MONTE CARLO SIMULATION OF MONETARY TRANSACTIONS: A SIMPLE MODEL FOR WEALTH DISTRIBUTION

FYS3150: COMPUTATIONAL PHYSICS

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Abstract. Lorem ipsum dolor sit amet, consectetur adipiscing elit. In id neque elementum, accumsan ligula at, lobortis tortor. Duis elementum pellentesque purus, sit amet euismod diam facilisis volutpat. Nulla facilisi. Mauris quis felis ante. Aliquam ac velit sit amet velit porta condimentum iaculis ut quam. Phasellus pretium libero nec lectus placerat, ac consectetur sem faucibus. Maecenas dictum porta finibus.

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1. Introduction

2. Theoretical Background

2.1. **The simplest model for an economy.** Arguably, the most famous equation in macroeconomics is the "autarky identity"

$$Y = C + G + I, \tag{1}$$

where Y is income, C is consumption, G is government spending and I is investment. The best-know, but not necessarily the best, measure of income Y is the Gross Domestic Product (GDP)[1]. Consumption C is the monetary value of all goods and services purchased in the private sector, while government spending G is the consumption of the government. Finally, investment I is the sum of private and public saving.

Equation 1 is autarkic because all interactions with other economies are excluded from the expression. There are no terms representing exports and captial inflow, for instance. We are dealing with a *closed* economy, alternatively the entire world as a whole. Moreover, let's assume that the economy we are studying is a peaceful anarchy, without a governing authority of any sort, in effect setting $G = 0$. To begin with, we will also forgo the agents the ability to save such that the worth of every individual, or agent, in the economy must be spent at once. Equation 1 is reduced to $Y = C$, everything one agent spends is the income of another.

2.2. **Monetary transactions.** To simulate monetary transactions in out model economy we expand employ the framework introduced in Patriarca et al.[1]. We assume there are N agents that exchange money in pairs (i, j). We assume also that all agents start with the same amount of money $m_0 > 0$. For every period an arbitrary pair of agents are picked at randomly and let them conduct business, id est a transaction takes place between them. Money is conserved during the transaction such that

$$m_i + m_j = m'_i + m'_j, \tag{2}$$

where the left hand side is the agent i and j's updated wealth and the right-hand side represents the amount of money agents i and j had before the transaction. The exchange is done via a random reassignment factor ϵ, such that

$$m'_i = \epsilon(m_i + m_j) \tag{3}$$

$$m'_j = (1 - \epsilon)(m_i + m_j) \tag{4}$$

No agent will ever have negative wealth, that is $m \geq 0$. Moreover, because of the conservation law in equation 2, the system eventually reaches an equilibrium state given by a Gibbs distribution

$$w_m = \beta e^{-\beta m}, \quad \beta = \frac{1}{\langle m \rangle}, \tag{5}$$

where $\langle m \rangle$ is the expected wealth, for which the arithmetic mean is an unbiased estimator. This implies that after an equilibrium has been reached the majority of agents is left with lower wealth than they had initially and the number of rich agents exponentially decrease.

2.3. **Transactions and savings.** We are can now expand upon the model by introducing a savings rate λ. The savings rate is defined as a fraction of an agent's wealth that does not partake in a transaction for every period. One can gather from the macroeconomic identity in equation 1 that income must still be the same and the transaction law in equation 2 still holds. Now one

[1]GDP = GNP (Gross National Product) in autarky.

3. Algorithm

4. Results

5. Discussion

6. Summary Remarks

References

[1] Patriarca, M., Chakraborti, A., & Kaski, K. (2004). Gibbs versus non-Gibbs distributions in money dynamics. *Physica A: Statistical Mechanics and its Applications*, 340(1), 334-339.