Assignment: Reductions and Prefix Sums

Passed but not verified · 10/10 points

Instructions (/learn/parprog1/programming/4rXwX/reductions-and-prefix-sums)

My submission (/learn/parprog1/programming/4rXwX/reductions-and-prefix-sums/submission)

Discussions (/learn/parprog1/programming/4rXwX/reductions-and-prefix-sums/discussions)

Reductions and Prefix Sums

You will find the starting files here:
http://chara.epfl.ch/~dockermoocs/parprog1/reductions.zip
(http://chara.epfl.ch/~dockermoocs/parprog1/reductions.zip)

In this assignment, you will implement several variants of reduction and prefix sum algorithms. Each of the three parts of the assignment will exercise a different aspect of parallel programming:

- choosing the right parallelization threshold
- identifying the correct reduction operator
- identifying the correct prefix sum operator

We will use the parallel construct, defined in the package common, as in the lecture to start parallel computations. Every parallel construct invocation takes two tasks as input and outputs the corresponding results as a tuple of two elements. It is not allowed to use the task construct in this assignment.

Parallel Counting Change

If you took the course Functional Programming in Scala, you surely recall the assignment in which you had to count the number of ways in which you can make the change for a given amount of money. The text of that assignment was as follows:

Write a recursive function that counts how many different ways you can make change for an amount, given a list of coin denominations. For example, there are 3 ways to give change for 4 if you have coins with denomination 1 and 2: 1+1+1+1, 1+1+2, 2+2.

In this assignment, you will repeat the same task, but this time, your implementation will be parallel. Start with the sequential version of this problem once more -- the countChange function takes the amount of money and the list of different coin denominations. It returns the total number of different ways you can give change:

```
def countChange(money: Int, coins: List[Int]): Int
```

Note that the solution to this problem is recursive. In every recursive call, we either decide to continue subtracting the next coin in the coins list from the money amount, or we decide to drop the coin from the list of coins. For example, if we have 4 CHF, and coin denominations of 1 and 2, the call graph, in which every node depicts one invocation of the countChange method, is as follows:

```
                              4,[1, 2]
                 3,[1, 2]         +            4,[2]
         2,[1, 2]    +     3,[2]          2,[2]   +   4,[]
    1,[1, 2] + 2,[2]   1,[2] + 2,[]    0,[2] + 2,[]    0
0,[1, 2] + 1,[2]   1       0      0       1       0
    1        0
```

We can take advantage of this recursive structure by evaluating different subtrees in parallel. This is the next part of the assignment -- implement the method parCountChange that counts the amount of change in parallel:

```
def parCountChange(money: Int, coins: List[Int], threshold: Threshold): Int
```

As we learned in the lectures, the parCountChange should not spawn parallel computations after reaching the leaf in the call graph -- the synchronization costs of doing this are way too high. Instead, we need to *agglomerate* parts of the computation. We do this by calling the sequential countChange method when we decide that the amount of work is lower than a certain value, called the *threshold*. To separate the concern of deciding on the threshold value from the implementation of our parallel algorithm, we implement the threshold functionality in a separate function, described by the Threshold type alias:

```
type Threshold = (Int, List[Int]) => Boolean
```

When a threshold function returns true for a given amount of money and the given list of coins, the sequential countChange implementation must be called.

Implement parCountChange!

Now that we have the parCountChange method, we ask ourselves what is the right implementation of the threshold function? Recall the examples from the lectures, such as summing the array values and computing the norm, where this was easy -- we exactly knew the amount of work required to traverse a subrange of the array, so threshold could return true when the length of the subrange was smaller than a certain value.

Sadly, the total amount of work for a given parCountChange invocation is hard to evaluate from the remaining amount of money and a list of coins. In fact, the amount of work directly corresponds to the count that parCountChange returns, which is the value that we are trying to compute. Counting change is a canonical example of a task-parallel problem in which the partitioning the workload across processors is *solution-driven* -- to know how to optimally partition the work, we would first need to solve the problem itself.

For this reason, many parallel algorithms in practice rely on heuristics to assess the amount of work in a subtask. We will implement several such heuristics in this exercise, and assess the effect on performance. First, implement the moneyThreshold method, which creates a threshold function that returns true when the amount of money is less than or equal to 2 / 3 of the starting amount:

```
def moneyThreshold(startingMoney: Int): Threshold
```

Now run the ParallelCountChange application and observe the speedup:

```
> runMain reductions.ParallelCountChangeRunner
```

The previous heuristic did not take into account how many coins were left on the coins list, so try two other heuristics. Implement the method totalCoinsThreshold, which returns a threshold function that returns true when the number of coins is less than or equal to the 2 / 3 of the initial number of coins:

```
def totalCoinsThreshold(totalCoins: Int): Threshold
```

Then, implement the method combinedThreshold, which returns a threshold function that returns true when the amount of money multiplied with the number of remaining coins is less than or equal to the starting money multiplied with the initial number of coins divided by 2:

```
def combinedThreshold(startingMoney: Int): Threshold
```

Which of the three threshold heuristics gives the best speedup? Can you think of a heuristic that improves performance even more?

Parallel Parentheses Balancing

In this part of the assignment, we recall the Parenthesis Balancing assignment that might be familiar to you from the Functional Programming in Scala course. Here, the task is to, given an array of characters, decide if the parentheses in the array are balanced.

Let us recall a few examples of strings in which parentheses are correctly balanced:

```
(if (zero? x) max (/ 1 x))
I told him (that it's not (yet) done). (But he wasn't listening)
```

Similarly, the parentheses in the following strings are not balanced:

```
(o_()
:-)
())(
```

Implement a sequential function balance, which returns true iff the parentheses in the array are balanced:

```
def balance(chars: Array[Char]): Boolean
```

Next, you will implement a parallel version of this method. By now, you're already an expert at implementing the structure of a reduction algorithm, so you should have no problem there. The tricky part in parallel parentheses balancing is choosing the reduction operator -- you probably implemented balance by keeping an integer accumulator, incrementing it for left parentheses and decrementing it for the right ones, taking care that this accumulator does not drop below zero. Parallel parentheses balancing will require a bit more ingenuity on your part, so we will give you a hint -- you will need two integer values for the accumulator.

Implement the parBalance method, which checks if the parentheses in the input array are balanced using two helper methods reduce and traverse. These methods implement the parallel reduction and the sequential traversal part, respectively:

```
def parBalance(chars: Array[Char], threshold: Int): Boolean = {
  def traverse(from: Int, until: Int, _???_: Int, _???_: Int): ???

  def reduce(from: Int, until: Int): ??? = ???

  reduce(0, chars.length) == ???
}
```

In this case, we again use the fixed threshold parameter, as we did in the lectures. For maximum performance, use a while loop in the traverse method, or make traverse tail-recursive -- do not use a Range.

Now, run the ParallelParenthesesBalancing application:

```
> runMain reductions.ParallelParenthesesBalancingRunner
```

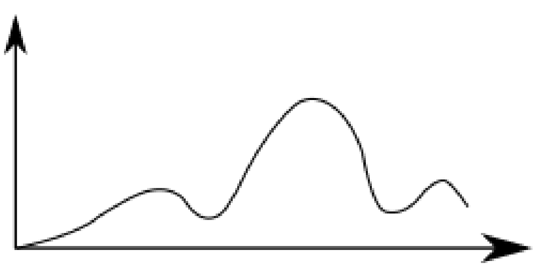
How large was your speedup?

If you are looking for additional challenges, prove that your reduction operator is associative!

Line of Sight

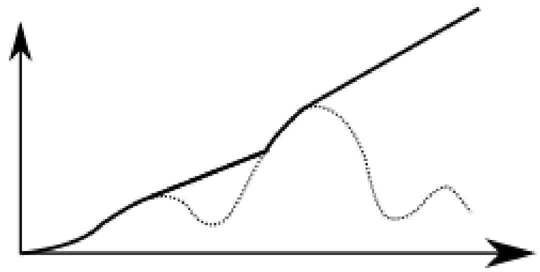
In the last part of the exercise, you will be implementing an entirely new parallel algorithm -- you will apply the prefix sum algorithm to computing the line-of-sight in two-dimensional terrain.

Imagine that you are standing at the zero of a coordinate system. The curve to your right describes the terrain that you are facing. This is shown in the following figure:



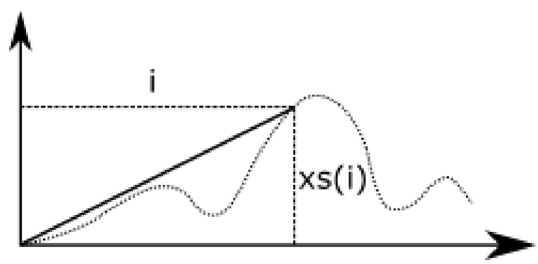
terrain

The task of the line-of-sight algorithm is to compute the visibility of each point of the terrain, as shown in the following figure, where the visible area is above of the full line, and the obscured terrain is shown with a dotted line.



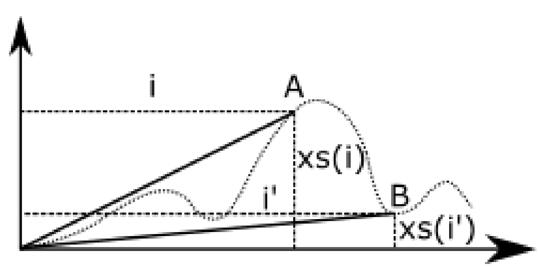
visibility

What is the necessary and sufficient condition for a point on the terrain to be visibile from the zero of the coordinate system, where you are standing? Imagine that the terrain heights are represented with an array of numbers. We can compute the viewing angle of each point on the terrain by dividing the height of the terraing xs(i) with the distance from the viewing point i, as shown in the following figure:



angle

It turns out that if the viewing angle of some point B is **lower** than the viewing angle of an earlier point A, then the point B is not visible, as shown in the following figure:



angle

This simple realization allows us to easily compute the line-of-sight on the terrain -- if you were a sequential programmer, you would traverse the array of height values from the beginning to the end, and write the maximum angle seen so far into the output array.

Implement the sequential lineOfSight method, which, for each height entry in the input array (except for input(0) which is the location of the observer and is always zero), writes the maximum angle until that point into the output array (output(0) should be 0):

```
def lineOfSight(input: Array[Float], output: Array[Float]): Unit
```

We keep things simple -- instead of outputting an array of booleans denoting the visibilities, we only output the angles.

When we see a sequential algorithm that produces a sequence of values by traversing the input from left to right, this is an indication that the algorithm might have a parallel prefix sum variant. So let's try to implement one!

Recall what you learned in the lectures -- the first phase of the parallel prefix sum algorithm is the *upsweep* phase. Here, the algorithm constructs the reduction tree by traversing parts of the input array in parallel. Implement the method upsweepSequential, which returns the maximum angle in a given part of the array, and the method upsweep, which returns the reduction tree over parts of the input array. If the length of the given part of the input array is less than or equal to threshold, then upsweep calls upsweepSequential. Note that the part of the input array that needs to traversed is represented using indices 'from' (inclusive) and 'until' (or 'end') (exclusive).

```
def upsweepSequential(input: Array[Float], from: Int, until: Int): Float

def upsweep(input: Array[Float], from: Int, end: Int, threshold: Int): Tree
```

The Tree data type is either a Leaf or an inner Node, and it contains the the maximum angle in the corresponding part of the array. Note that when the number of elements in the part of the input array, which is (end - from), falls below a given threshold, the sequential upsweepSequential has to be invoked, and you should return a Leaf. Otherwise, you should process the part of the input array in parallel, and return a Node. Make sure that the work is evenly distributed between the parallel computations.

The second phase is called *downsweep* -- here, the algorithm uses the tree to push the maximum angle in the corresponding *prefix* of the array to the leaves of the tree, and outputs the values. Implement the methods downsweep which processes parts of the tree in parallel, and the method downsweepSequential, which traverses the parts of the array corresponding to leaves of the tree and writes the final angles into the output array:

```
def downsweep(input: Array[Float], output: Array[Float],
  startingAngle: Float, tree: Tree): Unit

def downsweepSequential(input: Array[Float], output: Array[Float],
  startingAngle: Float, from: Int, until: Int): Unit
```

Finally, implement parLineOfSight using the upsweep and downsweep methods:

```
def parLineOfSight(input: Array[Float], output: Array[Float],
  threshold: Int): Unit
```

Now, run the LineOfSight application and observe the relative speedups:

```
> runMain reductions.LineOfSightRunner
```

How large is the speedup compared to the number of cores in your processor? Can you explain your results?

How to submit

Copy the token below and run the submission script included in the assignment download. When prompted, use your email address **gregoire.clement@epfl.ch**.

QSWtW0ynUXG8nQU0

Generate new token

Your submission token is unique to you and should not be shared with anyone. You may submit as many times as you like.