

Department of Computer Science
CS917 Foundations of Computing
Discrete Mathematics and Statistical Analysis Assignment

This assignment is due **at noon on 16th December, 2021**. The submission is on Tabula, and should comprise scanned copies of written work.

The work that you submit should be **your own work** and please show **full working** where appropriate, as this is necessary to gain full marks.

Marks for each question are indicated. The total marks you can get is 100.

If you have any questions then do please email me at long.tran-thanh@warwick.ac.uk.

1 Discrete Mathematics

1. For each of the following formulae, find a logically equivalent formula in which \wedge , \implies and \iff do not occur: (i) $(\neg p \implies q)$; (ii) $((p \vee q) \wedge r)$; (iii) $\neg((p \vee q) \iff r)$. Use the truth table to show that the proposed solution is indeed equivalent with the original one. [12 marks]
2. Write out paraphrases of the following, using \forall , \exists and $=$ (i) Arrakis has sandworms; (ii) sandworms are the only animals that can produce any spice; (iii) Every member of the Guild Navigator needs to have spice in order to navigate spaceships; (iv) The Kwisatz Haderach is the only one who can see the future. Auxiliary clauses: $\text{has}(x, y)$: x has/is in possession of y ; $\text{produce}(x, y)$: x produces y ; $\text{navigate}(x, y)$: x can navigate y ; and $\text{seesFuture}(x)$: x can see the future. [12 marks]
3. Write predicate logic formulae which state that the relation expressed by $R_{x,y}$ has the following properties: (i) $R_{x,y}$ is irreflexive; (ii) $R_{x,y}$ is intransitive; (iii) $R_{x,y}$ is not a partial order. Note that this formulation is a bit different from the one in the slides. To make this consistent, think about $R_{x,y}$ as R_p with relation p between x and y (you can assume that both x and y are from the same set A). So you can use $R_{x,y}$ as p in your formulations. Therefore, in your answer, you can use both notations, just be consistent (i.e., if you choose 1 notation, then use the same for all your answers). [9 marks]
4. Determine which of the following functions are injective and which are surjective (please provide explanations as well):
 - (i) $f: \mathbb{Z} \rightarrow \mathbb{N}$, where $\forall n \in \mathbb{Z}: f(n) = n^{2021} + 1$;
 - (ii) $g: \mathbb{N} \times \mathbb{N} \rightarrow \mathbb{N}$, where $\forall (n, k) \in \mathbb{N} \times \mathbb{N}: g(n, k) = 5^n 7^k 11^{n+k}$;
 - (iii) $h: \mathbf{P}(\mathbb{N}) \rightarrow \mathbf{P}(\mathbb{N})$, where $\forall A \in \mathbf{P}(\mathbb{N}): h(A) = \mathbb{N} \setminus A$ (recall what $\mathbf{P}(\mathbb{N})$ means);
 - (iv) $k: \mathbb{N} \rightarrow \mathbb{Z}$, where $\forall n \in \mathbb{N}: k(n) = (-1)^n$.[12 marks].

2 Statistical Analysis

5. On a vessel containing 3 white and 5 black balls, 4 balls are transferred into an empty vessel. From this vessel a ball is drawn and it is found to be white. What is the probability that out of 4 balls transferred 3 are white and 1 is black? [10 marks]
6. The probability of detecting tuberculosis in X-ray examination of a person suffering from the disease is $(1-b)$. The probability of diagnosing a healthy person as tubercular is a . The ratio of tubercular patient to the whole population is c . Find the probability that a person is healthy, if after the examination he is diagnosed as tubercular. [10 marks]
7. Consider all families with 2 children and assume that each child is equally likely to be a girl or a boy. If such a family is picked at random and found to have a boy, then what is the probability that it has another boy? [10 marks]
8.
 - a. The results below are the calorie content of the different brands of beef and poultry hotdogs.
Beef hotdogs: 186, 181, 176, 149, 184, 190, 158, 139, 175, 148, 152, 111
Poultry hotdogs: 129, 132, 102, 106, 94, 102, 87, 99, 170, 113
Can it be inferred that the calorie content of poultry hotdogs is lower than the calorie content of beef hotdogs? (use significance level 0.05) [10 marks]
 - b. After mixing a certain food for calves, calories observed on the same sample of 12 brands of beef hotdogs as used above are:
181, 191, 186, 129, 178, 194, 139, 122, 195, 158, 158, 104.
Can it be inferred that mix of food has an effect in calorie of the beef hotdog? (use significance level 0.05). [15 marks]