Lotka-Volterra Systems

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- Base Model
  - The foundation of predation modelling
  - Simple nonlinear system: 4 parameters
  - Easy to analyze
  - All solutions are **centers**
  - Accurate?... generally not
  - What can we do to improve it?

- Logistic Lotka-Volterra (LLV)
  - Logistic growth: Constrain resources of the prey (a more realistic assumption)
  - Still a somewhat simple model
  - Varied dynamics - **saddles**, **nodes**, **spirals**
  - No periodic solutions!

- Food Chain System
  - Extension to 3 dimensions
  - A given species almost always interacts with more than one other species
  - Varied dynamics - **centers**, **stable/unstable spirals**
  - Non-isolated fixed points!

$$\dot{x} = \alpha x - \beta xy$$

$$\dot{y} = \delta xy - \gamma y$$

$$\dot{x} = \alpha x \left(1 - \frac{x}{k}\right) - \beta xy$$

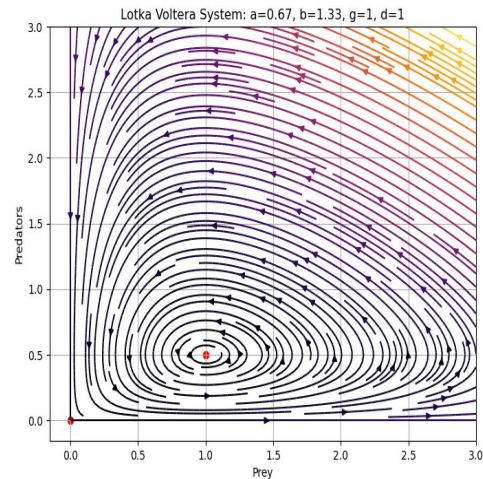$$\dot{y} = \delta xy - \gamma y$$

$$\dot{x} = \alpha x - \beta xy$$
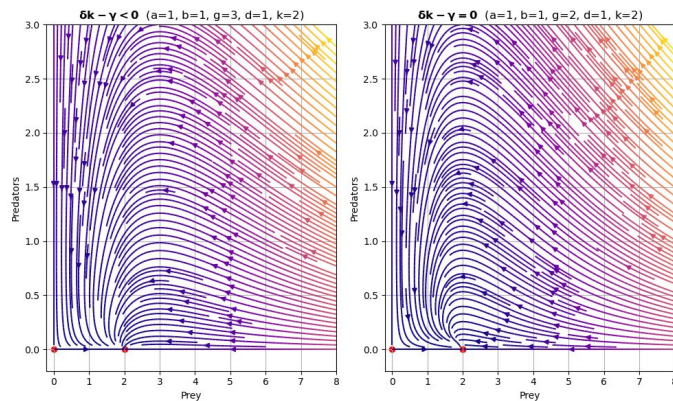
$$\dot{y} = \delta xy - \gamma y - \mu zy$$

$$\dot{z} = nyz - pz$$

- Linear analysis was applied to the fixed points of each system in order to determine stability and behavior

- To get a a better idea of the structure of the system, numerical integration was used to plot solutions and phase portraits

Base Model

Lotka Voltera System: a=0.67, b=1.33, g=1, d=1

LLV

δk − γ < 0 (a=1, b=1, g=3, d=1, k=2)

δk − γ = 0 (a=1, b=1, g=2, d=1, k=2)

Food Chain

$\alpha n < \beta p$ --- Parameters: [0.5, 1, 1, 1, 1, 1, 1]
- (0.1, 0.1, 0.1)
- (2, 2, 2)
- (3, 1, 3)

$\alpha n = \beta p$ --- Parameters: [1, 1, 1, 1, 1, 1, 1]
- (0.1, 0.1, 0.1)
- (2, 2, 2)
- (3, 1, 3)

$\alpha n > \beta p$ --- Parameters: [1.1, 1, 1, 1, 1, 1, 1]
- (0.1, 0.1, 0.1)
- (2, 2, 2)
- (3, 1, 3)

a < $\frac{4\delta k(\delta k - \gamma)}{\gamma}$ (a=1, b=1, g=1, d=1, k=2)

a = $\frac{4\delta k(\delta k - \gamma)}{\gamma}$ (a=8, b=1, g=1, d=1, k=2)

a > $\frac{4\delta k(\delta k - \gamma)}{\gamma}$ (a=12, b=1, g=1, d=1, k=2)