

0. PHYSICAL CONSTANTS

$k = 1.381 \times 10^{-23} \text{ J/K} = 8.617 \times 10^{-5} \text{ eV/K}$, $N_A = 6.022 \times 10^{23}$, $R = 8.315 \text{ J/mol} \cdot \text{K}$, $h = 6.626 \times 10^{-34} \text{ J} \cdot \text{s}$ $= 4.136 \times 10^{-15} \text{ eV} \cdot \text{s}$

1. ENERGY IN THERMAL PHYSICS

1.1. Thermal equilibrium. *Temperature* is a measure of the tendency of an object to spontaneously give up energy to its surroundings. When two objects are in thermal contact, the one that tends to spontaneously *lose* energy is at the *higher* energy. Room temperature 300K

1.2. The ideal gas. $PV = nRT = Nk_B T$. n is no of moles, $N = nN_A$ is number of molecules. $k_B = R/N_A$. Latter equation is valid when avg. space b/w molecules is larger than size of molecules. $\bar{E}_{K,trans} = \frac{3}{2}kT$.

1.3. Equipartition of energy. Theorem: at temperature T , the average energy of any quadratic degree of freedom is $\frac{1}{2}kT$. $U_{thermal} = Nf\frac{1}{2}kT$. Monoatomic gas: $f = 3$. Diatomic gas: $f = 5, 6$ (3 trans., 2-3, rot.) or $f = 8$ (3 trans., 3 rot., 2 vibr. K, P). Solid: $f = 6$ (6 vibr. 3K, 3P). Some vibrational energies may be "frozen out" at room temperature.

1.4. Heat and work. First law of thermodynamics $\Delta U = Q + W$. The change in energy is equal to the heat added and the work done. Heat transfer happens by *conduction*, *convection* and *radiation*.

1.5. Compression work. Consider a piston. The force is $F = PA$. Assumes that the pressure is uniform. Compression must be slow enough so the gas has time to continually equilibrate to the changing conditions \rightarrow *quasistatic*. A compressed gas, i.e. negative ΔV gives $W = F\Delta x = P\Delta x = -P\Delta V$.

1.5.1. Compression of ideal gas. Two idealised ways: *Isothermal* compression is so slow that the temperature of the gas does not rise (quasistatic). *Adiabatic* compression is so fast that no heat escapes during the compression. $VT^{f/2} = \text{constant}$, $V^\gamma P = \text{constant}$. $\gamma = \frac{f+2}{f}$ is the adiabatic exponent.

1.6. Heat capacities. Amount of heat needed to raise an object's temperature, per degree temperature increase: $C = \frac{Q}{\Delta T} = \frac{\Delta U - W}{\Delta T}$. $W = 0$ and $V = \text{constant}$ is called heat capacity at *constant volume*, else there would be compression work, $-P\Delta V$. $C_V = \left(\frac{\partial U}{\partial T}\right)_V$. If an object expand when heated and do work on surroundings, there is negative W . At constant P , Q is unambiguous \rightarrow heat capacity at *constant pressure*: $C_P = \left(\frac{\Delta U - (-P\Delta V)}{\Delta T}\right)_P = \left(\frac{\partial U}{\partial T}\right)_P + P\frac{\partial V}{\partial T}_P$.

1.6.1. Latent heat. During a phase transformation $C = \frac{Q}{\Delta T} = \frac{Q}{0} = \infty$. While $L = \frac{Q}{m}$ is the heat required to accomplish the transformation, the *latent heat*.

1.6.2. Enthalpy. To create a rabbit out of nothing, the sorcerer must summon up not only the energy U of the rabbit, but also some additional energy, equal to PV , to push the atmosphere out of the way to make room. The *enthalpy*: $H = U + PV$. At constant P , $\Delta H = Q + W_{other}$.

2. THE SECOND LAW

Entropy, and multiplicity, tends to increase.

2.1. Two-state systems. Multiplicity is given by the binomial coefficient. $\Omega(N, n) = \frac{N!}{n!(N-n)!} = \binom{N}{n}$. How many ways to pick n objects out of N . Permutations: $nP_k = n(n-1)(n-2)\dots(n-k) = \frac{n!}{(n-k)!}$. Unordered permutations: $nC_k = nP_k/k! = \binom{n}{k}$.

2.2. Einstein model of a solid. One energy unit is $h\nu = \hbar\omega$. Multiplicity of Einstein solid with N oscillators ($N/3$ atoms) and q energy units: $\Omega(N, q) = \binom{q+N-1}{q}$.

2.3. Interacting systems. Two solids are *weakly coupled* when flow of energy between them is much slower than flow of energy between atoms within each solid. Macrostate is the combined system, specified by temporarily constrained values U_A, U_B . Over time they will change, with the sum $U_{tot} = U_A + U_B$ remaining fixed. All parameters in such a system is $N_A, N_B, q_{tot} = q_A + q_B, \Omega_{tot} = \Omega_A \Omega_B$. Fundamental assumption of statistical mechanics: In an isolated system, all accessible microstates are equally probable.

2.4. Large systems. If $|x| \ll 1$, a Taylor expansion gives $\ln(x+1) \approx x$. If $N \gg 1$ one can apply *Stirling's approximation*: $N! \approx N^N e^{-N} \sqrt{2\pi N}$. If N is a large number, and $N!$ is very large, the square root factor can be omitted. This is usually good enough: $\ln N! = N \ln N - N$. In a large Einstein solid $q \gg N$ is the high temperature limit: $\Omega \approx \frac{(q+N)!}{q!N!}$, $\ln \Omega \approx (q+N) \ln(q+N) - q \ln q - N \ln N$, where $\ln(q+N) \approx \ln q + \frac{N}{q}$. S.T.: $\ln \Omega \approx N \ln \frac{q}{N} + N + \frac{N^2}{q}$.

2.5. The ideal gas. $\Omega(U, V, N) = f(N) V^N U^{3N/2}$, where $f(N)$ is a complicated function of N . $\Omega_N = \frac{1}{N!} \frac{V^N}{h^{3N}} \times$ (area of momentum hypersphere). "area" = $\frac{2\pi^{d/2}}{(d-1)} r^{d-1}$, where in general $d = 3N$ and $r = \sqrt{2mU}$.

2.6. Entropy. $S = k \ln \Omega$. Now you see why the logarithm of the multiplicity is nice to have. Entropy of an ideal gas: $S = Nk \left[\ln \left(\frac{V}{N} \left(\frac{4\pi m U}{3N h^2} \right)^{3/2} \right) + \frac{5}{2} \right]$ (The Sackur-Tetrode equation). Depends on V, E, N . Increasing any of them increases S .

2.6.1. *Mixing.* One gas into another chamber: $\Delta S_A = Nk \ln \frac{V_f}{V_i} = Nk \ln 2$. Two gases mixing, by removing a partition: $\Delta S_{tot} = \Delta S_A + \Delta S_B = 2Nk \ln 2$. Must be distinguishable gases. Gibbs Paradox.

2.6.2. *Irreversible.* Processes that create new entropy are said to be irreversible. A sudden expansion is irreversible. A reversible volume change must in fact be quasistatic S.T. $W = -P\Delta V$.

3. INTERACTIONS AND IMPLICATIONS

3.1. **Temperature.** Two Einstein solids: $\frac{\partial S_A}{\partial U_A} = \frac{\partial S_B}{\partial U_B}$ at equilibrium with N_A, N_B fixed. Temperature is defined as $\frac{1}{T} = \left(\frac{\partial S}{\partial U}\right)_{N,V}$. If the slope is large the temperature must be small and vice versa.

3.2. **Entropy and heat.** Algorithm to predict the heat capacity of a system: 1. Express the multiplicity Ω as a function of N, V and N . 2. Take the logarithm to find the entropy: $S = k \ln \Omega$. 3. Differentiate with respect to U and take the reciprocal to find the temperature, T as a function of U and other var's. 4. Solve for U as a function of T (and others). 5. Differentiate $U(T)$ to obtain a prediction of heat capacity (others held fixed). $C_V = \left(\frac{\partial U}{\partial T}\right)_{N,V}$. At very low temperatures all degrees of freedom must "freeze out", meaning $C_V \rightarrow 0$ as $T \rightarrow 0$. This is the *third law*.

3.3. **Paramagnetism.** A system consists of N spin- $\frac{1}{2}$ particles, often referred to as dipoles, immersed in a constant magnetic field \mathbf{B} . Total energy is $U = \mu B(N_\downarrow + N_\uparrow) = \mu B(N - 2N_\uparrow)$. Magnetisation is $M = \mu(N_\downarrow - N_\uparrow) = -\frac{U}{B}$. Multiplicity is $\Omega(N_\uparrow) = \binom{N}{N_\uparrow} = \frac{N!}{N_\uparrow!N_\downarrow!}$.

3.4. **Mechanical equilibrium and pressure.** Pressure is $P = \left(\frac{\partial S}{\partial V}\right)_{U,N}$. Another proof of ideal gas law: $\Omega = f(N)V^N U^{3N/2}$, $S = Nk \ln V + \frac{3}{2}Nk \ln U + k \ln f(N)$, $P = T \frac{\partial}{\partial V}(Nk \ln V) = \frac{NkT}{V} \rightarrow PV = NkT$.

3.4.1. *Thermodynamic identity.* Two steps: ΔU and ΔV . Sum of entropy change: $\Delta S = (\Delta S)_1 + (\Delta S)_2$. S.T. $dS = \left(\frac{\partial S}{\partial U}\right)_V \Delta U + \left(\frac{\partial S}{\partial V}\right)_U \Delta V$, then for a small change $dS = \left(\frac{\partial S}{\partial U}\right)_V dU + \left(\frac{\partial S}{\partial V}\right)_U dV$. Inserting definitions of temperature and pressure yields $dS = \frac{1}{T}dU + \frac{P}{T}dV$ which can be rearranged to $dU = TdS - PdV$.

3.4.2. *Entropy and heat again.* $dU = Tds - PdV = Q + W$ if any volume change is quasistatic. If $Q = 0$ the process is also adiabatic \rightarrow isentropic. We have $(\Delta S)_V = \int_{T_i}^{T_f} \frac{C_V}{T} dT$ and $(\Delta S)_P = \int_{T_i}^{T_f} \frac{C_P}{T} dT$

3.5. **Diffusive equilibrium and chemical potential.** The chemical potential is $\mu = -T \left(\frac{\partial S}{\partial N}\right)_{U,V} = \left(\frac{\Delta U}{\Delta N}\right)_S = -\epsilon$, where ϵ is the size of a unit of energy. The generalised thermodynamic identity becomes $dU = TdS - PdV + \sum \mu_i dN_i$

4. ENGINES AND REFRIGERATORS

4.1. **Heat engine.** A device that absorbs heat and converts part of that energy into work. The work produced is the difference between the heat absorbed and the waste expelled: $W = Q_h - Q_c$, where Q_h is heat absorbed from the hot reservoir and Q_c is heat expelled to cold reservoir. Efficiency: $e \equiv \frac{W}{Q_h} = \frac{Q_h - Q_c}{Q_h} = 1 - \frac{Q_c}{Q_h}$. First law: energy is conserved. Second law: entropy is a fluid that can be created but never destroyed, ergo $\frac{Q_c}{T_c} \geq \frac{Q_h}{T_h}$ and $\frac{Q_c}{Q_h} \geq \frac{T_c}{T_h}$. One concludes $e \leq 1 - \frac{T_c}{T_h}$.

4.1.1. *The Carnot cycle (max efficiency).* Four steps: 1. Isothermal expansion at T_h while absorbing heat. 2. adiabatic expansion to T_c . 3. Isothermal compression at T_c while expelling heat. 4. Adiabatic compression back to T_h . The system must be but in contact with the hot reservoir during step 1 and with the cold reservoir during step 3.

4.2. **Refrigerators.** A heat engine in reverse. Efficiency now called coefficient of performance $\text{COP} = \frac{Q_c}{W} = \frac{Q_c}{Q_h - Q_c} = \frac{1}{Q_h/Q_c - 1}$. Some inequalities: $\frac{Q_h}{T_h} \geq \frac{Q_c}{T_c}$ which gives $\frac{Q_h}{Q_c} \geq \frac{T_h}{T_c}$. One can conclude $\text{COP} \leq \frac{1}{T_h/T_c - 1} = \frac{T_c}{T_h - T_c}$.