Imperial College London

MSc Advanced Aeronautical Engineering Coursework

Imperial College London

Department of Aeronautics

Advanced Propulsion AERO97003

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Contents

1 Nomenclature

Table 1: Nomenclature

Nomenclature			
ρ	Density	ρ_0	Stagnation Density
P	Pressure	P_0	Stagnation Pressure
T	Temperature	T_0	Stagnation Temperature
γ	Heat Capacity Ratio	η_p	Propulsive Efficiency
R	Ideal Gas Constant	η_c	Thermodynamic Efficiency
η_t	Total Efficiency	F	Thrust
A	Cross-sectional Area	M	Mach Number
h	Enthalpy	U	Velocity

2 Ramjet Physical Architecture

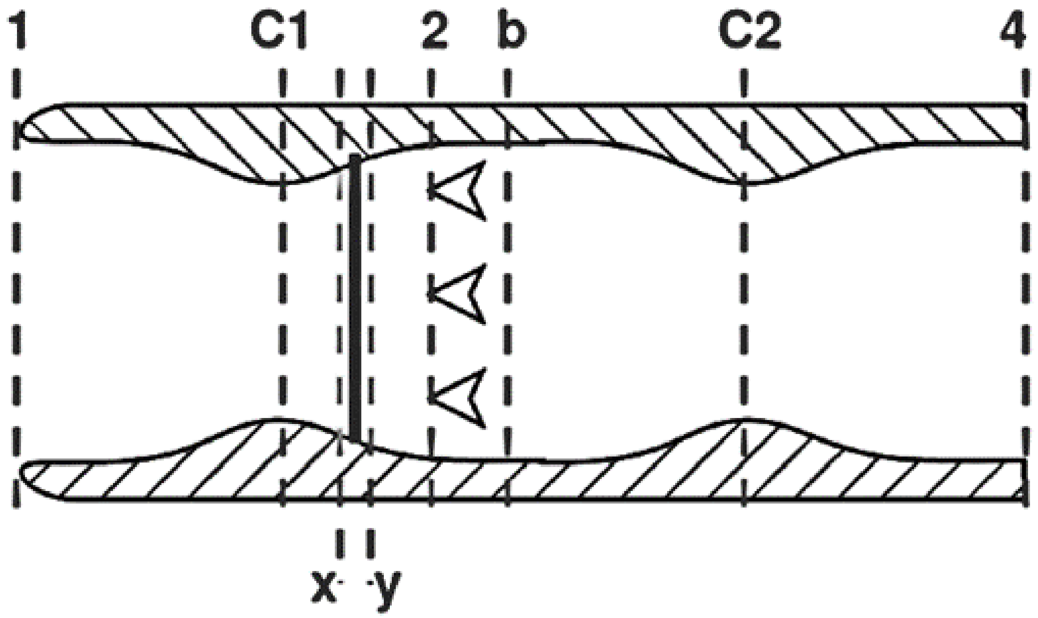

Figure 1: Ramjet Engine Stations

Table 2: Station Definition

Station Definition			
1	Intake(Freestream)	2	Burner Entry
C1	Inlet Throat	b	Burner Exit
x	Moment before Normal Shock	C2	Nozzle Throat
y	Moment after Normal Shock	4	Nozzle Exhaust

3 Assumptions

In our numerical analysis on ramjet engine, there were several assumptions made in order to perform the calculations:

- Exhaust gases have the same properties of air
- Ratio of specific heat capacity, γ =1.4 and is assumed to be constant throughout
- Assuming the compressions and expansions are isentropic everywhere except for shocks
- The stagnation properties remain constant through isentropic relations
- Flow is steady and inviscid
- All heat is added instantaneously between the combustion chamber, (2) and (b) and $P_2 = P_b$
- Burner area is constant and the size of the flame-holders are negligible
- Engine is assumed to be adiabatic except the burner
- The mass flow rate of fuel can be neglected as the fuel to air ratio is deemed small
- Shock is infinitely thin with $A_x = A_y$, hence A_s encapsulates both
- Ideal gas law is obeyed, $P = \rho RT$

4 Source Code

```
%% Advanced Propulsion Coursework %%
close all
clear
clc
%% Professionally Crafted By:
%% Gregory Foo
%% Jonathan Ting
%% Ngiam Yen Kui

%% Definitions of Geometry
%(1) - Intake(Freestream)
%(C1) - Inlet Throat
%(x) - Upstream of Shock
%(y) - Downstream of Shock
%(2) - Beginning of Burner
%(b) - End of Burner (Burn Complete)
%(C2) - Nozzle Throat
%(4) - Engine Exhaust

%% Code Organisation Parameters
font=13;
titl=15;
%% Notations
%Stagnation Temperature - T0
%Static Temperature - T
%Stagnation Pressure - P0
%Static Pressure - P
%Mach Number - M
%Area Ratio - A
%Reference Throat Area - A*
%% Assumptions for Design

%(1) - Exhaust has exactly the same properties as air
%(2) - Gamma = 1.4 and is constant throughout
%(3) - Compression and Expansion are Isentropic except for shocks
%(4) - All heat is added between the combustion chamber, (2) and ...
    (b), and P2=Pb
%(5) - Engine is Adiabatic
%(6) - Shock is infinitely thin, Ax = Ay, hence As encapsulates both
%(7) - Ideal Gas Law is obeyed P=rho*R*T
%(8) - Isentropic everywhere except for shock
%(9) - Flow is steady and inviscid
%(10) - Mass flow rate of fuel can be neglected as the fuel to air ...
    ratio is deemed small
%(11) - Stagnation properties remain constant through isentropic ...
    relations

%% Miscellaneous Equations Utilized in Derivations
%(1) - U = M*sqrt(gamma*R*T)

%% Declaration of input parameters
```

```
P1 = 70; %freestream pressure (kPa)
T1 = 210; %freestream temperature (K)
M1 = 3.24; %freestream mach number
Mx = 1.2; % Mach number at shock
M2 = 0.3; %Mach number at burner entry
Tb = 1400;%burner temperature (K)
F = 10; %engine thrust (kN)
gamma = 1.4; %Specific Heat Ratio of Air
R = 287; %Ideal Gas Constant
%% Computation of Engine Design Cross-sectional Areas for Selected ...
    Inputted Parameters from Tutorial 3
%-Inlet Area A1fixed
%-Inlet Throat Area AC1fixed
%-Burner Entry Area A2fixed
%-Burner Exit Area Abfixed
%-Nozzle Throat Area AC2fixed
%-Exit Area A4fixed
%-Propulsive Efficiency npfixed
%-Thermodynamic Efficiency ncyclefixed
%-Total Efficiency ntfixed
[ntfixed,ncyclefixed,npfixed,¬,¬,A1fixed,AC1fixed,AC2fixed,A2fixed...
,Abfixed,A4fixed] = design(P1,T1,M1,Mx,M2,Tb,F,gamma,R);
fprintf('The calculated values of areas are shown below in m^2:\n ...
    A1=%f\n AC1=%f\n AC2=%f\n A2=%f\n Ab=%f\n ...
    A4=%f\n',A1fixed,AC1fixed,AC2fixed,A2fixed,Abfixed,A4fixed);
fprintf('The calculated values of efficiencies are shown below:\n ...
    np=%f\n ncycle=%f\n nt=%f\n\n',npfixed,ncyclefixed,ntfixed);
%% For Clean Code Viewing
disp('This program allows the variation of parameters of:');
disp('1.Pressure');
disp('2.Temperature');
disp('3.Mach Number');
disp('4.Normal Shock Strength');
disp('5.Burner Entry Mach Number');
disp('6.Burner Temperature');
disp('7.Thrust');
cases = input('Please enter the corresponding number of the ...
    parameter which you wish to vary: ');
switch (cases)
case 1
%% ----- (1) Freestream pressure P1 variation ----- %%
initialP1 = P1*10^3; %Initial Freestream Pressure for Variation (Pa)
increP1 = 1000; %Pressure increment (Pa)
finalP1 = 400*10^3; %Final Freestream Pressure for Variation (Pa)
counterP1 = (finalP1-initialP1)/increP1; %Number of test ...
    variations of pressure

%-Utilizing the function to compute the varying efficiencies and ...
    putting them into matrices corresponding to their freestream ...
    pressure-%
%-Preallocating Matrices for Speed
P1mat=zeros(1,counterP1+1);
ntP=zeros(1,counterP1+1);
ncycleP=zeros(1,counterP1+1);
npP=zeros(1,counterP1+1);
```

```
for i = 1:1:counterP1+1
    P1 = initialP1 + (i-1)*increP1;
    %Extracting Prop, Thermo and Total efficiency from 'design' ...
        function
    [ntP(i), ncycleP(i), npP(i)] = ...
        design(P1,T1,M1,Mx,M2,Tb,F,gamma,R);
    P1mat(i) = initialP1 + (i-1)*increP1;
end

%-Plotting the variation of the efficiencies wrt Freestream Pressure-%
figure
hold on
plot (P1mat/1000,npP,'k-','LineWidth',1.2);
title ('Variation of Efficiencies with Freestream Pressure ...
    P_1','fontname','times new roman','fontsize',titl);
Hy=ylabel ('Efficiencies');
Hx=xlabel ('Freestream Pressure P_1 (kPa)');
set(Hy, 'fontname', 'times new roman', 'fontsize', font, ...
    'rotation', 0,'VerticalAlignment','middle', ...
    'HorizontalAlignment','right');
set(Hx, 'fontname', 'times new roman', 'fontsize', font, ...
    'rotation', 0);
hold on
plot (P1mat/1000,ncycleP,'k-.','LineWidth',1.2);
hold on
plot (P1mat/1000,ntP,'k:','LineWidth',1.2);
legend('Propulsive efficiency, n_p','Thermodynamic efficiency, ...
    n_c_y_c_l_e','Total efficiency, ...
    n_t_o_t_a_l','Location','northeastoutside');

 case 2
%% ----- (2) Freestream temperature T1 variation ----- %%
initialT1 = 100; %Initial Freestream Temperature for Variation (K)
increT1 = 1; %Temperature increment (K)
finalT1 = 500; %Final Freestream Temperature for Variation (K)
counterT1 = (finalT1-initialT1)/increT1; %Number of test ...
    variations of Temperature

%-Utilizing the function to compute the varying efficiencies and ...
    putting them into matrices corresponding to their freestream ...
    temperature-%
T1mat=zeros(1,counterT1+1);
for i = 1:1:counterT1+1
    T1 = initialT1 + (i-1)*increT1;
    %Extracting Prop, Thermo and Total efficiency, T2 and Mb from ...
        'design' function
    [ntT(i), ncycleT(i), npT(i), T2T(i),Mbnew(i)] = ...
        design(P1,T1,M1,Mx,M2,Tb,F,gamma,R);
    T1mat(i) = initialT1 + (i-1)*increT1;
end

%-Plotting the variation of the efficiencies wrt Freestream ...
    Temperature-%
figure
hold on
```

```
plot (T1mat,npT,'k-','LineWidth',1.2);
title ('Variation of Efficiencies with Freestream Temperature ...
    T_1','fontname','times new roman','fontsize',titl);
Hy=ylabel ('Efficiencies');
Hx=xlabel ('Freestream Temperature T_1 (K)');
set(Hy, 'fontname', 'times new roman', 'fontsize', font, ...
    'rotation', 0,'VerticalAlignment','middle', ...
    'HorizontalAlignment','right');
set(Hx, 'fontname', 'times new roman', 'fontsize', font, ...
    'rotation', 0);
hold on
plot (T1mat,ncycleT,'k-.','LineWidth',1.2);
hold on
plot (T1mat,ntT,'k:','LineWidth',1.2);
xlim([150,500]);
ylim([0,1.5]);

%-Plotting of Zone of 'Physical Impossibility'-%
for i=1:0.02:2.5 %for Propulsive efficiency higher than 1
    hold on
    yline(i,'r-');
end
for j=100:2:183 %for temperature ratio condition violated, ...
    (T2/Tb)*(M2+1/(gamma*M2))^2-(4/gamma) > 0
    hold on
    xline(j,'r-');
end
for k=455:2:500 %Burner Entry temperature would exceed Burner Exit ...
    Temperature
    xline(k,'r-');
end
text(200,1.1,'$Physically\ ...
    Impossible$','interpreter','latex','FontName','Times New ...
    Roman','FontSize',14);
h = text(160,0.9,'$Physically\ ...
    Impossible$','interpreter','latex','FontName','Times New ...
    Roman','FontSize',14);
set(h,'Rotation',90); %Rotate Text
h3 = text(475,0.1,'$Unrealistic$','interpreter','latex','FontName',...
'Times New Roman','FontSize',14);
set(h3,'Rotation',90); %Rotate Text
xline(210); %Chosen Operating Condition
xline(290); %Sea Level Condition
h1 = text(285,0.1,'$Sea\ Level\ ...
    Condition$','interpreter','latex','FontName','Times New ...
    Roman','FontSize',10);
set(h1,'Rotation',90); %Rotate Text
h2 = text(205,0.1,'$Chosen\ Operating\ ...
    Condition$','interpreter','latex','FontName','Times New ...
    Roman','FontSize',10);
set(h2,'Rotation',90); %Rotate Text
legend('Propulsive efficiency, n_p','Thermodynamic efficiency, ...
    n_c_y_c_l_e','Total efficiency, n_t','Location','North');

%-Check to ensure engine does not melt under excessive burner entry
```

```
%temperature-%
figure
hold on
title ('Variation of Burner Entry Temperature, T_2 with Freestream ...
    Temperature T_1','fontname','times new roman','fontsize',titl);
Hy=ylabel ('Burner Entry Temperature T_2 (K)');
Hx=xlabel ('Freestream Temperature T_1 (K)');
set(Hy, 'fontname', 'times new roman', 'fontsize', font, ...
    'rotation', 0,'VerticalAlignment','middle', ...
    'HorizontalAlignment','right');
set(Hx, 'fontname', 'times new roman', 'fontsize', font, ...
    'rotation', 0);
plot(T1mat,T2T);
ylim([0 2100]);
yline(1500,'k:');
text(350,1550,'$\downarrow\ Melting\ Point\ Lower\ ...
    limit$','interpreter','latex','FontName','Times New Roman');
yline(2000,'k-.');
text(350,1950,'$\uparrow\ Melting\ Point\ Higher\ ...
    limit$','interpreter','latex','FontName','Times New Roman');
yline(1400,'k--');
text(300,1350,'$\uparrow\ Defined\ Burner\ Exit\ ...
    Temperature$','interpreter','latex','FontName','Times New Roman');
xline(210); %Chosen Operating Condition
xline(290); %Sea Level Condition
h1 = text(285,200,'$Sea\ Level\ ...
    Condition$','interpreter','latex','FontName','Times New ...
    Roman','FontSize',10);
set(h1,'Rotation',90); %Rotate Text
h2 = text(205,200,'$Chosen\ Operating\ ...
    Condition$','interpreter','latex','FontName','Times New ...
    Roman','FontSize',10);
set(h2,'Rotation',90); %Rotate Text
legend('Variation of T_2 with T_1','Lower limit of metal melting ...
    point','Upper limit of metal melting ...
    point','Location','Southeast');

case 3
%% ----- (3) Freestream Mach Number M1 variation ----- %%
%-Notes: Based on Lecture 3, from the effect of burner temperature ...
    slides,
%it is understood that the engine would melt at M1=6 and beyond, ...
    therefore,
%the maximum limit of the freestream mach number variation would ...
    be 6.5 and
%the minimum limit is M1 = 1 as it is a ramjet and is built for ...
    supersonic
%flight

initialM1 = 1.0;
increM1 = 0.05;
finalM1 = 6.5;
counterM1 = (finalM1-initialM1)/increM1;

```

```
%-Utilizing the function to compute the varying efficiencies and ...
    putting them into matrices corresponding to their freestream ...
    mach number-%
M1mat=zeros(1,counterM1+1);
for i = 1:1:counterM1+1
    M1 = initialM1 + (i-1)*increM1;
    % '¬' is used to skip the outputs which are unnecessary for ...
        this analysis
    %Extracting Prop,Thermo and Total efficiency, T2, A1, T4 from ...
        'design' function
    [ntM(i), ncycleM(i), npM(i), ...
        T2M(i),¬,A1M(i),¬,¬,¬,¬,¬,¬,¬,¬,T4M(i)] = ...
        design(P1,T1,M1,Mx,M2,Tb,F,gamma,R);
    M1mat(i) = initialM1 + (i-1)*increM1;
end

%-Plotting the variation of the efficiencies wrt Freestream Mach ...
    Number-%
figure
hold on
plot (M1mat,npM,'k-','LineWidth',1.2);
title ('Variation of Efficiencies with Freestream Mach ...
    Number','fontname','times new roman','fontsize',titl);
Hy=ylabel ('Efficiencies');
Hx=xlabel ('Freestream Mach Number ');
set(Hy, 'fontname', 'times new roman', 'fontsize', font, ...
    'rotation', 0,'VerticalAlignment','middle', ...
    'HorizontalAlignment','right');
set(Hx, 'fontname', 'times new roman', 'fontsize', font, ...
    'rotation', 0);
hold on
plot (M1mat,ncycleM,'k-.','LineWidth',1.2);
hold on
plot (M1mat,ntM,'k:','LineWidth',1.2);
%-Plotting of Zone of 'Physical Impossibility'-%
for i=1:0.02:1.2 %for temperature ratio condition violated, ...
    (T2/Tb)*(M2+1/(gamma*M2))^2-(4/gamma) > 0
    hold on
    yline(i,'r-');
end
for i=1:0.05:2.9 %for propulsive efficiency higher than 1
    hold on
    xline(i,'r-');
end
for i=5.3:0.05:7 %Burner Entry temperature would exceed Burner ...
    Exit Temperature
    hold on
    xline(i,'r-');
end
legend('Propulsive efficiency, n_p','Thermodynamic efficiency, ...
    n_c_y_c_l_e','Total efficiency, n_t','Location','south');
text(3,1.1,'$Physically\ ...
    Impossible$','interpreter','latex','FontName','Times New ...
    Roman','FontSize',14);
```

```
h = text(1.5,0.7,'$Physically\ ...
    Impossible$','interpreter','latex','FontName','Times New ...
    Roman','FontSize',14);
set(h,'Rotation',90); %Rotate Text
h1 = text(6.5,0.05,'$Physically\ ...
    Impossible$','interpreter','latex','FontName','Times New ...
    Roman','FontSize',14);
set(h1,'Rotation',90); %Rotate Text
figure
hold on
plot(M1mat,T2M,'k-');
title ('Burner Entry Temperature T_2 with Freestream Mach ...
    Number','fontname','times new roman','fontsize',titl);
Hy=ylabel ('Burner Entry Temperature T_2');
Hx=xlabel ('Freestream Mach Number ');
set(Hy, 'fontname', 'times new roman', 'fontsize', font, ...
    'rotation', 0,'VerticalAlignment','middle', ...
    'HorizontalAlignment','right');
set(Hx, 'fontname', 'times new roman', 'fontsize', font, ...
    'rotation', 0);
yline(1500,'k:');
text(2,1550,'$\downarrow\ Melting\ Point\ Lower\ ...
    limit$','interpreter','latex','FontName','Times New Roman');
yline(2000,'k-.');
text(2,1950,'$\uparrow\ Melting\ Point\ Higher\ ...
    limit$','interpreter','latex','FontName','Times New Roman');
yline(1400,'k--');
text(2,1350,'$\uparrow\ Defined\ Burner\ Exit\ ...
    Temperature$','interpreter','latex','FontName','Times New Roman');
figure

%-Variation of Inlet Area with Freestream Mach Number
plot (M1mat,A1M,'k-');
title ('Variation of Inlet Area A_{1}','fontname','times new ...
    roman','fontsize',titl);
Hy=ylabel ('Inlet Area  A_{1} (m^2)');
Hx=xlabel ('Freestream Mach Number M_1');
set(Hy, 'fontname', 'times new roman', 'fontsize', font, ...
    'rotation', 0,'VerticalAlignment','middle', ...
    'HorizontalAlignment','right');
set(Hx, 'fontname', 'times new roman', 'fontsize', font, ...
    'rotation', 0);
xlim([3, 5]);
%-Variation of Temperature ratio T4/T1 with freestream mach number
figure
%-T1 is constant
plot (M1mat,T4M/T1,'k-');
title ('Variation of Temperature Ratio T_4/T_1','fontname','times ...
    new roman','fontsize',titl);
Hy=ylabel ('Temperature Ratio T_4/T_1');
Hx=xlabel ('Freestream Mach Number M_1');
set(Hy, 'fontname', 'times new roman', 'fontsize', font, ...
    'rotation', 0,'VerticalAlignment','middle', ...
    'HorizontalAlignment','right');
```

```
set(Hx, 'fontname', 'times new roman', 'fontsize', font, ...
    'rotation', 0);
xlim([3, 5]);

case 4
%% ----- (4) Normal Shock Strength Mx Variation ----- %%
%- A Normal shock is required to bring the flow to a subsonic ...
    speed in the
%- combustion chamber. It is also favourable to have a weak shock ...
    so that
%- there is minimum stagnation pressure loss
%- Vary Normal Shock Strength from 1.0 to 4.0
initialMx = 1.0;
increMx = 0.05;
finalMx = 4.0;
counterMx = (finalMx-initialMx)/increMx;

%-Utilizing the function to compute the varying efficiencies and ...
    putting them into matrices corresponding to their Normal Shock ...
    Strength-%
Mxmat=zeros(1,counterMx+1);
for i = 1:1:counterMx+1
    Mx = initialMx + (i-1)*increMx;
    %Extracting Prop,Thermo and Total efficiency, T2, A1, M4,U4,U1 ...
        from 'design' function
    [ntMx(i), ncycleMx(i), npMx(i), ...
        T2Mx(i),¬,A1Mx(i),¬,¬,¬,¬,¬,M4Mx(i),U4Mx(i),U1Mx(i)] = ...
        design(P1,T1,M1,Mx,M2,Tb,F,gamma,R);
    Mxmat(i) = initialMx + (i-1)*increMx;
end

%-Plotting the variation of the efficiencies wrt Normal Shock ...
    Strength-%
figure
hold on
plot (Mxmat,npMx,'k-','LineWidth',1.2);
title ('Variation of Efficiencies with Normal Shock ...
    Strength','fontname','times new roman','fontsize',titl);
Hy=ylabel ('Efficiencies');
Hx=xlabel ('Normal Shock Strength, M_x');
set(Hy, 'fontname', 'times new roman', 'fontsize', font, ...
    'rotation', 0,'VerticalAlignment','middle', ...
    'HorizontalAlignment','right');
set(Hx, 'fontname', 'times new roman', 'fontsize', font, ...
    'rotation', 0);
hold on
plot (Mxmat,ncycleMx,'k-.','LineWidth',1.2);
hold on
plot (Mxmat,ntMx,'k:','LineWidth',1.2);
legend('Propulsive efficiency, n_p','Thermodynamic efficiency, ...
    n_c_y_c_l_e','Total efficiency, n_t_o_t_a_l','Location','East');

%Plot to find the max shock strength that can be physically possible
figure
hold on
```

```
plot(Mxmat,U4Mx.^2-U1Mx.^2,'k-');
title ('(U_4)^2-(U_1)^2 against Normal Shock Strength ... M_x','fontname','times new roman','fontsize',titl);
Hy=ylabel ('(U_4)^2-(U_1)^2');
Hx=xlabel ('Normal Shock Strength, M_x ');
set(Hy, 'fontname', 'times new roman', 'fontsize', font, ... 'rotation', 0,'VerticalAlignment','middle', ... 'HorizontalAlignment','right');
set(Hx, 'fontname', 'times new roman', 'fontsize', font, ... 'rotation', 0);
hold on
yline(0,'k--');
xline(3.375,'k--');
case 5
%% ----- (5) Burner Entry Mach Number M2 Variation ----- %%
%- The flow within the combustion chamber has to be subsonic, ... hence the
%- variation was performed for 0.1 to 1.0

initialM2 = 0.1;
increM2 = 0.01;
finalM2 = 1.0;
counterM2 = (finalM2-initialM2)/increM2;

%-Utilizing the function to compute the varying efficiencies and ... putting them into matrices corresponding to their Burner Entry ... Mach Number-%
M2mat=zeros(1,counterM2+1);
for i = 1:1:counterM2+1
    M2 = initialM2 + (i-1)*increM2;
    %Extracting Prop,Thermo and Total efficiency, T2, T4 from ... 'design' function
    [ntM2(i), ncycleM2(i), ... npM2(i),T2M2(i),¬,¬,¬,¬,¬,¬,¬,¬,¬,¬,¬,T4M2(i)] = ... design(P1,T1,M1,Mx,M2,Tb,F,gamma,R);
    M2mat(i) = initialM2 + (i-1)*increM2;
end

%-Plotting the variation of the efficiencies wrt Burner Entry mach ... Number%

figure
hold on
plot (M2mat,npM2,'k-','LineWidth',1.2);
title ('Variation of Efficiencies with Burner Entry Mach Number, ... M_2','fontname','times new roman','fontsize',titl);
Hy=ylabel ('Efficiencies');
Hx=xlabel ('Burner Entry Mach Number, M_2');
set(Hy, 'fontname', 'times new roman', 'fontsize', font, ... 'rotation', 0,'VerticalAlignment','middle', ... 'HorizontalAlignment','right');
set(Hx, 'fontname', 'times new roman', 'fontsize', font, ... 'rotation', 0);
hold on
plot (M2mat,ncycleM2,'k-.','LineWidth',1.2);
```

```
hold on
plot (M2mat,ntM2,'k:','LineWidth',1.2);
%-Plotting of Zone of 'Physical Impossibility'-%
for i=0.33:0.0035:1 %Burner Entry temperature would exceed Burner ...
    Exit Temperature
    hold on
    xline(i,'r-');
end
h1 = text(0.355,0.55,'$Physically\ ...
    Impossible$','interpreter','latex','FontName','Times New ...
    Roman','FontSize',14);
set(h1,'Rotation',90); %Rotate Text
xlim([0.1, 0.4]);
legend('Propulsive efficiency, n_p','Thermodynamic efficiency, ...
    n_c_y_c_l_e','Total efficiency, ...
    n_t_o_t_a_l','Location','NorthEastOutside');

figure
hold on
plot(M2mat,T4M2,'k-');
title ('Variation of T_4 with Burner Entry Mach Number, ...
    M_2','fontname','times new roman','fontsize',titl);
Hy=ylabel ('Temperature (K)');
Hx=xlabel ('Burner Entry Mach Number, M_2');
set(Hy, 'fontname', 'times new roman', 'fontsize', font, ...
    'rotation', 0,'VerticalAlignment','middle', ...
    'HorizontalAlignment','right');
set(Hx, 'fontname', 'times new roman', 'fontsize', font, ...
    'rotation', 0);
legend('Temperature at Exhaust , T_4','Location', 'NorthEast');
xlim([0.1,0.35]);
figure
hold on
plot(M2mat,T2M2,'k-');
xlim([0.1,0.35]);
title ('Variation of T_2 with Burner Entry Mach Number, ...
    M_2','fontname','times new roman','fontsize',titl);
Hy=ylabel ('Temperature (K)');
Hx=xlabel ('Burner Entry Mach Number, M_2');
set(Hy, 'fontname', 'times new roman', 'fontsize', font, ...
    'rotation', 0,'VerticalAlignment','middle', ...
    'HorizontalAlignment','right');
set(Hx, 'fontname', 'times new roman', 'fontsize', font, ...
    'rotation', 0);
legend('Temperature at Exhaust , T_2','Location', 'NorthEast');
case 6
%% ----- (6) Burner Exit Temperature, Tb Variation ----- %%
%- Temperature range to observe effect after 1610K
initialTb = 1200;
increTb = 1;
finalTb = 1700;
counterTb = (finalTb-initialTb)/increTb;

%-Utilizing the function to compute the varying efficiencies and ...
    putting them into matrices corresponding to their Burner ...
```

```
    Temperature-%
Tbmat=zeros(1,counterTb+1);
for i = 1:1:counterTb+1
    Tb = initialTb + (i-1)*increTb;
    %Extracting Prop,Thermo and Total efficiency, T2, Mb, T4 from ...
        'design' function
    [ntTb(i), ncycleTb(i), npTb(i), ...
        T2Tb(i),Mbnewb(i),¬,¬,¬,¬,¬,¬,¬,¬,¬,¬,T4Tb(i)] = ...
        design(P1,T1,M1,Mx,M2,Tb,F,gamma,R);
    Tbmat(i) = initialTb + (i-1)*increTb;
end

%-Plotting the variation of the efficiencies wrt Burner Exit ...
    Temperature-%
figure
hold on
plot (Tbmat,npTb,'k-','LineWidth',1.2);
title ('Variation of Efficiencies with Burner Exit Temperature, ...
    T_b ','fontname','times new roman','fontsize',titl);
Hy=ylabel ('Efficiencies');
Hx=xlabel ('Burner Temperature, T_b');
set(Hy, 'fontname', 'times new roman', 'fontsize', font, ...
    'rotation', 0,'VerticalAlignment','middle', ...
    'HorizontalAlignment','right');
set(Hx, 'fontname', 'times new roman', 'fontsize', font, ...
    'rotation', 0);
hold on
plot (Tbmat,ncycleTb,'k-.','LineWidth',1.2);
hold on
plot (Tbmat,ntTb,'k:','LineWidth',1.2);
%-Plotting of Zone of 'Physical Impossibility'-%
for i=1609:2:1700 %Imaginary Roots
    hold on
    xline(i,'r-');
end
h1 = text(1650,0.5,'$Physically\ ...
    Impossible$','interpreter','latex','FontName','Times New ...
    Roman','FontSize',14);
set(h1,'Rotation',90); %Rotate Text
legend('Propulsive efficiency, n_p','Thermodynamic efficiency, ...
    n_c_y_c_l_e','Total efficiency, n_t','Location','southwest');

case 7
%% ----- (7) Thrust, F Variation ----- %%
%- Thrust was allowed to vary from 1kN to 200kN

initialF = 1*10^3;
increF = 1;
finalF = 200*10^3;
counterF = (finalF-initialF)/increF;

%-Utilizing the function to compute the varying efficiencies and ...
    putting them into matrices corresponding to their Burner ...
    Temperature-%
Fmat=zeros(1,counterF+1);
```

```
for i = 1:1:counterF+1
    F = initialF + (i-1)*increF;
    %Extracting Prop,Thermo and Total efficiency, T2 from 'design' ...
        function
    [ntF(i), ncycleF(i), npF(i), T2F(i)] = ...
        design(P1,T1,M1,Mx,M2,Tb,F,gamma,R);
    Fmat(i) = initialF + (i-1)*increF;
end

%-Plotting the variation of the efficiencies wrt Thrust-%
figure
hold on
plot (Fmat,npF,'k-','LineWidth',1.2);
title ('Variation of Efficiencies with Thrust','fontname','times ...
    new roman','fontsize',titl);
Hy=ylabel ('Efficiencies');
Hx=xlabel ('Thrust, F');
set(Hy, 'fontname', 'times new roman', 'fontsize', font, ...
    'rotation', 0,'VerticalAlignment','middle', ...
    'HorizontalAlignment','right');
set(Hx, 'fontname', 'times new roman', 'fontsize', font, ...
    'rotation', 0);
hold on
plot (Fmat,ncycleF,'k-.','LineWidth',1.2);
hold on
plot (Fmat,ntF,'k:','LineWidth',1.2);
legend('Propulsive efficiency, n_p','Thermodynamic efficiency, ...
    n_c_y_c_l_e','Total efficiency, ...
    n_t_o_t_a_l','Location','northeastoutside');

end

%% Design Code
%%- Able to provide Output of -%
%%- Total Efficiency, Thermodynamic Efficiency, Propulsive Efficiency,
%%- Burner Exit Mach, Inlet Area, Inlet Throat Area, Nozzle Throat ...
    Area,
%%- Burner Entry Area, Burner Exit Area, Exhaust Area, Exhaust Mach,
%%- Velocity at Exhaust, Velocity at Inlet, Temperature at Exhaust
%%- Respectively
 function [nt,ncycle,np,T2,Mbnew,A1,AC1,AC2,A2,Ab,A4,M4,U4,U1,T4] ...
     = design(P1,T1,M1,Mx,M2,Tb,F,gamma,R)

%% -----Analysis at Inlet Station (1)----- %%
%---Calculation of Isentropic Properties---%
%-Freestream Stagnation Temperature/Freestream Static Temperature, ...
    T01/T1-%
tempratio1 = 1 + ((gamma-1)/2).*(M1.^2);

%-Freestream Stagnation Pressure/Freestream Static Pressure, P01/P1-%
presratio1 = tempratio1.^(gamma/(gamma-1));

%-Area of Inlet/Area of Inlet Throat, A1/AC1-%
arearatio1 = (1/M1)*((2/(gamma+1))*(tempratio1)).^((gamma+1)/...
    (2*(gamma-1)));
```

```
%% -----Analysis at Inlet Throat Station (C1)----- %%
%-Area of Inlet Throat/Area of Inlet, AC1/A1-%
arearatioC1 = 1 / arearatio1;

%% -----Analysis at Normal Shock Station (x,y)----- %%
%Note: Stagnation Conditions change across the shock; Non-isentropic
%-Mach Number immediately downstream of Shock, My-%
My = sqrt(((gamma-1)*Mx.^2 + 2)/(2*gamma*Mx.^2 - (gamma-1)));

%-Static Pressure After Shock/Static Pressure Before Shock, Py/Px-%
presratioshock = (2*gamma*Mx.^2 - (gamma-1))/(gamma+1);

%-Density After Shock/Density Before Shock, rhoy/rhox-%
densityratioshock = ((gamma+1)*Mx.^2)/((gamma-1)*Mx.^2 + 2);

%%-Stagnation Conditions at (x)-%%
%-Stagnation Temperature/Static Temperature, T0x/Tx-%
tempratiox = 1+((gamma-1)/2)*Mx.^2;
%-Stagnation Pressure/Static Pressure, P0x/Px-%
presratiox = (tempratiox)^(gamma/(gamma-1));
%-Area of Shock/Area of Inlet Throat, Ax/AC1 = As/AC1-%
arearatiox = ...
    (1/Mx)*((2/(gamma+1))*tempratiox)^((gamma+1)/(2*(gamma-1)));

%%-Stagnation Conditions at (y)-%%
%-Stagnation Temperature/Static Temperature, T0y/Ty-%
tempratioy = 1+((gamma-1)/2)*My.^2;
%-Stagnation Pressure/Static Pressure, P0y/Py-%
presratioy = (tempratioy)^(gamma/(gamma-1));
%-Area of Shock/Area of Inlet Throat, Ay/AC1 = As/AC1 = As/Ay*-%
arearatioy = ...
    (1/My)*((2/(gamma+1))*tempratioy)^((gamma+1)/(2*(gamma-1)));

%% -----Analysis at Burner Entry (2)----- %%
%-Stagnation Temperature/Static Temperature, T02/T2-%
tempratio2 = 1+((gamma-1)/2)*M2^2;
%-Stagnation Pressure/Static Pressure, P02/P2-%
presratio2 = (tempratio2)^(gamma/(gamma-1));
%-Area of Burner Entry/Area of Inlet Throat, A2/A2* = A2/Ay*-%
arearatio2 = ...
    (1/M2)*((2/(gamma+1))*tempratio2)^((gamma+1)/(2*(gamma-1)));

%-Burner Entry Area/Inlet Area, A2/A1-%
%-Note: Isentropic, A2*/Ay* = A1*/Ax* = 1
%-Note: Due to infinitely thin shock, Ay/Ax = 1
%-A2/A1 = ...
    (A2/A2*)*(A2*/Ay*)*(Ay*/Ay)*(Ay/Ax)*(Ax/Ax*)*(Ax*/A1*)*(A1*/A1)
arearatio21 = ...
    (arearatio2)*(arearatioy^(-1))*(arearatiox)*(arearatio1^(-1));

%-Burner Entry Temperature/Inlet Temperature, T2/T1-%
%-Note: Isentropic, T02/T0y = T01/T0x
%-Note: Across Shock assumed to be adiabatic, T0y=T0x
%-Burner Entry Temperature/Inlet Temperature, T2/T1-%
```

```
tempratio21 = ((tempratio2)^-1)*tempratio1;

%Computation of Burner Entry Temperature, T2 for subsequent analysis
T2 = tempratio21*T1;

%% -----Analysis from Burner Entry to End of Burner (2,b)----- %%
%Note: Apply Conservation of Mass and Conservation of Momentum
%-Conservation of Mass, rho2*U2*A2 = rhob*Ub*Ab-%
%(P2*A2)/(Pb*Ab) = (Mb/M2)*sqrt(T2/Tb)

%-Conservation of Momentum, rho2*U2^2*A2 + P2A2 = rhob*Ub^2*Ab + PbAb
%(P2*A2)/(Pb*Ab) = (gamma*Mb^2+1)/(gamma*M2^2)+1

%-Combination of both expressions-%
%(Mb/M2)*sqrt(T2/Tb) = (gamma*MB^2+1)/(gamma*M2^2+1)
coeffMbsq = gamma*M2*sqrt(Tb/T2);%(gamma)/(gamma*M2^2+1);
coeffMb = -(gamma*M2^2+1);%-sqrt(T2/Tb)/M2;
coeffconstant = M2*sqrt(Tb/T2); %1/(gamma*M2^2+1);

eqn = [coeffMbsq coeffMb coeffconstant];
Mb = transpose(roots(eqn));
%-Supersonic solution is undesired, take subsonic Mb-%
  for i=1:1:2
      if Mb(i) < 1
          Mbnew =Mb(i);
      end
  end

%-Since the burning process is isobaric, Pb=P2-%
%-(P2*A2)/(Pb/Ab) = A2/Ab = (gamma*Mb^2+1)/(gamma*M2^2+1)-%
arearatio2b = (gamma*Mbnew^2+1)/(gamma*M2^2+1);

%-To calculate Ab/A1, Ab/A1 = (Ab/A2)*(A2/A1)-%
arearatiob1 = (arearatio2b)^(-1)*arearatio21;

%-New Stagnation Conditions at Burner Exit (b)-%
%-Stagnation Temperature at (b)/Static Temperature at (b), T0b/Tb-%
tempratiob = (1+((gamma-1)/2)*Mbnew^2);
%-Stagnation Pressure at (b)/Static Pressure at (b), P0b/Pb-%
presratiob = tempratiob^(gamma/(gamma-1));
%-Area of Burner Exit/Area of Throat, Ab/Ab* = Ab/AC2-%
arearatiob = ...
    (1/Mbnew)*((2/(gamma+1))*(tempratiob))^((gamma+1)/(2*(gamma-1)));

%-Nozzle Throat Area/Area of Inlet, Ab*/A1 = AC2/A1-%
%-Ab*/A1 = (Ab*/Ab)*(Ab/A1)-%
arearatiobstar1 = arearatiob^(-1)*arearatiob1;

%% -----Analysis from Burner Exit to Exhaust (b,4)----- %%
%-Stagnation Pressure at (4)/Static Pressure at (4), P04/P4-%
%- P04/P4 =
%- (P04/P0b)*(P0b/Pb)*(Pb/P2)*(P2/P02)*(P02/P0y)*(P0y/Py)...
%- *(Py/Px)*(Px/P0x)*(P0x/P01)*(P01/P1)*(P1/P4)-%
%-Note: Isentropic Flow, P04/P0b = P02/P0y = P0x/P01 = 1
%-Note: By Design, Pb/P2 = P1/P4 = 1
```

```
presratio4 = ...
    presratiob*((presratio2)^(-1))*presratioy*presratioshock...
*((presratiox)^(-1))*presratio1;

%-Mach Number at Exit, M4 for P04/P4-%
M4 = sqrt(((presratio4^((gamma-1)/gamma))-1)/((gamma-1)/2));
%-Stagnation Conditions at Exit (4)-%

%-Stagnation Temperature at (4)/Static Temperature at (4),T04/T4-%
tempratio4 = 1 + ((gamma-1)/2)*M4^2;

%-Stagnation Pressure at (4)/Static Pressure at (4), P04/P4-%
presratio4calc = tempratio4^(gamma/(gamma-1));

%-Area of Exit/Area of Nozzle Throat, A4/AC2 = A4/A4*-%
arearatio4 = ...
    (1/M4)*((2/(gamma+1))*(tempratio4))^((gamma+1)/(2*(gamma-1)));

%-Area of Exhaust/Area of Inlet, A4/A1-%
%-A4/A1 = (A4/A4*)*(A4*/Ab*)*(Ab*/A1)-%
%-Note: Isentropic flow from (b) to (4) hence, A4*/Ab* = 1
arearatio41 = arearatio4*arearatiobstar1;


%% -----Thrust and Cross-sectional Area Calculation----- %%
%- F ¬ rho4*U4^2*A4 - rho1*U1^2*A1 + P4*A4 - P1*A1
%- P4*A4 - P1*A1 is 0 due to control volume encompassing the whole ...
    engine-%
%- F/(P1*A1) = gamma*M1^2*((M4/M1)^2*(A4/A1)-1)
thrustPA = gamma*M1^2*((M4/M1)^2*arearatio41-1);

%-Inlet Area, A1-%
A1 = F/(thrustPA*P1);

%-Inlet Throat Area, AC1-%
AC1 = arearatioC1 * A1;

%-Burner Entry Area, A2-%
A2 = arearatio21 * A1;

%-Burner Exit Area, Ab-%
Ab = arearatiob1 * A1;

%-Nozzle Throat Area, AC2-%
AC2 = arearatiobstar1 * A1;

%-Exit Area, A4-%
A4 = arearatio41 * A1;

%% -----Efficiency Calculation----- %%
%Temperature at Exit (4)/Temperature at Inlet (1), T4/T1-%
tempratio41 = tempratio4^(-1)*tempratiob*(Tb/T1);
%-Propulsive Efficiency-%
np = (F/(P1*A1))*(2/gamma)*(1/((tempratio41*M4^2)-M1^2));
%-Thermodynamic Efficiency-%
```

```
%-Note:Assuming perfect efficiency of compressor and turbine
ncycle = 1 - (tempratio41*T1 - T1)/(Tb-T2);%1- tempratio21^(-1);
%-Total Efficiency-%
nt = ncycle .* np;

%-Calculation of Parameters for Post-Processing
T4 = tempratio41*T1; %temperature at exhaust
a4 = sqrt(gamma*R*T4); %speed of sound at exhaust
a1 = sqrt(gamma*R*T1); %speed of sound at inlet
U4 = (M4^2)*a4; %velocity at exhaust
U1 = (M1^2)*a1; %velocity at inlet
end
```

5 Results and Discussion

5.1 Freestream Pressure Variation

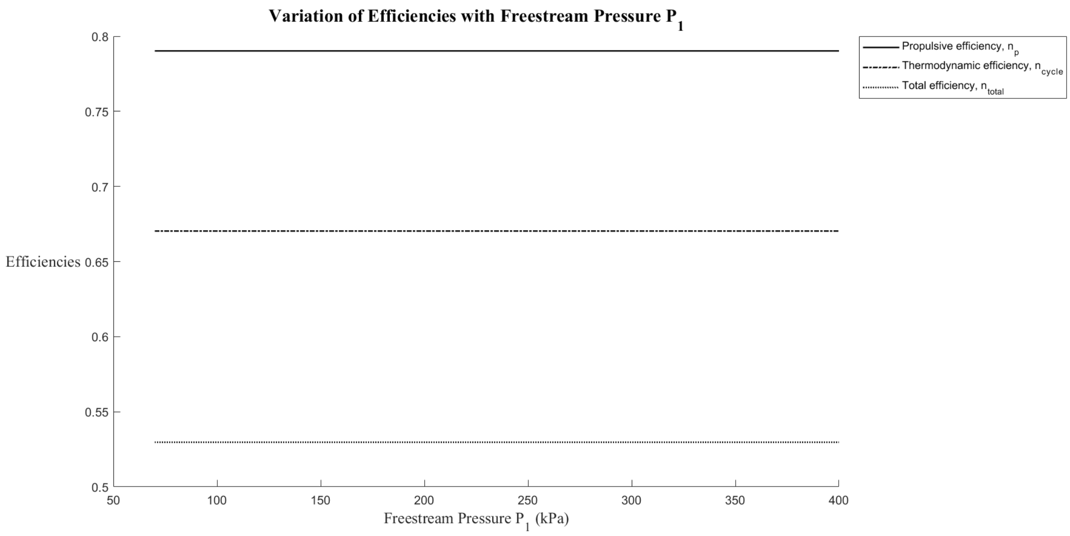

Figure 2: Variation of Efficiencies with Freestream Pressure, P_1

$$Non-Dimensional\ Thrust,\ NDT = \frac{F}{P_1 A_1} \tag{1}$$

$$A_1 = \frac{F}{NDT \cdot P_1} \tag{2}$$

$$\begin{aligned} Propulsive\ Efficiency,\ \eta_p &= \frac{Thrust\ Power}{Jet\ Momentum\ Power} \\ &= \frac{F}{P_1 A_1} \frac{2}{\gamma} \frac{1}{\frac{T_4}{T_1} M_4^2 - M_1^2} = \frac{F}{P_1 A_1} \frac{2RT_1}{U_4^2 - U_1^2} \end{aligned} \tag{3}$$

Since exhaust pressure equals to freestream pressure, and pressure is constant across the burner, the assumption that $h_4 - h_1 = C - p(T_4 - T_1)$ and $\Delta h_{burner} = C_p(T_b - T_2)$, resulting in Equation 4.

$$Thermodynamic\ Efficiency,\ \eta_{cycle} = 1 - \frac{T_1}{T_2} = 1 - \frac{h_4 - h_1}{\Delta h_{burner}} = 1 - \frac{T_4 - T_1}{T_b - T_2} \tag{4}$$

From the illustration of the variation of efficiencies with freestream pressure in Figure 2, it is established that all efficiencies remain constant and independent of the freestream pressure. This is fundamentally because inlet velocity M_1 and exhaust velocity M_4 remain constant with varying freestream pressure P_1 as there are no dependencies between them. As freestream pressure $P_!$ increases, the P_1 component within the computation of inlet area A_1 increases as seen from Equation 2, resulting in the reduction of A_1. The two resulting effects cancel out each other which thus kept propulsive efficiency η_p constant. As for thermodynamic efficiency η_{cycle}, it remains constant as P_1 is not involved in the computation of η_{cycle}.

5.2 Freestream Temperature Variation

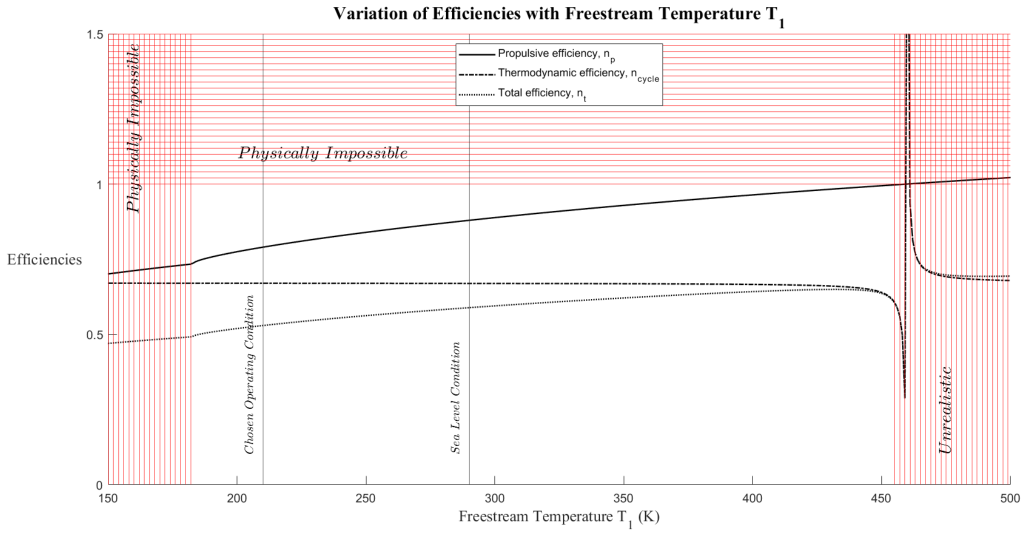

Figure 3: Variation of Efficiencies with Freestream Temperature, T_1

From Figure 3, it is established that the thermodynamic efficiency remains constant as freestream temperature is varied. On the other hand, propulsive efficiency increases with freestream temperature. It is observable that there is a discontinuity located at an approximate temperature of $180K$. This can be explained from the quadratic formula that was used to compute the Mach number at the burner exit.

$$M_b^2 - \sqrt{\frac{T_2}{T_b}}(M_2 + \frac{1}{\gamma M_2})M_b + \frac{1}{gamma} = 0 \tag{5}$$

$$M_b = \frac{1}{2}\sqrt{\frac{T_2}{T_b}}(M_2 + \frac{1}{\gamma M^2}) \pm \frac{1}{2}\sqrt{\frac{T_2}{T_b}(M_2 + \frac{1}{\gamma M^2})^2 - \frac{4}{\gamma}} \tag{6}$$

$$Term\ A = \frac{T_2}{T_b}(M_2 + \frac{1}{\gamma M^2})^2 - \frac{4}{\gamma} \tag{7}$$

It is observed from Term A that if it is smaller than 0, it would result in the Mach number at burner exit M_b to be non-real, which is undesired. Therefore, the discontinuity that occurs from $180K$ is an indication of this condition being violated and should not be taken into consideration.

Note that from inlet temperature of approximately $455K$, propulsive efficiency exceeds 1. This is because the static temperature at the burner entry T_2 would attain the designated burner exit temperature T_b, which is defined at $1400K$. This means that when inlet temperature is above $455K$, heat would have to be taken away from the flow instead of added into the flow, which violates the purpose of the engine. Therefore, these regions are marked as regions of impracticality and only the remaining region is correct.

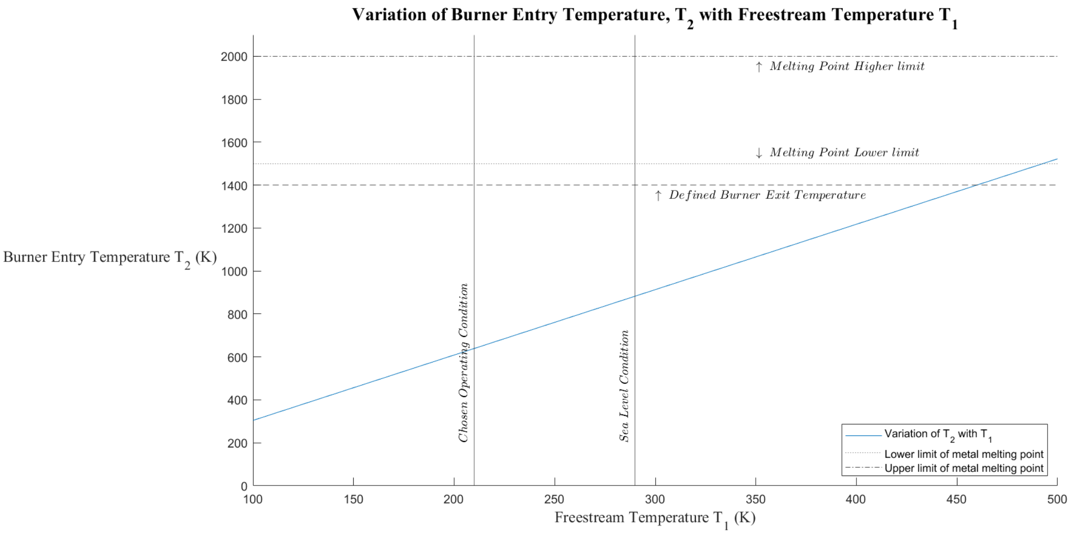

Figure 4: Variation of Burner Entry Temperature, T_2 with Freestream Temperature, T_1

From Figure 4, it is observed that the engine would start melting from $1500K$, based on the lower limit of the melting point. However, with the defined burner exit temperature at $1400K$, which is the highest temperature that the engine would experience, it is unlikely that the engine would experience temperatures of $1500K$. The freestream temperatures of the selected operating condition and at sea level are shown to provide clarify.

Thermodynamic efficiency remains fairly constant up to about $400K$. This is because as freestream temperature increases, burner inlet temperature increases as shown in Figure 4, resulting in a smaller Δh_{burner} value. However, increasing T_1 has little effect on exhaust temperature, meaning that the $T_4 - T_1$ term also decreases as T_1 increases, which cancels out the effect of the reduction in heat addition into the burner and result in a fairly constant thermodynamic efficiency.

A spike can be observed when inlet temperature approaches $460K$. This is because when inlet temperature increases to approximately $455K$, the temperature of the burner inlet increases as well and would eventually be equal to burner exit temperature ($1400K$), meaning that zero heat would be added to the flow, resulting in $\Delta h_{burner} = 0$. Therefore, the term $-\dfrac{h_4 - h_1}{\Delta h_{burner}}$ would tend to $-\infty$ before crossing over to $+\infty$ as T_1 approaches $455K$, resulting in the discontinuity. However, since the engine would not be able to operate realistically within this range of freestream temperature, the focus would only be on the range of temperatures much lower than $455K$.

5.3 Variation in Freestream Mach Number

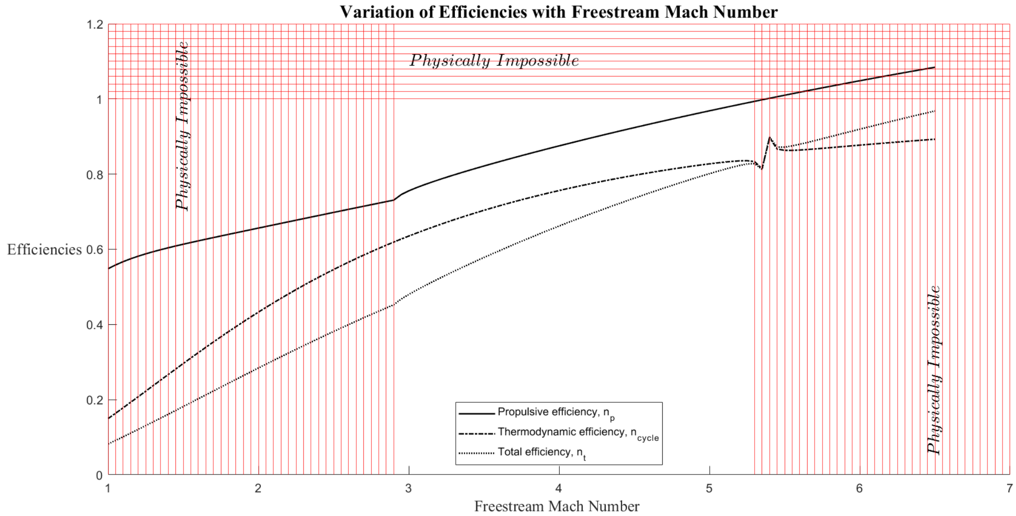

Figure 5: Variation of Efficiencies with Freestream Mach Number, M_1

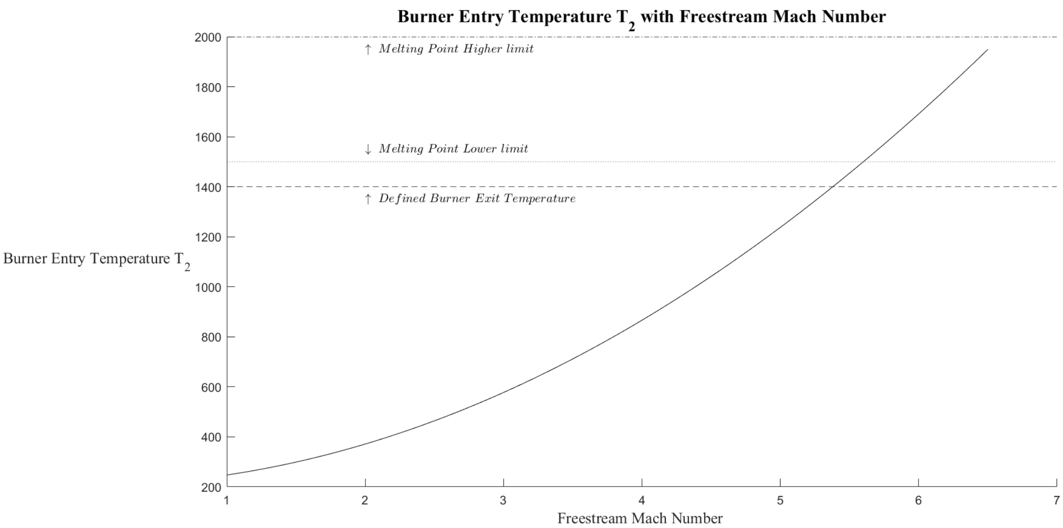

Figure 6: Burner Entry Temperature T_2 with Freestream Mach Number M_1

Both propulsive and thermodynamic efficiencies increase as freestream Mach number increases. A kink can be seen at inlet Mach of around 2.9 and a spike can be seen at around Mach 5.3. This is because at $M_1 < 2.9$, the quadratic equation used to calculate burner exit Mach number would result in imaginary numbers, denoting that the engine would simply not be able to operate under those sets of condition. At $M_1 > 5.3$, burner entry temperature would exceed the burner exit temperature of $1400K$, meaning that heat would have to be taken away from the flow instead of the heat addition to the flow, and therefore, the engine would simply not work when taken to those Mach number range. Therefore, the following analysis on engine efficiencies would only focus on the range $3 < M_1 < 5$.

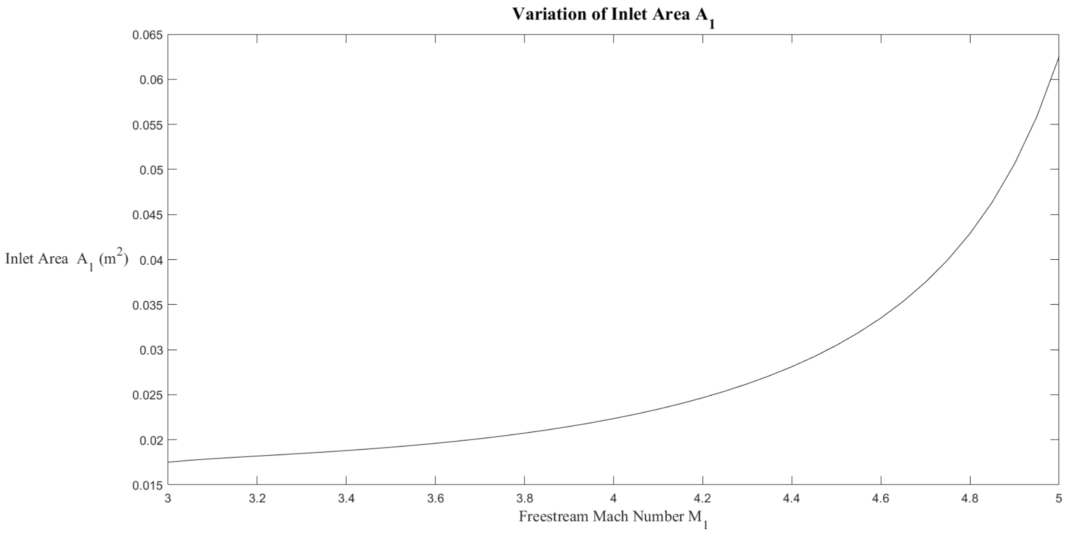

Figure 7: Variation of Inlet Area A_1 with Freestream Mach Number M_1

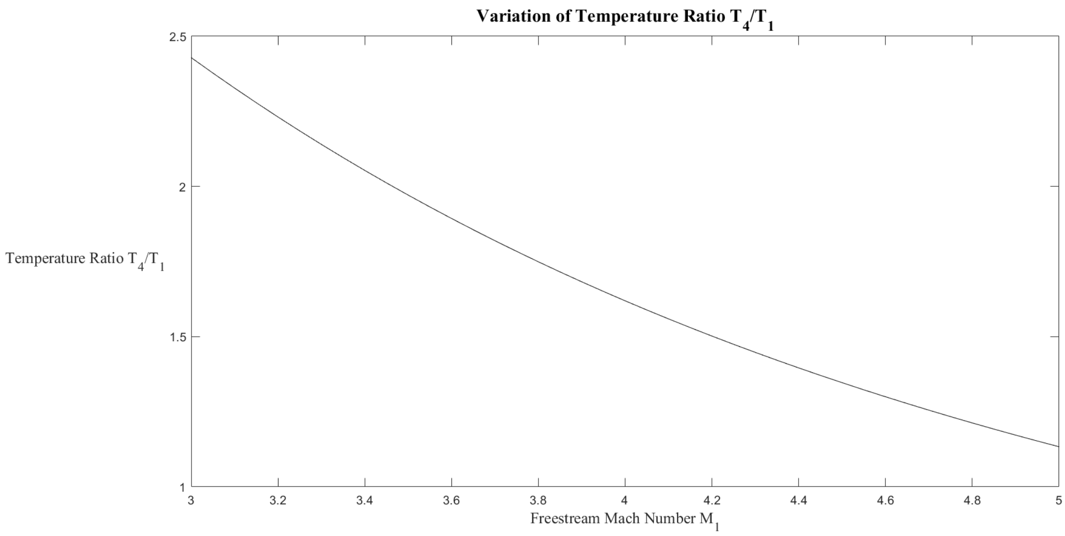

Figure 8: Variation of Temperature Ratio $\frac{T_4}{T_1}$ with Freestream Mach Number M_1

Increasing inlet Mach number reduces static temperature ratio between exhaust and inlet as illustrated by Figure 8. Therefore, by inspecting Equation 3, it is observed that the magnitude of the denominator would reduce as inlet Mach increases.

Also, increasing the freestream Mach number would result in an increase in required inlet area; however, the increase in inlet area is relatively small as shown in Figure 7 and therefore, the overall magnitude of the denominator would reduce. This implies that propulsive efficiency would increase.

As freestream Mach number increases, the burner entry temperature is going to increase, meaning less heat would need to be added to the flow; meanwhile, the temperature ratio between exhaust and inlet reduces as freestream Mach number increases, resulting in less heat loss to the environment. By examining Equation 4, this would result in an increase in thermodynamic efficiency.

From Figure 6, it is observed that the melting point is beyond the burner exit temperature that was defined at $1400K$. Therefore, engine melting would not be an issue if M_1 is kept at approximately < 5.3.

5.4 Variation in Normal Shock Strength

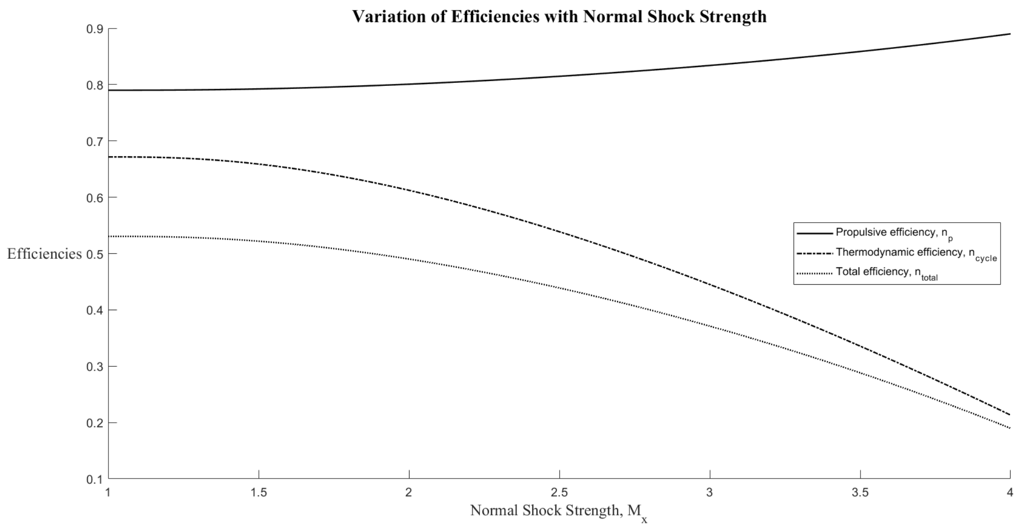

Figure 9: Variation of Efficiencies with Normal Shock Strength

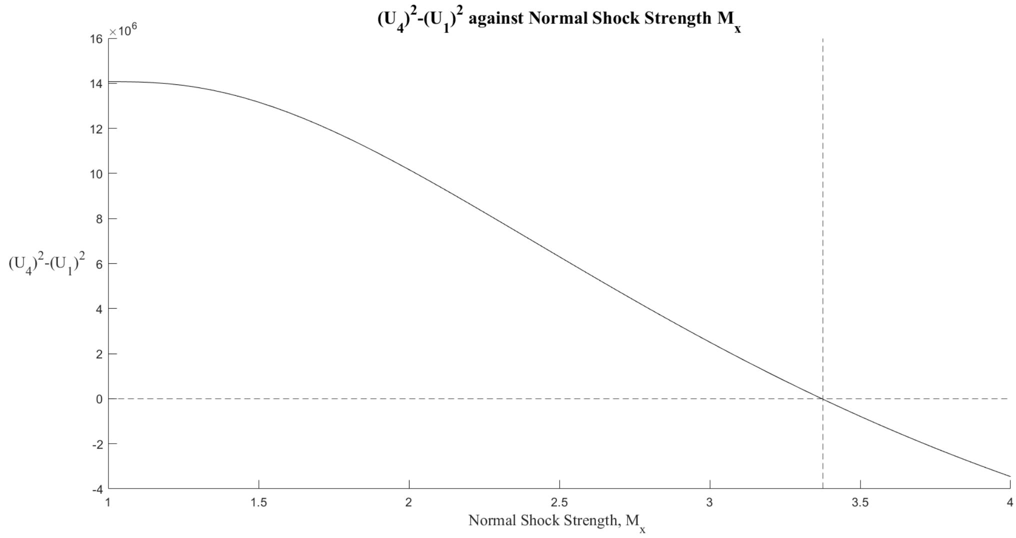

Figure 10: $U_4^2 - U_1^2$ against Normal Shock Strength M_x

In Figure 9, it can be observed that the thermodynamic efficiency reduces significantly as the normal shock strength increases. This is because the stagnation temperature across the shock is constant which leads to a constant thermodynamic efficiency that can be shown in Equation 4.

For the propulsive efficiency, Figure 9 shows that the propulsive efficiency increases gradually with the normal shock strength. In performing the analysis for propulsive efficiency, the thrust is kept constant which means that A_1 and U_4 will be the manipulated variables as shown in Equation 3. When normal shock strength increases, M_4 decreases quite significantly while T_4 increases slightly and thus decrease the magnitude of the term $\frac{T_4}{T_1}M_4^2 - M_1^2$. Increasing shock strength also increases A_1 slightly, but the increase in A_1 is relatively insignificant, and therefore, the result in the overall propulsive efficiency increasing with the normal shock strength.

In Figure 10, a graph of $U_4^2 - U_1^2$ against normal shock strength was plotted to derive a possible maximum normal shock strength magnitude and it illustrates that the maximum normal shock strength is approximately 3.375 as $U_4^2 - U_1^2$ would give a negative value when the normal shock goes beyond 3.375, which is not something desirable for a ramjet that is meant to propel an aircraft forward. The change in jet momentum reduces as shock strength increases, which means less proportion of energy is used to energize jet stream and a relatively higher proportion of energy is used to generate thrust. An engine is more efficient in the propulsive sense when a larger amount of air is moving relatively slower compare to a small amount of air moving relatively faster. Therefore, the engine's propulsive efficiency increases as shock strength increases due to the reduction in exhaust velocity.

5.5 Variation in Burner Entry Mach Number

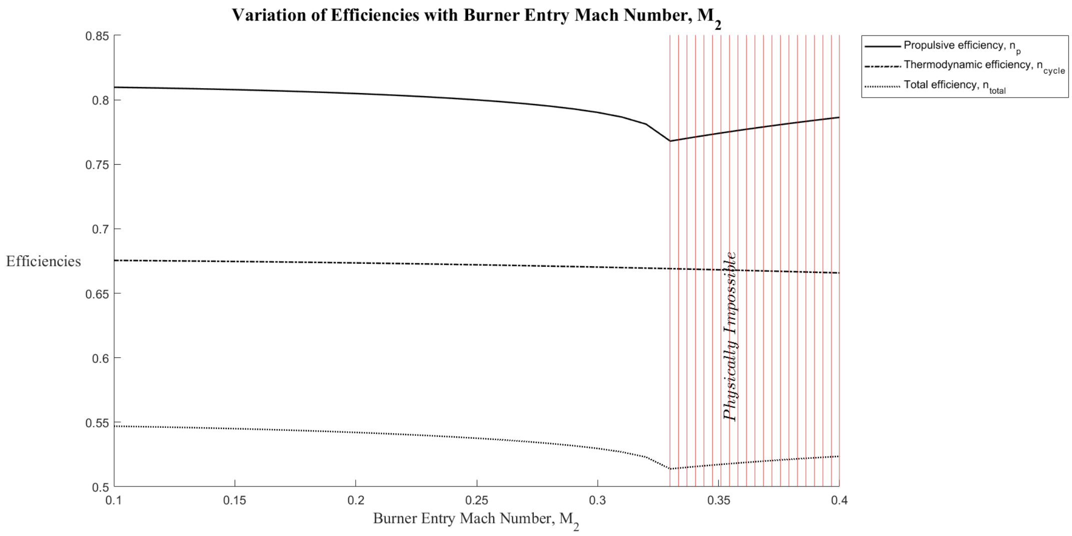

Figure 11: Variation of Efficiencies with Burner Entry Mach Number M_2

From Figure 11, both propulsive and thermodynamic efficiencies reduce as burner entry Mach number increases, resulting in a reduction in total efficiency as burner entry Mach increases. A kink can be observed at around $M_2 = 0.33$, which is caused by burner exit Mach number becoming imaginary number as M_2 was increased.

Examining Figure 12 shows that increasing M_2 shifts the M_b curve to the left. At the same time, increasing M_2 would result in a reduction in T_2, thus increasing the burner exit to entry temperature ratio. This shifts the vertical line representing the temperature ratio to be shifted towards the right. Therefore, at some point of M_2, there would be no real roots for the $\frac{T_b}{T_2}$ ratio that corresponds to the quadratic curve of M_b as there is no point of intersection. This means that the engine would not be able to operate under those conditions. Hence, this analysis will only focus on the range $M_2 < 0.33$.

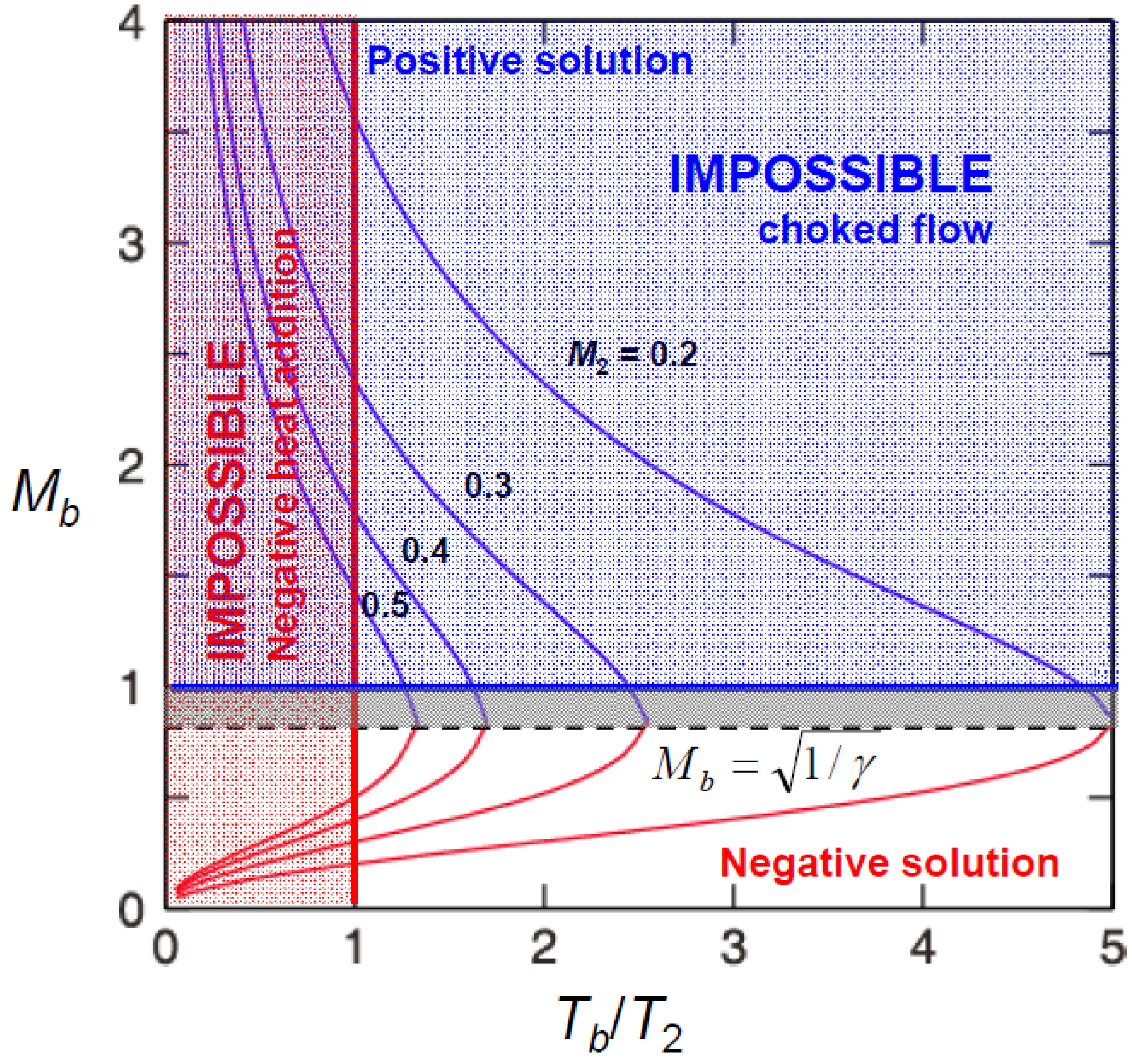

Figure 12: Illustration of M_b curve from Lecture 3(1)

Increasing burner entry Mach number M_2 reduces burner entry temperature T_2, which increases exhaust temperature T_4 as shown in Figure 13 and Figure 14. This means that more heat addition would be required at the burner and more heat would also be lost to the environment. Upon closer inspection, the increase in heat rejection of the system is more significant compared to the increase of heat addition to the system as M_2 increases, resulting in an overall reduction in thermodynamic efficiency.

Increasing burner entry Mach number M_2 increases M_4 and T_4 while the required inlet area reduces, which led to an overall increase in the denominator terms on the RHS of Equation 3, resulting in a reduction in propulsive efficiency.

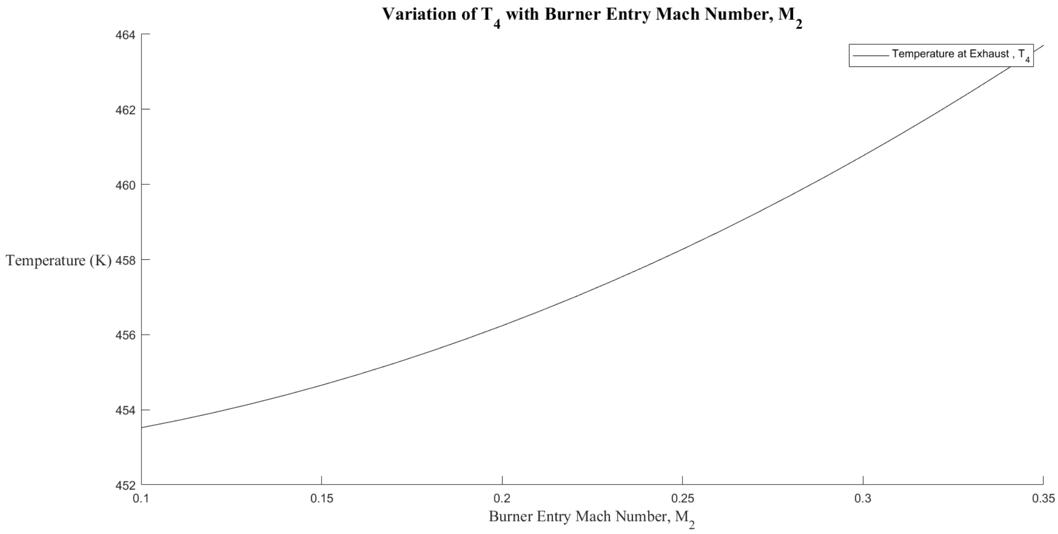

Figure 13: Variation of T_4 with Burner Entry Mach Number M_2

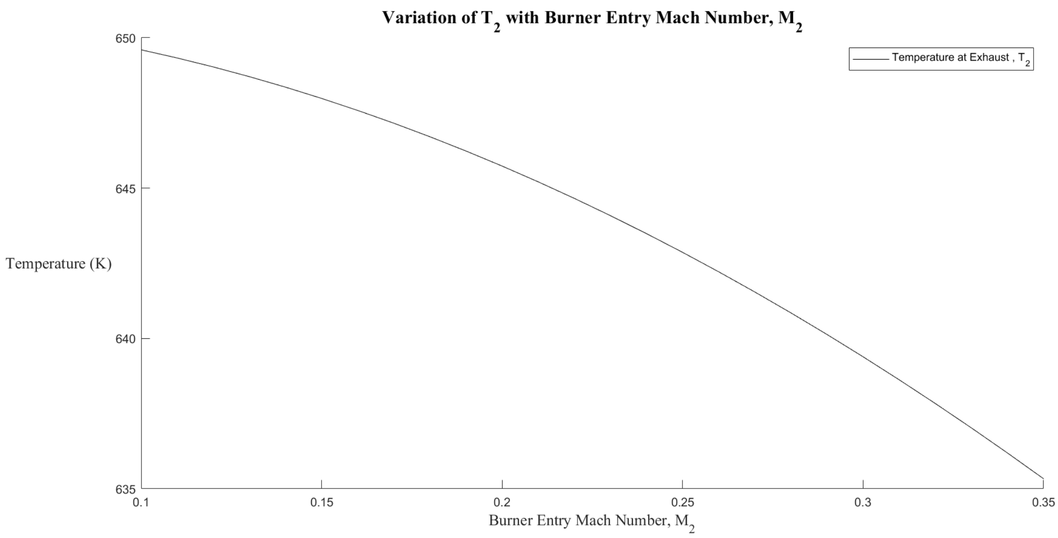

Figure 14: Variation of T_2 with Burner Entry Mach Number M_2

5.6 Variation in Burner Exit Temperature

From Figure 15, the general trend of the propulsive efficiency reflects a decreasing behavior as burner exit temperature is increased. This can be attributed to the fact that by increasing burner exit temperature, the coefficients utilized to solve for the Mach number at the burner exit would increase in magnitude, which ultimately causes the subsonic value of M_b to rise. Subsequently, the increase in burner exit temperature also increases the stagnation temperature and pressure ratio at the burner exit, which results in the magnitude of the stagnation pressure ratio at the exhaust to increase. This conclusively raises the magnitude of the exhaust Mach number M_4, which results in a decrease in propulsive efficiency. As for thermodynamic efficiency, it remains constant as burner exit temperature is varied. This is

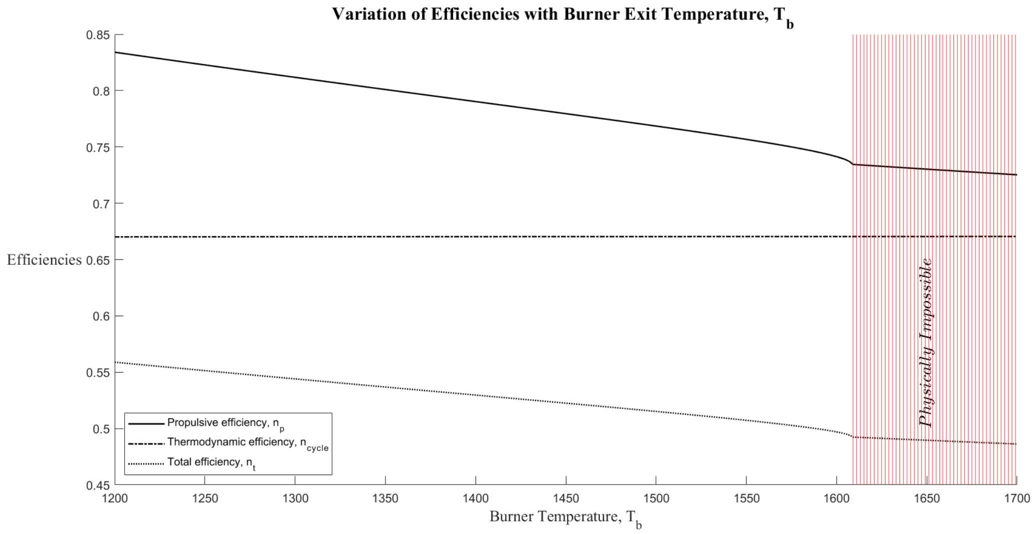

Figure 15: Variation of Efficiencies with Burner Temperature T_b

because T_4 increases linearly when T_b is increased, and referring back to Equation 4, it is observed that the change in numerator would be equal to the change in denominator.

Additionally, the plot illustrates a kink at approximately $1610K$ for the burner exit temperature. This is substantiated by the introduction of two subsonic roots of M_b when its quadratic equation if solved. As burner exit temperature is further increased, it would product non-real roots and the solution becomes physically wrong. The range of $1200K$ to $1700K$ was selected to illustrate the kink as well as the operating conditions below $1400K$.

5.7 Variation in Required Thrust

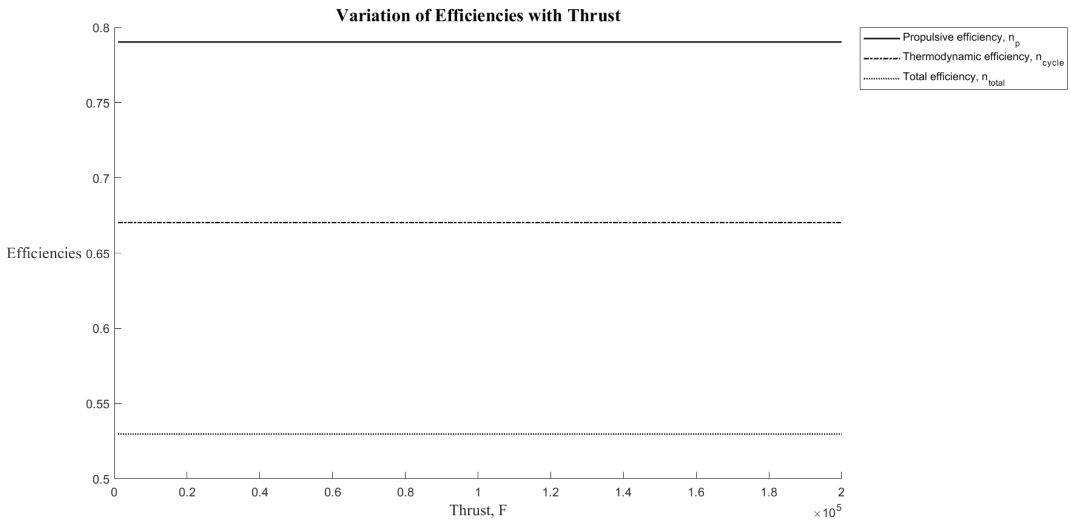

Figure 16: Variation of Efficiencies with Thrust F

From Figure 16, it shows that the required thrust does not affect the propulsive and thermodynamic efficiencies of the ramjet engine. By referring to Equation 2, it shows that the inlet area A_1 would be increasing proportionally with F. According to Equation 3, it shows that the propulsive efficiency is affected by both A_1 and F. With increasing F and A_1 in the equation, their effects would be cancelled out, which thus results in a constant propulsive efficiency.

6 Comparison with Tutorial Solutions

To validate the numerical simulation that was performed, the results calculated using Isentropic and Shock Tables in Tutorial 3 were utilized for comparison purposes. These results are stated in the table below.

Table 3: Comparison of Solutions

Variables	Units	Results(Numerical)	Results(Tables)	Difference
Inlet Area A_1	m^2	0.018308	0.019300	-5.14%
Inlet Throat Area A_{C1}	m^2	0.003442	0.003650	-5.70%
Burner Entry Area A_b	m^2	0.005403	0.005710	-5.38%
Nozzle Throat Area A_{C2}	m^2	0.004453	0.00471	-5.46%
Burner Exit Area A_2	m^2	0.007056	0.007470	-5.54%
Exhaust Area A_4	m^2	0.026239	0.027300	-3.89%
Propulsive Efficiency η_p	–	0.790224	0.759000	4.11%
Thermodynamic Efficiency η_{cycle}	–	0.671562	0.673000	-0.21%
Total Efficiency η_t	–	0.530685	0.510807	-3.89%

Utilizing the results computed with the tables as a reference, the difference between the two solutions were computed. It is observed that the magnitudes of the differences are all below 6%, which meant that the numerical solution is successfully validated with the tutorial solutions. The discrepancies between the computed values could be attributed to the method employed in their computations. For the numerical solution, the parameters were all calculated from mathematical equations, whereas the tutorial solutions utilized values from the Isentropic and Shock tables. Ultimately, utilizing equations to compute the parameters is undeniably the most accurate method due to the absence of interpolation while calculating. Therefore, with this comparison, the numerical solution is deemed to be reasonable for a preliminary ramjet design project.

7 Conclusion

The numerical calculations performed using the code is deemed to be correct in its theoretical computations of the magnitudes of the different variables as the trends correspond with the concepts that were imparted during the lecture. Based on the team's engineering judgement, there were regions of impracticality and violation which were duly highlighted and explained comprehensively. The numerical code was also validated with the solutions from Tutorial 3, with negligible discrepancies that could be attributed to rounding off errors.

It is however, vital to take note that this code is solely meant as a preliminary design of a ramjet with assumptions that rarely happen in reality. Parameters are also kept constant while a single variable is varied, which does not occur in reality as these variables are mostly coupled with each other. To top it up, there are many phenomena such as boundary layer interactions with the wall of the engine, flow separation caused by adverse pressure gradients, non-conformance with ideal gas law, varying constants such as specific heat ratio as well as variation nuances between 2-D and 3-D analyses.

Bibliography

[1] David M. Birch. *Advanced Propulsion Lecture Notes*. AERO97003, Imperial College London, Department of Aeronautics, 2020.