Problem 2.1

An operator $\hat{H}$ is represented in a particular orthonormal basis as the matrix

$$\hat{H} \simeq \begin{pmatrix} 1 & \frac{i}{2} & 0 \\ -\frac{i}{2} & 1 & 0 \\ 0 & 0 & \frac{1}{2} \end{pmatrix}$$

a) Show that $\hat{H}$ is hermitian.

b) Check that the kets $|1\rangle \simeq \frac{1}{\sqrt{2}}\left(\begin{smallmatrix} i \\ 1 \\ 0 \end{smallmatrix}\right)$, $|2\rangle \simeq \left(\begin{smallmatrix} 0 \\ 0 \\ 1 \end{smallmatrix}\right)$ and $|3\rangle \simeq \frac{1}{\sqrt{3}}\left(\begin{smallmatrix} -i \\ 1 \\ -1 \end{smallmatrix}\right)$ are eigenkets of $\hat{H}$. Find their eigenvalues.

c) Compute the 9 matrix elements of $\hat{H}$ in the eigenket basis used in b), that is compute $H_{ij} = \langle i|\hat{H}|j\rangle$ where $|j\rangle(j = 1, 2, 3)$ correspond to the kets in b). Is this matrix (H_{ij}) diagonal?

d) Construct an *orthonormal* set of eigenkets $|i'\rangle$ for $\hat{H}$ and compute the matrix elements $H_{i'j'} = \langle i'|\hat{H}|j'\rangle$. Is this matrix $(H_{i'j'})$ diagonal?

Problem 2.2

a) Find the hermitian conjugate of each of the following operators i, x^2 and $\frac{d}{dx}$.

b) Find the hermitian conjugate of the (composite) operator $\hat{H} = \hat{K}\hat{L}$.

c) Let $|\lambda\rangle$ be an eigenket of the hermitian operator $\hat{K}$ with eigenvalue λ, i.e. $\hat{K}|\lambda\rangle = \lambda|\lambda\rangle$. Use the definition of hermitian conjugate operator and the result in *b*) to show that

$$\langle\lambda|\hat{K}\hat{L}|g\rangle = \langle\lambda|\hat{L}|g\rangle\lambda$$

where $|g\rangle$ is an arbitrary state and $\hat{L}$ is an arbitrary operator.

Problem 2.3 (optional)

Look back at problem 1.3 (week 1 exercise) and find the eigenvalues and eigenstates of the Hamiltonian operator H. Assume that the states $|\psi\rangle$ and $|\phi\rangle$ satisfy the conditions, found in problem 1.3, such that the Hamiltonian is hermitian.