This is the first problem set. The problems are marked with different categories: (L) are problems that are referred to in the lectures and are meant to supplement the lecture material. Do them to get more out of the lectures! (H) are typical homework exercise problems that illustrate several concepts. (X) are exam type problems. At least one of these will appear, in a perhaps slightly different form, on the final exam. (E) are extra problems. Do them if you feel the need.

If you want access to the written solution and participate in getting and giving feedback on your work you must hand in your answers on Devilry by the deadline time (DUE time). Participation is recommended, but is not mandatory. The feedback session is held immediately after the deadline and will be concentrated on only a few of the problems marked (H) and (X).

Problem 1.1(L)

The complex inner product $\langle u|v\rangle$ is linear in its *second* factor, which means: Given $|v\rangle = \alpha|v_1\rangle + \beta|v_2\rangle$ where $\alpha, \beta \in \mathbb{C}$, then $\langle u|v\rangle = \alpha\langle u|v_1\rangle + \beta\langle u|v_2\rangle$. In contrast, the complex inner product is *not* linear in its *first* factor: Define $|u\rangle = \alpha|u_1\rangle + \beta|u_2\rangle$ and use the defining properties of the complex inner product to write out the inner product $\langle u|w\rangle$ for an arbitrary $|w\rangle$.

Problem 1.2(L)

Let $|v'\rangle = \alpha|v_1\rangle + \beta|v_2\rangle$ where $\alpha, \beta \in \mathbb{C}$. Use the properties of the complex inner product to show that $\langle v'| = \alpha^*\langle v_1| + \beta^*\langle v_2|$. Hint: Consider the inner product $\langle v'|u\rangle$ where $|u\rangle$ is an arbitrary ket. Is your answer consistent with what you got in Problem 1.1?

Problem 1.3(L)

Two different kets are represented by two different complex functions of one variable x which take values in $\mathbb{R}$: $|u\rangle \simeq f(x)$ and $|v\rangle \simeq g(x)$. We use the symbol $\simeq$ to mean represented by. (You may replace it by $=$ if you like.) Write down the expressions for the inner products $\langle u|v\rangle$ and $\langle u|u\rangle$ in terms of f and g. What is f if $\langle u|u\rangle = 0$?

Problem 1.4(L)

You are given an orthonormal basis set $|b_i\rangle$ where $i \in \{1, \dots, N\}$. A ket $|V\rangle = \sum_{i=1}^{N} v_i|b_i\rangle$. Show that $v_i = \langle b_i|V\rangle$.

Problem 1.5(L)

A basis set can be either discrete or continuous. For a continuous basis set $|x'\rangle$ the index x' which labels the basis vector takes values in a continuum, and an arbitrary ket is expressed as $|f\rangle = \int_0^L dx' f(x')|x'\rangle$ where the expansion coefficients become a function $f(x')$. We have put limits 0 and L on the values that the continuous variable takes, but these depends on the problem in question.
Show that the Dirac delta-normalization of the basis vectors: $\langle x|x'\rangle = \delta(x - x')$ implies that $\langle x|f\rangle = f(x)$. From this show that $\int_0^L dx'|x'\rangle\langle x'|$ is an operator (a mathematical object that maps a ket onto a ket) that maps a given ket onto itself (i.e. it is the identity operator).

Problem 1.6(H)

Consider the functions $\delta_\epsilon(x) = \frac{1}{\pi}\frac{\epsilon}{\epsilon^2+x^2}$ where ϵ is a parameter.

a) Show that $\int_{-\infty}^{\infty} dx \delta_\epsilon(x) = 1$

b) Verify that $\delta_{\epsilon/k}(x) = k\delta_\epsilon(kx)$ for a positive number k.

Such a sequence of functions is called a δ-sequence and constitutes one possible formal definition of a Dirac delta-function.

$$\int dx'\delta(x-x')f(x') \equiv \lim_{\epsilon\to 0}\int dx'\delta_\epsilon(x-x')f(x') = f(x)$$

c) Make numerical plots of $\delta_\epsilon(x)$ on the interval $x \in [-1, 1]$ for three values of ϵ, pick $\epsilon = \{0.01, 0.1, 1\}$.

d) Explain how you could obtain a plot of $\delta_{0.1}(x)$ from a table of many arguments (x) and function values $y = \delta_1(x)$.

Problem 1.7(H)

Consider an orthonormal basis set $|u_i\rangle$ where $i \in 1, 2, \ldots, N$ and where an arbitrary ket is expressed as $|A\rangle = \sum_{i=1}^{N}\alpha_i|u_i\rangle$ where $\alpha_i \in \mathbb{C}$. Let $\hat{P}_1 = |u_1\rangle\langle u_1|$. Compute $\hat{P}_1|A\rangle$ and $\hat{P}_1\hat{P}_1|A\rangle$. Justify in words why $\hat{P}_1$ is called a projection operator.

Problem 1.8(X)

What is a quantum state? In your answer, explain how a quantum state is different from a classical state and how it is represented mathematically.

Problem 1.9(E)

Two different kets are represented by two different column matrices, each with two entries: $|u\rangle \simeq \binom{u_1}{u_2}$ and $|v\rangle \simeq \binom{v_1}{v_2}$. Compute $\langle u|v\rangle$. Also compute $\langle u|\alpha|v\rangle$ where $\alpha \in \mathbb{C}$.

Problem 1.10(E)

Griffiths problem 3.22