Midterm "Take home"-exam
FYS3110

Unable to see candidate no

October 7, 2016

1 Spin-1/2 systems

The following is given:

$$\hat{S}^2 = \hat{S}_x^2 + \hat{S}_y^2 + \hat{S}_z^2, \quad \hat{S}^{\pm} = \hat{S}_x \pm i\hat{S}_y$$
$$|\uparrow\rangle \equiv \left| s = \frac{1}{2}, m_s = \frac{1}{2} \right\rangle, \quad |\downarrow\rangle \equiv \left| s = \frac{1}{2}, m_s = -\frac{1}{2} \right\rangle$$
$$\hat{S}^2 |\uparrow\rangle = \hbar^2 \frac{1}{2}\left(\frac{1}{2}+1\right)|\uparrow\rangle, \quad \hat{S}^2 |\downarrow\rangle = \hbar^2 \frac{1}{2}\left(\frac{1}{2}+1\right)|\downarrow\rangle$$
$$\hat{S}_z |\uparrow\rangle = \frac{\hbar}{2}|\uparrow\rangle, \quad \hat{S}_z |\downarrow\rangle = -\frac{\hbar}{2}|\downarrow\rangle$$
$$[\hat{S}_x, \hat{S}_y] = i\hbar\hat{S}_z, \quad [\hat{S}_y, \hat{S}_z] = i\hbar\hat{S}_x, \quad [\hat{S}_z, \hat{S}_x] = i\hbar\hat{S}_y$$

1.1

$$\hat{S}_z\hat{S}^+ |\downarrow\rangle = \hat{S}_z\hat{S}_x |\downarrow\rangle + i\hat{S}_z\hat{S}_y |\downarrow\rangle$$

rewriting commutation relations

$$[\hat{S}_z, \hat{S}_x] = \hat{S}_z\hat{S}_x - \hat{S}_x\hat{S}_z = i\hbar\hat{S}_y \rightarrow \hat{S}_z\hat{S}_x = i\hbar\hat{S}_y + \hat{S}_x\hat{S}_z$$
$$[\hat{S}_y, \hat{S}_z] = \hat{S}_y\hat{S}_z - \hat{S}_z\hat{S}_y = i\hbar\hat{S}_x \rightarrow \hat{S}_z\hat{S}_y = \hat{S}_y\hat{S}_z - i\hbar\hat{S}_x,$$

gives

$$\begin{aligned}\hat{S}_z\hat{S}^+ |\downarrow\rangle &= (i\hbar\hat{S}_y + \hat{S}_x\hat{S}_z + i\hat{S}_y\hat{S}_z + \hbar\hat{S}_x)|\downarrow\rangle \\ &= \left(i\hbar\hat{S}_y - \frac{\hbar}{2}\hat{S}_x - i\frac{\hbar}{2}\hat{S}_y + \hbar\hat{S}_x\right)|\downarrow\rangle \\ &= \left(\frac{\hbar}{2}\hat{S}_x + i\frac{\hbar}{2}\hat{S}_y\right)|\downarrow\rangle = \frac{\hbar}{2}\hat{S}^+ |\downarrow\rangle.\end{aligned}$$

This means that $\hat{S}^+ |\downarrow\rangle$ is an eigenstate of $\hat{S}_z$ with eigenvalue $\hbar/2$.