Problem Set II

FYS3110

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Problem 2.1

An operator $\hat{H}$ is represented in a particular orthonormal basis as the matrix

$$\hat{H} \simeq \begin{pmatrix} 1 & \frac{i}{2} & 0 \\ -\frac{i}{2} & 1 & 0 \\ 0 & 0 & \frac{1}{2} \end{pmatrix} \tag{1}$$

a)

$\hat{H}$ is Hermitian if it is equal to its own transpose conjugate, $\hat{H} = \hat{H}^\dagger$.

$$\hat{H}^\dagger = (\hat{H}^*)^T = \begin{pmatrix} 1 & -\frac{i}{2} & 0 \\ \frac{i}{2} & 1 & 0 \\ 0 & 0 & \frac{1}{2} \end{pmatrix}^T = \begin{pmatrix} 1 & \frac{i}{2} & 0 \\ -\frac{i}{2} & 1 & 0 \\ 0 & 0 & \frac{1}{2} \end{pmatrix} = \hat{H} \tag{2}$$

b)

Three ket vectors are given

$$|1\rangle \simeq \frac{1}{\sqrt{2}} \begin{pmatrix} i \\ 1 \\ 0 \end{pmatrix} \tag{3}$$

$$|2\rangle \simeq \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \tag{4}$$

$$|3\rangle \simeq \frac{1}{\sqrt{3}} \begin{pmatrix} -i \\ 1 \\ -1 \end{pmatrix} \tag{5}$$

$|1\rangle$ is and eigenvector of $\hat{H}$ with eigenvalue $\frac{3}{2}$:

$$\hat{H}\,|1\rangle = \begin{pmatrix} 1 & \frac{i}{2} & 0 \\ -\frac{i}{2} & 1 & 0 \\ 0 & 0 & \frac{1}{2} \end{pmatrix} \frac{1}{\sqrt{2}} \begin{pmatrix} i \\ 1 \\ 0 \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{3i}{2} \\ \frac{3}{2} \\ 0 \end{pmatrix} = \frac{3}{2}\,|1\rangle \tag{6}$$

$|2\rangle$ is and eigenvector of $\hat{H}$ with eigenvalue $\frac{1}{2}$:

$$\hat{H}|2\rangle = \begin{pmatrix} 1 & \frac{i}{2} & 0 \\ -\frac{i}{2} & 1 & 0 \\ 0 & 0 & \frac{1}{2} \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ \frac{1}{2} \end{pmatrix} = \frac{1}{2}|2\rangle \tag{7}$$

$|3\rangle$ is and eigenvector of $\hat{H}$ with eigenvalue $\frac{1}{2}$:

$$\hat{H}|3\rangle = \begin{pmatrix} 1 & \frac{i}{2} & 0 \\ -\frac{i}{2} & 1 & 0 \\ 0 & 0 & \frac{1}{2} \end{pmatrix} \frac{1}{\sqrt{3}} \begin{pmatrix} -i \\ 1 \\ -1 \end{pmatrix} = \frac{1}{\sqrt{3}} \begin{pmatrix} -\frac{i}{2} \\ \frac{1}{2} \\ -\frac{1}{2} \end{pmatrix} = \frac{1}{2}|3\rangle \tag{8}$$

c)

Computing the matrix elements of the linear operator

$$\langle 1|\hat{H}|1\rangle = \frac{3}{2}\langle 1|1\rangle = \frac{3}{2}\frac{1}{2}\begin{pmatrix} i & 1 & 0 \end{pmatrix}\begin{pmatrix} i \\ 1 \\ 0 \end{pmatrix} = \frac{3}{2}\frac{1}{2}2 = \frac{3}{2} \tag{9}$$

$$\langle 1|\hat{H}|2\rangle = \frac{1}{2}\langle 1|2\rangle = \frac{1}{2}\frac{1}{\sqrt{2}}\begin{pmatrix} -i & 1 & 0 \end{pmatrix}\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = 0 \tag{10}$$

$$\langle 1|\hat{H}|3\rangle = \frac{1}{2}\langle 1|3\rangle = \frac{1}{2}\frac{1}{\sqrt{2}}\frac{1}{\sqrt{3}}\begin{pmatrix} i & 1 & 0 \end{pmatrix}\begin{pmatrix} -i \\ 1 \\ -1 \end{pmatrix} = \frac{1}{2}\frac{1}{\sqrt{2}}\frac{1}{\sqrt{3}}2 = \frac{1}{\sqrt{6}} \tag{11}$$

$$\langle 2|\hat{H}|1\rangle = \frac{3}{2}\langle 2|1\rangle = \frac{3}{2}\frac{1}{\sqrt{2}}\begin{pmatrix} 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} i \\ 1 \\ 0 \end{pmatrix} = 0 \tag{12}$$

$$\langle 2|\hat{H}|2\rangle = \frac{1}{2}\langle 2|2\rangle = \frac{1}{2}\begin{pmatrix} 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = \frac{1}{2} \tag{13}$$

$$\langle 2|\hat{H}|3\rangle = \frac{1}{2}\langle 2|3\rangle = \frac{1}{2}\frac{1}{\sqrt{3}}\begin{pmatrix} 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} -i \\ 1 \\ -1 \end{pmatrix} = -\frac{1}{2\sqrt{3}} \tag{14}$$

$$\langle 3|\hat{H}|1\rangle = \frac{3}{2}\langle 3|1\rangle = \frac{3}{2}\frac{1}{\sqrt{2}}\frac{1}{\sqrt{3}}\begin{pmatrix} -i & 1 & -1 \end{pmatrix}\begin{pmatrix} i \\ 1 \\ 0 \end{pmatrix} = \frac{3}{2}\frac{1}{\sqrt{2}}\frac{1}{\sqrt{3}}2 = \sqrt{\frac{3}{2}} \tag{15}$$

$$\langle 3|\hat{H}|2\rangle = \tag{16}$$

$$\langle 3|\hat{H}|3\rangle = \tag{17}$$