1) 1×1 $\quad \det([a]) = a$

2) 2×2 $\quad \det\left(\begin{bmatrix} a & b \\ c & d \end{bmatrix}\right) = ad - bc$

3) 3×3

$$\begin{array}{cccccc} + & + & + & - & - & - \end{array}$$

$$\det \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \begin{matrix} a & b \\ d & e \\ g & h \end{matrix} = aei + bfg + cdh - ceg - afh - bdi$$

wzór Sarrusa

TO DZIAŁA TYLKO DLA 3×3

!

4) Rozwinięcie Laplace'a

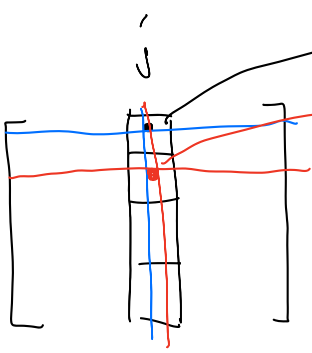
$A \quad n\times n$

(I) Niech j będzie ustalonym numerem kolumny

$A_{ij} \quad (n-1)\times(n-1)$

$$\det A = \sum_{i=1}^{n} (-1)^{i+j} a_{ij} \det A_{ij}$$

macierz, która powstaje z macierzy A przez usunięcie i-tego wiersza i j-tej kolumny.

det  $= (-1)^{1+j} a_{1j} \cdot \det A_{1j} + (-1)^{2+j} a_{2j} \det A_{2j} + \dots$

$$n\times n \longrightarrow n \cdot (n-1)\times(n-1) \longrightarrow n \cdot (n-1)\cdot(n-2)\times(n-2) \longrightarrow$$

$$\dots \longrightarrow n\cdot(n-1)\cdot(n-2)\cdot\ldots\cdot 2\cdot 1 = n! \approx \frac{n^n}{e^n}$$

(II) i – ustalony w wierszu

$$\det A = \sum_{j=1}^{n} (-1)^{i+j} a_{ij} \det A_{ij}$$

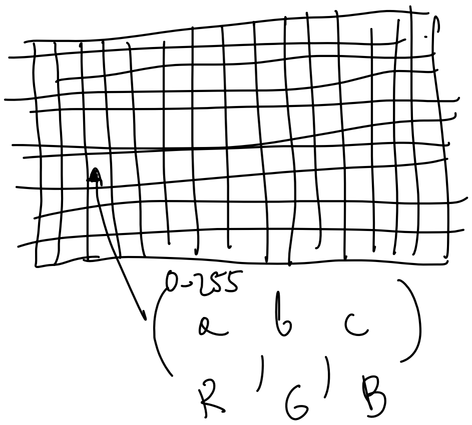
Tw. Cauchy'ego

$A, B \in \mathbb{R}_{n\times n}$

$$\Rightarrow \quad \det(AB) = \det A \cdot \det B$$

GRAFIKA

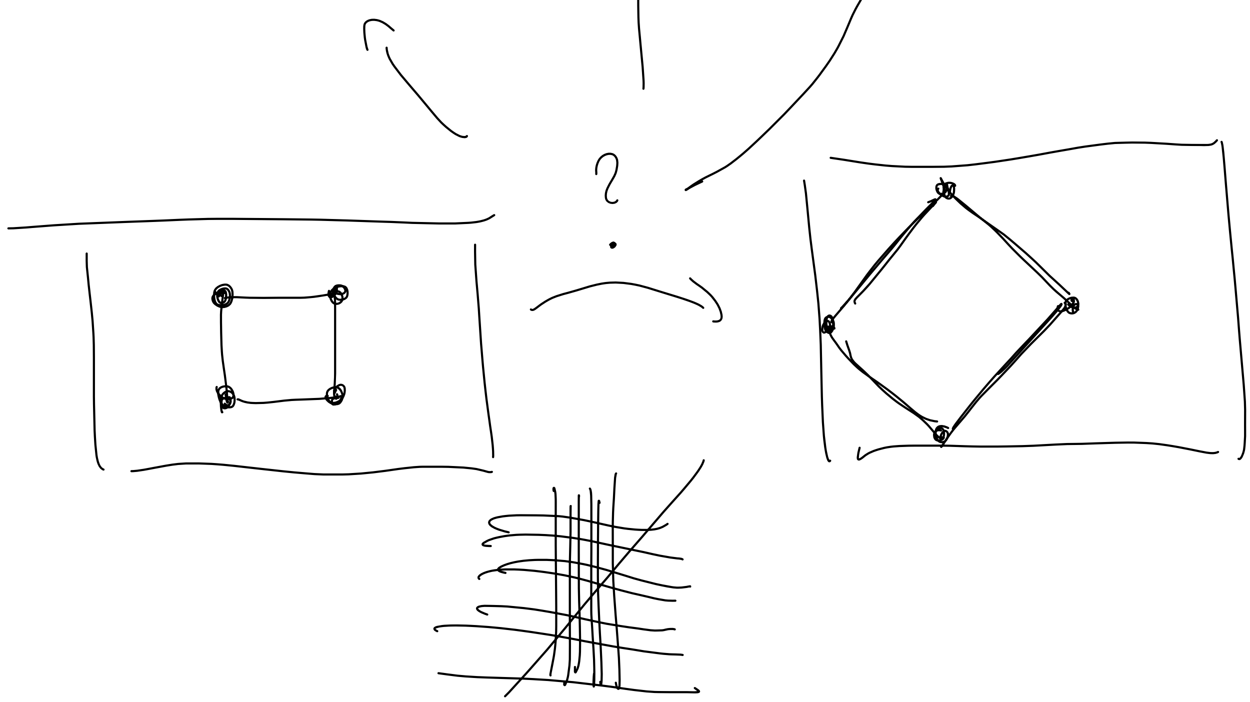
BITMAPOWA



WEKTOROWA

OPIS MATEMATYCZNY

- punkty
- proste
- okręgi
- odcinki



Przekształcenie liniowe

$$A: \mathbb{R}^m \to \mathbb{R}^n$$

1) $\bigwedge_{\vec{x},\vec{y}} \quad A(\vec{x}+\vec{y}) = A(\vec{x}) + A(\vec{y}) = A\vec{x} + B\vec{y}$

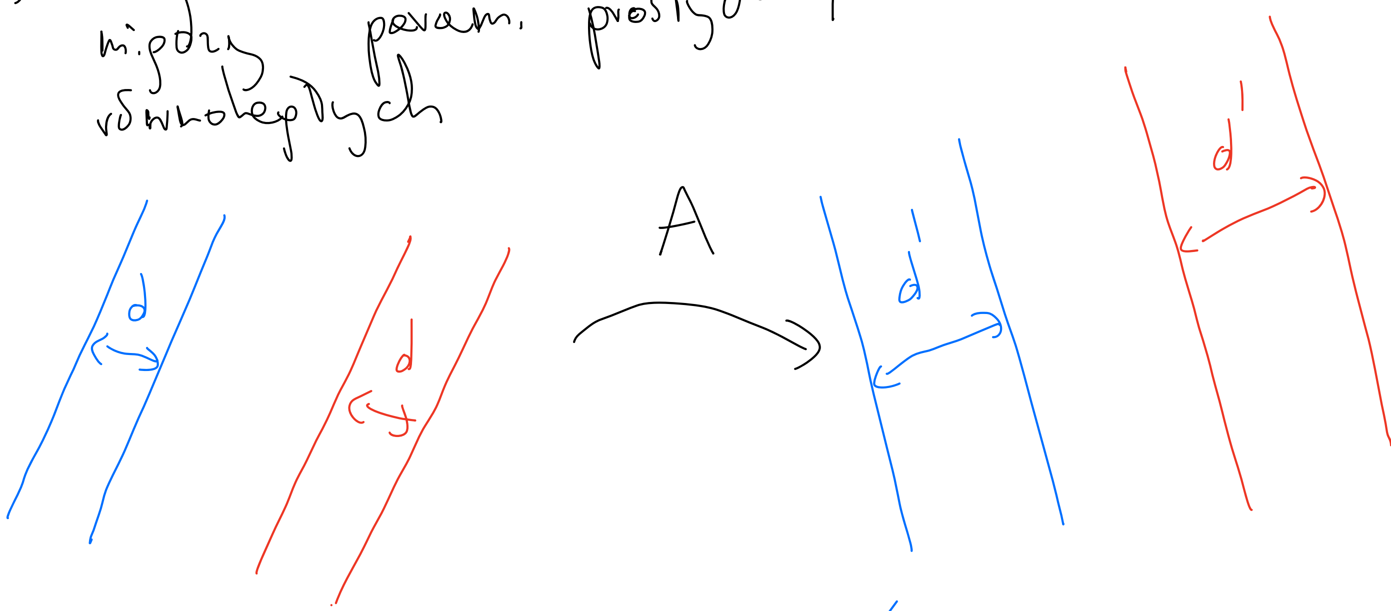
ADDYTYWNOŚĆ

2) $\bigwedge_{\alpha \in \mathbb{R}} \bigwedge_{\vec{x}} \quad A(\alpha\vec{x}) = \alpha A\vec{x}$

JEDNORODNOŚĆ

$$A: \mathbb{R}^2 \to \mathbb{R}^2$$

1) $O \to O$
2) proste → proste
3) proste równoległe → proste równoległe
4) zachowuje odległość między parami prostych równoległych



funkcja liniowa

$$\boxed{f(x) = ax + \underline{b}}$$

$$f: \mathbb{R}^1 \to \mathbb{R}^1$$

Funkcja liniowa jest prz. liniowym $\iff$ $b=0$

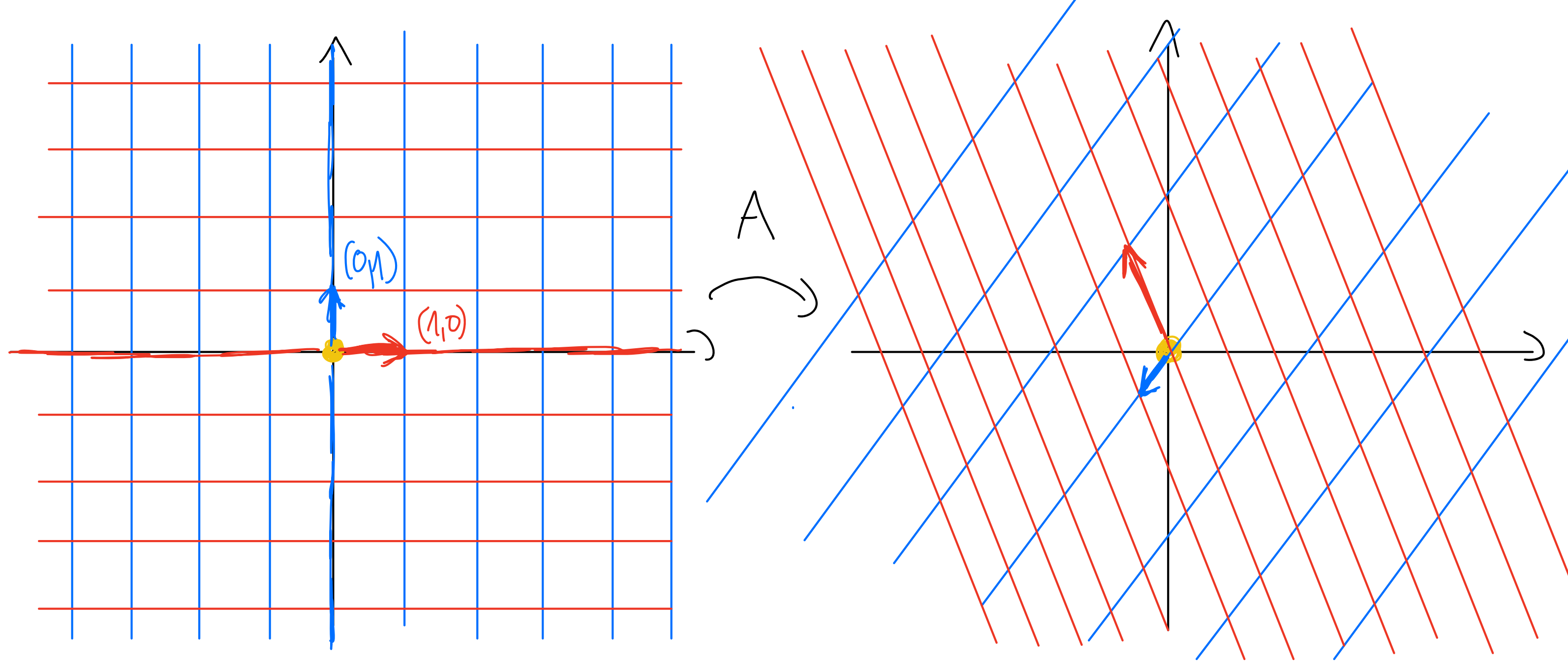
$b=0$

$$f(x+y) = a(x+y) = ax + ay = f(x) + f(y)$$

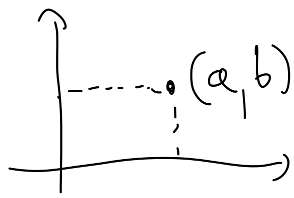
$$f(\alpha x) = a(\alpha x) = \alpha(ax) = \alpha f(x)$$

$$f(x) = ax$$

$$f \leftrightarrow a$$



$$A : \mathbb{R}^2 \to \mathbb{R}^2$$



$$\boxed{A(a,b)} = A\big((a,0)+(0,b)\big) =$$

$$= A(a,0) + A(0,b) =$$

$$= A\big(a(1,0)\big) + A\big(b(0,1)\big) =$$

$$= a\,\boxed{A(1,0)} + b\,\boxed{A(0,1)}$$

$$A \longleftrightarrow \underset{\mathbb{R}^2}{\underset{\cap}{A(1,0)}},\ \underset{\mathbb{R}^2}{\underset{\cap}{A(0,1)}}$$

$f(x) = ax$

$f \longleftrightarrow a$

$$A(1,0) = (\alpha, \beta) \qquad A(0,1) = (\gamma, \delta)$$

$$A \longleftrightarrow \alpha, \beta, \gamma, \delta$$

$$A \longleftrightarrow A(1,0),\ A(0,1) \longleftrightarrow \alpha, \beta, \gamma, \delta$$

$$A \longleftrightarrow \begin{array}{c} \begin{bmatrix} \alpha & \gamma \\ \beta & \delta \end{bmatrix} \\ A(1,0) \quad A(0,1) \end{array}$$

$$\begin{aligned} A(a,b) &= a\,A(1,0) + b\,A(0,1) = \\ &= (a\alpha, a\beta) + (b\gamma, b\delta) = \\ &= \boxed{(a\alpha + b\gamma,\ a\beta + b\delta)} \end{aligned}$$

$$\Updownarrow$$

$$\begin{bmatrix} \alpha & \gamma \\ \beta & \delta \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} \alpha a + \gamma b \\ \beta a + \delta b \end{bmatrix}$$

$A, B : \mathbb{R}^2 \to \mathbb{R}^2$ $\qquad$ A - macierz T

$\qquad\qquad\qquad\qquad\qquad\quad$ B - macierz S

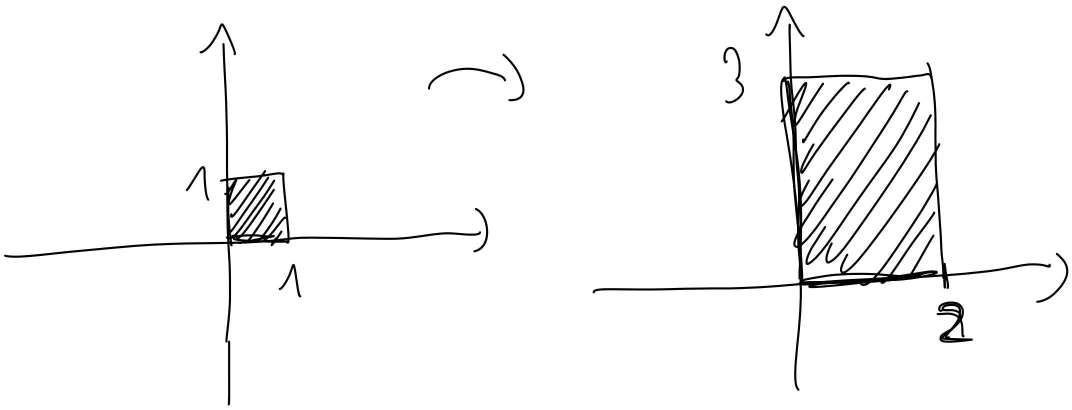
$$\begin{aligned} (A \circ B)(a,b) &= A(B(a,b)) = \\ &= A\Big(\underbrace{S \cdot \begin{bmatrix} a \\ b \end{bmatrix}}_{2\times 1}\Big) = \\ &= T \cdot \Big(S \cdot \begin{bmatrix} a \\ b \end{bmatrix}\Big) = \\ &= (T \cdot S) \begin{bmatrix} a \\ b \end{bmatrix} \iff \boxed{A \circ B \longleftrightarrow T \cdot S} \end{aligned}$$

1) $A \longleftrightarrow \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$

jednokładność o skali 2

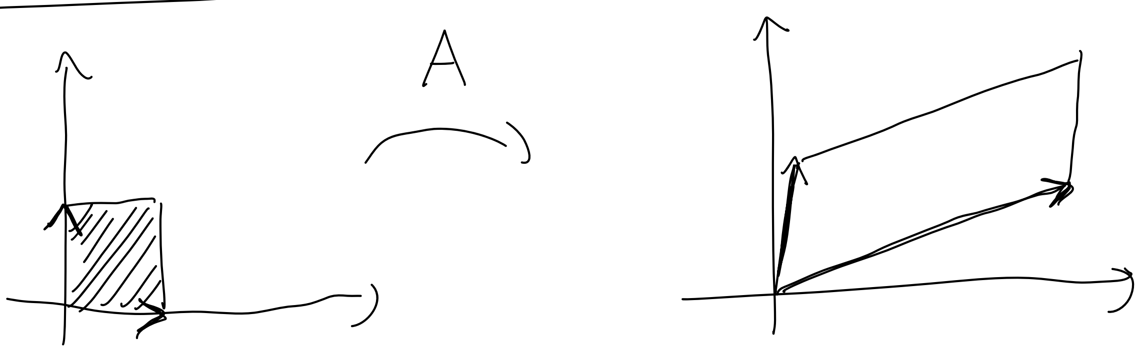
$$\begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 2a \\ 2b \end{bmatrix}$$

2) $\begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix}$

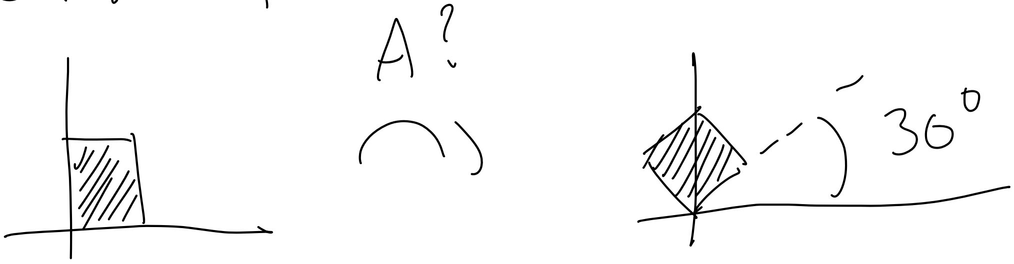


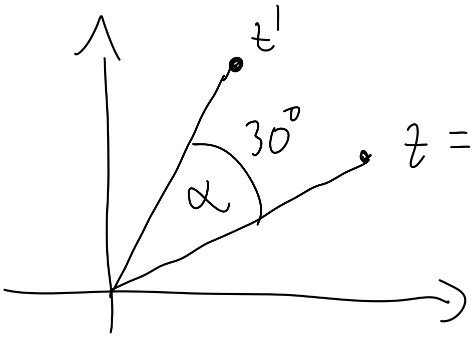
3) $\begin{bmatrix} k_x & 0 \\ 0 & k_y \end{bmatrix}$, $k_x, k_y \in \mathbb{R}$

JEDNOKŁADNOŚĆ



4) OBROTY $\alpha = 30^\circ$





$z = (a, b) = a + bi$

$$z' = z \cdot e^{i30^\circ} =$$

$$= (a+bi)\cdot\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right) =$$

$$= \underbrace{a\cos\frac{\pi}{6} - b\sin\frac{\pi}{6}} + i\underbrace{\left(a\sin\frac{\pi}{6} + b\cos\frac{\pi}{6}\right)}$$

$$a + bi \xrightarrow{pn.} a\cos\alpha - b\sin\alpha + i(a\sin\alpha + b\cos\alpha)$$

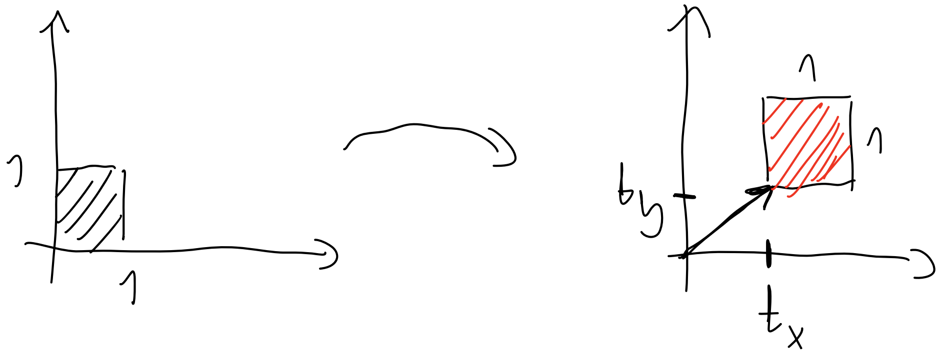
$$(a, b) \to (\cos\alpha\cdot a - \sin\alpha\cdot b, \sin\alpha\cdot a + \cos\alpha\cdot b)$$

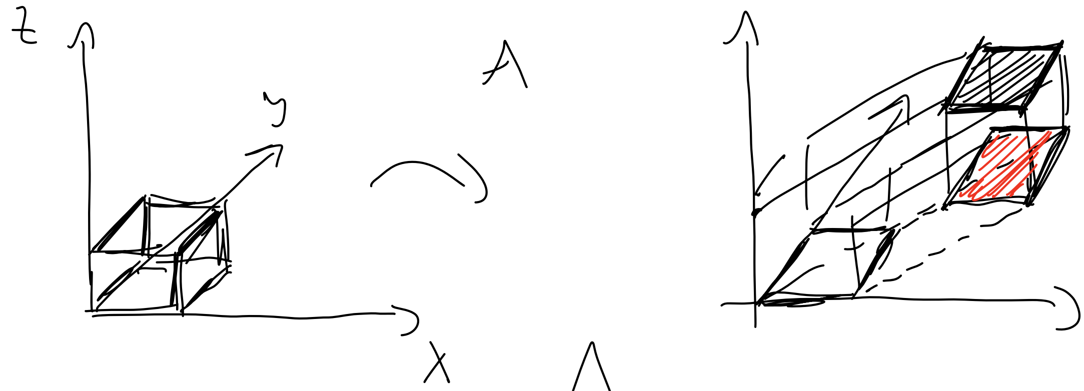
$$\begin{bmatrix} \cos\alpha, & -\sin\alpha \\ \sin\alpha, & \cos\alpha \end{bmatrix}\begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} \cos\alpha\cdot a - \sin\alpha\cdot b \\ \sin\alpha\cdot a + \cos\alpha\cdot b \end{bmatrix}$$

$$\alpha = \frac{\pi}{6} \to \begin{bmatrix} \frac{\sqrt{3}}{2} & -\frac{1}{2} \\ \frac{1}{2} & \sqrt{3}/2 \end{bmatrix}$$

5) TRANSLACJA (PRZESUNIĘCIE)

$$? \cdot \begin{bmatrix} a \\ b \end{bmatrix} \cancel{+ \begin{bmatrix} t_x \\ t_y \end{bmatrix}} \to \begin{bmatrix} a + t_x \\ b + t_y \end{bmatrix}$$





A

$$\begin{bmatrix}1&0&t_x\\0&1&t_y\\0&0&1\end{bmatrix}\begin{bmatrix}a\\b\\1\end{bmatrix}=\begin{bmatrix}a+t_x\\b+t_y\\1\end{bmatrix}$$

$\boxed{\mathbb{R}^2}$

1)

$$\begin{bmatrix}k_x&0\\0&k_y\end{bmatrix}$$

$$\begin{bmatrix}k_x&0&0\\0&k_y&0\\0&0&1\end{bmatrix}$$

2)

$$\begin{bmatrix}\cos\alpha&-\sin\alpha\\\sin\alpha&\cos\alpha\end{bmatrix}$$

$$\begin{bmatrix}\cos\alpha&-\sin\alpha&0\\\sin\alpha&\cos\alpha&0\\0&0&1\end{bmatrix}$$

3)

$$\begin{bmatrix}1&0&t_x\\0&1&t_y\\0&0&1\end{bmatrix}$$

$\downarrow$