MAE 3260 Final Group Work: Exploring a System of Interest

Report

Title: Shocked: The Art of Suspense and the Science of Measuring it

Topic of Interest: Off-road Suspension System

Abstract: We are choosing to analyze the shock from a small off-road vehicle's suspension system (Baja SAE). Using a quarter-car suspension model, our objective is to determine the key parameters of the system, including the effective damping coefficient, sprung and unsprung masses, and spring constant from prior shock dynamometer data. With these parameters, we will construct a mathematical model of the suspension, derive the system's transfer function, and generate Bode plots to evaluate its frequency response characteristics. We will then compare this model to the systems we've seen in coursework that analyze passenger vehicle suspensions to see how our assumptions might change given this specific application and the smaller size of the vehicle whose suspension we are studying.

Students/Roles:

Student	**Task/Role**	**Portfolio**
Greg Svensson	I will do the mathematical modelling of the system. I will use skills like passive design analysis, system modeling, and analytical data comparison. The final result should be a simplified model of the shock and a comprehensive comparison.	
Ava Rosenow	I plan to work on shock dissection and dynamometer data analysis.	
Aniket Martins	I plan to work specifically on making/analyzing the bode plots for our shock system. As we have discussed, we will work on everything together as a group so I will work on taking the shock apart as well.	
Tavan Bhatia	I plan to help write the equations of motion for the shock absorber system, with 1 DOF and help with data analysis.	

List of MAE 3260 concepts or skills used in this group work:

- Models:
  - ODEs
  - Transfer Functions
  - Bode Plots
- Open-loop system:
  - Parameter estimation
  - Passive design

Data Analysis

Figure 1 shows the raw dynamometer data for our shock. A force of increasing magnitude was applied to the full shock system and velocity was measured. The slope of this graph gives the damping coefficient of the shock in $\frac{lb \cdot s}{in}$.

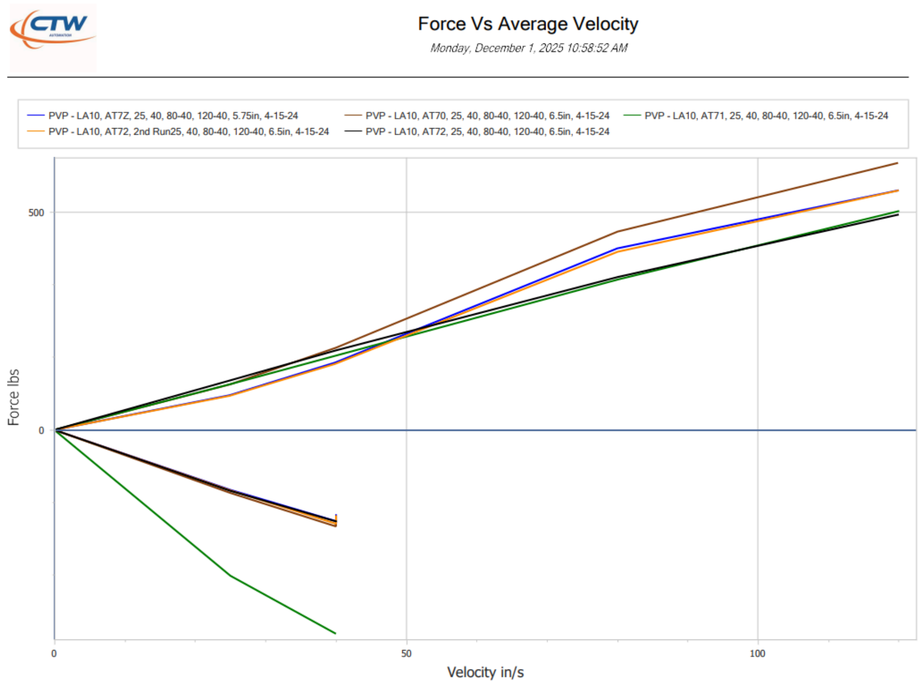


Figure 1: Raw dynamometer data for 5 different tunes, F vs. avg. velocity

Figure 2 shows a simplified version of our absolute force versus velocity, done by averaging all data points and linearizing the data to get a more confident estimate of the damping coefficient. From this graph, we get: $b = 19.2\, \frac{lb \cdot s}{in} = 3360\, \frac{N \cdot s}{m}$

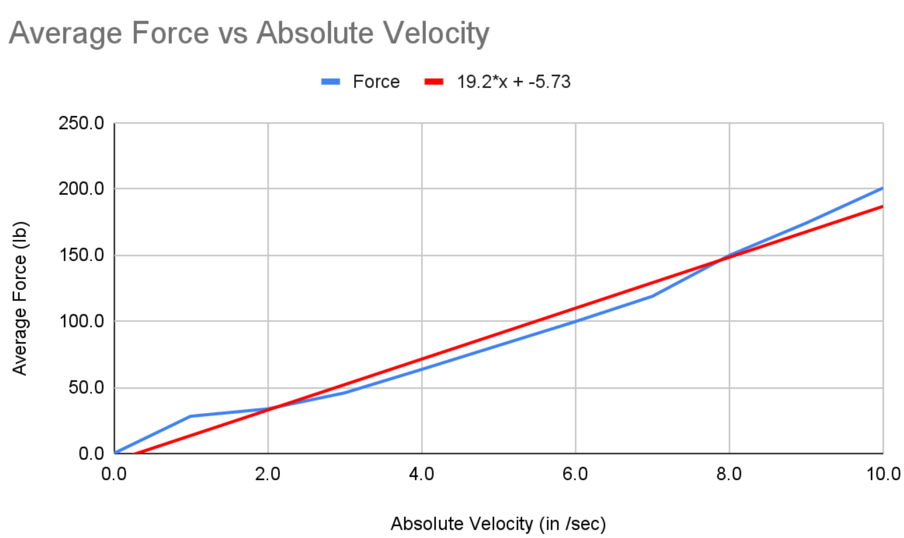


Figure 2: Linearized Graph

The spring constant was experimentally determined from the shock spring using the setup shown in Figure 3. The spring was compressed and displacement was measured with a ruler while the force was measured using the scale. $k_1 = 184.15\ lb/in\ = N \cdot m$

Figure 3: Experimental setup used to determine spring constants

Vehicle mass is known to be 370lbs. We divide this by 4 to obtain the relative mass for a quarter car model: $m\ =\ 92.5lbs\ =\ 41.957kg$. This value was confirmed by putting corner scales under each wheel of the car to double check that the weight distribution wasn't significantly different front to rear or left to right.

$$k_1 = \frac{29\ lb}{0.4\ cm} = 32345\frac{N}{m}$$
$$k_2 = \frac{20\ lb}{1\ cm} = 8879\frac{N}{m}$$
$$\frac{1}{k_{eq}} = \frac{1}{k_1} + \frac{1}{k_2} = \frac{1}{32345} + \frac{1}{8879}$$
$$k_{eq} = 6975$$

Mathematical Model:

$$m\ddot{x} + b\dot{x} + kx = b\dot{r} + kr$$

Laplace Transform:

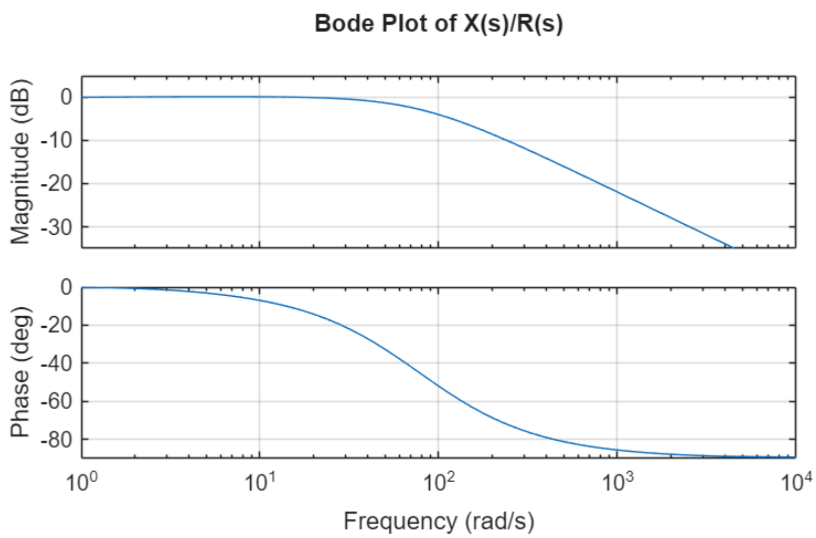
$$(ms^2 + bs + k)X(s) = (bs + k)R(s)$$

$$G(s) = \frac{X(s)}{R(s)} = \frac{bs+k}{ms^2+bs+k}$$

Comparison:

	m kg	b $\frac{N\cdot s}{m}$	k $\frac{N}{m}$
Baja SAE Suspension	41.95	3360	6897
Group Work	325	300	500

Bode Plot:



Code to generate Bode Plot:

```
m = 41.95;          % kg
b = 3362;          % N*s/m
k = 6975;          % N/m

s = tf('s');
G = (b*s + k)/(m*s^2 + b*s + k);

bode(G); grid on;
title('Bode Plot of X(s)/R(s)');
```

Sources:

[tf - Transfer function model - MATLAB]

[bode - Bode frequency response of dynamic system - MATLAB]

Conclusion:

The mass of our car is far smaller than the mass of the car from the group work. The group work specifies that the car modeled is a passenger car, whereas our car is a single-seat buggy. Thus we reasonably expect the mass of our car to be substantially smaller than the group work car. This is consistent with our car's mass being 41.95kg and the mass of the groupwork car being 325kg (both measurements per corner, quarter car).

From data we acquired by compressing the shock stack up used in pace with our dyno, we arrived at a k value of approximately 7000 $\frac{N}{m}$ or about 40 $\frac{lb}{in}$. This is in the range of target stiffness values for the application of a rally car rather than an on-road car. For those applications, the standard spring stiffness of 100-200 $\frac{lb}{in}$ provides a much tighter suspension. In the off-road case, much more bump case and collision can be expected, as such this validates the stiffness in our model suspension. This is much greater than the exemplar suspension, but we find that (500 $\frac{N}{m}$, or 2.8 $\frac{lb}{in}$) is an inaccurate estimation as it would not provide enough grip or restoring force for a vehicle.

For b, we extracting a damping coefficient of 3363 $\frac{N \cdot s}{m}$ from the dynamometer data, as compared to 300 $\frac{N \cdot s}{m}$ from the in class group work, which also seems reasonable. The passenger vehicle suspension design would be aimed at making the ride as smooth as possible for the passengers. On the other hand, the off-road vehicle suspension needs to handle much more aggressive terrain, so it's designed to safely and quickly handle obstacles, which necessitates a much higher damping coefficient. These shocks in particular even have a custom machined piston inside of them to further modify the tune beyond just things like the spring stack up to ensure that the damping is optimal for the specific application, which is what the dynamometer data was originally used to validate.