18-875/19-739 Homework 4

Engineering and Economics of Electric Power Systems

Due on 02/25/2020 midnight (Canvas submission only) Student Name: Xi Chen

Key Equation:

$$\boldsymbol{S_i = V_i \times \sum_{j=1}^{busNum} V_j{}^* \times Y_{ij}{}^*} \qquad \textbf{eq. 1}$$

$$\boldsymbol{P_{cal} = \mathbf{real}(S_i)} \qquad \textbf{eq. 2}$$

$$\boldsymbol{Q_{cal} = imag(S_i)} \qquad \textbf{eq. 3}$$

$$\boldsymbol{\begin{bmatrix} P_{k+1} \\ Q_{k+1} \end{bmatrix} - \begin{bmatrix} P_L \\ Q_L \end{bmatrix} - \begin{bmatrix} P_{cal.k+1} \\ Q_{cal.k+1} \end{bmatrix} = \begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix}} \qquad \boldsymbol{eq.4}$$

$$\boldsymbol{J_1 = \frac{\partial P}{\partial \theta} \qquad J_2 = \frac{\partial P}{\partial |V|} \qquad J_3 = \frac{\partial Q}{\partial \theta} \qquad J_4 = \frac{\partial Q}{\partial |V|}}$$

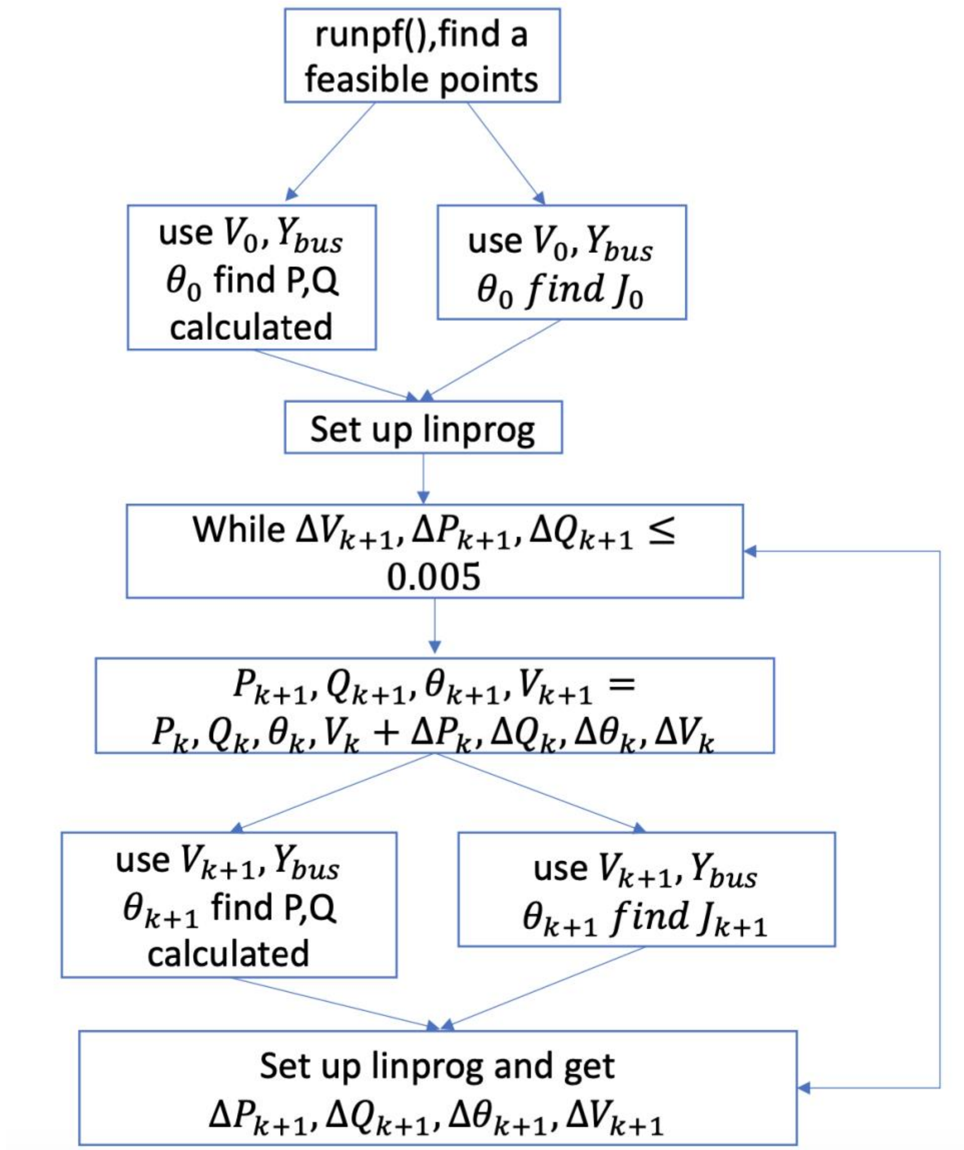
$$\boldsymbol{\begin{bmatrix} J_1 & J_2 \\ J_3 & J_4 \end{bmatrix} \times \begin{bmatrix} \Delta \theta_k \\ \Delta |V|_k \end{bmatrix} = \begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix}} \qquad \textbf{eq.5}$$

$$P, Q, V, \Theta^{k+1} = P, Q, V, \Theta^k + \Delta P, \Delta Q, \Delta V, \Delta \Theta_{\boldsymbol{k}} \qquad \textbf{eq.6}$$

Since the voltage and angle of the slack bus are constant so I set my lower bounds and upper bounds 0. Also, the voltage of the PV bus also constant, so I set the lower and upper bound of the voltage to zero.

$$\boldsymbol{LB} = \begin{bmatrix} {P_G}^{min} - P^{k+1} \\ {Q_G}^{min} - Q^{k+1} \\ -\frac{\pi}{2} - \theta^{k+1} \\ |V|^{min} - |V|^{k+1} \end{bmatrix} \qquad \boldsymbol{UB} = \begin{bmatrix} {P_G}^{max} - P^{k+1} \\ {Q_G}^{max} - Q^{k+1} \\ +\frac{\pi}{2} - \theta^{k+1} \\ |V|^{max} - |V|^{k+1} \end{bmatrix}$$

Code Algorithm:



Main Idea

$$\sum_{i=1}^{N_{\text{bus}}} F_i\left(P_{\text{gen}_i}\right) = \sum_{i=1}^{N_{\text{bus}}} \left(a_i + b_i P_{\text{gen}_i} + c_i P_{\text{gen}_i}^2\right)$$

Given a quadratic cost function, we can turn it into linear function by using

$$\min \sum_{j=1}^{N_{\text{gen}}} \left[F_j(P_{\text{gen}\,j}^0) + \frac{\mathrm{d}F_j(P_{\text{gen}\,j})}{\mathrm{d}P_{\text{gen}\,j}} \Delta P_{\text{gen}\,j} \right]$$

Since $F_j(P_{\text{gen}j}^0)$ is constant, we can ignore it in the objective function. So, the quadratic function is now in a linear form.

$$\min \sum_{j=1}^{N_{\text{gen}}} \left[\frac{\mathrm{d}F_j(P_{\text{gen}\,j})}{\mathrm{d}P_{\text{gen}\,j}} \Delta P_{\text{gen}\,j} \right]$$

Which $\frac{\mathrm{d}F_j(P_{\text{gen}\,j})}{\mathrm{d}P_{\text{gen}\,j}}$ is evaluated at Pgenj

Now, our objective function is related to the $\Delta \boldsymbol{P}$. Next, we will turn P, Q, V, Θ into $\Delta P, \Delta Q, \Delta V, \Delta\Theta_{\boldsymbol{k}}$.

$$P_{\text{gen}_j}^{\min} - P_{\text{gen}_j} \le \Delta P_{\text{gen}_j} \le P_{\text{gen}_j}^{\max} - P_{\text{gen}_j}$$
$$Q_{\text{gen}_j}^{\min} - Q_{\text{gen}_j} \le \Delta Q_{\text{gen}_j} \le Q_{\text{gen}_j}^{\max} - Q_{\text{gen}_j}$$
$$\left|V_i\right|^{\min} - \left|V_i\right| \le \Delta\left|V_i\right| \le \left|V_i\right|^{\max} - \left|V_i\right|$$

By doing that, each variable will be linear now.

Result

I cannot remember how many days I have devoted to this homework and I learn a lot from it, especially I get a deeper understanding of how the jacobian works and how to code the Ybus and nodal power flow equation which I have never done before. I also have learned how to manipulate the matrix in MATPOWER.

However, I didn’t get a feasible result because my code gets into an infinite loop after the first iteration and I cannot find a way to solve it. After the first iteration, my generation 1 in case9 will drop to 0.1 pu which is the minimum output. It cannot be that small. I guess there may be something wrong with my jacobian but I cannot figure out where is wrong. Since it is close to 11:59 now so I decided to submit.