RELATIVISTIC KINEMATICS

FYS3120: PROBLEM SET 8

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1. Mirror Mirror on the (Moving) Wall

A monocromatic light source is at rest in the laboratory and sends photons with frequency ν_0 towards a mirror which has its reflective surface perpendicular to the beam direction. The mirror moves away from the light source with velocity v. The transformation formula for four-momentum is given by $p^\mu = (E/c, \mathbf{p})$ and the Planck relation is $E = h\nu$.

1.a. **Light Frequency in Rest Frame of Mirror.** The relativistic energy of a moving particle is

$$E = \sqrt{p^2c^2 + m^2c^4}. \tag{1}$$

Because a photon is without mass, the energy of a photon according to the formula above is

$$E = pc, \tag{2}$$

which can be inserted into Planck relation yielding

$$p = \frac{h\nu_0}{c}. \tag{3}$$

This provides an expression for the four vector

$$p^\mu = \left(\frac{E}{c}, \mathbf{p}\right) = \left(\frac{h\nu_0}{c}\right) = (p, p, 0, 0). \tag{4}$$

To get from emitted frequency ν_0 in lab reference frame S, to frequency ν in mirror reference frame S' one needs to take the Lorentz transform

$$p'^\mu = L^\mu{}_\rho p^\rho, \tag{5}$$

because the mirror reference frame is just a boost along the x-axis, relative to the lab reference frame.

$$\begin{pmatrix} p'^0 \\ p'^1 \\ p'^2 \\ p'^3 \end{pmatrix} = \begin{pmatrix} \gamma & -\beta\gamma & 0 & 0 \\ -\beta\gamma & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} p^0 \\ p^1 \\ p^2 \\ p^3 \end{pmatrix} = \gamma(1-\beta) \begin{pmatrix} p \\ p \\ 0 \\ 0 \end{pmatrix}, \tag{6}$$

so

$$p' = \gamma(1-\beta)p. \tag{7}$$

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The de Broglie relations gives

$$p = \frac{h}{\lambda} = \frac{h\nu}{c}, \tag{8}$$

so the frequency of the emitted and reflected light in the rest frame of the mirror must be

$$\nu' = \gamma(1-\beta)\nu. \tag{9}$$

The frequency of the emitted and reflected light must necessarily be the same, due to conservation of momentum.

1.b. **Frequency of Reflected Light in Lab System.** Denoting frequency of reflected light as ν_R and frequency of emitted light as ν_0, we already have that

$$\nu'_R = \gamma(1-\beta)\nu_0, \tag{10}$$

in the mirror rest frame. Similarly, the frequency of reflected light in laboratory rest frame is

$$\nu_R = \gamma(1-\beta)\nu'_R. \tag{11}$$

Inserting 10 into 11 yields

$$\nu_R = \gamma^2(1-\beta)^2\nu_0 = \frac{(1-\beta)^2}{1-\beta^2}\nu_0 = \frac{(1-\beta)^2}{(1+\beta)(1-\beta)}\nu_0 = \frac{1-\beta}{1+\beta}\nu_0 \tag{12}$$