ComS 3310 Spring 2025 Name:_____

HW 3 Due: 14 feb 2025

1. Draw a DFA, simplified to the best of your abilities, that recognizes the language:

   $L = \{w \in \{0,1\}^* : w \text{ ends with } 000 \text{ or contains } 111, \text{ but not both}\}$. 50

2. Consider the n-bit binary representation of a natural number x:

$$\text{the binary representation of } x \text{ is } (x_{n-1}x_{n-2}\cdots x_1x_0)_2 \iff x = \sum_{i=0}^{n-1} x_i 2^i$$

   where each bit x_i is a binary digit, either zero or one. For example, $(00000101)_2$ is the 8-bit binary representation of the number 5, since $0\cdot2^7+0\cdot2^6+0\cdot2^5+0\cdot2^4+0\cdot2^3+1\cdot2^2+0\cdot2^1+1\cdot2^0 = 4+1=5$. This is the format normally employed by digital computers to store nonnegative integers. Now, consider the language

$$\begin{aligned} L \;=\; & \{a_0b_0c_0\cdots a_{n-1}b_{n-1}c_{n-1} \;:\; n \in \mathbb{N} \wedge \forall i, 0 \le i < n, a_i \in \{0,1\}, b_i \in \{0,1\}, c_i \in \{0,1\} \wedge \\ & (a_{n-1}\cdots a_0)_2 + (b_{n-1}\cdots b_0)_2 = (c_{n-1}\cdots c_0)_2\} \end{aligned}$$

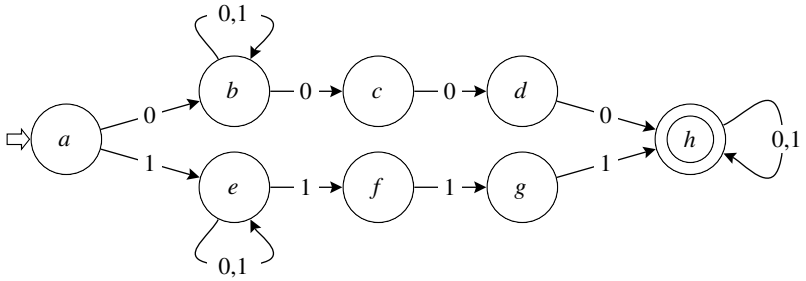
   For example, since $5+3=8$, $5=(000101)_2$, $3=(000011)_2$, and $8=(001000)_2$, then

$$110\ 010\ 100\ 001\ 000\ 000 \in L$$

   (the string is spaced every three digits for readability's sake only).

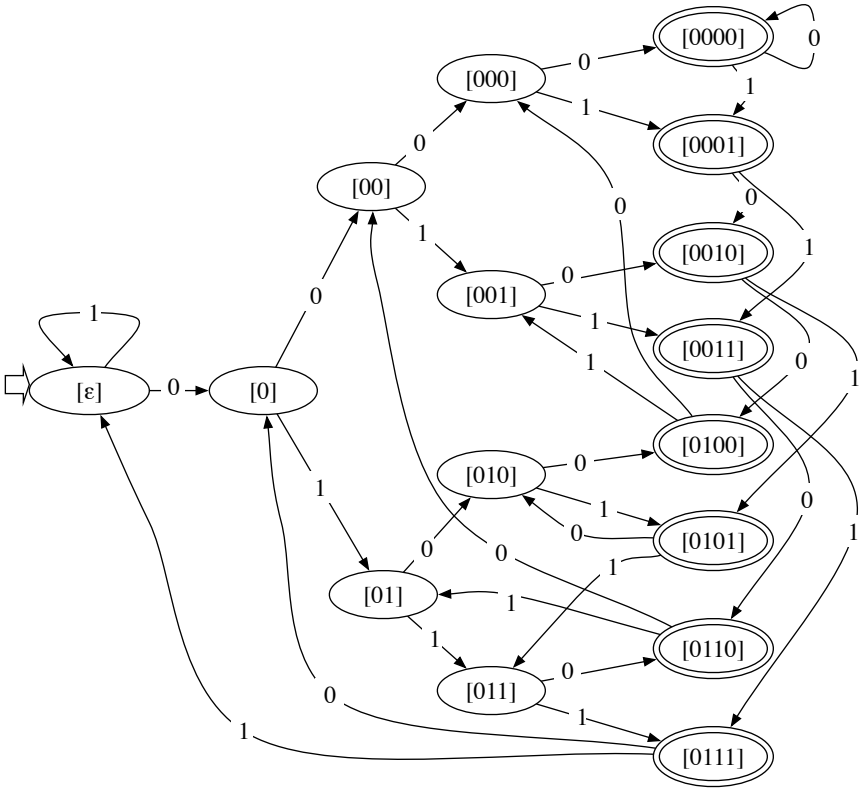
   Draw a DFA that recognizes L. 100

3. Precisely describe in English the language accepted by this NFA.



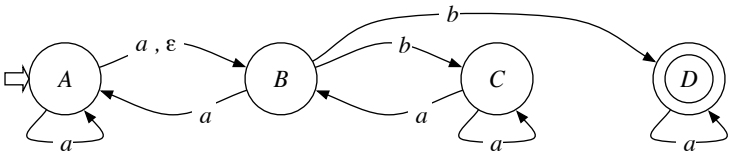
   Then, give a regular expression for it. 25

4. Describe in a short English sentence the language accepted by the following DFA, and give a regular expression for it (hint: the "names" of the states reflect their meaning). Then, draw a 5-state NFA that accepts the same language.



100

5. Consider the following NFA M:



- Using the algorithm described in class, derive an equivalent (non-minimized) DFA M'.
- Using the algorithm described in class, minimize M' and obtain the minimized DFA M''.
- Describe in English, as succintly as you can, the essential characteristics of the language accepted by these automata.

100