Algoritmos y Estructuras de Datos I

Digesto de Funciones de Listas y Propiedades

Definiciones

a. Largo de una lista:

$$\begin{array}{|l} \#: [A] \to Nat \\ \hline \#[\,] \doteq 0 \\ \#(x \triangleright xs) \doteq \#xs + 1 \end{array}$$

b. Indexar:

$$\begin{array}{|l} !: [A] \to Nat \to A \\ \hline (x \triangleright xs)\,!\,0 \doteq x \\ (x \triangleright xs)\,!\,(n+1) \doteq xs\,!\,n \end{array}$$

c. Concatenar:

$$\begin{array}{|l} +\!\!+ : [A] \to [A] \to [A] \\ \hline [\,] +\!\!+ ys \doteq ys \\ (x \triangleright xs) +\!\!+ ys \doteq x \triangleright (xs +\!\!+ ys) \end{array}$$

d. Tirar:

$$\begin{array}{|l} \downarrow: [A] \to Nat \to [A] \\ \hline [\,] \downarrow n \doteq [\,] \\ (x \triangleright xs) \downarrow 0 \doteq x \triangleright xs \\ (x \triangleright xs) \downarrow (n+1) \doteq xs \downarrow n \end{array}$$

e. Tomar:

$$\begin{array}{|l} \uparrow: [A] \to Nat \to [A] \\ \hline [\,] \uparrow n \doteq [\,] \\ (x \triangleright xs) \uparrow 0 \doteq [\,] \\ (x \triangleright xs) \uparrow (n+1) \doteq x \triangleright (xs \uparrow n) \end{array}$$

f. Head (cabeza):

$$\begin{array}{|l} hd: [A] \to A \\ \hline hd.(x \triangleright xs) \doteq x \end{array}$$

g. Tail (cola):

$$\begin{array}{|l} tl: [A] \to [A] \\ \hline tl.(x \triangleright xs) \doteq xs \end{array}$$

Propiedades

1. Constructores de lista $([\,], \triangleright)$:

$$x \triangleright xs \neq [\,]$$
$$(x \triangleright xs) = (y \triangleright ys) \equiv x = y \wedge xs = ys$$

2. Concatenación:

$$(xs +\!\!+ ys) +\!\!+ zs = xs +\!\!+ (ys +\!\!+ zs)$$
$$(xs +\!\!+ ys)\,!\,i = (\ i < \#xs \to xs\,!\,i \ \square\ i \geq \#xs \to ys\,!\,(i - \#xs)\ )$$
$$(xs +\!\!+ ys) = [\,] \equiv xs = [\,] \wedge ys = [\,]$$
$$\langle\, \exists\, as, bs \ ::\ ys = as +\!\!+ bs \,\rangle$$
$$\neg \langle\, \exists\, as, bs \ ::\ [\,] = as +\!\!+ x \triangleright xs +\!\!+ bs \,\rangle$$

3. Largo:

$$\begin{aligned} \#(xs +\!\!+ ys) &= \#xs + \#ys \\ \#(xs{\uparrow}n) &= n \min \#xs \\ \#(xs{\downarrow}n) &= (\#xs - n) \max 0 \end{aligned}$$