18-875/19-739 Engineering and Economics of Electric Energy Systems Take Home Final Exam **Due May 15, 2020**

Question 1. **(100 pts)**

Code in Matlab DCOPF with flexible demand using modified Matpower case (MPC) format. MPC format allows for a single fixed load connected to a bus so you will need to add an additional matrix describing the flexible load utility functions similar to generation cost functions matrix. You should allow for quadratic demand utility and generation cost functions so you must use quadprog() optimization function from Matlab or OPTI toolbox that you can download from the internet. Remember that a quadratic generator cost function must be strictly increasing between the generator output limits and the demand utility function must be strictly decreasing between its limits. MPC can define multiple generators on a bus but a single load. Although it is trivial to allow for multiple loads connected to a bus, because of MPC format limitations you can assume a single load connected to a bus. Note that there might be buses with no load or generators or with a single or multiple generators and a single load. This might look complicated and confusing but it is straight forward to code.
You should submit your solution through Canvas only and it should contain well commented code and test cases to demonstrate the functionality. Make sure that your solution is reproducible and with instructions how to run it to receive full credit.

In this project, we are asked to write DCOPF with flexible demand using a modified MATPOWER case. We can add a matrix to describe the demand curve which similar to the generation cost function matrix.

I used the formulation provided by the lecture slides to code my DCOPF with fixed generation first.

$$\min_{P_{g_i}} \sum_{i=1}^{N_g} C_i\left(P_{g_i}\right) \qquad \text{subject to}$$

$$-P_G + P_D + B \times \theta = 0$$

$$P_{g_i}^{\min} \le P_{g_i} \le P_{g_i}^{\max}$$

$$|F_{ij}| \le F_{ij}^{\max}$$

C_i	generator cost function	P_{g_i}	generator output
$P_{g_i}^{\min}, P_{g_i}^{\max}$	generator output limits	$F_{ij}^{\max}$	line power flow limit
F_{ij}	power flow between node i and node j		
B	imaginary part of admittance	$\theta^T = \begin{bmatrix} 0 & \theta_2 & \cdots & \theta_{N_b} \end{bmatrix}$	

To make sure my DCOPF algorithm is correct, I compared my result with MATPOWER DCOPF results with a fixed load first. I used three cases to verify my answer, case 9, case 14, and case 30.

Case 9:

x_my_case9_u

27x1 double

	1
1	86.5645
2	134.3776
3	94.0579
4	0
5	0
6	0
7	0
8	0
9	0

x_mat_case9.gen

	1	2
1	1	86.5645
2	2	134.3776
3	3	94.0579

Case 14:

x_my_case14_unvar

42x1 double

	1	2
1	220.9677	
2	38.0323	
3	1.1364e-...	
4	0	
5	0	
6	1.1749e-...	
7	0	
8	1.2343e-...	
9	0	
10	0	
11	0	
12	0	
13	0	
14	0	

x_mat_case14.gen

1	2
1	220.9677
2	38.0323
3	1.2524e-08
6	1.2524e-08
8	1.2524e-08

Case 30

x_my_case30_unvar

90x1 double

	1	2
1	44.7299	
2	58.2628	
3	0	
4	0	
5	0	
6	0	
7	0	
8	0	
9	0	
10	0	
11	0	
12	0	
13	15.7839	
14	0	
15	0	
16	0	
17	0	
18	0	
19	0	
20	0	
21	0	
22	22.3136	
23	15.7839	
27	32.3259	

x_mat_case30.gen

	1	2
1	1	44.7299
2	2	58.2628
3	22	22.3136
4	27	32.3259
5	23	15.7839
6	13	15.7839

We can find the results are matched. Next, I adapted this algorithm to fit the flexible load function. We just need to make small adaptions to the original algorithm by changing the demands from fixed constants to variables.

In previous formulation, the demand is fixed value. Now, Pd is flexible and it becomes one of the variables. Our variables are [Pg, Pd, θ].
The objective function is:

$$\min_{P_{g_i}, P_{D_i}} \left(\sum_{j=1}^{N_g} C_i\left(P_{g_i}\right) + \sum_{i=1}^{N_d} -U_i\left(P_{D_i}\right) \right)$$

Since the demand function is strictly decreasing in our case, we don't need the negative sign now. And the objective becomes:

$$\text{Min } \sum_{j=1}^{Ng} A * Pg_j^2 + \mathrm{B} * Pg_j + C\} + \sum_{i=1}^{Nd} \{\mathrm{a} * \mathrm{Pd}_i^2 + \mathrm{b} * Pd_i + c\}$$

Where Pg is power generation for each generator and Pd is the demand for each load, and C,c is constant which can be ignored for optimization purposes.

The total generation cost = $A * Pg^2 + \mathrm{B} * Pg + C$ and it is strictly increasing.
The total demand cost = $a * Pd^2 + \mathrm{b} * Pd + c$ and it is strictly decreasing.

If we take a derivative with respect to the generation, we can get the generation price. It represents what is the generation price for the next 1 MW produced by the current generator. Same way, we can get the demand price for another 1 MW at the current bus.

The followings are the flexible load utility functions I added for each case.

```
%|bus|load_max|load_min|n||a|   |b|  |c|
case9.load = [
    5  90 0 3 0.006 -11.4 2000;
    7 100 0 3 0.005 -11.5 2000;
    9 125 0 3 0.004   -12 2000;
];
```

```
case14 =loadcase(case14);
case14.load = [
    2 135 0 3 0.006 -31.7 4000;
    3 100 0 3 0.005   -32 4000;
    4 125 0 3 0.004 -32.5 4000;
];
```

```
case30 =loadcase(case30);
case30.load = [
    7 610 0 3 0.006 -13.5 6000;
    8 600 0 3 0.005   -13 6000;
   21 530 0 3 0.004 -13.3 6000;
];
```

The generation price will increase as the generation increase. On the contrary, the demand price will decrease as the demand increase. It's plausible because the generation will use more expensive energy sources and the demand will increase since price become cheaper.

Next, the results will be presented.

```
current case is:case9
The generation cost is $1941.035304 each hour
The demand cost is $4504.493607 each hour
The total cost is $6445.528911 each hour
Generator 1 output is 28.435652 MW, the price is $11.255843/MW
Generator 2 output is 59.152021 MW, the price is $11.255843/MW
Generator 3 output is 41.860586 MW, the price is $11.255843/MW
Load 5 output is 12.013043 MW, the price is $11.255843/MW
Load 7 output is 24.415651 MW, the price is $11.255843/MW
Load 9 output is 93.019564 MW, the price is $11.255843/MW
```

If the generators are not at limits and the transmission line is not congestion, we can see the demand price and generator price are all the same when the problem is optimal.

```
current case is:case14
The generation cost is $4079.992467 each hour
The demand cost is $6944.624805 each hour
The total cost is $11024.617271 each hour
Generator 1 output is 134.894550 MW, the price is $31.608825/MW
Generator 2 output is 23.217651 MW, the price is $31.608825/MW
Generator 3 output is 0.000000 MW, the price is $40.000000/MW
Generator 6 output is 0.000000 MW, the price is $40.000000/MW
Generator 8 output is 0.000000 MW, the price is $40.000000/MW
Load 2 output is 7.597892 MW, the price is $31.608825/MW
Load 3 output is 39.117470 MW, the price is $31.608825/MW
Load 4 output is 111.396838 MW, the price is $31.608825/MW
```

A similar thing happened to case14. Generator 1 and 2 are the only two generators are open, and they have the same price as the demand price.

```
current case is:case30
The generation cost is $914.233065 each hour
The demand cost is $14657.858126 each hour
The total cost is $15572.091191 each hour
Generator 1 output is 80.000000 MW, the price is $5.200000/MW
Generator 2 output is 80.000000 MW, the price is $4.550000/MW
Generator 22 output is 18.953327 MW, the price is $3.369166/MW
Generator 27 output is 47.409892 MW, the price is $4.040797/MW
Generator 23 output is 0.000000 MW, the price is $3.000000/MW
Generator 13 output is 40.000000 MW, the price is $5.000000/MW
Load 7 output is 186.953075 MW, the price is $11.256563/MW
Load 8 output is 44.595844 MW, the price is $12.554042/MW
Load 21 output is 34.814300 MW, the price is $13.021486/MW
```

Since the transmission congestion and generator 1 ,2 are exhausted in case 30, the prices of each load and generator are not the same.

```
% if no congestion in the line and generator never reach its limitation
rows = length(case30.branch(:,1));
maxLine = repmat(999,rows,3);
case30.branch(:,6:8) = maxLine;
rows2 = length(case30.gen(:,9));
maxGen = repmat(999,rows2,1);
case30.gen(:,9) = maxGen;
```

Then, I tried to remove all the constraints and run it again.

```
current case is:case30
The generation cost is $6847.701803 each hour
The demand cost is $5106.853819 each hour
The total cost is $11954.555622 each hour
Generator 1 output is 189.985040 MW, the price is $9.599402/MW
Generator 2 output is 224.268617 MW, the price is $9.599402/MW
Generator 22 output is 68.795213 MW, the price is $9.599402/MW
Generator 27 output is 380.659569 MW, the price is $9.599402/MW
Generator 23 output is 131.988032 MW, the price is $9.599402/MW
Generator 13 output is 131.988032 MW, the price is $9.599402/MW
Load 7 output is 325.049866 MW, the price is $9.599402/MW
Load 8 output is 340.059839 MW, the price is $9.599402/MW
Load 21 output is 462.574799 MW, the price is $9.599402/MW
```

We can find the prices are the same for all the generators and loads and my thought has proved.

It is good idea to set the load as a variable without separate them in different periods and it is quite useful and can be applied in many situations.