CS 188 Spring 2019	Introduction to Artificial Intelligence	Written HW 6

Due: Monday 3/11/2019 at 11:59pm (submit via Gradescope).

Leave self assessment boxes blank for this due date.

Self assessment due: Monday 3/18/2019 at 11:59pm (submit via Gradescope)

Instructions for self-assessment:

Take your original submission and annotate any differences from the provided solutions. **For each subpart** where your original answer was correct, **write "correct"** to demonstrate that you have checked your work. For each subpart where your original answer was incorrect, write out the correct answer and comment on the difference between your answer and the explanation provided in the solutions. You should complete your self-assessment using a **different color** of ink from your original work. If you need to, you can download a PDF copy of your submission from Gradescope.

Your submission must be a PDF that follows the template (page 1 has name/collaborators, question 1 begins on page 2, etc.). Do not reorder, split, combine, or add extra pages. If your original homework submission did not follow the correct format, **you must fix the format to receive credit on your self-assessment.**

If you did not complete some questions in your original submission, first complete those questions without consulting the solutions and then use a different color of ink to conduct a self-assessment.

Policy: Can be solved in groups (acknowledge collaborators) but must be written up individually

Submission: Your submission should be a PDF that matches this template. Each page of the PDF should align with the corresponding page of the template (page 1 has name/collaborators, question 1 begins on page 2, etc.). **Do not reorder, split, combine, or add extra pages.** The intention is that you print out the template, write on the page in pen/pencil, and then scan or take pictures of the pages to make your submission. You may also fill out this template digitally (e.g. using a tablet.)

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Collaborators	

0,0	0,1	
1,0	1,1	1,2
	2,1	

Q1. Minesweeper

Minesweeper, the well-known computer game, is played on a rectangular grid of N squares with M invisible mines scattered among them. Any square may be probed by the agent; instant death follows if a mine is probed. Minesweeper indicates the presence of mines by revealing, in each probed square, the number of mines that are directly or diagonally adjacent. The goal is to probe every unmined square.

(a) Let $X_{i,j}$ be true iff square $[i, j]$ contains a mine. Write down the assertion that exactly two mines are adjacent to $[1, 1]$ as a sentence involving some logical combination of $X_{i,j}$ propositions. (The upper left most corner is $[0, 0]$. You can write the first disjunct and explain how to generalize to the rest.)

$[X_{0,1} \land X_{1,0} \land \neg X_{2,1} \land \neg X_{1,2}] \lor \ldots$ so through all permutations of $X_{i,j}$ $i \in [0,2], j \in [0,2]$ where exactly 2 $X_{i,j}$ are true.

✗ needed to do other squares

(b) Generalize your assertion from (a) by explaining how to construct a CNF sentence asserting that k of n neighbors contain mines.

k of the $X_{i,j}$ must be true and $n-k$ of the $X_{i,j}$ neighbors of the tile must be false. ✓

k mines are adjacent ≡ at least k mines are adjacent ∧ at most k mines are adjacent

(c) Say you have successfully probed l squares, each of which is separated by a Manhattan Distance of at least 2. Each square has n_i neighbors and the game reveals that the square is surrounded by k_i mines ($i = 1 \ldots l$). How can an agent use DPLL to infer whether a given square $[i, j]$ contains a mine, ignoring the global constraint that there are exactly M mines in all? Explain

(i) the query

$\sum_{n=i-1}^{i+1} k_i \geq 4 \Rightarrow X_{i,j}$ ✓ $X_{i,j}$ is query. Goal $KB \models X_{i,j}$

(ii) the knowledge base

$\neg X_{i,j} \Rightarrow$ (k_i mines around $\Leftrightarrow$ $n_i - k_i$ free squares around)

Sentence for each probed square

(iii) how to combine the sentences of the knowledge base into CNF

($X_{i,j} \lor \neg k_i$ mines $\lor$ $n_i - k_i$ free) $\land$ ($X_{i,j} \lor \neg n_i - k_i$ free $\lor$ k_i mines)

Conjunction of all sentences already in CNF

~~**(iv)**~~ the number of disjuncts in the CNF.

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(d) Explain how to write the global constraint using the notation from part (a). How does the number of clauses in the constraint depend on M and N? Suggest a way to modify DPLL so that the global constraint does not need to be represented explicitly.

✗ $\sum_{i=1}^{n} \sum_{j=1}^{m} X_{i,j} = M \land X_{i,j} = 1$. Sum of all ~~exported~~ successfully probed squares divided by l = M ⇒ done w/ DPLL

~~[illegible] w/ $\binom{N}{M}$ disjuncts, each~~ ~~w/ [illegible]~~

1) add min (min # of unassigned symbols that must be true) max (max # of " ") to DPLL function

2) M is used as both min and max values

3) within DPLL: fail if min < rem. symbols or max < 0

4) For each recursive call, update min/max by subtracting 1 when we assign a true value to a symbol

... minimum ... w/ ...

(e) Are any conclusions derived by the method in part (c) invalidated when the global constraint is taken into account?

No. ✓

Q2. DPLL

Convert the following set of sentences to clausal form.

(a) S1: $A \Leftrightarrow (B \wedge E)$. $A \Rightarrow (B \wedge E) \wedge (B \wedge E) \Rightarrow A$

$\neg A \vee (B \wedge E) \wedge \neg(B \wedge E) \vee A$

$\neg A \vee (B \wedge E) \wedge (\neg B \vee \neg E) \vee A$ ⇝ $(\neg A \vee B) \wedge (\neg A \vee E) \wedge (\neg B \vee \neg E \vee A)$ ✓

(b) S2: $E \Rightarrow D$.

$\neg E \vee D$ ✓

(c) S3: $C \wedge F \Rightarrow \neg B$.

$\neg (C \wedge F) \vee \neg B$

$\neg C \vee \neg F \vee \neg B$ ✓

(d) S4: $E \Rightarrow B$.

$\neg E \vee B$ ✓

(e) S5: $B \Rightarrow F$.

$\neg B \vee F$ ✓

(f) S6: $B \Rightarrow C$.

$\neg B \vee C$ ✓

(g) Give a trace of the execution of DPLL on the conjunction of these clauses. (erased soln. for room)

$(\neg A \vee (B \wedge E)) \wedge (\neg B \vee \neg E \vee A)$

$\neg E \vee D$

$\neg C \vee \neg F \vee \neg B$

$\neg E \vee B$

$\neg B \vee F$

$\neg B \vee C$

1) $(\neg A \vee B) \wedge (\neg A \vee E) \vee \ldots$

2) No pure symbol or unit clause → $A = true$

$(B) \wedge (E) \wedge (\neg E \wedge D) \wedge (\neg C \vee \neg F \vee \neg B) \wedge (\neg E \vee B) \wedge (\neg B \vee F) \wedge (\neg B \vee C)$

3) Assign unit clauses to be true → $B = true$, $E = true$

4) Assign unit clauses to be true → $C = true$, $D = true$

5) Assign unit clause $F = false$ → terminate

6) Backtrack, assign $A = false$.

Pure symbols $D = true$, $E = false$

7) Assign unit clause $E = false$

8) Assign pure symbol $B = false$

9) Alg terminates ✓

A = 0	A = F
B = 0	B = F
C = 1	C = Anything
D = 1	D = true
E = 0	E = true
F = 0	F = anything

Q3. Inference with First Order Logic

Suppose you are given the following axioms:

1. $0 \leq 3$.
2. $7 \leq 9$.
3. $\forall x,\ x \leq x$.
4. $\forall x,\ x \leq x + 0$.
5. $\forall x,\ x + 0 \leq x$.
6. $\forall x, y,\ x + y \leq y + x$.
7. $\forall w, x, y, z,\ w \leq y \land x \leq z \Rightarrow w + x \leq y + z$.
8. $\forall x, y, z,\ x \leq y \land y \leq z \Rightarrow x \leq z$.

(a) Give a backward-chaining proof of the sentence $7 \leq 3 + 9$. (Be sure, of course, to use only the axioms given here, not anything else you may know about arithmetic.) Show only the steps that leads to success, not the irrelevant steps.

$0 \leq 3$

$7 \leq 9$

$\Rightarrow\ 0 + 7 \leq \cancel{7}+3$

$\forall w, x, y, z\ ,\ w \leq y \land x \leq z \Rightarrow w + x \leq y + z$

Goal: $7 \leq 3+9$. From (8) $\{x/7, z/3+9\}$, derive 2 subgoals $7 \leq y_1$, $y_1 \leq 3+9$

Goal: $7 \leq y_1$. Resolve w/ (4) and substitution $\{y_1/7+0\}$

Goal: $7+0 \leq 3+9$. From (8) and $\{x_2/7+0, z_2/3+9\}$ derive 2 subgoals: $7+0 \leq y_2$, $y_2 \leq 3+9$

Goal: $7+0 \leq y_2$. Resolve w/ (6) ; $\{y_2/0+7, x_3/7, y_3/0\}$

Goal: $0+7 \leq 3+9$. From (7) ; $\{w_4/0, x_4/7, y_4/3, z_4/9\}$ derive 2 subgoals:

Goal: $0 \leq 3$. Resolve w/ (1)

Goal: $0 \leq 3$. Resolve w/ (2)

(b) Give a forward-chaining proof of the sentence $7 \leq 3 + 9$. Again, show only the steps that lead to success.

$0 \leq 3$

$7 \leq 9$

$\Rightarrow\ 0 + 7 \leq 3 + 9$

$w \leq y \land x \leq z \Rightarrow w + x \leq y + z$

i) From (7) $\{w/0, y/3, x/7, z/9\}$, $\Rightarrow 0+7 \leq 3+9$

ii) From (6) $\{y_1/0, x_1/7\}$ $\Rightarrow 7+0 \leq 0+7$

iii) From (4) $\{x_2/7\}$ $\Rightarrow 7 \leq 0+7$

iv) From (8), (ii), (iii) $\{x_3/7, y_3/7+0, z_3/0+7\}$ $\Rightarrow 7 \leq 0+7$

v) From (8) (i), (iv) $\{x_4/7, y_4/0+7, z_4/3+9\}$ $\Rightarrow 7 \leq 3+9$