MIDTERM EXAM
FYS3120

CANDIDATE 15137

1. A (boring) Lagrangian

A non-relativistic particle (no-potential) of mass m is moving in three dimensions.

1.a.

1.b.

1.c.

1.d.

1.e.

1.f.

1.g. Consider a Lorentz transformation where the Lorentz transformation tensor is given as

$$L^{\mu}{}_{\nu} = \delta^{\mu}{}_{\nu} + \omega^{\mu}{}_{\nu}. \tag{1}$$

Any particular Lorentz transformation must leave the line element $ds^2 = dx_{\mu}dx^{\mu}$ invariant,

$$g_{\mu\nu}dx'^{\mu}dx'^{\nu} = g_{\mu\nu}L^{\mu}{}_{\rho}L^{\mu}{}_{\sigma}dx^{\rho}dx^{\sigma} = g_{\rho\sigma}dx^{\rho}dx^{\sigma}$$
$$g_{\mu\nu}L^{\mu}{}_{\rho}L^{\nu}_{\sigma} = g_{\rho\sigma}$$

Not to see if the Lorentz transformation in 1 statisfies this requirement

$$\begin{aligned}
g_{\mu\nu} &= q_{\mu\nu}L^{\mu}{}_{\rho}L^{\nu}{}_{\sigma} \\
&= g_{\mu\nu}(\delta^{\mu}{}_{\rho} + \omega^{\mu}{}_{\rho})(\delta^{\nu}{}_{\sigma} + \omega^{\nu}{}_{\sigma}) \\
&= (\delta_{\nu\rho} + \omega_{\nu\rho})(\delta^{\nu}{}_{\sigma} + \omega^{\nu}{}_{\sigma}) \\
&= \delta_{\nu\rho}\delta^{\nu}{}_{\sigma} + \delta_{\nu\rho}\omega^{\nu}{}_{\sigma} + \omega_{\nu\rho}\delta^{\nu}{}_{\sigma} + \omega_{\nu\rho}\omega^{\nu}{}_{\sigma} \\
&= g_{\nu\rho}\delta^{\nu}{}_{\sigma} + g_{\nu\rho}\omega^{\nu}{}_{\sigma} + \omega_{\nu\rho}g^{\nu\gamma}g_{\gamma\sigma} + \omega^2_{\rho\sigma} \\
&= \delta_{\rho\sigma} + \omega_{\rho\sigma} + \omega_{\sigma\rho} = g_{\rho\sigma} + g_{\nu\rho}(\omega^{\nu}{}_{\sigma} + \omega_{\sigma}{}^{\nu}),
\end{aligned}$$

which only works if ω^{μ}_{ν} is antisymmetric, that is if $\omega^{\mu}{}_{\nu} = -\omega_{\nu}{}^{\mu}$.

Date: March 27, 2017.