

Problem Set II

FYS3110

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Problem 2.1

An operator \hat{H} is represented in a particular orthonormal basis as the matrix

$$\hat{H} \simeq \begin{pmatrix} 1 & \frac{i}{2} & 0 \\ -\frac{i}{2} & 1 & 0 \\ 0 & 0 & \frac{1}{2} \end{pmatrix} \quad (1)$$

a)

\hat{H} is Hermitian if it is equal to its own transpose conjugate, $\hat{H} = \hat{H}^\dagger$.

$$\hat{H}^\dagger = (\hat{H}^*)^T = \begin{pmatrix} 1 & -\frac{i}{2} & 0 \\ \frac{i}{2} & 1 & 0 \\ 0 & 0 & \frac{1}{2} \end{pmatrix}^T = \begin{pmatrix} 1 & \frac{i}{2} & 0 \\ -\frac{i}{2} & 1 & 0 \\ 0 & 0 & \frac{1}{2} \end{pmatrix} = \hat{H} \quad (2)$$

b)

Three ket vectors are given

$$|1\rangle \simeq \frac{1}{\sqrt{2}} \begin{pmatrix} i \\ 1 \\ 0 \end{pmatrix} \quad (3)$$

$$|2\rangle \simeq \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \quad (4)$$

$$|3\rangle \simeq \frac{1}{\sqrt{3}} \begin{pmatrix} -i \\ 1 \\ -1 \end{pmatrix} \quad (5)$$

$|1\rangle$ is and eigenvector of \hat{H} with eigenvalue $\frac{3}{2}$

$$\hat{H} |1\rangle = \begin{pmatrix} 1 & \frac{i}{2} & 0 \\ -\frac{i}{2} & 1 & 0 \\ 0 & 0 & \frac{1}{2} \end{pmatrix} \frac{1}{\sqrt{2}} \begin{pmatrix} i \\ 1 \\ 0 \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{3i}{2} \\ \frac{3}{2} \\ 0 \end{pmatrix} = \frac{3}{2} |1\rangle \quad (6)$$

$|2\rangle$ is and eigenvector of \hat{H} with eigenvalue $\frac{1}{2}$

$$\hat{H} |2\rangle = \begin{pmatrix} 1 & \frac{i}{2} & 0 \\ -\frac{i}{2} & 1 & 0 \\ 0 & 0 & \frac{1}{2} \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ \frac{1}{2} \end{pmatrix} = \frac{1}{2} |2\rangle \quad (7)$$

$|3\rangle$ is and eigenvector of \hat{H} with eigenvalue $\frac{1}{2}$

$$\hat{H} |3\rangle = \begin{pmatrix} 1 & \frac{i}{2} & 0 \\ -\frac{i}{2} & 1 & 0 \\ 0 & 0 & \frac{1}{2} \end{pmatrix} \frac{1}{\sqrt{3}} \begin{pmatrix} -i \\ 1 \\ -1 \end{pmatrix} = \frac{1}{\sqrt{3}} \begin{pmatrix} -\frac{i}{2} \\ \frac{1}{2} \\ -\frac{1}{2} \end{pmatrix} = \frac{1}{2} |3\rangle \quad (8)$$