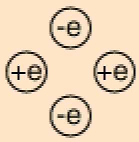
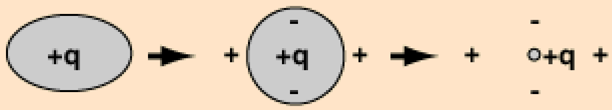
Electric Quadrupole

A general distribution of electric charge may be characterized by its net charge, by its dipole moment, its quadrupole moment and higher order moments. An elementary quadrupole can be represented as two dipoles oriented antiparallel.



An elementary quadrupole would be seen as having zero charge and zero dipole moment at a great distance. Its interaction with an electric field can be quantified in terms of its quadrupole moment.



An ellipsoidal charge distribution can be represented by a spherical charge plus a quadrupole, and the spherical charge can be represented by a point charge for assessing its field and potential outside the volume of the charge. It follows that a spherically symmetric charge has no quadrupole moment.

One of the most common uses of the electric quadrupole is in the characterization of nuclei. The nucleus has charge, but not dipole moment since it is all positive. But if the nucleus is not spherically symmetric, it will have a quadrupole moment.

Quadrupole and higher order multipoles are not important for the characteriztion of dielectric materials. Dipole fields are much smaller than the fields of isolated charges, but in dielectrics where there are no free charges, the dipole effects are dominant. There is no such circumstance favoring the quadrupole effects, since they must arise from the same number of molecules as the dipole effects. Scott says that the macroscopic quadrupole effects are smaller than dipole effects by about the ratio of atomic dimensions to the distances of experimental observation.

Field of a linear electric quadrupole

Quadrupole moments of nuclei

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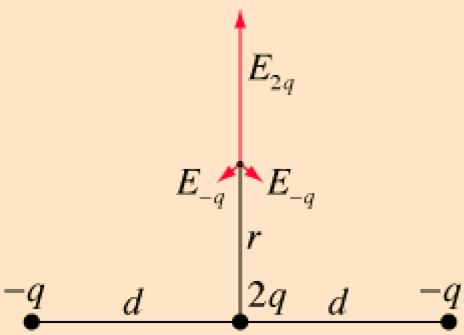
Electric dipole concepts

References
Cohen
Concepts of Nuclear Physics, Ch 1

Scott
Sec 3.3

Linear Electric Quadrupole

A linear electric quadrupole can be created by superimposing two electric dipoles of opposite orientation so that their positive charges overlap. This case can be treated analytically and gives some insights into the nature of quadrupole fields. The electric field from any collection of charges can be obtained from Coulomb's law by vector addition of the fields from the individual charge elements.



Linear quadrupole

$$E_{2q} = \frac{2q}{4\pi\varepsilon_0 r^2}$$

$$E_{-q} = \frac{-q}{4\pi\varepsilon_0 (r^2 + d^2)}$$

By symmetry, the field components parallel to the quadrupole cancel. The resultant field perpendicular is

$$E_r = \frac{2q}{4\pi\varepsilon_0}\left[\frac{1}{r^2} - \frac{r}{(r^2+d^2)^{3/2}}\right]$$

Factoring out the $\frac{1}{r^2}$ term puts it in the form $E_r = \frac{2q}{4\pi\varepsilon_0 r^2}\left[1 - \left(1 + \frac{d^2}{r^2}\right)^{-3/2}\right]$

For $\frac{d^2}{r^2}$ << 1, the binomial expansion gives $\left(1 + \frac{d^2}{r^2}\right)^{-3/2} \approx 1 - \frac{3}{2}\frac{d^2}{r^2}$

and the quadrupole electric field at large distances simplifies to $E_r \approx \frac{3qd^2}{4\pi\varepsilon_0 r^4}$

This has shown that the distant electric field perpendicular to the quadrupole drops off like $1/r^4$. This $1/r^4$ dependence applies to other directions as well. In fact, it is characteristic of quadrupoles in general, although we have not shown that here. It is also a general characteristic of quadrupoles that the electric field depends upon the magnitude of one of the charges times the square of the dimension of the quadrupole, qd^2.

Binomial

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Electric dipole concepts

Reference
Schwarz
Sec 1.4

[expansion]

Using maxima, I calculated :

```
(%i181) load("quadrupole.mac");
(%o181) C:/Users/warwar/maxima/quadrupole.mac
(%i182) ans;
```

$$\text{(\%o182)}\quad \frac{d^2\,Q}{r^4}-\frac{d^4\,Q}{r^6}+\frac{d^6\,Q}{r^8}-\frac{d^8\,Q}{r^{10}}$$

```
(%i183) fns;
```

$$\text{(\%o183)}\quad \frac{2\,d^3\,Q}{r^5}-\frac{5\,d^4\,Q}{r^6}+\frac{7\,d^5\,Q}{r^7}-\frac{7\,d^6\,Q}{r^8}+\frac{4\,d^7\,Q}{r^9}+\frac{3\,d^8\,Q}{r^{10}}$$

/*
This file calculates the field from two dipoles, where each falls
off a1 Q/r2
*/

/*
The dipoles are arranged in at (0,0) and (0,d), and we go to the point
(-r, 0) and calculate the field

```
*/



/*
let a=d/r, then take the taylor series limit as a --> 0, and then substitute back in

distance^2 to the first point is r^2
distance^2 to the second point is k^2 = r^2 + d^2 = r^2(1+a^2)

*/

eqq : Q/r^2 - Q/(r^2*(1+a^2));


epp : taylor(eqq, a, 0, 8);


ans : subst(d/r, a, epp);


/*
now instead of putting +Q and -Q, we put +Q/2, -Q/2, +Q/2, -Q/2
at (0,0) (0, d) (d, 0) (d, d)

k0^2 = r^2
k1^2 = r^2*(1+a^2)
k2^2 = r^2*(1+a)^2
k3^2 = r^2*((1+a)^2 + a^2)
*/

fqq : (Q/2)/r^2 - (Q/2)/(r^2*(1+a^2)) - (Q/2)/(r^2*(1+a)^2) + (Q/2)/(r^2*((1+a)^2+a^2));
fpp : taylor(fqq, a, 0, 8);

fns : subst(d/r, a, fpp);

/*
stringout("/Users/warwar/Desktop/Open/quadupole.txt", ans);
*/
```