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1.1. Heating of a Copper Ball

- heating from 100°C to 150°C
- time : 30 minutes
- density $\rho = 8950\ kg/m^3$
- specific heat $c_p = 0{,}395\ kJ/kg \cdot °C$
- ball diameter : 10cm ⟷ $D = 10cm$

Determine:

a) the total amount of heat transfer to the ball

b) the average rate of heat transfer to the ball

c) the average heat flux.

a) it is simply the change in its internal energy

energy transfer to system = energy increase of the system

$$Q = \Delta U = m \cdot c_{avg} (T_2 - T_1)$$

$\boxed{1}$ FINITE CHANGES $\Delta U = m \cdot c_{v,avg} \cdot \Delta T$
* for solids and liquids we can use just „c"

* density $\rho = \frac{m}{V}$, so $m = \rho V$ → $m = \rho \cdot \frac{\pi}{6} D^3$

* volume of ball : $V = \frac{\pi}{6} D^3$

$$Q = \rho \cdot \frac{\pi}{6} D^3 \cdot c \cdot 50°C = 92{,}6\ kJ$$

$\boxed{2}$ $Q = \int_0^{\Delta t} \dot{Q}\, dt$, but $\dot{Q} = const$ so $Q = \dot{Q} \Delta T$
so $\dot{Q} = \frac{Q}{\Delta T}$

b) $\dot{Q}_{avg} = \frac{Q}{\Delta t} = \frac{92{,}6\ kJ}{1800s} = 51{,}4\ W$

c) $\dot{q}_{avg} = \frac{\dot{Q}}{A}$, $A = \pi D^2$, so $\dot{q}_{avg} = \frac{51{,}4W}{(0{,}1)m^2 \cdot \pi} = 1636\ \frac{W}{m^2}$

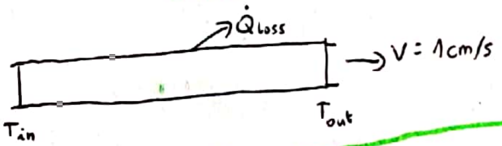
$\boxed{2}$ Note that heat flux may vary with location on a surface. This is average value
A = surface area, not volume !

1.2. Cooling a stainless steel sheets.

- convection at a constant speed 1 cm/s into a chamber to be cooled.
- sheet : 5mm thick × 2m wide
- temperature : 500K → 300 K

Determine:

a) the rate of heat loss from the stainless steel. sheet inside the chamber.



a) Assumptions:

1. STEADY OPERATING CONDITIONS EXISTS
2. THE STEEL SHEET HAS CONSTANT PROPORTIES
3. CHANGES IN POTENTIAL & KINETIC ENERGY ARE NEGLIGIBLE

$\boxed{3}$ no change with time at specified location

* the constant pressure specific heat of this steel
AVERAGE TEMPERATURE $\frac{500+300}{2} K = 400K$
so $c_p = 515\ \frac{J}{kg K}$

* density $\rho = 7900\ kg/m^3$

* rate of steel sheet entering and exiting the chamber

$$\dot{m} = \rho V A_c = \left(7900 \frac{kg}{m^3}\right)\left(0{,}01 \frac{m}{s}\right)(2m)(0{,}005m) = 0{,}79\ kg/s$$

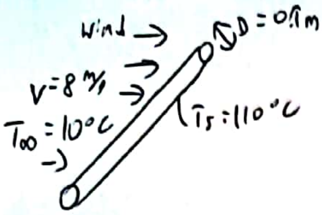
$\boxed{3}$ mass flow rate

$\boxed{1}$ $\Delta U = m \cdot c_{avg} \cdot \Delta T$ implies that: energy balance for steady-flow system: $\dot{Q} = \dot{m} \cdot \Delta h = \dot{m} \cdot c_p \cdot \Delta T$ ($\boxed{3}$)

* the rate of heat loss

$$\dot{Q}_{loss} = \dot{m} \cdot c_p \cdot (T_{in} - T_{out}) = \left(0{,}79 \frac{kg}{s}\right)\left(515 \frac{J}{kg K}\right)(500-300\ K) = 81370\ J/s = 81{,}4\ kW$$

7.5 Heat Loss from a Steam Pipe



I Film temperature $T_f = \frac{(T_s + T_\infty)}{2} = \frac{110 + 10}{2} = 60\,°C$, 1 atm :

II Properties for Tf: $k = 0.02802\ \frac{W}{mK}$, $Pr = 0.7202$, $\nu = 1.896 \cdot 10^{-5}\ \frac{m^2}{s}$

III Reynolds

$$Re = \frac{V \cdot D}{\nu} = \frac{(8\frac{m}{s})(0.1m)}{1.896 \cdot 10^{-5}} = 4.219 \cdot 10^4$$

IV Nusselt $\to h$

$$Nu = \frac{hD}{k} = 0.3 + \ldots\ldots\ldots = 124$$

~ simpler and less accurate $Nu = 0.027\ Re^{0.805}\ Pr^{\frac{1}{3}}$

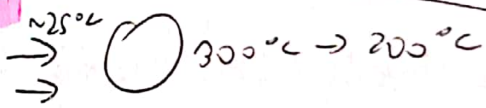
$$h = \frac{k}{D} Nu = 34.8\ \frac{W}{m^2K}$$

V Newton

$$A_s = \pi D L = \pi (0.1m)(1m) = 0.314\ m^2$$

$$\dot{Q} = h \cdot A_s (T_s - T_\infty) = 1093\ W$$

7.6. Cooling of a Steel Ball by Forced Air



≈ T_f → props. → Reynolds → Nusselt → Newton

$\dot{Q}_{avg} = h \cdot A_s (T_{s,avg} - T_\infty) = 610\ W$ – average rate of heat transfer

I Total heat transferred from the ball (change of its energy as it cools from 300°C to 200°C)

$$m = \rho V = \rho \cdot \frac{1}{6}\pi D^3 = 65.9\ kg$$

$$Q_{total} = m \cdot c_p (T_2 - T_1) = 3\,163\,000\ J$$

$Q = \int_0^{\Delta t} \dot{Q}\, dt$, $\dot{Q} = const \to Q = \dot{Q} \int_0^{\Delta t} dt \to Q = \dot{Q} \Delta t \to \Delta t = \frac{Q}{\dot{Q}} = \frac{3\,163\,000\ J}{610\ J/s} = 1h26$

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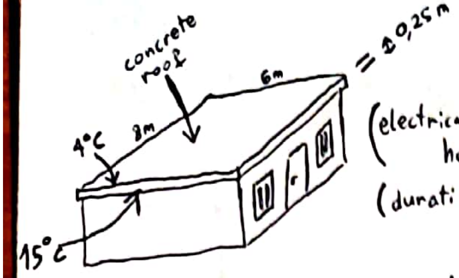
1.5. The cost of heat loss through a roof



(electrically heated home)

(duration: 10h)

- concrete thermal conductivity $k = 0{,}8 \frac{W}{mK}$

Determine:

a) the rate of heat loss through the roof that night

b) the cost of that heat loss to the home owner if the cost of electricity is \$0,08 / kWh.

Assumptions

1: STEADY OPERATING CONDITIONS (constant surface temperatures during night)

1: CONSTANT ROOF PROPERTIES

$\boxed{4}$ Fourier's Law

a) $\dot{Q} = kA \frac{T_1 - T_2}{L} = 0{,}8K \cdot 48m^2 \frac{(15-4)°C}{0{,}25m} = 1690W = 1{,}69kW$

b) The amount of heat lost through the roof during a 10-hour period:

$\boxed{2}$ $Q = \int_0^{\Delta t} \dot{Q}\, dt \rightarrow Q = \dot{Q}\Delta t$

$Q = \dot{Q}\Delta t = 1{,}69kW \cdot 10h = 16{,}9 kWh$

Cost = $16{,}9 kWh \cdot \$0{,}08/kWh = \$1{,}35$ ▪

1.11 Heat Transfer Between Two Isothermal Plates

$T_2 = 200K$

$L = 1cm$ $\quad \varepsilon = 1$, no convection!

$T_1 = 300K$

1. Average temperature $= \frac{300+200 [K]}{2} = 250K$

2. k from Table

a) air $k = 0{,}0219 \frac{W}{mK}$, b) vacuum, c) urethane insulation $k = 0{,}026 \frac{W}{mK}$, d) superinsulation $k = 0{,}00002 \frac{W}{mK}$

$$\begin{cases} \dot{Q}_{cond} = kA \frac{T_1 - T_2}{L} = 219W \\ Q_{rad} = \varepsilon \sigma A (T_1^4 - T_2^4) = 369W \end{cases}$$

$Q_{total} = 219W + 369W = 588W$

only radiation!

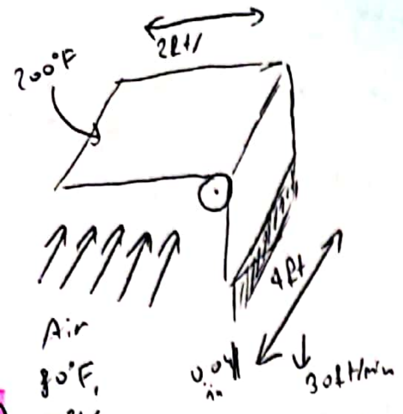
$Q_{rad} = \ldots$

. . . . .

. . . . .

7.3 Cooling of plastic sheets by Forced Air

FORCED



Determine:

a) the rate of heat transfer from the plastic sheet to air by forced convection and radiation

b) the temperature of plastic sheet at the end of the cooling section

$\rho = 75\ lbm/ft^3$, $c_p = 0.4\ Btu/lbm°F$, $\varepsilon = 0.9$

air - ideal gas

a)

I Film temperature and properties $T_f = \frac{T_s + T_\infty}{2} = \frac{200+80}{2} = 140°F$, 1 atm pressure

1 atm, 140°F
$k = 0.01623\ Btu/h \cdot ft\,F$
$Pr = 0.7202$
$\nu = 0.204 \cdot 10^{-3}\ ft^2/s$

II Reynolds at the end of the air flow across the plastic

$$Re_L = \frac{VL}{\nu} = \frac{(10\,ft/s)(4\,ft)}{0.204 \cdot 10^{-3}\,ft^2/s} = 1.961 \cdot 10^5 < 5 \cdot 10^5 \Rightarrow \text{Laminar flow.}$$

III Nusselt for laminar flat plate flow → calculate h

$$Nu = \frac{hL}{k} = 0.664\, Re_L^{0.5} \cdot Pr^{1/3} = 263.6$$

then

$$h = \frac{k}{L} Nu = 1.07\ Btu/h \cdot ft^2 °F$$

$$A_s = (2\,sides)(4\,ft)(2\,ft) = 16\,ft^2$$

IV $\dot{Q}_{conv}$ & $\dot{Q}_{rad}$ for the whole plate

$$\dot{Q}_{conv} = h \cdot A_s (T_s - T_\infty) = (1.07)(16\,ft^2)(200-80°F) = 2054\ Btu/h$$

$$\dot{Q}_{rad} = \varepsilon \cdot \sigma \cdot A_s (T_s^4 - T_\infty^4) = 2585\ Btu/h$$

$$\dot{Q}_{total} = \dot{Q}_{conv} + \dot{Q}_{rad} = 2054 + 2585\ (Btu/h) = \underline{4639\ Btu/h}$$

$\dot{Q}_{total}$ FROM THE PLASTIC SHEET TO AIR

b) I mass of the plastic rolling out per unit time (mass flow rate)

$$\dot{m} = \rho \cdot A_c \cdot V_{plastic} = \left(75\left[\frac{lbm}{ft^3}\right]\right)\left(\frac{4 \cdot 0.04}{12}[ft^2]\right)\left(\frac{30}{60}[ft/s]\right) = 0.5\ lbm/s$$

II Energy balance on the cooled section

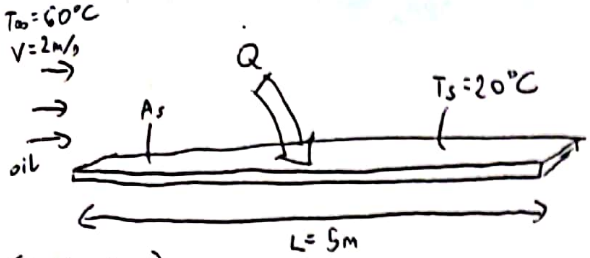
$\boxed{\dot{Q} \text{ is negative! HEAT LOSS}}$

$$\dot{Q} = \dot{m}\, c_p (T_2 - T_1) \rightarrow T_2 = T_1 + \frac{\dot{Q}}{c_p \cdot \dot{m}}$$

$$T_2 = 200°F + \frac{-4639}{0.5 \cdot 0.4} \cdot \left(\frac{1h}{3600s}\right) = 193.6°F.$$

FORCED

7.1 Flow of Hot Oil over a Flat Plate



Determine:

a) The total drag force ?

b) The rate of heat transfer per unit width of the entire plate ?

(Critical Reynolds) = flow becomes turbulent → for flow over a (flat) plate $Re = \frac{V x_{cr}}{\nu} = 5 \cdot 10^5$

1. Properties of oil at film temperature $\left(\frac{60° + 20°}{2}\right) = 40°C$:

$\rho = 876 \, kg/m^3$, $Pr = 2962$, $k = 0{,}1444 \, \frac{W}{mK}$, $\nu = 2{,}485 \cdot 10^{-4} \, \frac{m^2}{s}$

2. Reynolds:

$$Re_L = \frac{VL}{\nu} = \frac{(2 m/s)(5 m)}{2{,}485 \cdot 10^{-4} \frac{m^2}{s}} = 4{,}024 \cdot 10^4$$, so $< \underbrace{Re_{critical}}$ ⇒ LAMINAR FLOW

(a)

so average friction coefficient:

$$C_f = 1{,}33 \; Re_L^{-0{,}5} = 0{,}00663 \quad \text{and} \quad F_D = C_f \, A \, \frac{\rho V^2}{2} = 58.1 N$$

(b)

The Nusselt Number - laminar flow for a flat plate

$$Nu = \frac{hL}{k} = 0{,}664 \; Re_L^{0.5} \; Pr^{\frac{1}{3}} = 1913 ,$$

then:

$$h = \frac{Nu \cdot k}{L} = 55{,}25 \, \frac{W}{m^2 K}$$

$$\dot{Q} = h \, A_s (T_\infty - T_s) = 11{,}050 \, W$$

7.2. Prevention of Fire Hazard in the Event of Oil Leakage

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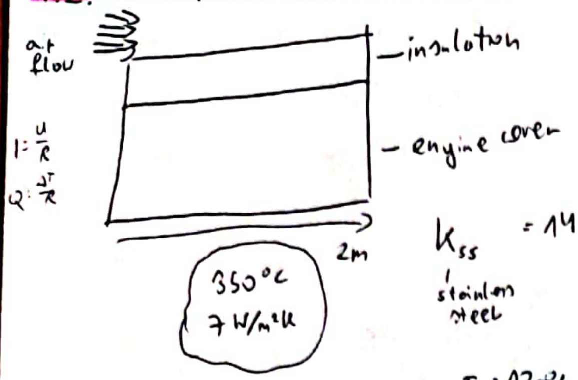

$I = \frac{U}{R}$

$Q = \frac{\Delta T}{R}$

Test: increase the cooling air velocity by 10%.

$k_{ss} = 14 \frac{W}{mK}$ (stainless steel), $k_{ins} = 0.5 \frac{W}{mK}$ (insulation)

$V = 7 \frac{m}{s}$

1. The properties of air: at $T = 120°C \Rightarrow k = 0.03235 \frac{W}{mK}$, $\nu = 2.522 \cdot 10^{-5} \frac{m^2}{s}$, $Pr = 0.7073$

2. Reynolds for 110% cooling air velocity

$$Re_L = \frac{VL}{\nu} = \frac{(7.7\,m/s) \cdot 2m}{2.522 \cdot 10^{-5}} = 610{,}626 > 5 \cdot 10^5, \text{ but } < 10^7,$$

so

$$Nu = \frac{hL}{k} = \left(0{,}037\, Re_L^{0.8} - 871\right) \cdot Pr^{1/3} = 625{,}77.$$

then

$$\boxed{h = h_o = \frac{Nu \cdot k}{L} = \frac{625{,}77 \cdot 0.03235}{2} = 10{,}122 \frac{W}{m^2K}.}$$

3. Thermal resistances

$R_{conv,o}$	$= \frac{1}{h_o A}$
R_{ins}	$= \frac{L_{ins}}{k_{ins} \cdot A}$
R_{ss}	$= \frac{L_{ss}}{k_{ss} \cdot A}$
$R_{conv,i}$	$= \frac{1}{h_i A}$

$$A R_{total} = A\left(R_{conv,i} + R_{ss} + R_{ins} + R_{conv,o}\right) = A\left(\frac{1}{h_i} + \frac{L_{SS}}{k_{SS}} + \frac{L_{ins}}{k_{ins}} + \frac{1}{h_o}\right)$$

$$AR_{total} = \frac{1}{7\,W/m^2K} + \frac{0.01\,m}{14\,W/mK} + \frac{0.005\,m}{0.5\,W/mK} + \frac{1}{10{,}122 \frac{W}{m^2K}} = \boxed{0.25237\, m^2K/W}$$

$$AR_{conv,o} = \frac{1}{h_o} = \frac{1}{10.122\,W/m^2K} = 0.09879 \frac{m^2K}{W}$$

4. Heat flux through the layers.

$$\dot{q} = \frac{\dot{Q}}{A} = \left(\cdot \frac{R_{total}}{R_{total}}\right) \cdot \frac{\dot{Q}}{A} = \boxed{\frac{T_{\infty,i} - T_{\infty,o}}{A \cdot R_{total}} = \frac{T_{s,o} - T_{\infty,o}}{A \cdot R_{conv,o}}} \rightarrow T_{s,o} = \frac{R_{conv,o}}{R_{total}}\left(T_{\infty,i} - T_{\infty,o}\right) + T_{\infty,o}$$

$\boxed{Q = \frac{\Delta T}{R}}$

$$\boxed{T_{s,o} = \frac{0.09879}{0.25237}(350-60)°C + 60°C = 173.5\,°C} \quad < 180°C$$

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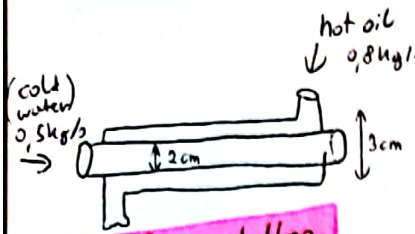
11.1 Overall Heat Transfer Coefficient of a Heat Exchanger



- negligible inner tube thickness

$T_{water} = 45°C$

$T_{oil} = 80°C$

I Read from tables:

Properties for water at 45°C & Properties of oil at 80°C

ρ, Pr, k, ν $\qquad$ ρ, Pr, k, ν

II U can be determined from: $\left[\frac{1}{U} \approx \frac{1}{h_i} + \frac{1}{h_o} \right]$

III Find h_i and h_o

$Re = \frac{VD}{\nu}$, and you don't know V? Don't worry, use $\dot{m}$ $\left(\dot{m} = \rho A_c V\right) \to V = \frac{\dot{m}}{\rho A_c}$

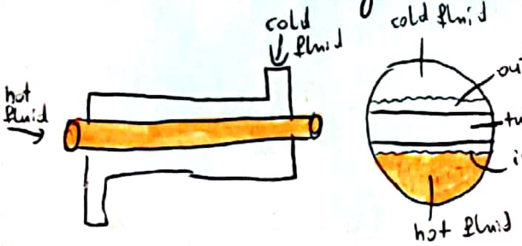
$\to$ Nusselt $\to h$ ■

IV Paste h_i & h_o to II

$$U = \frac{1}{\frac{1}{h_i} + \frac{1}{h_o}} = \frac{1}{\frac{1}{7663 \frac{W}{m^2K}} + \frac{1}{75{,}2 \frac{W}{m^2K}}} = 74{,}5 \frac{W}{m^2K}$$

very close to smaller h

11.2. Effect of Fouling on the Overall Heat Transfer Coefficient



inner tube:

$D_i = 1{,}5\,cm$ inner } diameter, $h_i = 800\ W/m^2K$

$D_o = 1{,}9\,cm$ outer } diameter, $h_o = 1200\ W/m^2K$

outer shell $\to$ inner diameter 3,2cm

Fouling factor $R_{f,i} = 0{,}0004\ m^2K/W$ on the tube side

$R_{f,o} = 0{,}0001\ m^2K/W$ on the shell side

Find a) the thermal resistance of the heat exchanger per unit length

b) the overall heat transfer coefficients U_i, U_o (based on inner, outer (surfaces) of the tube)

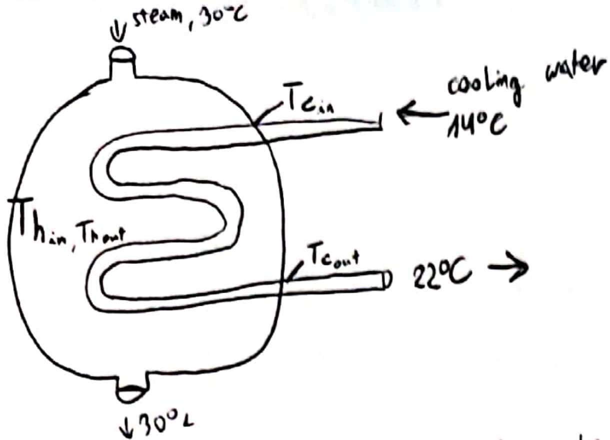
I Use this eq.

$$R = \frac{1}{UA_s} = \underbrace{\frac{1}{U_iA_i}} = \frac{1}{U_oA_o} = \frac{1}{h_iA_i} + \frac{R_{fi}}{A_i} + \frac{\ln(D_o/D_i)}{2\pi kL} + \frac{R_{fo}}{A_o} + \frac{1}{h_oA_o}$$, where $A_i = \pi D_i L$

to $R = 0{,}0532\ °C/W$

$$R = \frac{1}{U_iA_i} \to U_i = \frac{1}{R \cdot A_i} = 399 \frac{W}{m^2K}$$

11.3 The Condensation of Steam in a Condenser



Determine
- the mass flow rate of the cooling water needed
- the rate of condensation of the steam in the condenser

* can be treated as a counter-flow heat exchanger, since the temperature of one of the fluids remains constant (the steam)

The temperature difference between the steam and the cooling water at the two ends of the condenser is:

$$\Delta T_1 = T_{h,in} - T_{c,out} = 30 - 22 = 8 [°C]$$

$$\Delta T_2 = T_{h,out} - T_{c,in} = 30 - 14 = 16 [°C]$$

but the proper average temperature is... (not the arithmetic!), a little less.

$$\Delta T_{lm} = \frac{\Delta T_1 - \Delta T_2}{\ln(\Delta T_1/\Delta T_2)} = \frac{8-16}{\ln(8/16)} = 11.5°C$$

Then $\dot{Q} = U A_s \Delta T_{lm} = \left(2100 \frac{W}{m^2 K}\right)(45 m^2)(11,5°C) = 1087 kW$

* so, steam will lose heat at a rate of 1087 kW as it flows through the condenser, and the cooling water will gain practically all of it, since the condenser is well insulated

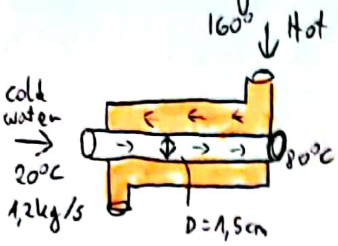
$$\dot{Q} = [\dot{m} c_p (T_{out} - T_{in})]_{\text{cooling water}} = [\dot{m} h_{fg}]_{\text{steam}}$$, so:

$$\dot{m}_{\text{cooling water}} = \frac{Q}{c_p (T_{out} - T_{in})} = \frac{1087 kJ/s}{(4,184 kJ/kgK)(22-14)°C} = 32,5 kg/s$$

$$\dot{m}_{steam} = \frac{\dot{Q}}{h_{fg}} = \frac{1087 kJ/s}{2431 kJ/kg} = 0,45 kg/s$$

so we have to circulate about 72 kg of cooling water for each kg of steam condensing to remove the heat released during the condensation process.

11.4 Heating Water in a Counter-Flow Heat Exchanger



Determine: the length of the heat exchanger required to achieve the desired heating

$c_{p\,water} = 4{,}18 \frac{kJ}{kgK}$

$\dot{m}_{water} = 1{,}2 kg/s$ $\quad \dot{m}_{geothermal} = 2 kg/s$

I The rate of heat transfer we can determine from:

$$\dot{Q} = [\dot{m}\, c_p (T_{out} - T_{in})]_{water} = (1{,}2\, kg/s)(4{,}18 \tfrac{kJ}{kgK})(80-20)°C = 301\, kW$$

II The outlet temperature of geothermal water:

$$\dot{Q} = [\dot{m}\, c_p \cdot (T_{in} - T_{out})]_{geothermal} :$$

$$T_{out} = T_{in} - \frac{\dot{Q}}{\dot{m} c_p} = 125°C$$

III Temperatures

$$\Delta T_1 = T_{h\,in} - T_{c\,out} = 160 - 80 = 80°C$$

$$\Delta T_2 = T_{h\,out} - T_{c\,in} = 125 - 20 = 105°C$$

[hot IN − cold OUT]
[hot OUT − cold IN]

IV Logarithmic mean temperature

$$\Delta T_{lm} = \frac{\Delta T_1 - \Delta T_2}{\ln(\Delta T_1/\Delta T_2)} = \frac{80-105}{\ln(80/105)} = 91{,}9°C$$

V Surface Area of Heat Exchanger

from $\dot{Q} = U A_s \Delta T_{lm} \rightarrow A_s = \frac{\dot{Q}}{U \cdot \Delta T_{lm}} = \frac{301\,000}{(640 \frac{W}{m^2 K})(91{,}9°C)} = 5{,}12 m^2$

then: $A_s = \pi D L \rightarrow L = \frac{A_s}{\pi D} = \frac{5{,}12[m^2]}{\pi \cdot 0{,}015[m]} = 109\ m.$ ∎