Greg Sood
AERO 215-03
HW #3

1. $\mathcal{E} = \frac{v^2}{2} - \frac{\mu}{R}$

$v = \sqrt{(1.59)^2 + (6.65)^2 + (6.5)^2} \approx 9.434 \text{ km/s}$

$R = \sqrt{(6533)^2 + (1570)^2 + (42)^2} \approx 6719.13 \text{ km}$

$\mu_\oplus = 398\,600 \text{ km}^3/\text{s}^2$

$\mathcal{E} = \frac{9.434^2}{2} - \frac{398600}{6719.13} = -14.823$

$a = \frac{-\mu}{2\mathcal{E}} = \frac{-398600}{2(-14.823)} \approx \boxed{13445 \text{ km}}$

$\vec{e} = \frac{1}{\mu}\left[\left(v^2 - \frac{\mu}{R}\right)\vec{R} - (\vec{R}\cdot\vec{v})\vec{V}\right]$

$= \frac{1}{398600}\left[\left(9.434^2 - \frac{398600}{6719.13}\right)(-6533\hat{i} + 1570\hat{j} + 42\hat{k}) - (-6533\hat{i} + 1570\hat{j} + 42\hat{k} \cdot -1.59\hat{i} - 6.65\hat{j} + 6.5\hat{k})(-1.59\hat{i} - 6.65\hat{j} + 6.5\hat{k})\right]$

$= \frac{1}{398600}\left[(-193881\hat{i} + 46593\hat{j} + 1246\hat{k}) - (-349.7523\hat{i} - 1462.8\hat{j} + 1429.805\hat{k})\right] = -.4855\hat{i} + .1206\hat{j} - .00046\hat{k}$

$e = \sqrt{(-.4855)^2 + (.1206)^2 + (-.00046)^2} \approx \boxed{.5003}$

$i = \cos^{-1}\left(\frac{\hat{k}\cdot\vec{h}}{kh}\right) \qquad \vec{h} = \vec{R} \times \vec{V}$

$\vec{h} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -6533 & 1570 & 42 \\ -1.59 & -6.65 & 6.5 \end{vmatrix} \begin{matrix} \text{km} \\ \text{km/s} \end{matrix} = 10484.3\hat{i} + 42397.72\hat{j} + 45940.75\hat{k}$

$h = \sqrt{(10484.3)^2 + (42397.72)^2 + (45940.75)^2}$

$\approx 63388.01 \text{ km}^2/\text{s}$

$i = \cos^{-1}\left(\frac{45940.75}{63388.01}\right) \approx \boxed{43.55^\circ}$

$\Omega = \cos^{-1}\left(\frac{\hat{I}\cdot\vec{n}}{In}\right) \qquad \vec{n} = \hat{K} \times \vec{h}$

$\vec{n} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 0 & 1 \\ 10484.3 & 42397.72 & 45940.75 \end{vmatrix} = -42397.72\hat{i} + 10484.3\hat{j}$

$n = \sqrt{(-42397.72)^2 + (10484.3)^2} \approx 43674.79$

$\Omega = \cos^{-1}\left(\frac{-42397.72}{43674.79}\right) \approx \boxed{166.11^\circ}$

$\omega = \cos^{-1}\left(\frac{\vec{n}\cdot\vec{e}}{ne}\right) = \cos^{-1}\left(\frac{-42397.72\hat{i} + 10484.3\hat{j} \cdot -.4855\hat{i} + .1206\hat{j} - .0046\hat{k}}{43674.79 \cdot .5003}\right)$

$\approx .775^\circ \Rightarrow \boxed{359.23^\circ}$