

GALOIS PARSIMONY

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ABSTRACT. We prove and interpret the Galois connection induced by satisfaction between sets of sentences and classes of structures in first-order logic.

Let \mathcal{L} be a first-order language, and \mathcal{S} be a set of \mathcal{L} -sentences, and \mathcal{M} be a class of \mathcal{L} -structures.

Definition 1. Let $\Phi \subseteq \mathcal{S}$.

$$\text{Mod}(\Phi) = \{m \in \mathcal{M} \mid \forall \varphi \in \Phi (m \models \varphi)\}$$

Definition 2. Let $M \subseteq \mathcal{M}$.

$$\text{Th}(M) = \{\varphi \in \mathcal{S} \mid \forall m \in M (m \models \varphi)\}$$

Theorem 1. Let $\Phi \subseteq \mathcal{S}$, and $M \subseteq \mathcal{M}$.

$$\Phi \subseteq \text{Th}(M) \iff M \subseteq \text{Mod}(\Phi)$$

Proof.

$$\Phi \subseteq \text{Th}(M) \iff \forall \varphi \in \Phi, \forall m \in M (m \models \varphi) \iff M \subseteq \text{Mod}(\Phi)$$

□

The syntax–semantics correspondence reverses inclusion: more axioms yield fewer models. This is a precise sense in which more constraints yield fewer possible worlds.