ELECTRODYNAMICS
FYS3120: PROBLEM SET 11

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1. Simple Lagrangian Dynamics

A non-relativistic particle, with electric charge q and mass m moves in a magnetic dipole field, given by the vector potential

$$\vec{\mathbf{A}} = \frac{\mu_0}{4\pi r^3}(\vec{\mu} \times \vec{\mathbf{r}}), \tag{1}$$

where $\vec{\mu}$ is the magnetic dipole moment of a static charge distribution centered at the origin.

1.a. **Lagrangian.** The Lagrangian is given by

$$L = T + q\vec{\mathbf{v}} \cdot \vec{\mathbf{A}}. \tag{2}$$

The kinetic energy is simply $T = \frac{1}{2}m\vec{\mathbf{v}}^2$ while the potential is

$$\begin{aligned} q\vec{\mathbf{v}} \cdot \vec{\mathbf{A}} &= \frac{q\mu_0}{4\pi r^3}\vec{\mathbf{v}} \cdot (\vec{\mu} \times \vec{\mathbf{r}}) \\ &= \frac{q\mu_0}{4\pi r^3}\vec{\mu} \cdot (\vec{\mathbf{r}} \times \vec{\mathbf{v}}) \\ &= \frac{q\mu_0}{4\pi m r^3}\vec{\mu} \cdot \vec{\ell}, \end{aligned}$$

using a vector triple product identity and $\vec{\ell} = m\vec{\mathbf{r}} \times \vec{\mathbf{v}}$. Inserting the parts into 2 the Lagrangican becomes

$$L = \frac{1}{2}m\vec{\mathbf{v}}^2 + \frac{q\mu_0}{4\pi m r^3}\vec{\mu} \cdot \vec{\ell}. \tag{3}$$

1.b. **Alternative Lagrangian.** We now make the assumption that the magnetic dipole moment is oriented along the z-axis and that the particle moves in the (x, y)-plane. In the following, $r = |\vec{\mathbf{r}}|$ and the angle ϕ between the x-axis and the position vector var are chosen as generalised coordinates.

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