Supporting Information: Alan E. Gelfand and Shinichiro Shirota. 2019. Preferential sampling for presence/absence data and for fusion of presence/absence data with presence-only data. *Ecological Monographs.*

Appendix S3. Additional preferential sampling models

Following "Preferential sampling: Preferential sampling models for presence/absence data" in the main manuscript, we can propose additional preferential sampling specifications. We can extend model (i) to

(iii), $\log \lambda(\mathbf{s}) = \mathbf{w}^T(\mathbf{s})\boldsymbol{\beta} + \psi\omega(\mathbf{s})$.

In this notation, with model (b) for $\mathcal{Y}$, $\omega(\mathbf{s})$ is a shared process for both $\mathcal{Y}$ and $\mathcal{S}$ so $\mathcal{Y}$ and $\mathcal{S}$ are not independent. Working with (b) and (iii), if $\psi = 0$, then, following Diggle et al. (2010), we have non-preferential sampling while if $\psi \neq 0$, we have *strong* preferential sampling.

Cecconi et al. (2016) add another GP to the intensity for $\mathcal{S}$, i.e.,

(iv): $\log \lambda(\mathbf{s}) = \mathbf{w}^T(\mathbf{s})\boldsymbol{\beta} + \eta(\mathbf{s}) + \xi(\mathbf{s})$.

That is, using model (iv) with model (c), there is a shared GP for $\mathcal{Y}$ and $\mathcal{S}$ as well as individual GP's for each, a total of three independent GP's altogether. They acknowledge identifiability problems in model fitting with the three latent Gaussian fields. Table 2 in the main manuscript summarizes the modeling choices we have offered for $\mathcal{Y}$ and $\mathcal{S}$.

References

Cecconi, K., L. Grisotto, D. Catelan, C. Lagazio, V. Berrocal, and A. Biggeri (2016). Preferential sampling and Bayesian geostatistics: statistical modeling and examples. *Statistical Methods in Medical Research 25*, 1224–1243.

Diggle, P., R. Menezes, and T. Su (2010). Geostatistical inference under preferential sampling. *Journal of the Royal Statistical Society, Series C 59*, 191–232.