18-875 Homework 4 hints

Linearized ACOPF (not DC OPF)

$$\min \sum_{i=1}^{N} C_i(P_{Gi})$$

N – number of buses

$h(x) = 0$ – node balancing equations

$g(x) \le 0$ – line flow limits (ignore)

$x^{min} \le x \le x^{max}$ – simple variable inequalities

Linearize the cost function:

$$C_i(P_{Gi}) = C_i(P_{Gi}^{k}) + \left.\frac{dC_i(P_{Gi})}{dP_{Gi}}\right|_{P_{Gi}^{k}} \cdot \Delta P_{G}$$

so, LP cost function is (ignoring the const term) on the next page.

Linearize the node balancing equations:

$$\begin{bmatrix} J_{11} & J_{12} \\ J_{21} & J_{22} \end{bmatrix} \begin{bmatrix} \Delta\theta \\ \Delta|V| \end{bmatrix} = \begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix}$$

You can arrange the Jacobian and Δx & $[\Delta P, \Delta Q]^T$ however you want but it must be CONSISTENT

$$\begin{bmatrix} J_{11} & J_{12} \\ J_{21} & J_{22} \end{bmatrix} \begin{bmatrix} \Delta\theta \\ \Delta|V| \end{bmatrix} = \begin{bmatrix} P^{k} - P_{load} - P_{calc} \\ Q^{k} - Q_{load} - Q_{calc} \end{bmatrix}$$

Jacobian has to be recalculated at each iteration.

Using $x = [\underset{N}{\Delta P_G} \; \underset{N}{\Delta Q_G} \; \underset{N}{\Delta\theta} \; \underset{N}{\Delta V}]$

So, input data for linprog is

$$C^T = \left[\left.\frac{\partial C_1(P_{G1})}{\partial P_{G1}}\right|_{P_{G1}^k} \cdots \left.\frac{\partial C_N(P_{GN})}{\partial P_{GN}}\right|_{P_{GN}^k} ; 0_{1\times N}, 0_{1\times N}, 0_{1\times N} \right]$$

<u>NOTE:</u> C^T has to be recalculated at each iteration. If there is no gen on a bus that entry is zero.

$$A_{eq} = {}_{2N}\left[\begin{array}{c|c|c} 0_{N\times N} & 0_{N\times N} & J_{2N\times 2N} \end{array} \right]$$

$$b_{eq} = \begin{bmatrix} P^k_{N\times 1} \\ Q^k_{N\times 1} \end{bmatrix} - \begin{bmatrix} P_{load\,N\times 1} \\ Q_{load\,N\times 1} \end{bmatrix} - \begin{bmatrix} P_{calc\,N\times 1} \\ Q_{calc\,N\times 1} \end{bmatrix}$$

You can assume unlimitted Q generation capacity but if you don't you NEED TO do PV $\leftrightarrow$ PQ bus conv.

$$lb = \begin{bmatrix} (P_G^{min} - P^k)_{N\times 1} \\ (Q_G^{min} - Q^k)_{N\times 1} \\ (-\frac{\pi}{2} - \theta^k)_{N\times 1} \\ (|V|^{min} - |V|^k)_{N\times 1} \end{bmatrix} \qquad ub = \begin{bmatrix} (P_G^{max} - P^k)_{N\times 1} \\ (Q_G^{max} - Q^k)_{N\times 1} \\ (+\frac{\pi}{2} - \theta^k)_{N\times 1} \\ (|V|^{max} - |V|^k)_{N\times 1} \end{bmatrix}$$

IGNORE Line flow Limits.

You can use Matpower ACPF (not ACOPF) to get

$$x^0 = [P^0 \; Q^0 \; \theta^0 \; |V|^0]$$

These are all the inputs you need to code a linearized ACOPF using Matlab and linprog().

Just follow steps 1–6 on page 372 in the textbook.

Remember that you are solving for Δx^k

and $x^{k+1} = x^k + \Delta x^k$