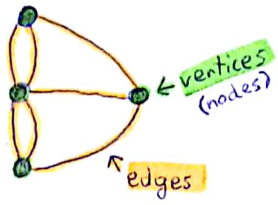
GRAPH THEORY

* Seven bridges of Konigsberg problem



Ok. Why it is impossible to go through every edge only one time?
(EULERIAN WALK)

- Because every vertex (that is not a start or end vertex), MUST HAVE even degree.

* so, the number of odd vertices should be exactly 0 or 2.

0 ↓ when start = end

2 ↓ when start ≠ end

Graph G - ordered pair $G = (V, E)$ where:

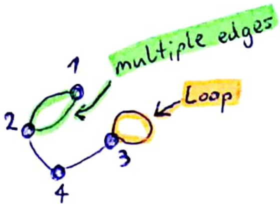
V - finite set of elements

E - set of pairs of V

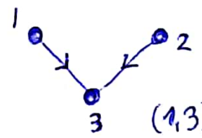
Example: $V = \{1, 2, 3\}$

$E = \{\{1,2\}, \{2,3\}, \{1,3\}\}$

* simple graph:
  - no loops
  - no multiple edges



* directed graph: each edge has a direction associated to it



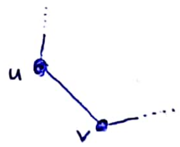
(1,3), (2,3)

Size & Order:

$|V|$ - order of the graph

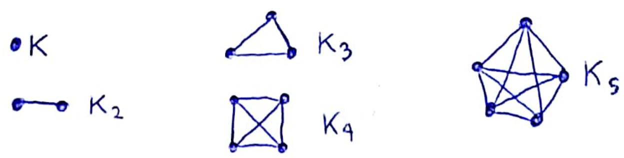
$|E|$ - size of the graph

* adjacent vertices: I can simplify the notation $\{u,v\} \in E$ to $uv = vu$, then



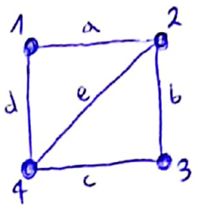
* if $e = uv$
  - u and v are ends of the edge e
  - u and v are adjacent (neighbours)
* if two edges have a common end, then they are adjacement

* complete graph - (K_n on n vertices)
  - simple graph
  - has an edge between every pair of vertices



*** You can represent any graph using a matrix**

Graph



Adjacency matrix

$$A = \begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & 0 & 1 & 0 & 1 \\ 2 & 1 & 0 & 1 & 1 \\ 3 & 0 & 1 & 0 & 1 \\ 4 & 1 & 1 & 1 & 0 \end{array}$$

rows & columns - vertices
row sum - degree of the vertex that the row represents

Incidence matrix

$$\begin{array}{c|ccccc} & a & b & c & d & e \\ \hline 1 & 1 & 0 & 0 & 1 & 0 \\ 2 & 1 & 1 & 0 & 0 & 1 \\ 3 & 0 & 1 & 1 & 0 & 0 \\ 4 & 0 & 0 & 1 & 1 & 1 \end{array}$$

rows - vertices
columns - edges

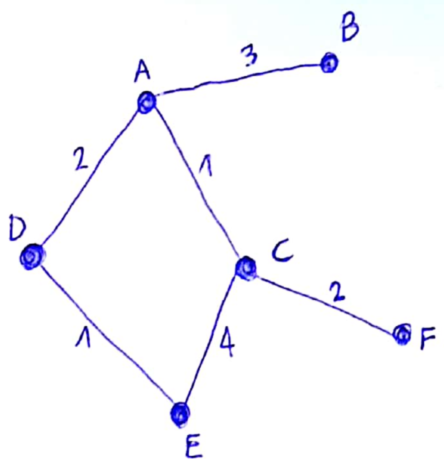
* **k-regular graph** - graph where every vertex has degree $= k$.

Dijkstra's Algorithm

1. Set $S = \emptyset$ - it will store the vertices of G for which a shortest path has been found.
2. Set: $t(u) = 0$
   $t(v) = \infty$ for all $v \in V(G)$, $v \neq u$
   Add: u to S
3. Let w be the newest member of S.
   For each $v \notin S$, $v \in N_G(w)$ set $t(v) = \min\{t(v), t(w) + \alpha(vw)\}$
4. Pick any $w \notin S$ with minimum $t(w)$ and add w to S.
   Repeat step 3.

u - source vertex
α - function that assigns a weight to every edge of the graph.

Connected edge weighted graph G



Task:

* find the minimum weight of a path from u to every other vertex in G.

Task: solution source vertex : A

1 $S = \emptyset$

2 $t(A) = 0$

$t(B) = t(C) = t(D) = t(E) = t(F) = \infty$

$S = \{A\}$ (A added to S)

3 For each vertex $v \notin S$ which is in neighbourhood of A (A is the newest member of S) write the condition, so:

$t(B) = \min\{\infty, 0+3\} = 3$ ($v = B$, $w = A$)

$t(C) = \min\{\infty, 0+1\} = 1$ ($v = C$, $w = A$)

$t(D) = \min\{\infty, 0+2\} = 2$ ($v = D$, $w = A$)

4 Pick a new w, the one with the smallest $t(w)$. In this case it is $t(C) = 1$.

Add C to S.

$S = \{A, C\}$

3 Repeat previous activities considering that you operate on C.

$t(E) = \min\{\infty, 1+4\} = 5$ ($v = E$, $w = C$)

$t(F) = \min\{\infty, 1+2\} = 3$ ($v = F$, $w = C$)

4 Find the smallest $t(w)$. Yes, it's $t(D)$.

Add D to S.

$S = \{A, C, D\}$

3 Update the only one left vertex.

$t(E) = \min\{5, 2+1\} = 3$ ($v = E$, $w = D$)

4 If there are few vertices with the same smallest value it doesn't matter which one you choose.

For instance: I will take B. $S = \{A, B, C, D\}$

3 It's nothing to update. So we are already done with B.

4 Add E to S. $S = \{A, C, D, B, E\}$. The same scenario as above - no change.

Task: results

These are the minimum weight paths from vertex A to each of other vertices on the graph:

A - 0 , C - 1 , D - 2 , B - 3 , E - 3 , F - 3 ;