the empty set notation entirely.

**Final answer:** $a^*bb$

**(e)** $aaa^*(ab^*)^*a^*$

To start, we can combine the ending $a^*$ with $(ab^*)^*$ getting simply $(ab^*)^*$. Additionally, at the start, we can combine $aa^*$ into $a^+$. So, at this point we have $aa^+(ab^*)^*$. Next, we can drop the $a^+$ for a solo $a$ as the expression following this can be repeated for any number of $a$'s already. Overall, we get $aa(ab^*)^*$.

**Final answer:** $aa(ab^*)^*$

**Question 3:**
Give a regular expression, simplified to the best of your abilities, for the language of all strings of $a$'s, $b$'s, and $c$'s where $a$ is never immediately followed by $b$.

I believe that the answer is $b^*(a^* + cb^*)^*$. The reason that this works is because $a$ can never follow $b$. The $b^*$ allows any amount of $b$'s at the start or 0. The $(a^* + cb^*)^*$ then allows any number of $a$'s and $c$'s. If we want to add any $b$'s after the fact, we can do so enforcing that they either follow a $c$ or another $b$, but never an $a$.

**Final answer:** $b^*(a^* + cb^*)^*$

**Question 4:**
Give a regular expression, simplified to the best of your abilities, for the language of all strings of $a$'s, $b$'s, and $c$'s that contain an even number of $b$'s.

I believe that the answer is $(a + c)^*(b(a + c)^*b(a + c)^*)^*$. The reason that this works is because $(a + c)^*$ allows us to start with any number of $a$'s or $c$'s. The next piece, $(b(a + c)^*b(a + c)^*)^*$ guarantees that $b$'s will always come in doubles. They do not necessarily need to be next to each other either due to $(a + c)^*$. Additionally, we can end in any number of $a$'s or $c$'s. Additionally, we can have 0 $b$'s (which is even) because of the final $*$.

**Final answer:** $(a + c)^*(b(a + c)^*b(a + c)^*)^*$