Problem set 3
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Problem 1

a) $z - z_0 = a\left((x-x_0)^2 + (y-y_0)^2\right)$

$$\Rightarrow 0 = ax^2 - 2ax_0x + ax_0^2 + ay^2 - 2ay_0y + ay_0^2 + z_0 - z$$

$$\Rightarrow (ax^2 - 2ax_0x + ay^2 - 2ay_0y - z) = -(z_0 + ay_0^2 + ax_0^2)$$

$$\Rightarrow \frac{-1}{(z_0 + ay_0^2 + ax_0^2)}(ax^2 - 2ax_0x + ay^2 - 2ay_0y - z) = 1$$

We can write that as: $c_1x^2 + c_2x + c_3y^2 + c_4y + c_5z = 1$

with:

$c_1 = -a/(z_0 + ay_0^2 + ax_0^2)$

$c_2 = 2x_0a/(z_0 + ay_0^2 + ax_0^2)$

$c_3 = -a/(z_0 + ay_0^2 + ax_0^2)$

$c_4 = 2ay_0/(z_0 + ay_0^2 + ax_0^2)$

$c_5 = 1/(ax_0^2 + y_0^2a + z_0)$

Then:

$a = -c_1/c_5$

$x_0 = (c_2/c_5)\cdot(1/2a) = (c_2/c_5)(-c_5/2c_1) = (-c_2/2c_1)$

$y_0 = (c_4/c_5)(1/2a) = -(c_4/2c_1)$

$$z_0 = (1/c_5) - a(y_0^2 + x_0^2) = \frac{1}{c_5} + \frac{c_1}{c_5}\left(\frac{(-c_2)^2}{(c_1)^2} + \frac{(-c_4)^2}{(c_1)^2}\right)$$

$$\Rightarrow z_0 = \frac{1}{c_5} + \frac{1}{c_5}\left(\frac{c_2^2 + c_4^2}{c_1}\right) = \frac{1}{c_5}\left(1 + \frac{c_2^2 + c_4^2}{c_1}\right)$$

b) See attached picture

c) With $N \equiv$ noise, we have $N = Am - b$, that we put as a diagonal matrix. We can use that to calculate the "left hand side" matrix and then compute the uncertainty on on parameters. Since a is $-c_1/c_5$, we propagate the uncertainties on both and find:

$$\sigma_a = a\left[\frac{\sigma_{c_1}^2}{c_1^2} + \frac{\sigma_{c_2}^2}{c_2^2}\right]^{1/2} = 3,491 \cdot 10^{-9}$$

We find: $f = \frac{1}{4a}$. We propagate the error in the same way and find the uncertainty on f. (See images attached).

<u>Problems 3-5</u>: See attached files