

* Basic concepts

- thermodynamics deals with the amount of heat transfer, makes no reference how long the process will take.

= HEAT - the form of energy that can be transferred from one system to another as a result of temperature difference.
(AT THE MICROSCALE: IT IS THE INCREASED MOTION OF MOLECULES)

* LAWS OF THERMODYNAMICS:

- first law: requires that the rate of the energy transfer into a system be equal to the rate of increase of the energy of that system
- second law: requires that heat be transferred in the direction of decreasing temperature. (like a car parked on a inclined road must go downhill)
IN THE DIRECTION OF DECREASING ELEVATION

* the basic requirement for heat transfer is the presence of temperature difference
(it is the driving force, just as the voltage difference for electric current flow)

* E - total energy of a system

* U - internal energy - the sum of all microscopic forms of energy (the forms related to the molecular structure of a system and the degree of molecular activity)

* unit: joule (J) or kilojoule (kJ)

flow energy of the fluid = energy needed to push a fluid and to maintain flow.

* enthalpy: $h = u + Pv$



* IDEAL GAS - gas that obeys the relation:

$$Pv = RT$$

OR

$$P = \rho RT$$

WHERE:

P - absolute pressure

v - specific volume

ρ - density ($\rho = \frac{m}{v}$)

T - thermodynamic / absolute temperature

R - gas constant

Specific heat - the energy required to raise the temperature of a unit mass of a substance by one degree.

C_p - at constant pressure

C_v - at constant volume

* $C_p > C_v$ because at constant pressure the system is allowed to expand and the energy for this expansion work must also be supplied to the system.

* for ideal gases: $C_p = C_v + R$

* unit for specific heats: $\frac{kJ}{kg \cdot ^\circ C}$, also: $\frac{kJ}{kg \cdot K}$

* the specific gases depend on two independent properties: temperature & pressure

- but for an ideal gas they depend on TEMPERATURE ONLY

- at low pressures all real gases approach ideal gas behaviour, therefore their specific heats depend on temperature only

* THE DIFFERENTIAL CHANGES in the internal energy and enthalpy h of an ideal gas:

$$du = C_v dT, dh = C_p dT$$

* THE FINITE CHANGES:

$$\Delta U = C_{V,\text{avg}} \Delta T, \Delta h = C_{P,\text{avg}} \Delta T \quad (\Delta U = m \cdot C_{V,\text{avg}} \Delta T, \Delta H = m \cdot C_{P,\text{avg}} \Delta T) \quad \begin{matrix} \text{m-mass of} \\ \text{the system} \end{matrix}$$

[J/g]

* for solids and liquids the subscripts C_p, C_v can be represented by single symbol c

* ENERGY TRANSFER

* energy can be transferred to or from a given mass by two mechanisms

- heat transfer Q

- work W

* (energy driving force is temperature difference) = heat transfer, otherwise it is work.

* energy driving force is temperature difference = heat transfer, otherwise it is work.

* work done per unit time = POWER

* energy of a system decreases, as it does work and increases as work is done on it

* Scientifically we should replace 'heat' with 'thermal energy' in most everyday-use cases (BODY HEAT \rightarrow THERMAL ENERGY OF A BODY), but keeping in line with current practice we will refer to thermal energy as heat and the transfer of thermal energy as heat transfer.

↳

Q - the amount of heat (thermal energy) transferred during the process.

\dot{Q} - the amount of heat transferred per unit time (heat transfer rate).
(the overdot stands for time derivative, or 'per unit time')

- \dot{Q} has the unit [J/s], which is equivalent to W .

* When we know \dot{Q} , we can get Q

$$Q = \int_0^{\Delta t} \dot{Q} dt [J]$$

- when $\dot{Q} = \text{const}$, then $Q = \dot{Q} \int_0^{\Delta t} dt \rightarrow Q = \dot{Q} \Delta t$

* **heat flux** - the rate of heat transfer per unit area normal to the direction of heat transfer

$$\dot{q}_v = \frac{\dot{Q}}{A} [W/m^2] \quad \text{, where}$$

A - heat transfer area.

EXAMPLE:



$$\dot{Q} = 24 W = \text{const}$$

$$\dot{q}_v = \frac{\dot{Q}}{A} = \frac{24 W}{6 m^2} = 4 \frac{W}{m^2}$$

The first law of thermodynamics (conservation of energy principle)

- energy can neither be created nor destroyed during a process; it can only change forms

$$\left(\begin{array}{l} \text{TOTAL ENERGY} \\ \text{ENTERING THE} \\ \text{SYSTEM} \end{array} \right) - \left(\begin{array}{l} \text{Total Energy} \\ \text{leaving the} \\ \text{system} \end{array} \right) = \left(\begin{array}{l} \text{CHANGE IN THE} \\ \text{TOTAL ENERGY} \\ \text{OF THE SYSTEM} \end{array} \right)$$

* energy can be transferred to or from the system by heat, work and mass flow

Energy balance for any system:

$$\dot{E}_{in} - \dot{E}_{out} = \Delta E_{system} [J]$$

Rate form:

$$\dot{E}_{in} - \dot{E}_{out} = \frac{d}{dt} E_{system} [W]$$

* the process is steady when $\dot{E}_{in} = \dot{E}_{out}$

Heat balance:

$$\dot{Q}_{in} - \dot{Q}_{out} + \dot{E}_{gen} = \Delta E_{\text{therm, system}}$$

\dot{E}_{gen} = heat generation = conversion of all energies into thermal energy

Energy balance for closed systems (FIXED MASS)

$$\rightarrow \dot{E}_{in} - \dot{E}_{out} = \dot{\Delta U} = \frac{1}{m} \cdot c_v \cdot \Delta T \quad [J]$$

* but when there is NO WORK! (heat transfer only)

$$\dot{Q} = m \cdot c_v \cdot \Delta T \quad [J]$$

Energy balance for steady-flow systems

STEADY = NO CHANGE WITH TIME at specified location

UNIFORM = NO CHANGE WITH POSITION at a specified time

* the amount of energy entering a control volume in all forms (heat, work, mass transfer) for a steady-flow process must be equal to the energy leaving it.

$$E_{cv} = \text{const} , \Delta E_{cv} = 0$$

\dot{m} = **mass flow rate** = the amount of mass flowing through a cross section of a flow device per unit time.

$$\dot{m} = \rho V A_c \quad \text{, where: } \begin{aligned} \rho &= \text{fluid density} \\ V &= \text{average fluid velocity} \\ A_c &= \text{cross-sectional area of the pipe or duct} \end{aligned}$$

\dot{V} = **volume flow rate** = the volume of a fluid flowing through a pipe or duct per unit time

$$\dot{V} = V A_c = \frac{\dot{m}}{\rho}$$

$$[\text{m}^3/\text{s}]$$

$$\text{Energy balance: } \dot{Q} = \dot{m} \Delta h = \dot{m} c_p \Delta T \quad [\text{kJ}/\text{s}]$$

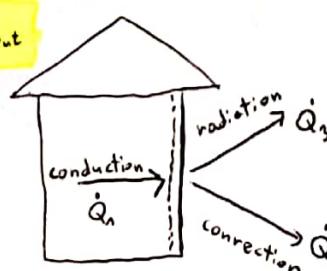
Surface energy balance

* surface contains no energy (no mass / no volume)

* Therefore we can assume energy content remains constant during a process (just like a steady-state or steady-flow process)

$$\text{so: } \dot{E}_{in} = \dot{E}_{out}$$

Example:



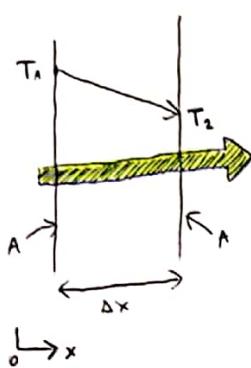
$$\text{then } \dot{Q}_1 = \dot{Q}_2 + \dot{Q}_3$$

Conduction

TRANSFER BY DIRECT CONTACT

- is the transfer of energy from the more energetic particles of a substance to the adjacent less energetic ones as a result of interactions between particles.

Example: Heat conduction through a plane wall of thickness Δx and area A

**Proportions:**

rate of heat conduction

$$\propto \frac{(\text{AREA})(\text{TEMPERATURE DIFFERENCE})}{\text{THICKNESS}}$$

Scientifically:

$$\dot{Q}_{\text{cond}} = k \cdot A \frac{T_1 - T_2}{\Delta x} = -k \cdot A \frac{\Delta T}{\Delta x} \quad [\text{W}]$$

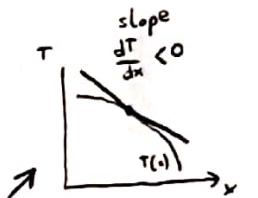
where:

k - thermal conductivity = measure of the ability of a material to conduct heat

In the limiting case of $\Delta x \rightarrow 0$:

THE EQUATION REDUCES TO DIFFERENTIAL FORM

$$\dot{Q}_{\text{cond}} = -k \cdot A \frac{dT}{dx} \quad [\text{W}]$$

~ Fourier's Law of heat conduction

- * $\frac{dT}{dx}$ is temperature gradient. Heat is conducted in the direction of decreasing temperature, and the temperature gradient becomes negative when temperature decreases with increasing x . The negative sign ensures that heat transfer in positive x -direction is positive quantity

Convection - is the mode of energy transfer between a solid surface and the adjacent liquid or gas that is in motion.

* the faster the fluid motion = the greater the convection heat transfer.

* in the absence of any bulk fluid motion heat transfer between a solid surface and the adjacent fluid is by pure conduction

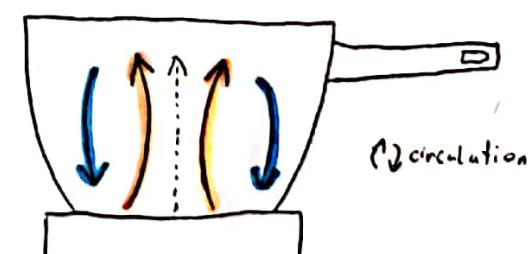
* Rate of convection heat transfer is proportional to the temperature difference (Newton's Law of cooling)

$$\dot{Q}_{\text{conv}} = h \cdot A_s (\underset{\substack{\text{surface area} \\ \text{heat transfer coefficient}}}{T_s - T_{\infty}}) \quad [\text{W}]$$

temp. of the fluid sufficiently far from the surface.

Example: the warmer soup rises upward and colder soup up above it falls down to take its place.

- in the same way we heat up our homes



Radiation: is the energy emitted by matter in the form of electromagnetic waves, as the result of the changes in the electronic configurations of the atoms.

- does not require the presence of an intervening medium
- is the fastest (speed of light)
- it suffers no attenuation in a vacuum.

* We are interested in thermal radiation (not x-rays, microwaves etc. that are not related to temperature)

↓

radiation emitted by bodies → all bodies at a temperature $> 0^\circ$

* The MAXIMUM RATE of radiation that can be emitted from a surface at a temperature T_s [K] is given by Stephan-Boltzmann Law.

$$\dot{Q}_{\text{emit, max}} = \sigma \cdot A_s \cdot T_s^4$$

where: σ - Stephan Boltzmann constant

- **blackbody** - the idealized surface that emits radiation at this maximum rate.

* The radiation emitted by real surfaces is less than blackbody radiation at the same temperature and is expressed as:

$$\dot{Q}_{\text{emit}} = \epsilon \sigma A_s T_s^4$$

where ϵ - emissivity of the surface ($0 \leq \epsilon \leq 1$)

- **Emissivity** - measure of how closely a surface approximates a blackbody (for which $\epsilon = 1$)

- **Absorptivity** - (α) - is the fraction of the radiation energy incident on a surface that is absorbed by the surface.

blackbody is also a perfect absorber ($\alpha = 1$)

$$\dot{Q}_{\text{absorbed}} = \alpha \cdot \dot{Q}_{\text{incident}}$$

Both ϵ and α of a surface depend on the temperature and the wavelength of the radiation

* **Kirchoff's Law of Thermal Radiation** - states that:

"the emissivity and the absorptivity of a surface are equal [at given temperature and wavelength]"

- so if an object have high emissivity then it have also high absorptivity.

* absolute zero = -273.15°C [0K] - no kinetic energy left
- nothing could be colder

- * the lowest temperature anything can ever reach
- * in practice - impossible to cool anything down that much.

Radiation - continuation

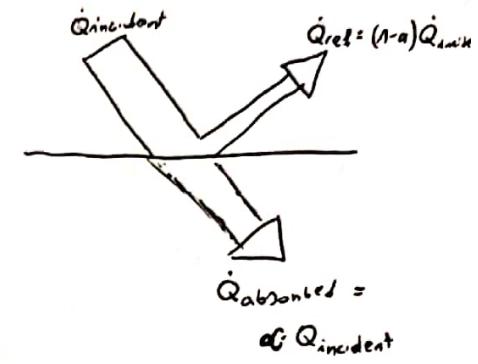
* THE RATE at which A SURFACE ABSORBS RADIATION

$$\dot{Q}_{\text{absorbed}} = \alpha \cdot \dot{Q}_{\text{incident}}$$

(so $\alpha=1$, then $\dot{Q}_{\text{abs}} = \dot{Q}_{\text{incident}}$ (BLACKBODY))

NO REFLECTION

(but when $\alpha < 1$ then the portion of incident radiation not absorbed by the surface is reflected back)



* when a surface is completely enclosed by a much larger (or black) surface at

temperature T_{surr} separated by gas, that does not interfere with radiation the net rate transfer between these surfaces is given by:

$$\dot{Q}_{\text{rad}} = \epsilon \sigma A_s (T_s^4 - T_{\text{surr}}^4)$$

- emissivity and the surface area of the surrounding surface do not have any effect on net radiation heat transfer.

* net radiation heat transfer - the difference between the rates of radiation emitted by the surface and radiation absorbed.

- gaining energy - when the rate of radiation absorption is greater than the rate of radiation emission.

- otherwise the surface is said to be losing energy by radiation

Heat conduction in a medium is said to be:

* **steady** - the temperature does not vary with time

* **unsteady / transient** - it does.

also:

one-dimensional - conduction is significant in one dimension only and negligible in the other two primary dimensions

two-dimensional - ... conduction in the third dimension is negligible

three-dimensional - ... conduction in all dimensions is significant

The temperature in a medium varies with position as well as time $T = T(x, y, z, t)$

Heat generation - conversion process from mechanical, electrical, nuclear or chemical energy into heat

examples:

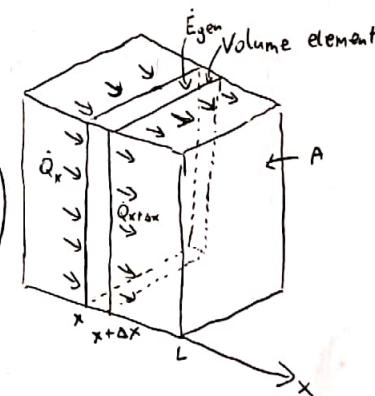
- the temperature of a resistance wire rises rapidly when electric current passes through it as a result of electrical energy being converted at rate of I^2R .
- the heat generated in the sun as the result of the fusion of hydrogen into helium makes the sun a large nuclear reactor that supplies heat to the earth.

The total rate of heat generation in a medium [when the variation of heat generation with position is known]

$$\dot{E}_{\text{gen}} = \int_V \dot{e}_{\text{gen}} dV$$

↓ the rate of heat generation

(when heating throughout a homogeneous material)
the relation reduces to $\dot{E}_{\text{gen}} = \dot{e}_{\text{gen}} \cdot V$



* Heat Equation in a Large Plane Wall

Energy balance on the thin element during a small time interval Δt

$$\left(\text{Rate of heat conduction at } x \right) - \left(\text{Rate of heat conduction at } x + \Delta x \right) + \left(\text{Rate of heat generation inside the element} \right) = \left(\text{Rate of change of the energy content of the element} \right)$$

"rate of ..." ~ per unit time

$$\left(\dot{Q}_x \right) - \left(\dot{Q}_{x+\Delta x} \right) + \left(\dot{E}_{\text{gen, element}} \right) = \left(\frac{\Delta E_{\text{element}}}{\Delta t} \right)$$

$\Delta E = m \cdot c \cdot \Delta T$ so I can write that:

$$\Delta E_{\text{element}} = E_{t+\Delta t} - E_t = mc(T_{t+\Delta t} - T_t) = \rho c \Delta x (T_{t+\Delta t} - T_t)$$

$$\dot{E}_{\text{gen, elem.}} = \dot{e}_{\text{gen}} V_{\text{elem.}} = \dot{e}_{\text{gen}} A \Delta x$$

Substituting to main eq and dividing by $A \Delta x$ gives....

$$-\frac{1}{A} \frac{\dot{Q}_{x+\Delta x} - \dot{Q}_x}{\Delta x} + \dot{e}_{\text{gen}} = \rho c \frac{T_{t+\Delta t} - T_t}{\Delta t}$$

$\frac{\partial \dot{Q}}{\partial x}$ $\frac{\partial T}{\partial x}$

when we limit $\Delta x \rightarrow 0, \Delta t \rightarrow 0$
we can rewrite this using the
definition of derivative

$$\rightarrow \frac{1}{A} \cdot \frac{\partial}{\partial x} \left(kA \frac{\partial T}{\partial x} \right) + \dot{e}_{\text{gen}} = \rho c \frac{\partial T}{\partial t}$$

and from Fourier's law: $\frac{\partial Q}{\partial x} = \frac{\sigma}{\partial x} \left(-kA \frac{\partial T}{\partial x} \right)$

Noting that the area A is constant for a plane wall the one-dimensional transient heat conduction equation in a plane wall becomes:

$$\frac{\partial}{\partial x} \left(K \frac{\partial T}{\partial x} \right) + \dot{e}_{gen} = \rho c \frac{\partial T}{\partial t}$$

when the thermal conductivity K of material is constant (in most practical applications!)

it can be taken out of the derivative:

$$\frac{\partial^2 T}{\partial x^2} + \frac{\dot{e}_{gen}}{K} = \frac{1}{\alpha} \cdot \frac{\partial T}{\partial t}, \text{ where } \alpha = \frac{K}{\rho \cdot c} \text{ is thermal diffusivity of the material}$$

\downarrow
represents how fast heat propagates through a material.

It reduces to the following forms under specified conditions:

$$\frac{\partial^2 T}{\partial x^2} + \frac{\dot{e}_{gen}}{K} = \frac{1}{\alpha} \cdot \frac{\partial T}{\partial t}$$

$= 0$ when **NO ENERGY GENERATION**

$= 0$ when **STEADY STATE**

steady-state, no heat generation:

$$\frac{\partial^2 T}{\partial x^2} = 0$$

\Rightarrow when the function depends on a single variable only $T = T(x)$
 \Rightarrow can replace partial derivatives with ordinary derivatives.

Boundary conditions - mathematical expressions of the thermal conditions at the boundaries.

- two boundary conditions must be given for each direction of coordinate system along which heat transfer is significant
(because heat conduction equation is second order)

Initial condition - mathematical expression for the temperature distribution of the medium initially

* is usually specified at time $t=0$; the temperature at any point on the wall at a specified time also depends on the condition of the wall at the beginning of the heat conduction process.

* we need only one initial condition for a heat conduction problem regardless of the dimension since the conduction equation is first order in time (it involves the first derivative of temperature with respect to time)

$T(x,y,z,0) = f(x,y,z)$ - the function f represents the temperature distribution throughout the medium at time $t=0$

* under steady conditions, the heat conduction equation does not involve any time derivatives, thus we do not need to specify an initial condition

for example: $T(x,0) = T_i = 600^\circ\text{C}$ ~initial temperature that oven has been heated to.

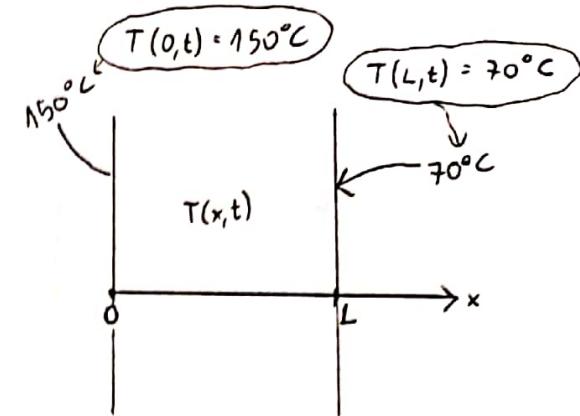
Boundary Conditions:

I Specified temperature

- the temperature of an exposed surface can usually be measured directly and easily, so this is one of the easiest ways to specify the thermal conditions on a surface.

$$\begin{aligned} T(0,t) &= T_1 \quad \text{- specified temperature at surface at } x=0 \\ T(L,t) &= T_2 \quad \text{--- // ---} \quad x=L \end{aligned}$$

- When T_1, T_2 are constant we have steady heat conduction, when they vary with time - unsteady.



II Specified Heat Flux (Neumann)

- when we have sufficient information about interactions at a surface
- the heat flux in the positive x-direction anywhere in the medium can be expressed by Fourier's Law:

$$\dot{q} = -k \frac{\partial T}{\partial x} [\text{W/m}^2]$$

- so the boundary condition is obtained by setting the specified heat flux equal to $-k \frac{\partial T}{\partial x}$ at that boundary!
- the sign is determined by inspection (positive = positive x-axis direction)

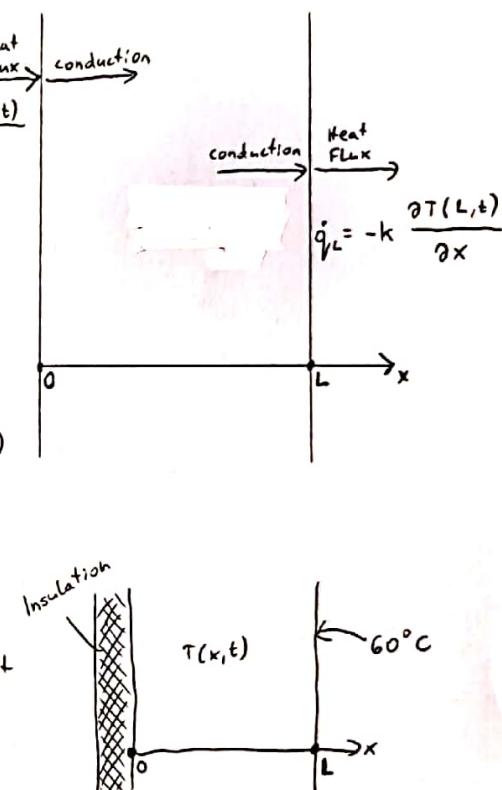
$$\dot{q}_0 = -k \frac{\partial T(0,t)}{\partial x}, \quad \dot{q}_L = -k \frac{\partial T(L,t)}{\partial x}$$

Special case: Insulated Boundary

- insulation reduces heat transfer but does not totally eliminate it unless its thickness is infinity
- heat transfer can be taken to be zero since adequate insulation reduces heat transfer through a surface to negligible levels.

$$\left(k \frac{\partial T(0,t)}{\partial x} = 0 \right) \text{ or } \frac{\partial T(0,t)}{\partial x} = 0$$

so, on an insulated surface, the first derivative of temperature with respect to the space variable (the temperature gradient) in the direction normal to the insulated surface is zero.

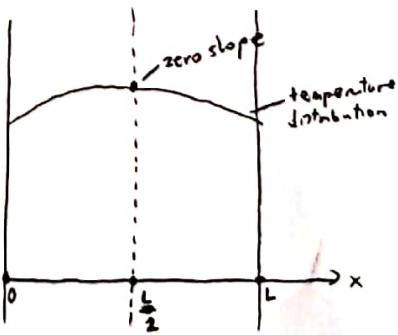


$$\frac{\partial T(0,t)}{\partial x} = 0, \quad T(L,t) = 60^\circ\text{C}$$

Another special case: Thermal symmetry

- the two surfaces of a large hot plane of thickness L suspended vertically in air is subjected to the same thermal conditions
- the temperature distribution in one half of the plate is the same as that in the other half
- thermal symmetry about the center plane at $x = \frac{L}{2}$
- there is not heat flow across the center plane.
- so the center plane can be viewed as an insulated surface

$$\frac{\partial T(\frac{L}{2}, t)}{\partial x} = 0$$



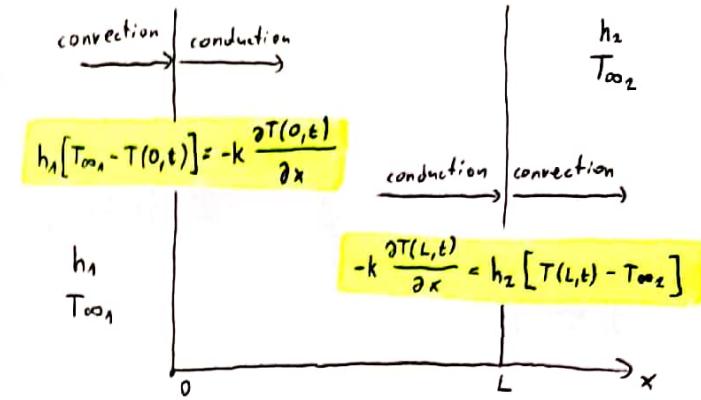
Convection Boundary Condition

- probably the most common one encountered in practice since most heat transfer surfaces are exposed to an environment at a specified temperature.

- is based on a 'surface energy balance' expressed as:

$$(\text{Heat conduction}) = (\text{Heat convection})$$

[at the surface
in a selected direction]



h_1, h_2 - convection heat transfer coefficients

$T_{\infty 1}, T_{\infty 2}$ - the temperatures of the surrounding mediums

* Note that a surface has zero thickness and thus no mass and it cannot store any energy!

Therefore, the entire net heat entering the surface from one side must leave the surface from the other side ...

* The convection boundary condition simply states that heat continues to flow from a body to the surrounding medium at the same rate (it just changes vehicles at the surface from conduction to convection (or vice versa in the other direction))

[Like people traveling on buses on land and transferring to the ships at the shore]

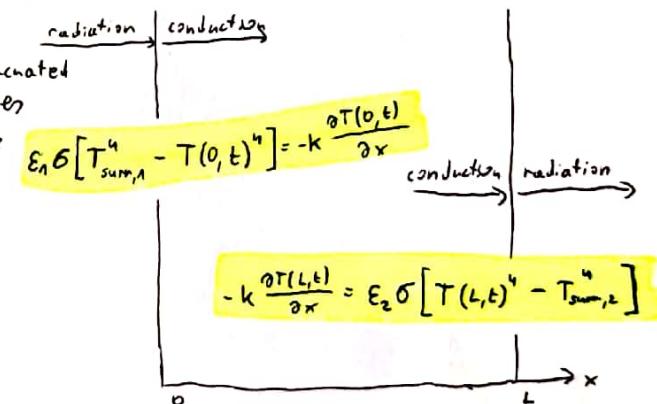
* The surface temperatures $T(0,t)$ and $T(L,t)$ are not known (if they were known, we would simply use them as the specified temperature boundary condition and not bother with convection)
(We know only h and T_{∞})

Radiation Boundary Condition

- when a heat transfer surface is surrounded by an evacuated space and thus there is no convection radiation becomes the only mechanism of heat transfer between the surface under consideration and the surroundings.

$$(\text{Heat conduction}) = (\text{Radiation exchange})$$

[AT THE SURFACE
IN A SELECTED DIRECTION]



E_1, E_2 - emissivities of the boundary surfaces

$\sigma = 5,67 \cdot 10^{-8} \frac{W}{m^2 K^4}$ - Stephan-Boltzmann constant

$T_{sur,1}, T_{sur,2}$ - average temperatures of the surfaces surrounding the two sides of the plate

* this boundary condition involves the fourth power of temperature, and thus it is a nonlinear condition.

* When heat transfer at the surface is dominated by convection, and the role of radiation is minor it is tempting to ignore radiation exchange at a surface.

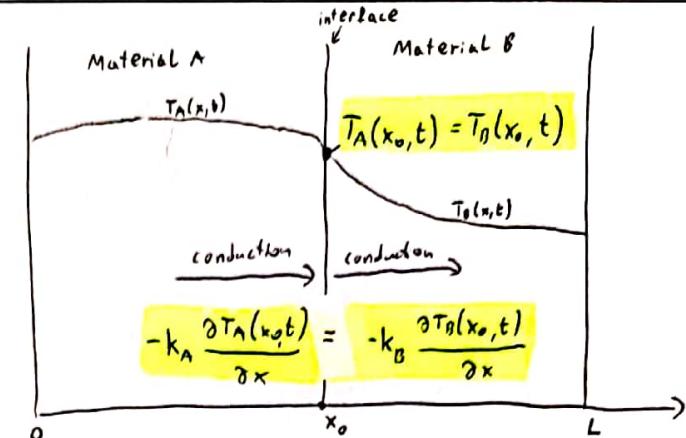
Interface Boundary Conditions

- some bodies are made up of layers of different materials, so we require the solution of heat transfer problem in each layer
- this requires the specification of the boundary conditions at each interface.

Requirements :

- (1) Two bodies in contact must have the same temperature at the area of contact
- (2) an interface (surface at $x=x_0$) cannot store any energy, and thus the heat flux on the two sides of an interface must be the same

k_A, k_B - thermal conductivities of the layers A and B

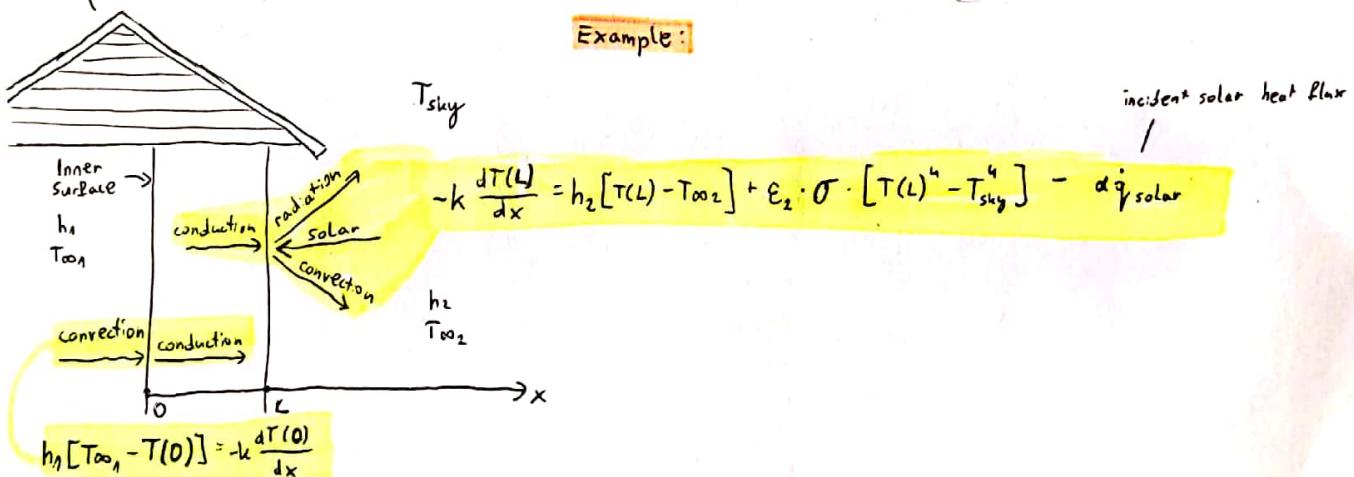


Generalized Boundary Conditions

- a surface may involve convection, radiation and specified heat flux simultaneously. the boundary condition is again obtained from energy balance :

$$\begin{pmatrix} \text{Heat transfer} \\ \text{to the surface} \\ \text{in all modes} \end{pmatrix} = \begin{pmatrix} \text{Heat transfer} \\ \text{from the surface} \\ \text{in all modes} \end{pmatrix}$$

Example:



General notes:

- * the two boundary conditions in a direction may be specified at the same boundary, while no condition is imposed on the other boundary

For example : specifying the temperature and heat flux at $x=0$ of a plate of thickness L will result in a unique solution for the one-dimensional steady temperature distribution in the plate, including the temperature at the surface $x=L$.

- * There is nothing wrong with specifying more than two boundary conditions in a specified direction, provided that there is no contradiction. The extra conditions can be used to verify the results.

Convection : explained

- requires the presence of material medium (like conduction), but they are different in that **convection requires the presence of fluid motion**. When there is no bulk fluid motion the heat transfer is by conduction. Conduction in a fluid can be viewed as the limiting case of convection. (case of quiescent fluid).
- the fluid motion enhances heat transfer, since it brings warmer and cooler chunks of fluid into contact, initiating higher rates of conduction at a greater number of sites in a fluid. Therefore, the rate of heat transfer through a fluid is much higher by convection than it is by conduction. The higher the fluid velocity, the higher the rate of heat transfer.

Convection depends on:

* the fluid properties

μ - dynamic viscosity

k - thermal conductivity

ρ - density

C_p - specific heat

* fluid velocity V

* geometry and roughness of the solid surface

* type of fluid flow

- streamlined
- turbulent

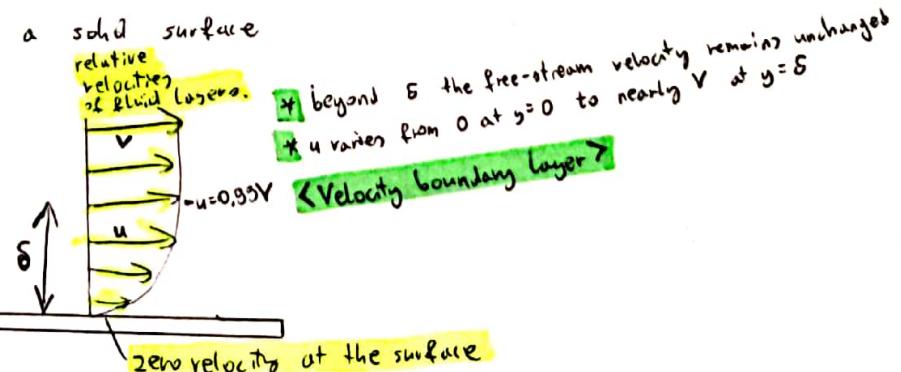
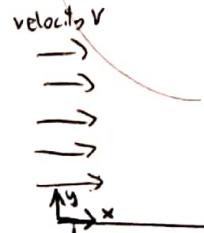
The rate of convection: Newton's Law of cooling:

$$q_{\text{conv}} = h(T_s - T_\infty) \left[\frac{W}{m^2} \right]$$

, where **h - convection heat transfer coefficient** (the heat transfer rate between a solid surface and a fluid per unit surface area per unit temperature difference) $\left[\frac{W}{m^2 K} \right]$

Boundary Layer

- A fluid in direct contact with a solid "sticks" to the surface due to viscous effects, and there is no slip.
- the layer that sticks to the surface slows the adjacent fluid layer because of viscous forces between the fluid layers, which slows the next layer, and so on.
- Therefore the "**no-slip condition**" is responsible for the development of the velocity profile
- The flow region adjacent to the wall in which the viscous effects (and thus the velocity gradients) are significant is called **the boundary layer**.
- the fluid property responsible for the no-slip condition and the development of the boundary layer is viscosity
- a fluid layer adjacent to a moving surface has the same velocity as the surface
- all velocity profiles must have zero values with respect to the surface at the points of contact between a fluid and a solid surface



- heat transfer from solid surface to the fluid layer adjacent to the surface is by pure conduction, since the fluid layer is motionless, and can be expressed as:

$$\dot{q}_{\text{conv}} = \dot{q}_{\text{cond}} = -k_{\text{fluid}} \frac{\partial T}{\partial y} \Big|_{y=0} \quad \left[\frac{W}{m^2 K} \right], \text{ where } T - \text{temperature distribution in the fluid}$$

Heat is then convected away from the surface as a result of fluid motion

Convection heat transfer from the solid surface to a fluid is merely the conduction from the solid surface to the fluid layer adjacent to the surface.

We can equate:

$$h [T_s - T_\infty] = -k_{\text{fluid}} \frac{\partial T}{\partial y} \Big|_{y=0} \Rightarrow h = \frac{-k_{\text{fluid}} \frac{\partial T}{\partial y} \Big|_{y=0}}{T_s - T_\infty} \quad \left[\frac{W}{m^2 K} \right]$$

Nusselt Number

- it is common practice to nondimensionalize the heat transfer coefficient h with the Nusselt number.

$$Nu = \frac{h \cdot L}{k} \quad \begin{aligned} L &= \text{characteristic length} \\ k &= \text{thermal conductivity} \end{aligned} \quad \left(\frac{\dot{q}_{\text{conv}}}{\dot{q}_{\text{cond}}} = \frac{h \Delta T}{k \frac{\Delta T}{L}} = \frac{h L}{k} = Nu \right)$$

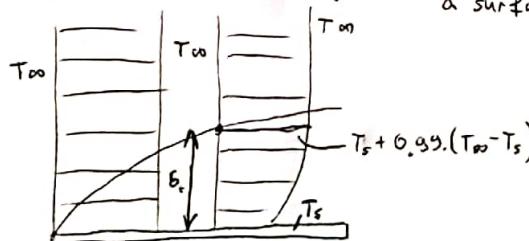
- represents the enhancement of heat transfer through a fluid layer as a result of convection relative to conduction across the same fluid layer.

* THE LARGER the Nusselt number THE MORE EFFECTIVE THE CONVECTION

* $Nu = 1$ ~ pure conduction

* **velocity boundary layer** - region in which the fluid velocity varies from 0 to $0.99 V$

* **thermal boundary layer** - develops when a fluid at a specified temperature flows over a surface that is at a different temperature



Prandtl Number

$$Pr = \frac{v}{\alpha} = \frac{\text{momentum diffusivity}}{\text{thermal diffusivity}}, \quad \alpha = \frac{k}{c_p \cdot \rho} \quad \begin{aligned} v &= \text{kinematic viscosity} \left[\frac{m^2}{s} \right] \\ &\quad / \text{dynamic viscosity} \left[\frac{kg}{m \cdot s} \right] \\ &\quad / \text{density} \left[\frac{kg}{m^3} \right] \\ &\quad / \text{specific heat} \left[\frac{J}{kg \cdot K} \right] \end{aligned} \quad \begin{aligned} k &= \text{thermal conductivity} \left[\frac{W}{mK} \right] \\ \rho &= \text{density} \left[\frac{kg}{m^3} \right] \end{aligned}$$

[for gases ~ 1]

[liquid metals $Pr \ll 1$
(heat diffuses quickly)]

[oils $\gg 1$]

So the thermal boundary layer is much thicker for liquid metals and much thinner for oils relative to the velocity boundary layer.

- describes relative thickness of the velocity and thermal boundary layers.

- dependent only on the fluid and the fluid state / no length /

Reynolds Number

$$Re = \frac{\text{inertia forces}}{\text{viscous forces}} = \frac{V \cdot L_c}{\nu}$$

- is used to predict flow patterns in different fluid-flow situations. (Laminar / turbulent)
- (Laminar flow < 2100 & turbulent flow for Newton fluids)

Natural convection

Example: hot egg exposed to cooler air (disregarding heat transfer by radiation!)

- * the temperature of the outer surface of the egg shell drops somewhat and the temperature of the air adjacent to the shell rises as a result of heat conduction from the shell to the air.
- * heat is transferred from this warmer layer to the outer layers of the air. (egg is surrounded by a thin layer of warmer air.)
- * the temperature of the layer adjacent to the egg is higher and thus its density is lower [at constant pressure the density of a gas is inversely proportional to its temperature].
[at constant pressure the density of a gas is inversely proportional to its temperature]
- * low-density gas is surrounded by a high-density gas, so the lighter gas rises.
- * the space vacated by the warmer air is replaced by the cooler air nearby.
- * the space vacated by the warmer air is replaced by the cooler air nearby.
- * the motion that results from the continual replacement of the heated air by the cooler air nearby is called a natural convection current.

Grashof Number

- represents the natural convection effects.

$$Gr = \frac{L^3 \cdot g \cdot \beta (T_s - T_\infty)}{\nu^2}$$

g - gravitational acceleration $\left[\frac{m}{s^2}\right]$

β - coefficient of volume expansion $\left[\frac{1}{K}\right]$

T_s - temperature of the surface $[K]$

T_∞ - temperature of the fluid sufficiently far from the surface $[K]$

L - characteristic length

ν - kinematic viscosity of the fluid

Laminar/turbulent

* governs the flow regime in natural convection (like Reynolds in forced convection)

* represents the ratio of the buoyancy force to the viscous force acting on the fluid.

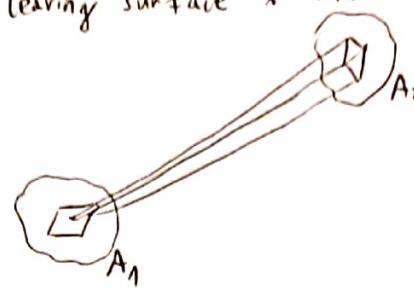
Rayleigh Number

* below a certain critical value, there is no fluid motion and heat transfer is by conduction rather than convection

$$Ra = Gr \cdot Pr$$

The view factor \ configuration factor

F_{ij} = the fraction of the radiation leaving surface i that strikes j directly



Reciprocity relation $A_1 F_{12} = A_2 F_{21}$

(allow calculation of a view factor from)
(a knowledge of the other)

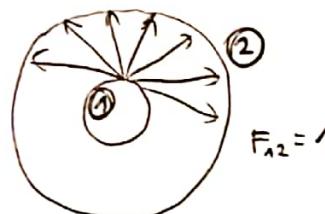
Special case

F_{ii} - strikes itself

$$\textcircled{2} \rightarrow F_{22} = 0, \quad \textcircled{3} \rightarrow F_{33} \neq 0$$

* When $F_{ij} = 0$ surfaces don't have a direct view of each other
(radiation from i cannot strike j)
* values 0 - 1

* $F_{ij} = 1$ surface j completely surrounds surface i .
(so the entire radiation leaving surface i is intercepted by surface j)



* view factors depend only on geometry

* the radiation that strikes a surface does not need to be absorbed by that surface, also radiation that strikes a surface after being reflected by other surfaces is not considered in the evaluation of view factors.

* calculating view factors are usually very complex so we often use table values for selected geometries.

Convection - definition (one more)

Heat transfer by the movement of fluid between areas of different temperature.

Warm air is less dense than cold air, so convection currents can form in the presence of a temperature gradient

→ only by the temperature-derived density differences in the fluid → NATURAL
currents are produced
→ due to external factor such as pump or fan → FORCED.

Forced: examples.

Natural: Example 2

Sea Breeze: occurs during the day
* sun heats up both the sea surface and land, but the sea has a greater heating capacity, it absorbs most of the sun energy, but warms up much slower than the land



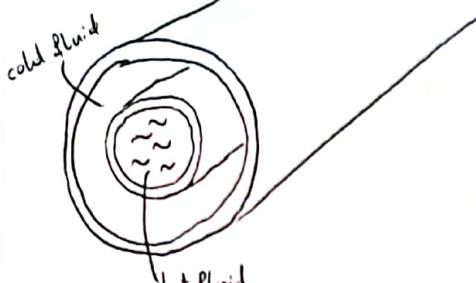
* the temperature above the land rises and heats up the air in the atmosphere above it, but it is less dense, so it expands creating a low-pressure area over the land near the coast

* meanwhile there is high-pressure over the sea, the difference in air pressure causes the air to flow from sea to land.

* the sudden gush of wind felt is known as sea breeze.



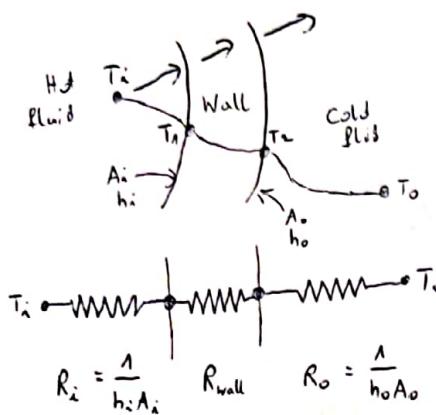
Thermal resistance network example (heat exchanger)



$$R_{\text{wall}} = \frac{\ln(D_o/D_i)}{2\pi L \cdot k} \quad (\text{resistance of cylinder})$$

, also

$$\dot{Q}_{\text{cond, wall}} = \frac{T_1 - T_2}{R_{\text{wall}}} , \text{ which is equal } \frac{T_i - T_o}{R_{\text{out}}}$$

(A_i - area of inner surface) (A_o - outer..)

$$A_i = \pi D_i L \quad , \quad A_o = \pi D_o L$$

In the analysis of heat exchangers it is convenient to combine all the thermal resistances into a single R.

then...

$$\dot{Q} = \frac{\Delta T}{R} = U A_s \Delta T = U_i A_i \Delta T = U_o A_o \Delta T ,$$

where

A_s - surface area

U - overall heat transfer coefficient

canceling ΔT...

$$\frac{1}{U A_s} = \frac{1}{U_i A_i} = \frac{1}{U_o A_o} = R = \frac{1}{h_i A_i} + R_{\text{wall}} + \frac{1}{h_o A_o}$$

Note that $U_i \neq U_o$, but $U_i A_i = U_o A_o$
Therefore the U is meaningless unless the area on which it is based is specified.

Therefore the U is meaningless unless the area on which it is based is specified.
When the wall thickness is small and h is high R_{wall} is negligible, then $A_i \approx A_o \approx A_s$, so

$$\frac{1}{U} \approx \frac{1}{h_i} + \frac{1}{h_o}$$

here, U is dominated by the smaller convection coefficient (inverse)

when one of them is 'much smaller' $U \approx h_{\text{smaller}}$

* fins are commonly used on the gas side (gas-liquid exchange) to enhance the product UA
and thus the heat transfer on that side [A is greater when finned]

$$(A_s = A_{\text{total}} = A_{\text{fin}} + A_{\text{unfinned}})$$

or $n_{\text{fin}} A_{\text{fin}} + A_{\text{unfinned}}$, where n_{fin} fin efficiency

Fouling Factor

 R_f - represents the net effect of deposits on heat transfer surfaces.

* for an unfinned double-pipe heat exchanger:

$$\frac{1}{U A_s} = \frac{1}{U_i A_i} = \frac{1}{U_o A_o} = R = \frac{1}{h_i A_i} + \frac{R_{f,i}}{A_i} + \frac{\ln(D_o/D_i)}{2\pi k L} + \frac{R_{f,o}}{A_o} + \frac{1}{h_o A_o}$$

The first law of thermodynamics requires that the rate of heat transfer from the hot fluid be equal to the rate of heat transfer to the cold one.

$$\dot{Q} = \underline{m_c C_p c} (T_{c,out} - T_{c,in}) , \quad \dot{Q} = \underline{m_h C_p h} (T_{h,in} - T_{h,out})$$

[c-cold] [h-hot]

m_c mass flow rate
 C_p specific heat
 C_c heat capacity rate

the rate of heat transfer needed to change the temperature of the fluid stream by 1°C . as it flows through a heat exchanger.

only when the heat capacity rates of the two fluids are equal to each other

* fluid with a large heat capacity rate \rightarrow small temperature change

so doubling the mass flow rate of a fluid will halve the temperature change

* two types of heat exchangers commonly used in practice are condensers and boilers.

- one of the fluid undergoes a phase-change process

the rate of heat transfer: $\dot{Q} = m \cdot h_{fg}$, m - rate of evaporation/condensation
 h_{fg} - enthalpy of vaporization

analogous manner to Newton:

$$\dot{Q} = UA_s \Delta T_m$$

The log mean temperature difference!

$$\Delta T_m = \frac{\Delta T_1 - \Delta T_2}{\ln(\Delta T_1 / \Delta T_2)}$$