Problem Sheet 6

FYS3110

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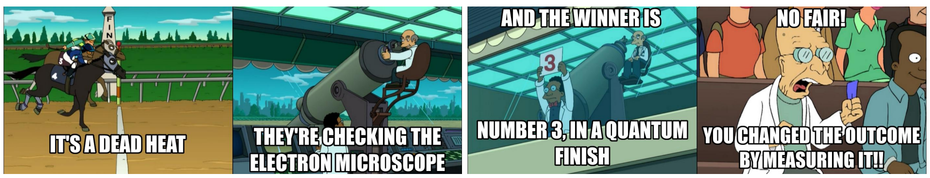

Figure 1: Quantum joke from Futurama.

Problem 6.1

An electron which has spin-1/2 is in the states

$$|\psi\rangle = \sqrt{\frac{2}{5}}\,|3,2,1\rangle \otimes |{\downarrow_z}\rangle + \sqrt{\frac{3}{5}}\,|2,1,1\rangle \otimes |{\uparrow_z}\rangle\,, \tag{1}$$

of the hydrogen atom. The state with quantum numbers n, l, m and spin $s_z = \{\uparrow_z = \hbar/2, \quad \downarrow_z = -\hbar/2\}$ along the z-axis is denoted $|n,l,m\rangle \otimes |s_z\rangle$.

a)

The probability that the electron is measured to be in the spin up state along the z-axis can be calculated in a very difficult manner by computing $\langle\psi| (\mathbb{1} \otimes |{\uparrow_z}\rangle\langle{\uparrow_z}|) |\psi\rangle$. However, one can simply look at the coefficients in the superposition represesenation of the state in equation 1 and realize that

$$P(\uparrow_z) = \frac{3}{5}, \quad P(\downarrow_z) = \frac{2}{5}. \tag{2}$$

The probabilities add up to one, implying that the state is normalized.

b)

To find which values can be measured for L^2 and for what probabilities one need simply to apply the $\hat{L}^2$ operator to the state of the electron in equation 1.

$$\begin{aligned}
\hat{L}^2 |\psi\rangle &= \sqrt{\frac{2}{5}} \hat{L}^2 |3,2,1\rangle \otimes |\downarrow_z\rangle + \sqrt{\frac{3}{5}} \hat{L}^2 |2,1,1\rangle \otimes |\uparrow_z\rangle \\
&= \sqrt{\frac{2}{5}} (\hbar^2 2(2+1) |3,2,1\rangle \otimes |\downarrow_z\rangle) + \sqrt{\frac{3}{5}} (\hbar^2 1(1+1) |2,1,1\rangle \otimes |\uparrow_z\rangle) \\
&= \sqrt{\frac{2}{5}} (6\hbar |3,2,1\rangle \otimes |\downarrow_z\rangle) + \sqrt{\frac{2}{5}} (2\hbar^2 |2,1,1\rangle \otimes |\uparrow_z\rangle)
\end{aligned}$$

which means that you can measure $6\hbar^2$ with probability $2/5$ and $2\hbar^2$ with probability $3/5$ for L^2.

The same computation for L_z is

$$\begin{aligned}
\hat{L}_z |\psi\rangle &= \sqrt{\frac{2}{5}} \hat{L}_z |3,2,1\rangle \otimes |\downarrow_z\rangle + \sqrt{\frac{3}{5}} \hat{L}_z |2,1,1\rangle \otimes |\uparrow_z\rangle \\
&= \sqrt{\frac{2}{5}} |3,2,1\rangle \otimes |\downarrow_z\rangle + \sqrt{\frac{3}{5}} |2,1,1\rangle \otimes |\downarrow_z\rangle
\end{aligned}$$