

Problem Sheet 4

FYS3110

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The hamiltonian of particle with mass m in a one-dimensional oscillator potential having a characteristic frequency ω is

$$\hat{H} = \frac{1}{2m}\hat{p}^2 + \frac{1}{2}m\omega^2\hat{X}^2 \quad (1)$$

The ladder operators for the harmonic oscillator potential are

$$\hat{a} = \frac{1}{\sqrt{2\hbar m\omega}}(m\omega\hat{X} + i\hat{P}) \quad (\text{lowering operator}) \quad (2)$$

$$\hat{a}^\dagger = \frac{1}{\sqrt{2\hbar m\omega}}(m\omega\hat{X} - i\hat{P}) \quad (\text{raising operator}) \quad (3)$$

Problem 4.1

a)

I want to find an expression for \hat{X} in terms of \hat{a} and \hat{a}^\dagger . This can be done by first rewriting equation 2

$$\begin{aligned} \hat{a} &= \frac{1}{\sqrt{2\hbar m\omega}}(m\omega\hat{X} + i\hat{P}) \\ \hat{a} &= \sqrt{\frac{m\omega}{2\hbar}}\hat{X} + \frac{i}{\sqrt{2\hbar m\omega}}\hat{P} \end{aligned}$$