# Drawings That We Use in Many-Body Physics

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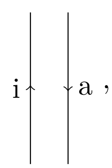
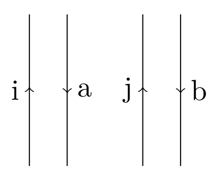
## 1 Slater determinants

Drawing the reference state will result in a drawing of nothing. A single-excited reference state is two vertical arrows

$$\Phi_i^a = $$ 

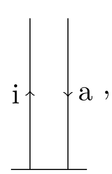
, (1)

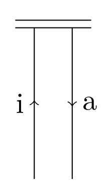
while the double-excited Slater determinant consists of four vertical arrows,

$$\Phi_{ij}^{ab} = $$ 

. (2)

The horizontal positions of the lines have no significance. If we want to indicate a bra or ket form we draw a couple of horizontal lines,

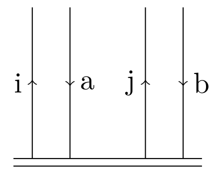
$$|\Phi_i^a\rangle = \{\hat{a}^\dagger \hat{i}\}\,|0\rangle = $$ 

, $$\langle\Phi_i^a| = \langle 0|\,\{\hat{i}^\dagger \hat{a}\} = $$ 

, (3)

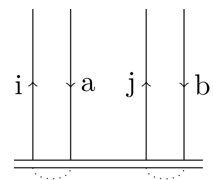
where $\{ABC\ldots\}$ is a normal ordered product relative to the Fermi vacuum. A double-excited ket state could be drawn like

$$\left|\phi_{ij}^{ab}\right\rangle = \{\hat{a}^\dagger \hat{b}^\dagger \hat{j}\hat{i}\}\left|0\right\rangle = \{(\hat{a}^\dagger \hat{i})(\hat{b}^\dagger \hat{j})\}\left|0\right\rangle = \tag{4}$$




This drawing could, however, also mean $\left|\phi_{ij}^{ba}\right\rangle$. The use of diagrams will be independent of this ambiguity, as long as one remains consistent. To be precise one can introduce dotted/dashed lines,

$$\left|\phi_{ij}^{ab}\right\rangle = \{\hat{a}^\dagger \hat{b}^\dagger \hat{j}\hat{i}\}\left|0\right\rangle = \{(\hat{a}^\dagger \hat{i})(\hat{b}^\dagger \hat{j})\}\left|0\right\rangle = \tag{5}$$




These indicate what index letters should be above and below one another.

## 2 One-Particle Operators

The one-electron operator on normal-ordered from is given by

$$\hat{U}_N = \sum_{pq} \left\langle p\right| \hat{u} \left|q\right\rangle \{\hat{p}^\dagger \hat{q}\}, \tag{6}$$

acting on a singly excited Slater determinant

$$\left|\Phi_i^a\right\rangle = \{\hat{a}^\dagger \hat{i}\}\left|0\right\rangle, \tag{7}$$

id est

$$\sum_{pq} \left\langle p\right| \hat{u} \left|q\right\rangle \{\hat{p}^\dagger \hat{q}\}\{\hat{a}^\dagger \hat{i}\}\left|0\right\rangle. \tag{8}$$

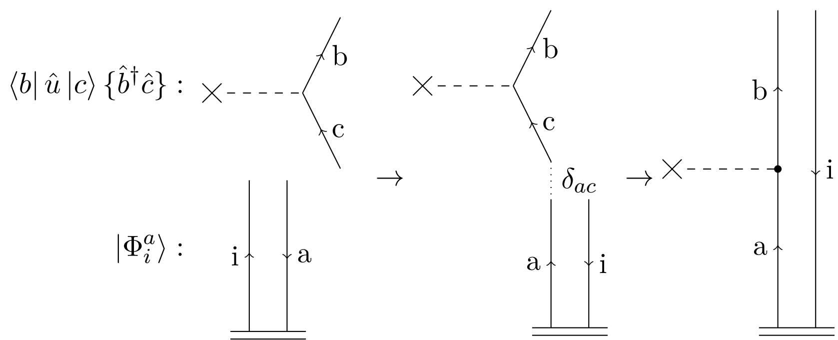
There are four different terms arising from this expression, depending on whether $p$ and $q$ represents particles or holes. Beginning with a *particle-particle* term,

$$\begin{aligned}
\left\langle b\right| \hat{u} \left|c\right\rangle \{\hat{b}^\dagger \hat{c}\}\{\hat{a}^\dagger \hat{i}\}\left|0\right\rangle &= \left\langle b\right| \hat{u} \left|c\right\rangle \{\hat{b}^\dagger \hat{c}\hat{a}^\dagger \hat{i}\}\left|0\right\rangle + \left\langle b\right| \hat{u} \left|c\right\rangle \{\hat{b}^\dagger \overbrace{\hat{c}\hat{a}^\dagger} \hat{i}\}\left|0\right\rangle \\
&= \left\langle b\right| \hat{u} \left|c\right\rangle \hat{b}^\dagger \hat{a}^\dagger \hat{i}\hat{c}\left|0\right\rangle + \left\langle b\right| \hat{u} \left|c\right\rangle \delta_{ac}\{\hat{b}^\dagger \hat{i}\} \\
&= 0 + \left\langle b\right| \hat{u} \left|c\right\rangle \delta_{ac}\left|\Phi_i^a\right\rangle,
\end{aligned} \tag{9}$$

giving non-zero contributions of the type

$$\langle b|\,\hat{u}\,|a\rangle \,\{\hat{b}^\dagger \hat{a}\}\,|\Phi_i^a\rangle = \langle b|\,\hat{u}\,|a\rangle \,|\Phi_i^b\rangle. \tag{10}$$

We can draw a graphical representation of this contraction process,




(11)

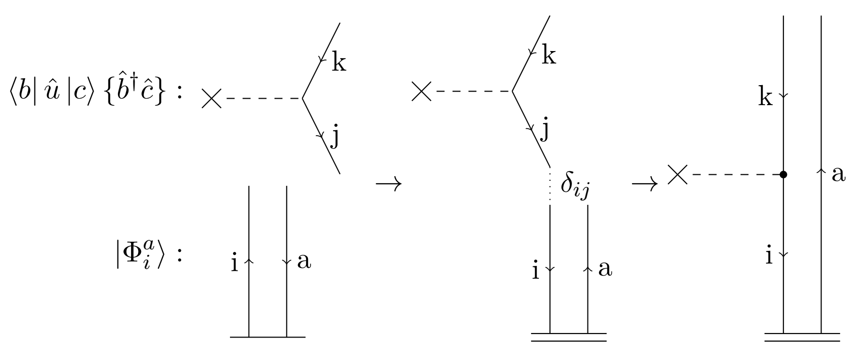
Now, let's consider a *hole-hole* term acting on the same single-excited Slater determinant,

$$\begin{aligned}
\langle j|\,\hat{u}\,|k\rangle \,\{\hat{j}^\dagger \hat{k}\}\{\hat{a}^\dagger \hat{i}\}\,|0\rangle &= \langle j|\,\hat{u}\,|k\rangle \,\{\hat{j}^\dagger \hat{k}\hat{a}^\dagger \hat{i}\}\,|0\rangle + \langle j|\,\hat{u}\,|k\rangle \,\{\hat{j}^\dagger \hat{k}\hat{a}^\dagger \hat{i}\}\,|0\rangle \\
&= -\,\langle j|\,\hat{u}\,|k\rangle \,\{\hat{k}\hat{a}^\dagger \hat{i}\hat{j}^\dagger\}\,|0\rangle + \delta_{ij}\,\langle i|\,\hat{u}\,|k\rangle \,\{\hat{k}\hat{a}^\dagger\}\,|0\rangle \\
&= 0 - \delta_{ij}\,\langle i|\,\hat{u}\,|j\rangle \,\{\hat{a}^\dagger \hat{k}\}\,|0\rangle \\
&= -\delta_{ij}\,\langle i|\,\hat{u}\,|j\rangle \,|\Phi_k^a\rangle\,,
\end{aligned} \tag{12}$$

meaning we are only left with non-zero contributions of the type,

$$\langle i|\,\hat{u}\,|j\rangle \,\{\hat{i}^\dagger \hat{k}\}\,|\Phi_i^a\rangle = -\,\langle i|\,\hat{u}\,|k\rangle \,|\Phi_k^a\rangle\,. \tag{13}$$

One can make a diagrammatic representation of this contraction as well,




(14)