

CS 188  
Spring 2019

Introduction to  
Artificial Intelligence

Written HW 6

**Due:** Monday 3/11/2019 at 11:59pm (submit via Gradescope).

Leave self assessment boxes blank for this due date.

**Self assessment due:** Monday 3/18/2019 at 11:59pm (submit via Gradescope)

**Instructions for self-assessment:**

Take your original submission and annotate any differences from the provided solutions. For each subpart where your original answer was correct, write "correct" to demonstrate that you have checked your work. For each subpart where your original answer was incorrect, write out the correct answer and comment on the difference between your answer and the explanation provided in the solutions. You should complete your self-assessment using a **different color** of ink from your original work. If you need to, you can download a PDF copy of your submission from Gradescope.

Your submission must be a PDF that follows the template (page 1 has name/collaborators, question 1 begins on page 2, etc.). Do not reorder, split, combine, or add extra pages. If your original homework submission did not follow the correct format, **you must fix the format to receive credit on your self-assessment.**

If you did not complete some questions in your original submission, first complete those questions without consulting the solutions and then use a different color of ink to conduct a self-assessment.

**Policy:** Can be solved in groups (acknowledge collaborators) but must be written up individually

**Submission:** Your submission should be a PDF that matches this template. Each page of the PDF should align with the corresponding page of the template (page 1 has name/collaborators, question 1 begins on page 2, etc.). **Do not reorder, split, combine, or add extra pages.** The intention is that you print out the template, write on the page in pen/pencil, and then scan or take pictures of the pages to make your submission. You may also fill out this template digitally (e.g. using a tablet.)

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Collaborators	



# Q1. Minesweeper

0,0	0,1
1,0	1,1
2,1	1,2

Minesweeper, the well-known computer game, is played on a rectangular grid of  $N$  squares with  $M$  invisible mines scattered among them. Any square may be probed by the agent; instant death follows if a mine is probed. Minesweeper indicates the presence of mines by revealing, in each probed square, the number of mines that are directly or diagonally adjacent. The goal is to probe every unmined square.

- (a) Let  $X_{i,j}$  be true iff square  $[i,j]$  contains a mine. Write down the assertion that exactly two mines are adjacent to  $[1,1]$  as a sentence involving some logical combination of  $X_{i,j}$  propositions. (The upper left most corner is  $[0,0]$ . You can write the first disjunct and explain how to generalize to the rest.)

$[X_{0,1} \wedge X_{1,0} \wedge \neg X_{2,1} \wedge \neg X_{1,2}] \vee \dots$  go through all permutations of  $X_{i,j}$   
 $i \in [0,2], j \in [0,2]$  where exactly 2  $X_{i,j}$  are true.

- (b) Generalize your assertion from (a) by explaining how to construct a CNF sentence asserting that  $k$  of  $n$  neighbors contain mines.

$k$  of the  $X_{i,j}$  must be true and  $n-k$  of the  $X_{i,j}$  neighbors of the tile must be false.

- (c) Say you have successfully probed  $l$  squares, each of which is separated by a Manhattan Distance of at least 2. Each square has  $n_i$  neighbors and the game reveals that the square is surrounded by  $k_i$  mines ( $i = 1 \dots l$ ). How can an agent use DPLL to infer whether a given square  $[i,j]$  contains a mine, ignoring the global constraint that there are exactly  $M$  mines in all? Explain

- (i) the query

$$\sum_{n=i-1}^{i+1} k_i \geq 4 \Rightarrow X_{i,j}$$

- (ii) the knowledge base

$$\neg X_{i,j} \Rightarrow (k_i \text{ mines around} \Leftrightarrow n_i - k_i \text{ free squares around})$$

- (iii) how to combine the sentences of the knowledge base into CNF

$$(\neg X_{i,j} \vee \neg k_i \text{ mines} \vee n_i - k_i \text{ Free}) \wedge (X_{i,j} \vee \neg n_i - k_i \text{ Free} \vee k_i \text{ mines})$$

- (iv) the number of disjuncts in the CNF.

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- (d) Explain how to write the global constraint using the notation from part (a). How does the number of clauses in the constraint depend on  $M$  and  $N$ ? Suggest a way to modify DPLL so that the global constraint does not need to be represented explicitly.

$$\sum_{i=1}^n \sum_{j=1}^m X_{i,j} = M \quad \wedge \quad X_{i,j} = 1 \quad \text{Sum of all successfully probed squares}$$

divided by  $l = M \Rightarrow$  done w/ DPLL

- (e) Are any conclusions derived by the method in part (c) invalidated when the global constraint is taken into account?

No.



## Q2. DPLL

Convert the following set of sentences to clausal form.

(a) S1:  $A \Leftrightarrow (B \wedge E)$ .  $A \Rightarrow (B \wedge E) \wedge (B \wedge E) \Rightarrow A$

$$\neg A \vee (B \wedge E) \wedge \neg(B \wedge E) \vee A$$

$$\neg A \vee (B \wedge E) \wedge (\neg B \vee \neg E) \vee A$$

(b) S2:  $E \Rightarrow D$ .

$$\neg E \vee D$$

(c) S3:  $C \wedge F \Rightarrow \neg B$ .

$$\neg(C \wedge F) \vee \neg B$$

$$\neg C \vee \neg F \vee \neg B$$

(d) S4:  $E \Rightarrow B$ .

$$\neg E \vee B$$

(e) S5:  $B \Rightarrow F$ .

$$\neg B \vee F$$

(f) S6:  $B \Rightarrow C$ .

$$\neg B \vee C$$

(g) Give a trace of the execution of DPLL on the conjunction of these clauses.

$$(\neg A \vee (B \wedge E)) \wedge (\neg B \vee \neg E \vee A)$$

$$\neg E \vee D \quad \checkmark$$

$$\neg C \vee \neg F \vee \neg B \quad \checkmark$$

$$\neg E \vee B \quad \checkmark$$

$$\neg B \vee F \quad \checkmark$$

$$\neg B \vee C$$

$$A = 0$$

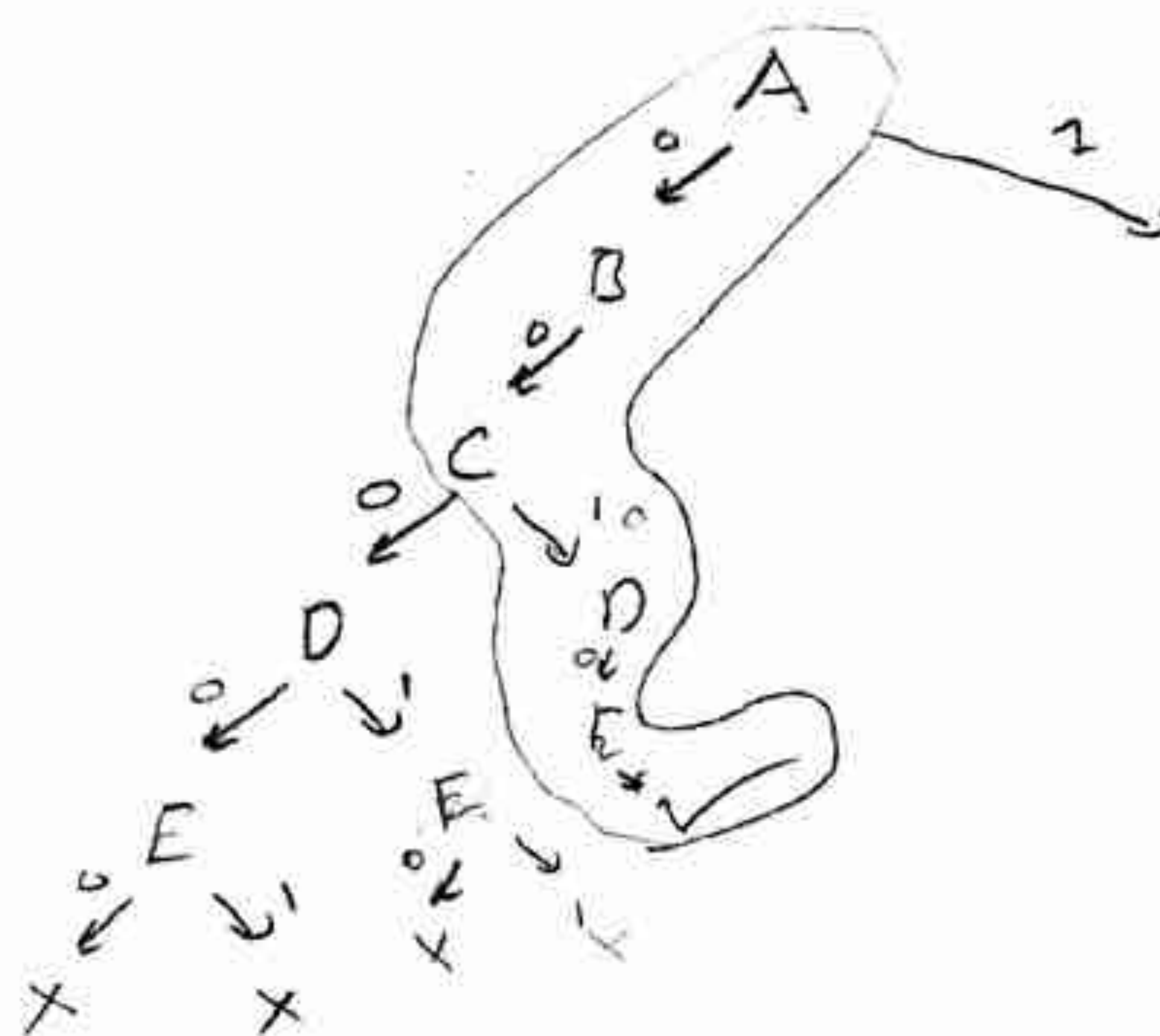
$$B = 0$$

$$C = 1$$

$$D = 0$$

$$E = 0$$

$$F = 0$$





### Q3. Inference with First Order Logic

Suppose you are given the following axioms:

1.  $0 \leq 3$ .
2.  $7 \leq 9$ .
3.  $\forall x, x \leq x$ .
4.  $\forall x, x \leq x + 0$ .
5.  $\forall x, x + 0 \leq x$ .
6.  $\forall x, y, x + y \leq y + x$ .
7.  $\forall w, x, y, z, w \leq y \wedge x \leq z \Rightarrow w + x \leq y + z$ .
8.  $\forall x, y, z, x \leq y \wedge y \leq z \Rightarrow x \leq z$ .

- (a) Give a backward-chaining proof of the sentence  $7 \leq 3 + 9$ . (Be sure, of course, to use only the axioms given here, not anything else you may know about arithmetic.) Show only the steps that leads to success, not the irrelevant steps.

$$0 \leq 3$$

$$7 \leq 9$$

$$\forall w, x, y, z, w \leq y \wedge x \leq z \Rightarrow w + x \leq y + z$$

$$\Rightarrow 0 + 7 \leq 7 + 3$$

- (b) Give a forward-chaining proof of the sentence  $7 \leq 3 + 9$ . Again, show only the steps that lead to success.

$$0 \leq 3$$

$$7 \leq 9$$

$$x \leq y \wedge y \leq z$$

$$\Rightarrow 0 + 7 \leq 3 + 9$$

$$w \leq y \wedge x \leq z \Rightarrow w + x \leq y + z$$