CS 188 Spring 2019 | Introduction to Artificial Intelligence | Written HW 7

Due: Monday 4/1/2019 at 11:59pm (submit via Gradescope).

Leave self assessment boxes blank for this due date.

Self assessment due: Monday 4/8/2019 at 11:59pm (submit via Gradescope)

For the self assessment, **fill in the self assessment boxes in your original submission** (you can download a PDF copy of your submission from Gradescope – be sure to delete any extra title pages that Gradescope attaches). For each subpart where your original answer was correct, write "correct." Otherwise, write and explain the correct answer. **Do not leave any boxes empty.**

If you did not submit the homework (or skipped some questions) but wish to receive credit for the self-assessment, we ask that you first complete the homework without looking at the solutions, and then perform the self assessment afterwards.

Policy: Can be solved in groups (acknowledge collaborators) but must be written up individually

Submission: Your submission should be a PDF that matches this template. Each page of the PDF should align with the corresponding page of the template (page 1 has name/collaborators, question 1 begins on page 2, etc.). **Do not reorder, split, combine, or add extra pages.** The intention is that you print out the template, write on the page in pen/pencil, and then scan or take pictures of the pages to make your submission. You may also fill out this template digitally (e.g. using a tablet.)

First name	Gregory
Last name	Uezono
SID	3031467186
Collaborators	

Q1. Probability

(a) For the following questions, you will be given a set of probability tables and a set of conditional independence assumptions. Given these tables and independence assumptions, write an expression for the requested probability tables. Keep in mind that your expressions cannot contain any probabilities other than the given probability tables. If it is not possible, mark "Not possible."

(i) Using probability tables $\mathbf{P(A), P(A \mid C), P(B \mid C), P(C \mid A, B)}$ and no conditional independence assumptions, write an expression to calculate the table $\mathbf{P(A, B \mid C)}$.

$$\mathbf{P(A, B \mid C)} = \frac{P(B,C \mid A)\,P(A)}{\sum_a P(A \mid C)\,P(C)}$$

no $P(C)$ $\quad P(A \mid B,C)\,P(B \mid C)$

● Not possible.

(ii) Using probability tables $\mathbf{P(A), P(A \mid C), P(B \mid A), P(C \mid A, B)}$ and no conditional independence assumptions, write an expression to calculate the table $\mathbf{P(B \mid A, C)}$.

$$\mathbf{P(B \mid A, C)} = \frac{P(A)\,P(B \mid A)\,P(C \mid A,B)}{\sum_b P(A)\,P(B \mid A)\,P(C \mid A,B)}$$

● Not possible.

(iii) Using probability tables $\mathbf{P(A \mid B), P(B), P(B \mid A, C), P(C \mid A)}$ and conditional independence assumption $\mathbf{A \perp\!\!\!\perp B}$, write an expression to calculate the table $\mathbf{P(C)}$.

$P(A \mid B) = P(A) \leftarrow A \perp\!\!\!\perp B$

$$\mathbf{P(C)} = \sum_a P(C \mid A)\,P(A \mid B)$$

○ Not possible.

(iv) Using probability tables $\mathbf{P(A \mid B, C), P(B), P(B \mid A, C), P(C \mid B, A)}$ and conditional independence assumption $\mathbf{A \perp\!\!\!\perp B \mid C}$, write an expression for $\mathbf{P(A, B, C)}$.

$$\mathbf{P(A, B, C)} = P(A \mid B,C)\,P(B \mid A,C)\,\sum_b P(C \mid B,A)\,P(B)$$

● Not possible.

Self assessment If correct, write "**correct**" in the box. Otherwise, **write and explain** the correct answer.

i) no $P(C)$ so not possible

ii) $\frac{P(A)P(B \mid A)P(C \mid A,B)}{\sum_b P(A)P(B \mid A)P(C \mid A,B)} = \frac{P(A,B,C)}{P(A,C)} = P(B \mid A,C)$

iii) forgot summation

iv) Not possible

(b) For each of the following equations, select the *minimal set* of conditional independence assumptions necessary for the equation to be true.

(i) $\mathbf{P(A, C) = P(A \mid B)\, P(C)}$

- ■ $A \perp\!\!\!\perp B$ ✓
- ☐ $A \perp\!\!\!\perp B \mid C$
- ■ $A \perp\!\!\!\perp C$
- ☐ $A \perp\!\!\!\perp C \mid B$
- ☐ $B \perp\!\!\!\perp C$
- ☐ $B \perp\!\!\!\perp C \mid A$
- ☐ No independence assumptions needed.

(ii) $\mathbf{P(A \mid B, C) = \frac{P(A)\, P(B|A)\, P(C|A)}{P(B|C)\, P(C)}}$

- ☐ $A \perp\!\!\!\perp B$
- ☐ $A \perp\!\!\!\perp B \mid C$
- ☐ $A \perp\!\!\!\perp C$
- ☐ $A \perp\!\!\!\perp C \mid B$
- ☐ $B \perp\!\!\!\perp C$ ✓
- ■ $B \perp\!\!\!\perp C \mid A$
- ☐ No independence assumptions needed.

$P(A, B \mid C)\, P(C)$

(iii) $\mathbf{P(A,B) = \sum_c P(A \mid B, c)\ P(B \mid c)\ P(c)}$

- ☐ $A \perp\!\!\!\perp B$
- ☐ $A \perp\!\!\!\perp B \mid C$
- ☐ $A \perp\!\!\!\perp C$
- ☐ $A \perp\!\!\!\perp C \mid B$
- ☐ $B \perp\!\!\!\perp C$ ✓
- ☐ $B \perp\!\!\!\perp C \mid A$
- ☒ No independence assumptions needed.

(iv) $\mathbf{P(A,B \mid C,D) = P(A \mid C,D)\ P(B \mid A,C,D)}$ ✓

- ☐ $A \perp\!\!\!\perp B$
- ☐ $A \perp\!\!\!\perp B \mid C$
- ☐ $A \perp\!\!\!\perp B \mid D$
- ☐ $C \perp\!\!\!\perp D$
- ☐ $C \perp\!\!\!\perp D \mid A$
- ☐ $C \perp\!\!\!\perp D \mid B$
- ☒ No independence assumptions needed.

Self assessment If correct, write "**correct**" in the box. Otherwise, **write and explain** the correct answer.

Correct except forgot $P(A,C) = P(A)P(C)$ only when $A \perp\!\!\!\perp C$

(c) (i) Mark **all** expressions that are equal to $\mathbf{P(A \mid B)}$, **given no independence assumptions.**

- ☐ $\sum_c P(A \mid B, c)$
- ☒ $\sum_c P(A, c \mid B)$ ✓
- ☐ $\frac{P(B|A)\ P(A|C)}{\sum_c P(B,c)}$
- ☒ $\frac{\sum_c P(A,B,c)}{\sum_c P(B,c)}$ ✓
- ☐ $\frac{P(A,C|B)}{P(C|B)}$
- ☐ $\frac{P(A|C,B)\ P(C|A,B)}{P(C|B)}$
- ☐ None of the provided options.

(ii) Mark **all** expressions that are equal to $\mathbf{P(A,B,C)}$, given that $\mathbf{A \perp\!\!\!\perp B}$.

- ☐ $P(A \mid C)\ P(C \mid B)\ P(B)$
- ☒ $P(A)\ P(B)\ P(C \mid A, B)$ ✓
- ☐ $P(C)\ P(A \mid C)\ P(B \mid C)$
- ☐ $P(A)\ P(C \mid A)\ P(B \mid C)$
- ☒ $P(A)\ P(B \mid A)\ P(C \mid A, B)$ ✓
- ☒ $P(A, C)\ P(B \mid A, C)$ ✓
- ☐ None of the provided options.

(iii) Mark **all** expressions that are equal to $\mathbf{P(A,B \mid C)}$, given that $\mathbf{A \perp\!\!\!\perp B \mid C}$.

- ☒ $P(A \mid C)\ P(B \mid C)$
- ☐ $\frac{P(A)\ P(B|A)\ P(C|A,B)}{\sum_c P(A,B,c)}$
- ☐ $P(A \mid B)\ P(B \mid C)$
- ☐ $\frac{P(C)\ P(B|C)\ P(A|C)}{P(C|A,B)}$
- ☐ $\frac{\sum_c P(A,B,c)}{P(C)}$
- ☒ $\frac{P(C,A|B)\ P(B)}{P(C)}$
- ☒ None of the provided options.

Self assessment If correct, write "**correct**" in the box. Otherwise, **write and explain** the correct answer.

iii) Conditional $\perp\!\!\!\perp$ ~~was~~

$A \perp\!\!\!\perp B, C \Rightarrow P(A|C)P(B|C) = P(A,B|C)$

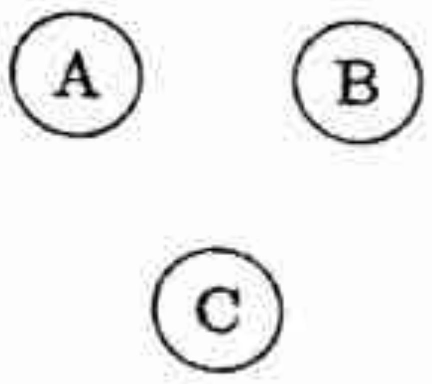
$\frac{P(C,A|B)\,P(B)}{P(C)} = \frac{P(A,B,C)}{P(C)}$

$= P(A,B|C)$

Q2. Bayes' Nets: Representation

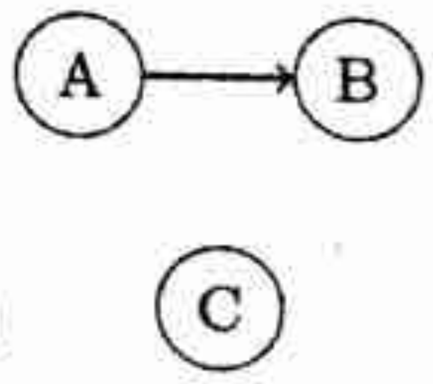
Assume we are given the following ten Bayes' nets, labeled $\mathbf{G_1}$ to $\mathbf{G_{10}}$:

P(A)
P(B)
P(C)



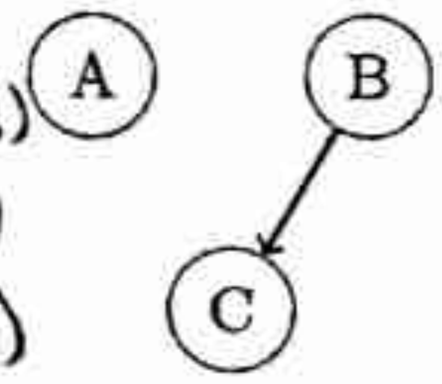
$\mathbf{G_1}$

P(B|A)
P(A)
P(C)

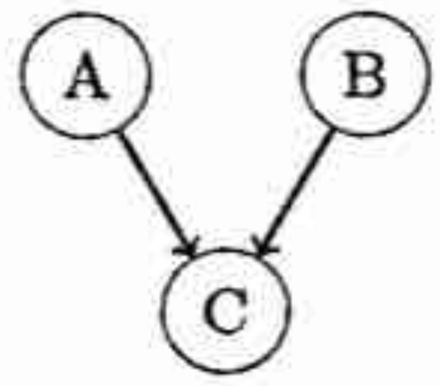


$\mathbf{G_2}$

P(C|B)
P(B)
P(A)

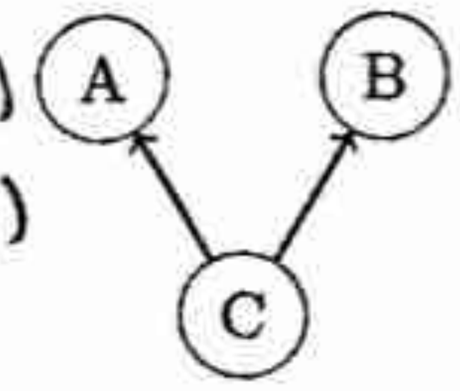


$\mathbf{G_3}$



$\mathbf{G_4}$

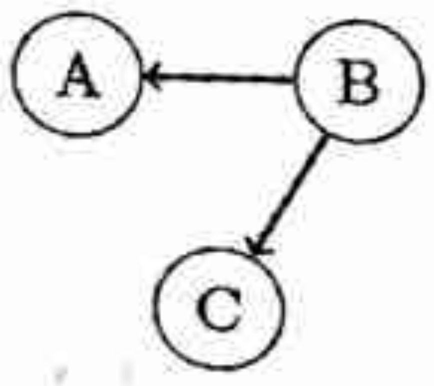
P(C|A)
P(C|B)
P(A)
P(B)



$\mathbf{G_5}$

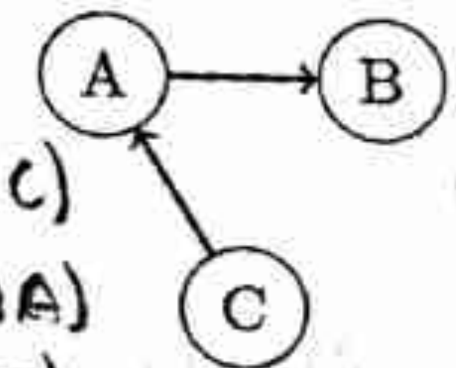
P(A|C)
P(B|C)
P(C)

P(A|B)
P(C|B)
P(C)



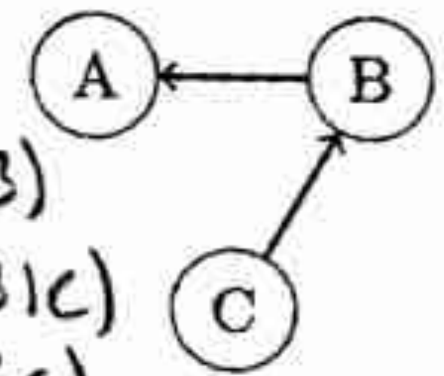
$\mathbf{G_6}$

P(A|C)
P(B|A)
P(C)



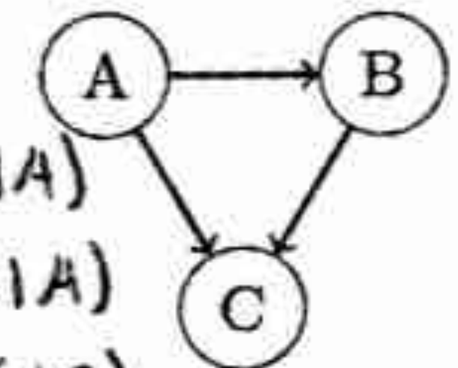
$\mathbf{G_7}$

P(A|B)
P(B|C)
P(C)

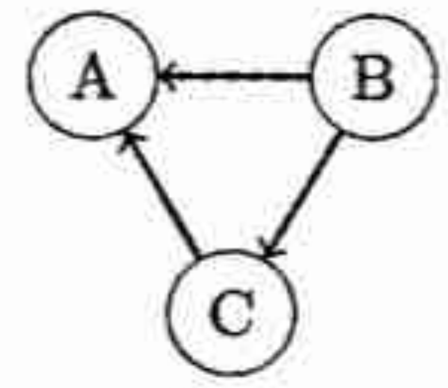


$\mathbf{G_8}$

P(B|A)
P(C|A)
P(C|B)



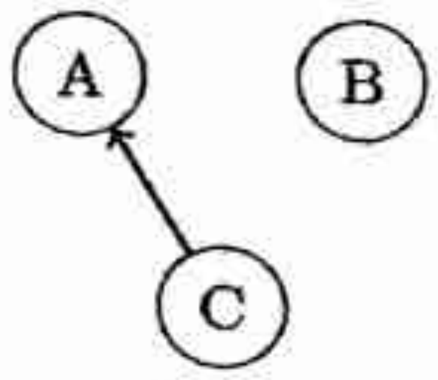
$\mathbf{G_9}$



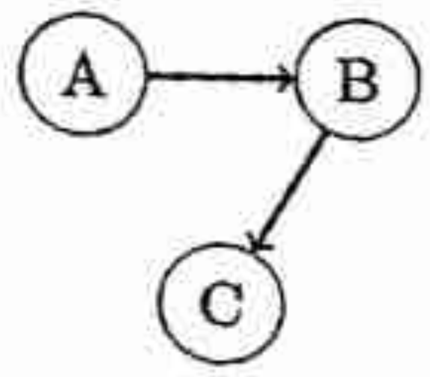
$\mathbf{G_{10}}$

P(A|B)
P(A|C)
P(C|B)

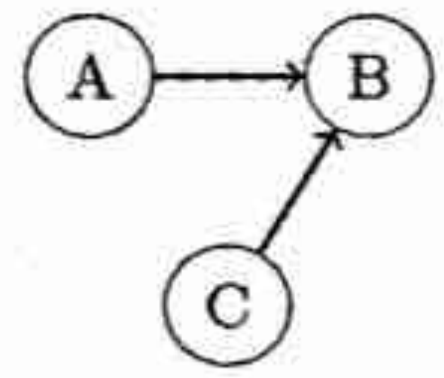
Assume we are also given the following three Bayes' nets, labeled $\mathbf{B_1}$ to $\mathbf{B_3}$:



$\mathbf{B_1}$



$\mathbf{B_2}$



$\mathbf{B_3}$

(continued on next page)

(a) Assume we know that a joint distribution $\mathbf{d_1}$ (over $\mathbf{A}, \mathbf{B}, \mathbf{C}$) can be represented by Bayes' net $\mathbf{B_1}$. Mark all of the following Bayes' nets that are guaranteed to be able to represent $\mathbf{d_1}$.

☐ $\mathbf{G_1}$ ☐ $\mathbf{G_2}$ ☐ $\mathbf{G_3}$ ☒ $\mathbf{G_4}$ ☒ $\mathbf{G_5}$

☐ $\mathbf{G_6}$ ☒ $\mathbf{G_7}$ ☐ $\mathbf{G_8}$ ☒ $\mathbf{G_9}$ ☒ $\mathbf{G_{10}}$

☐ None of the above.

Self assessment If correct, write "**correct**" in the box. Otherwise, **write and explain** the correct answer.

(b) Assume we know that a joint distribution $\mathbf{d_2}$ (over $\mathbf{A}, \mathbf{B}, \mathbf{C}$) can be represented by Bayes' net $\mathbf{B_2}$. Mark all of the following Bayes' nets that are guaranteed to be able to represent $\mathbf{d_2}$.

☐ $\mathbf{G_1}$ ☐ $\mathbf{G_2}$ ☐ $\mathbf{G_3}$ ☐ $\mathbf{G_4}$ ☐ $\mathbf{G_5}$

☒ $\mathbf{G_6}$ ☐ $\mathbf{G_7}$ ☒ $\mathbf{G_8}$ ☒ $\mathbf{G_9}$ ☒ $\mathbf{G_{10}}$

☐ None of the above.

Self assessment If correct, write "**correct**" in the box. Otherwise, **write and explain** the correct answer.

(c) Assume we know that a joint distribution $\mathbf{d_3}$ (over $\mathbf{A}, \mathbf{B}, \mathbf{C}$) *cannot* be represented by Bayes' net $\mathbf{B_3}$. Mark all of the following Bayes' nets that are guaranteed to be able to represent $\mathbf{d_3}$.

☐ $\mathbf{G_1}$ ☐ $\mathbf{G_2}$ ☐ $\mathbf{G_3}$ ☐ $\mathbf{G_4}$ ☐ $\mathbf{G_5}$

☒ $\mathbf{G_6}$ ☐ $\mathbf{G_7}$ ☒ $\mathbf{G_8}$ ☒ $\mathbf{G_9}$ ☒ $\mathbf{G_{10}}$

☐ None of the above.

Self assessment If correct, write "**correct**" in the box. Otherwise, **write and explain** the correct answer.
Did not read <u>cannot</u>

(d) Assume we know that a joint distribution $\mathbf{d_4}$ (over $\mathbf{A}, \mathbf{B}, \mathbf{C}$) can be represented by Bayes' nets $\mathbf{B_1}$, $\mathbf{B_2}$, and $\mathbf{B_3}$. Mark all of the following Bayes' nets that are guaranteed to be able to represent $\mathbf{d_4}$.

☒ $\mathbf{G_1}$ ☒ $\mathbf{G_2}$ ☒ $\mathbf{G_3}$ ☒ $\mathbf{G_4}$ ☒ $\mathbf{G_5}$

☒ $\mathbf{G_6}$ ☒ $\mathbf{G_7}$ ☒ $\mathbf{G_8}$ ☒ $\mathbf{G_9}$ ☒ $\mathbf{G_{10}}$

☐ None of the above.

Self assessment If correct, write "**correct**" in the box. Otherwise, **write and explain** the correct answer.
All of the above. The union of assumptions made by B_1, B_2, B_3: AB; AB|C; BC; BC|A; AC; AC|B. This encompasses all possible

assumptions w/ 3 RV ⇒ any Bayes' Net will do.