FYS3120 Classical Mechanics and Electrodynamics

Problem set 8

March 10, 2017

Problem 1 A monocromatic light source is at rest in the laboratory and sends photons with frequency ν_0 towards a mirror which has its reflective surface perpendicular to the beam direction. The mirror moves away from the light source with velocity v. Use the transformation formula for four-momentum $p^\mu = (E/c, \vec{p})$ and the Planck relation $E = h\nu$ to:

a) Find the frequency of the emitted and reflected light in the rest frame of the mirror.

b) Find the frequency of reflected light in the lab system.

Problem 2 Figure 1 shows a particle with mass m and (relativistic) kinetic energy K in the laboratory frame S. The particle is moving towards another particle, with the same mass m, which is at rest in S.

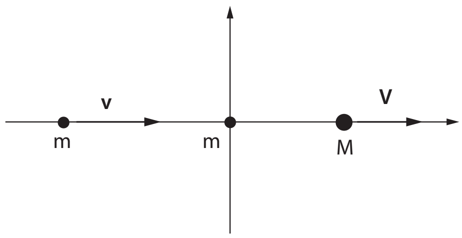

Figure 1: Collision between two particles of mass m resulting in a particle with mass M.

a) Find the velocity v of the first particle expressed in terms of the dimensionless quantity $\alpha = K/mc^2$ (and the speed of light).

First we will assume that the particles collide in such a way that they form one particle after the collision (a totally inelastic collision).

b) Determine the compound particle's energy E, momentum P, velocity V and mass M. Find the change in the total kinetic energy of the system due to the collision.

In the rest of the exercise we will assume that the situation before the collision is as described earlier, but that the particles now collide elastically, *i.e.* after the collision the two particles are the same as before the collision, with no change in their masses. The collision happens in such a way that the particles after the collision make the same angle, θ, with the x-axis in the lab frame S, see Fig. 2.

c) Show that after the collision the particles have the same magnitude of momentum ($|\vec{p}_1| = |\vec{p}_2|$) and energy ($E_1 = E_2$).

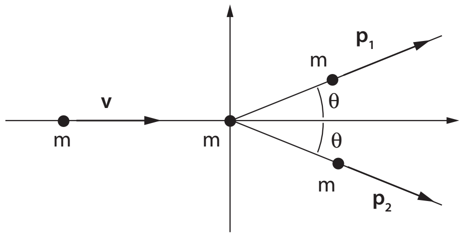

Figure 2: Collision between two particles of mass m resulting in scattering at angle θ.

d) Determine $E \equiv E_1 = E_2$ and $p \equiv |\vec{p}_1| = |\vec{p}_2|$.

e) Determine the angle θ. Find θ in the limiting cases when $\alpha = T/mc^2$ goes to zero and to infinity. Show that $\theta < \pi/4$.

Problem 3 Midterm exam 2006

A space ship leaves Earth orbit at local time $t = 0$ and head for the closest star, *Proxima Centauri*, at a distance of $d = 4.2$ light years. The space ship starts with a velocity $v = 0$ relative to earth, and during the first part of the journey it has a constant proper acceleration $a = g$ in the direction of Proxima Centauri, with $g = 9.8\ \mathrm{m/s^2}$ as the gravitational acceleration at the Earth surface.

At time τ_0 measured in the proper time of the space ship, the acceleration is reversed, so that $a = -g$ and the velocity decreases until it reaches Proxima Centauri with final velocity $v = 0$. The first part of the trip, with acceleration $a = g$, we refer to as part I, the second part, with $a = -g$, as part II. The space ship visits Proxima Centauri only for a short time, and we neglect this time interval in our description of the journey. The return travel to Earth is carried out in the same way as the travel towards Proxima Centauri, and we refer to these parts of the journey as parts III and IV.

During the first part of the journey (part I), the coordinates of the space ship in the Earth fixed reference frame S, define a hyperbolic space-time orbit, given by

$$x - x_I = \frac{c^2}{a}\cosh\left(\frac{a}{c}(\tau - \tau_I)\right), \quad t - t_I = \frac{c}{a}\sinh\left(\frac{a}{c}(\tau - \tau_I)\right), \qquad (1)$$

where x_I, t_I and τ_I are constants. Similar expressions are valid for the other parts of the journey, but with other constants. The coordinates of the Earth fixed reference frame are set to $x = 0$, $t = 0$ at departure of the space ship. Also the time parameter τ is set to 0 at this event.

a) Show that the time parameter τ of Eq. (1) can be identified as the proper time of the space ship. Find the four-velocity and four-acceleration for Part I of the journey as a function of τ and check that the proper acceleration (the acceleration measured in the instantaneous inertial rest frame) defined by the path (1) is a. Explain why the space-time path is called hyperbolic.

b) Determine the constants of Eq. (1) for Part I of the journey, and write the form of the equations with the correct constants and with $a = g$. Draw a two-dimensional space-time diagram (Minkowski diagram) which shows the full journey to Proxima Centauri and back.

c) Determine the proper time value τ_0. Find the total time of the journey as measured on Earth and on the space ship.

d) What is the maximum speed reached by the space ship on the journey, as measured on Earth?