Problem 3

Cosine function: The rms errors, for the cubic interpolation, the cubic spline and the rational interpolation are, respectively: $1,64 \cdot 10^{-5}$; $5,41 \cdot 10^{-6}$ and $1,01 \cdot 10^{-6}$. Those values are in tune with our expectations regarding those methods.

Lorentzian function: The rms errors are: $0,00205$; $0,00154$ and $68,64$ for the cubic interpolation, the cubic spline and the rational interpolation, respectively. The first two are ordered as expected, but it is much less accurate than the cosine function. For the rational interpolation, the value is very high. Those discrepancies are due to a pole in the complexe plane, near the values we use.

When we go to a higher order for the rational interpolation, the error becomes much bigger. Using np.linalg.pinv instead of np.linalg.inv fixes the issues and the rms is $1,37 \cdot 10^{-16}$ $(5;\xi)$

With $n=3$; $m=5$, we get:

$p = [1 \quad 0 \quad -1/3]$ for np.linalg.pinv
$q = 0 \quad 0,\bar{6} \quad 4,44 \cdot 10^{-16} \quad -0,\bar{3}]$ for np.linalg.pinv.

$$\left.\begin{array}{l} p = [-7,98 \quad -6 \quad -1] \\ q = [-6 \quad 1 \quad -6 \quad 2] \end{array}\right\} \text{for np.linalg.inv}$$

linalg.inv case { ~~Looking~~ Looking at those, we see that the denominator, dominated by the x coefficient gets really small for $0 < x < 1$. It is also clear that q, being very small in that interval, amplifies the values of p.

On the other hand $q = 1 + \frac{2}{3}x^2 + \xi x^3 - (1/3)x^4$. The bottom term is thus superior to 0 and we avoid the erratic behavior seen in the inv case.

Problem 4

The integral goes to infinity at $t = R$, after it is evaluated.

The quad assigned a finite value at $t=0$ and stayed equal to 0 for $t < R$. My integrator goes away from 0 when t gets near R, from the left.