MIDTERM EXAM

FYS3120

CANDIDATE 15137

1. A (boring) Lagrangian

A non-relativistic particle (no-potential) of mass m is moving in three dimensions.

1.a.

1.b.

1.c.

1.d.

1.e.

1.f.

1.g. Consider a Lorentz transformation where the Lorentz transformation tensor is given as

$$L^\mu{}_\nu = \delta^\mu{}_\nu + \omega^\mu{}_\nu. \tag{1}$$

Any particular Lorentz transformation must leave the line element $ds^2 = dx_\mu dx^\mu$ invariant,

$$\begin{aligned} g_{\mu\nu} dx'^\mu dx'^\nu = g_{\mu\nu} L^\mu{}_\rho L^\mu{}_\sigma dx^\rho dx^\sigma &= g_{\rho\sigma} dx^\rho dx^\sigma \\ g_{\mu\nu} L^\mu{}_\rho L^\nu_\sigma &= g_{\rho\sigma} \end{aligned}$$

Not to see if the Lorentz transformation in 1 statisfies this requirement

$$\begin{aligned} g_{\mu\nu} &= q_{\mu\nu} L^\mu{}_\rho L^\nu{}_\sigma \\ &= g_{\mu\nu} (\delta^\mu{}_\rho + \omega^\mu{}_\rho)(\delta^\nu{}_\sigma + \omega^\nu{}_\sigma) \\ &= (\delta_{\nu\rho} + \omega_{\nu\rho})(\delta^\nu{}_\sigma + \omega^\nu{}_\sigma) \\ &= \delta_{\nu\rho}\delta^\nu{}_\sigma + \delta_{\nu\rho}\omega^\nu{}_\sigma + \omega_{\nu\rho}\delta^\nu{}_\sigma + \omega_{\nu\rho}\omega^\nu{}_\sigma \\ &= g_{\nu\rho}\delta^\nu{}_\sigma + g_{\nu\rho}\omega^\nu{}_\sigma + \omega_{\nu\rho} g^{\nu\gamma} g_{\gamma\sigma} + \omega^2_{\rho\sigma} \\ &= \delta_{\rho\sigma} + \omega_{\rho\sigma} + \omega_{\sigma\rho} = g_{\rho\sigma} + g_{\nu\rho}(\omega^\nu{}_\sigma + \omega_\sigma{}^\nu), \end{aligned}$$

which only works if ω^μ_ν is antisymmetric, that is if $\omega^\mu{}_\nu = -\omega_\nu{}^\mu$.

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2. Relativistics

3. Finding the Shortest Way

The shortest path between two points on a sphere. At some contstant radius r, some small movement in some direction on the sphere is

$$ds^2 = r^2 d\theta^2 + r^2 \sin^2\theta\phi^2 \tag{2}$$

inserting for $d\phi = (d\phi/d\theta)d\theta = \dot{\phi}d\theta$ gives

$$ds = r\sqrt{1+\sin^2\theta\dot{\phi}^2}d\theta \tag{3}$$

A path is given by

$$S = \int ds = r\int_{\theta_A}^{\theta_B}\sqrt{1+\sin^2\theta\dot{\phi}^2}d\theta \tag{4}$$

where the integrand $F(\theta, \phi, \dot{\phi}) = \sqrt{1+\sin^2\theta\dot{\phi}^2}$ does not depend explicitly on ϕ. This implies that $\partial F/\partial\dot{\phi}$ is constant, yielding

$$\frac{\partial F}{\partial\dot{\phi}} = \frac{2\sin^2\theta\dot{\phi}}{\sqrt{1+\sin^2\theta\dot{\phi}^2}} = C' \rightarrow \frac{\sin^2\theta\dot{\phi}}{\sqrt{1+\sin^2\theta\dot{\phi}^2}} = C \tag{5}$$

This can be rearranged

$$\begin{aligned}
C^2 &= \frac{\sin^4\theta\dot{\phi}^2}{1+\sin^2\theta\dot{\theta}^2} \\
C^2 + C\sin^2\theta\dot{\phi}^2 &= \sin^4\theta\dot{\phi}^2 \\
C^2 &= (\sin^4\theta - C\sin^2\theta)\dot{\phi}^2 \\
\dot{\phi}^2 &= \frac{c^2}{(\sin^4\theta - C\sin^2\theta)} \\
\dot{\phi} &= \frac{C}{\sin\theta\sqrt{\sin^2 - C}}
\end{aligned}$$

INTEGRATE!!