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http://www.uio.no/studier/emner/matnat/fys/FYS4170/h17/

Mid-term exam

Lecture autumn 2017: Relativistic quantum field (FYS4170)

⇝ *Answers must be handed in latest* **Monday, 23 October 2016, 10:00am***; you can use the corresponding box (labelled with the course name and code) at the administrative office of the Physics Department. Please write your candidate number (not name!) on the top of the front page.*

Maximal number of available points: 50.

Problem 1 (*5 points*)
Evaluate the following traces, and simplify the resulting expressions, by using properties of the Dirac matrices that we have derived in this course:

a) $\mathrm{Tr}\left[\gamma^\mu\gamma^\nu\right]$

b) $\mathrm{Tr}\left[\gamma^\mu\gamma^\nu\gamma^5\right]$

c) $\mathrm{Tr}\left[\gamma^\mu\gamma^\nu\gamma^\rho\right]$

d) $\mathrm{Tr}\left[\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma\gamma^5\right]$

Problem 2
One of our central results in the course so far can be written in the form

$$d\sigma = \frac{1}{4vE_AE_B}\left|\mathcal{M}\right|^2 d\Pi_n$$

a) What are the definitions of the various components that enter in this expression, and what is their physical meaning? (*6 points*)

b) Recap, in your own words, how we arrived at this expression, and state the most important intermediate results! [try to be complete, but concise – point subtractions for answers longer than 1 page!] (*5 points*)

Problem 3
One of the central ingredients that enter in the calculation of a cross section or decay rate is the invariant amplitude $\mathcal{M}$.

a) Briefly recap the algorithm to calculate the quantity $\mathcal{M}$ from a given interaction Hamiltonian! (*4 points*)

b) Apply this algorithm explicitly, for a suitable set of external states, to derive the Feynman rule for vertices appearing in the following theory of two real scalar fields ϕ_a and ϕ_b (what is the dimension of the parameter κ?): (*5 points*)

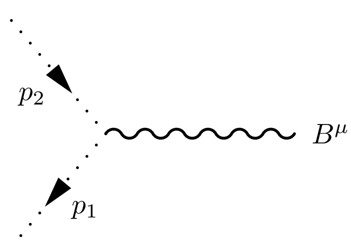
$$\mathcal{L} = \frac{1}{2}(\partial\phi_a)^2 - \frac{1}{2}m_a^2\phi_a^2 + \frac{1}{2}(\partial\phi_b)^2 - \frac{1}{2}m_b^2\phi_b^2 - \kappa\phi_a^2\phi_b^4 \,.$$

<u>Problem 4</u> (*5 points*)
Under what conditions is the axial vector current $j_A^\mu \equiv \bar{\psi}\gamma^\mu\gamma^5\psi$ conserved (divergence-free) for a Dirac field $\psi(x)$? Calculate the corresponding charge in terms of creation and annihilation operators!

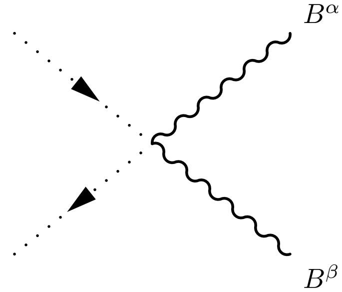
<u>Problem 5</u>
Consider a complex scalar field ϕ whose interaction with massless 'photons' B^μ is described by the following interaction Lagrangian:

$$\mathcal{L}_{\text{int}} = g^2 B_\mu B^\mu \left|\phi\right|^2 + igB_\mu(\phi\partial^\mu\phi^* - \phi^*\partial^\mu\phi)$$

This results in the following Feynman rules for the two vertices in this theory: (you don't have to derive this, but will be able to do so after the 2nd part of the course)



$$= \quad -ig(p_1 + p_2)^\mu$$



$$= \quad ig^2 g^{\alpha\beta}$$

Note that the arrows on the scalar lines are important, and indicate a 'charge flow' in the same way as for fermion propagators. For the first rule, the momenta are meant to be assigned in the same direction as these arrows. This implies that the momenta in this rule should be taken as incoming (p_2) and outgoing (p_1) momentum of particles ϕ. If you want instead to describe two incoming particles, you have to assign the momentum p_1 in the opposite direction: you then describe an incoming *anti-* particle, ϕ^*, for which you have to use $p_1 \to -p_1$ in the stated rule.

Using these Feynman rules,

a) write down an expression for the amplitude of $\phi\,\phi^* \to \gamma\gamma$ (i.e. two scalar particles of opposite charge annihilating to a pair of photons), to lowest order in g, and demonstrate that it satisfies the Ward identity! What does this imply when summing over all photon polarizations? (*4 points*)

b) calculate the differential cross section for two scalar particles annihilating into photons with arbitrary polarization! (*5 points*)

Problem 6

In the lecture, we have started to look at the process of Compton scattering in QED. We considered a photon with momentum k scattering on an electron with momentum p, into a final state consisting of a photon with momentum k' and an electron with momentum p'. With some straight-forward – but still lengthy – algebra, the expression for the spin-averaged squared matrix element that we arrived at can be simplified to

$$\overline{|\mathcal{M}|}^2 = 2e^4 \left[\frac{p\cdot k'}{p\cdot k} + \frac{p\cdot k}{p\cdot k'} + \left(1 + \frac{m_e^2}{p\cdot k} - \frac{m_e^2}{p\cdot k'}\right)^2 - 1 \right] .$$

In this problem, you will finalize the cross-section calculation for this process.

a) Consider the 'lab frame', i.e. the frame in which the electron initially is at rest. In this frame, calculate all contractions of 4-momenta that appear above in terms of the initial (ω) and final (ω') photon energies! (*3 points*)

b) Express ω' in terms of ω and θ, the scattering angle of the photon in the lab frame! (*2 points*)

c) Evaluate the 2-body relativistically invariant phase-space in this frame, writing it as $d\Pi_2 = f(\omega', \omega)\, d(\cos\theta)$ (*3 points*)

d) Calculate the differential cross section, $d\sigma/d(\cos\theta)$, take the non-relativistic limit and integrate it to get the full cross section! (*3 points*)