TAKE-HOME EXAM
FYS4170: QUANTUM FIELD THEORY

CANDIDATE NUMBER: [?????]

1. Trace of Dirac Matrix Products

The Dirac gamma matrices are in the chiral representation given in 2×2 block form as,

$$\gamma^0 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}; \quad \gamma^i = \begin{pmatrix} 0 & \sigma^i \\ \sigma & 0 \end{pmatrix}, \tag{1}$$

where σ^i are the Pauli sigma matrices,

$$\sigma^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma^2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. \tag{2}$$

The gamma matrices satisfy the anti-commutation relations

$$\{\gamma^\mu, \gamma^\nu\} \equiv \gamma^\mu \gamma^\nu = 2g^{\mu\nu} \times \mathbb{1}. \tag{3}$$

a. The trace of the product of two gamma matrices can be evaluated using the anti-commutator relation and the cyclic property of the trace of a matrix product,

$$\operatorname{Tr}[\gamma^\mu \gamma^\nu] = \operatorname{Tr}[2g^{\mu\nu} \times \mathbb{1} - \gamma^\nu \gamma^\mu] \tag{4}$$

$$= 2g^{\mu\nu} \operatorname{Tr} \mathbb{1} - \operatorname{Tr}[\gamma^\nu \gamma^\mu] \tag{5}$$

$$= 8g^{\mu\nu} - \operatorname{Tr}[\gamma^\mu \gamma^\nu]. \tag{6}$$

In this particular case $g^{\mu\nu}$ is a matrix element, not the metric, and can therefore be moved outside the trace operator as in 5. In 6 the cyclic property of the trace is employed. This yields

$$\operatorname{Tr}[\gamma^\mu \gamma^\nu] = 4g^{\mu\nu}. \tag{7}$$

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