

MIDTERM EXAM
FYS3120

CANDIDATE 15137

1. A (BORING) LAGRANGIAN

A non-relativistic particle (no-potential) of mass m is moving in three dimensions.

1.a.

1.b.

1.c.

1.d.

1.e.

1.f.

1.g. Consider a Lorentz transformation where the Lorentz transformation tensor is given as

$$(1) \quad L^\mu{}_\nu = \delta^\mu{}_\nu + \omega^\mu{}_\nu.$$

Any particular Lorentz transformation must leave the line element $ds^2 = dx_\mu dx^\mu$ invariant,

$$g_{\mu\nu} dx'^\mu dx'^\nu = g_{\mu\nu} L^\mu{}_\rho L^\nu{}_\sigma dx^\rho dx^\sigma = g_{\rho\sigma} dx^\rho dx^\sigma$$
$$g_{\mu\nu} L^\mu{}_\rho L^\nu{}_\sigma = g_{\rho\sigma}$$

To see if the Lorentz transformation in 1 is invariant it must satisfy this requirement

$$\begin{aligned} g_{\rho\sigma} &= g_{\mu\nu} L^\mu{}_\rho L^\nu{}_\sigma \\ &= g_{\mu\nu} (\delta^\mu{}_\rho + \omega^\mu{}_\rho) (\delta^\nu{}_\sigma + \omega^\nu{}_\sigma) \\ &= (\delta_{\nu\rho} + \omega_{\nu\rho}) (\delta^\nu{}_\sigma + \omega^\nu{}_\sigma) \\ &= \delta_{\nu\rho} \delta^\nu{}_\sigma + \delta_{\nu\rho} \omega^\nu{}_\sigma + \omega_{\nu\rho} \delta^\nu{}_\sigma + \omega_{\nu\rho} \omega^\nu{}_\sigma \\ &= g_{\nu\rho} \delta^\nu{}_\sigma + g_{\nu\rho} \omega^\nu{}_\sigma + \omega_{\nu\rho} g^{\nu\gamma} g_{\gamma\sigma} + \omega_{\rho\sigma}^2 \\ &= \delta_{\rho\sigma} + \omega_{\rho\sigma} + \omega_{\sigma\rho} = g_{\rho\sigma} + g_{\nu\rho} (\omega^\nu{}_\sigma + \omega_\sigma{}^\nu), \end{aligned}$$

which only works if $\omega^\mu{}_\nu$ is antisymmetric, that is if $\omega^\mu{}_\nu = -\omega_\nu{}^\mu$.

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2. RELATIVISTICS

3. FINDING THE SHORTEST WAY

The shortest path between two points on a sphere. At some constant radius r , some small movement in some direction on the sphere is

$$(2) \quad ds^2 = r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2$$

inserting for $d\phi = (d\phi/d\theta)d\theta = \dot{\phi}d\theta$ gives

$$(3) \quad ds = r\sqrt{1 + \sin^2 \theta \dot{\phi}^2} d\theta$$

A path is given by

$$(4) \quad S = \int ds = r \int_{\theta_A}^{\theta_B} \sqrt{1 + \sin^2 \theta \dot{\phi}^2} d\theta$$

where the integrand $F(\theta, \phi, \dot{\phi}) = \sqrt{1 + \sin^2 \theta \dot{\phi}^2}$ does not depend explicitly on ϕ . This implies that $\partial F / \partial \dot{\phi}$ is constant, yielding

$$(5) \quad \frac{\partial F}{\partial \dot{\phi}} = \frac{2 \sin^2 \theta \dot{\phi}}{\sqrt{1 + \sin^2 \theta \dot{\phi}^2}} = C' \rightarrow \frac{\sin^2 \theta \dot{\phi}}{\sqrt{1 + \sin^2 \theta \dot{\phi}^2}} = C$$

This can be rearranged

$$\begin{aligned} C^2 &= \frac{\sin^4 \theta \dot{\phi}^2}{1 + \sin^2 \theta \dot{\phi}^2} \\ C^2 + C \sin^2 \theta \dot{\phi}^2 &= \sin^4 \theta \dot{\phi}^2 \\ C^2 &= (\sin^4 \theta - C \sin^2 \theta) \dot{\phi}^2 \\ \dot{\phi}^2 &= \frac{C^2}{(\sin^4 \theta - C \sin^2 \theta)} \\ \dot{\phi} &= \frac{C}{\sin \theta \sqrt{\sin^2 \theta - C}} \end{aligned}$$

INTEGRATE!!