SOLVING THE POISSON-EQUATION IN ONE DIMENSION
FYS3150: COMPUTATIONAL PHYSICS

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ABSTRACT. Lorem ipsum dolor sit amet, consectur adipiscing lit. Nullam ut lacus eget lorem...

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1. Introduction

2. Theory

2.1. The Poisson Equation. The Poisson equation is a classical equation from electromagnetism. The electrostatic potential Φ is generated by a localized charge distribution $\rho(\mathbf{r})$. In three dimensions the equation reads

$$\nabla^2\Phi = -4\pi\rho(\mathbf{r}) \tag{1}$$

where ∇^2 is the Laplace operator. In three dimensions the Laplace operator can be expressed using spherical coordinates, but in this study I am assuming that Φ and ρ are spherically symmetric, thus reducing the equation to a one-dimensional problem. only dependent on radius r.

$$\nabla^2 = \frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{d\Phi}{dr}\right) \tag{2}$$

By substituting $\Phi(r) = \phi(r)/r$ the Poisson equation is reduced to

$$\frac{d^2\phi}{dr^2} = -4\pi r\rho(r) \tag{3}$$

and by letting $\phi \to u$ and $r \to x$ one is left with the very simple equation

$$-u''(x) = f(x) \tag{4}$$

The inhomogenous term f, or source term, is given by the charge distribution ρ multiplied by r and the constant -4π. In this study, however, the source term will be $f(x) = 100e^{-10x}$ and the results can be compared to the analytical solution $u(x) = 1 - (1 - e^{-10})x - e^{-10x}$.

2.2. Approximation of the Second Derivative. In this study the one-dimensional Poisson equation will be solved with Dirichlet boundary conditions by rewrting