Problem 6.1

This problem is about a composite system having several degrees of freedom.

An electron which has spin-1/2 is in the state

$$|\psi\rangle = \sqrt{\frac{2}{5}}\,|3,2,1\rangle \otimes |{\downarrow_z}\rangle + \sqrt{\frac{3}{5}}\,|2,1,1\rangle \otimes |{\uparrow_z}\rangle\,,$$

of the hydrogen atom. The state with quantum numbers n, l, m and spin $s_z = \{\uparrow_z = \hbar/2, \downarrow_z = -\hbar/2\}$ along the z-axis is denoted $|n,l,m\rangle \otimes |s_z\rangle$.

a) What is the probability that the electron is measured to be in the spin up state along the z-axis? Find also the probability for measuring spin down.

b) Which values can you measure for L^2 and with what probabilities? What about L_z and S^2?

Consider now the total angular momentum $\vec{J} = \vec{L} + \vec{S}$ (really $\vec{L} \otimes I + I \otimes \vec{S}$). For this problem you need to consult the Clebsch-Gordan tables.

c) What values of total angular momentum squared J^2 can you measure for the electron and with what probabilities?

d) Find also measurement values and probabilitites for J_z?

e) Compute the radial probability density $P_{\uparrow_z}(r)$ of finding the electron in the state with $s_z = \hbar/2$ at a radial position r.

Problem 6.2 (optional)

Griffiths: Chapter 4, Problem 49