

### Problem 1

My method gets me 2 calls for a cosine between 0 and  $\pi/2$ .

It gives me 4 calls for exp between 0 and 5.

5 calls for the Lorentzian between -10 and 10. And 1 call for sine between  $-\pi$  and  $\pi$ .

The "lazy method" gives, for the same integrals, between the same values 3, 33, 43 and 1 calls respectively.

However, my method is dependent on the lower bound of the size that a step in a numpy array can have. For a too small tolerance, or a too big interval of integration, the code may stop functioning.

### Problem 2

My best results came at an order of 5. They can be found in the appropriate folder.

### Problem 3

a) The difference in the half-time between the Uranium 238 and the Plonium 214 makes this a set of "stiff equations". I used `scipy.integrate.solve_ivp` with the method set to "Radau" to solve implicitly.

b) I showed the evolution of U-238 in time against that of Pb-206. The time scale was multiply of  $10^{17}$  s. While U-238 decreases exponentially at a constant rhythm (as expected analytically), the Pb-206 has 3 phases in its evolution. At first it grows rapidly (almost vertically on a log scale), slows down, then the slope becomes very stable again. It is expected that Pb-206 would only grow in number, as it is the bottom of the chain. The steady slope indicates that we reached a time scale where all the half lives have been passed. Hilroy