Unique List of "Reactions"

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Abstract

1 Introduction

The list below is a unique list provided so as to generate a protocol for deleting "reactions". "Reactions" are defined as distinct formulae forming the RHS of the relevant o.d.e. As a way of getting to a reduced system of NVC we have not altered the neuron model nor the ??

2 Synaptic Cleft and Astrocyte

THE LIST

Table 1

Reaction	Description	
		Astrocyte and Synaptic Cleft.....
1	J_{K_k}	potassium (K^+) channel
2	$2J_{NaK_k}$	K^+ sodium (Na^+)-ATPase pump
3	J_{NKCC1_k}	Na^+, K^+ and Cl flux through the NKCC1 channel
4	J_{KCC1_k}	Cl and K^+ flux through the KCC1 channel
5	$J_{KNEtoSC}$	flux of K^+ into the SC based on the extracellular K^+
6	J_{Na_k}	Na^+ flux through the Na^+ channel
7	J_{NBC_k}	Na^+ and HCO3 flux through the NBC channel
8	$J_{NaNEtoSC}$,	flux of Na^+ into the SC based on the extracellular K^+ = $J_{KNEtoSC}$
9	J_{IP3_k}	flux of calcium (Ca^{2+}) through the IP3R channel
10	J_{pump_k}	flux of Ca^{2+} through the uptake pump
11	J_{ERleak_k}	flux of Ca^{2+} through the leak channel
12	$\frac{J_{TRPV_k}}{r_{buff}}$	flux of Ca^{2+} through TRPV4 channel
13	J_{CICR_k}	flux of Ca^{2+} through CICR into astrocytic cytosol
14	$r_h G$	rate of IP3 production in astrocyte due to glutamate receptors
15	$k_{deg} IP3_k$	Rate constant for IP3 degradation in astrocyte
16	$V_{eet} \max(Ca_k - c_{k_{min}}, 0)$	production of astrocytic epoxyeicosatrienoic acid (EET) concentration
17	$k_{eet} eet_k$	degredation of astrocytic epoxyeicosatrienoic acid (EET) concentration
18	J_{BK_k}	
19	J_{Cl_k}	
20	$\phi_n w_\infty$	

21	$-\phi_n w_k$	
22	$k_{on}K_{inh}$	inactivation variable h_k of the astrocytic IP_3R channel
23	$-k_{on}(Ca_k + K_{inh})h_k$	inactivation variable h_k of the astrocytic IP_3R channel
24	$\frac{m_{inf}}{\tau_{TRPV4}}$	TRPV4 channel open probability
25	$\frac{-m_k}{\tau_{TRPV4}}$	TRPV4 channel open probability
26	$\frac{(AA_i - AA_k)}{\tau_{AA}}$	concentration of arachidonic acid in the astrocyte AA_k.
27	$\frac{AA_m AA_{max}}{(AA_m + max(Ca_k - Ca_0))^2 \frac{dCa_k}{dt}}$	concentration of arachidonic acid in the astrocyte AA_k.
PVS.....		
28	$\frac{J_{BK_k}}{VR_{pk}}$	K^+ concentration in the perivascular space (PVS)
29	$\frac{J_{KIR_i}}{VR_{pi}}$	K^+ concentration in the PVS
30	$K_{decay_p}(K_p - K_{min_p})$	K^+ concentration in the PVS
31	$\frac{J_{TRPV_k}}{VR_{pk}}$	Ca^{2+} concentration in the PVS
32	$\frac{J_{VOCC_i}}{VR_{pi}}$	Ca^{2+} concentration in the PVS
33	$-Ca_{decay_p}(Ca_p - Ca_{min_p})$	Ca^{2+} concentration in the PVS
34	$\frac{m_{\infty_k}}{t_{TRPV_k}}$	The open probability of the transient receptor potential vanniloid-related 4 (TRPV4) channel
35	$\frac{-m_k}{t_{TRPV_k}}$	The open probability of the TRPV4 channel
SMC		
36	$J_{IP_{3i}}$	Cytosolic Ca^{2+} in the smooth muscle cell (SMC)
37	$-J_{SR_{uptake_i}}$	Cytosolic Ca^{2+} in the SMC
38	$+J_{CICR_i}$	Cytosolic Ca^{2+} in the SMC
39	$-J_{extrusion_i}$	Cytosolic Ca^{2+} in the SMC
40	$J_{SR_{leak_i}}$	Cytosolic Ca^{2+} in the SMC
41	$-J_{VOCC_i}$	Cytosolic Ca^{2+} in the SMC
42	J_{Na/Ca_i}	Cytosolic Ca^{2+} in the SMC
43	$-0.1J_{stretch_i}$	Cytosolic Ca^{2+} in the SMC
44	$J^{SMC-EC}_{Ca^{2+}-coupling_i}$	Cytosolic Ca^{2+} in the SMC
45	$-\gamma_v J_{NaK_i}$	Membrane potential of the SMC
46	$-\gamma_v J_{Cl_i}$	Membrane potential of the SMC
47	$-\gamma_v J_{K_i}$	Membrane potential of the SMC
48	$-\gamma_v J_{KIR_i}$	Membrane potential of the SMC
49	$V^{SMC-EC}_{coupling_i}$	Membrane potential of the SMC
50	$\lambda_i\,(K_{act_i})$	Open state probability of Ca^{2+}-activated K^+ channels
51	$-\lambda_i w_i$	Open state probability of Ca^{2+}-activated K^+ channels
		fluxes for K^+ in SMC are 45, 47, 48 divided by γ_v
52	$-J_{degrad_i}$	inositol trisphosphate (IP_3) concentration in the SMC
53	$J^{SMC-EC}_{IP_3-coupling_i}$	IP_3 concentration in the SMC
54	$\frac{AA_k}{\tau_{AA}}$	Arachidonic acid in the SMC
55	$\frac{-AA_i}{\tau_{AA}}$	Arachidonic acid in the SMC
56	$\frac{1}{1+exp(\frac{(NO_i - NO_{rest})}{R_{NO}})}\frac{V_a AA_i}{K_a + AA_i}$	20-HETE in the SMC

57	$\frac{V_f AA_i}{K_f + AA_i}$	20-HETE in the SMC
58	$-\lambda_h H_i$	20-HETE in the SMC
EC		
59	$J_{IP_{3j}}$	Cytosolic Ca^{2+} concentration in the endothelial cell (EC)
60	$-J_{ER_{uptake_j}}$	Cytosolic Ca^{2+} concentration in the EC
61	J_{CICR_j}	Cytosolic Ca^{2+} concentration in the EC
62	$-J_{extrusion_j}$	Cytosolic Ca^{2+} concentration in the EC
63	$J_{ER_{leak_j}}$	Cytosolic Ca^{2+} concentration in the EC
64	J_{cation_j}	Cytosolic Ca^{2+} concentration in the EC
65	J_{0_j}	Cytosolic Ca^{2+} concentration in the EC
66	$-J_{stretch_j}$	Cytosolic Ca^{2+} concentration in the EC
67	$-\frac{1}{C_{m_j}}(I_{K_j} + I_{R_j})$	Membrane potential of the EC
68	$-V_{coupling_j}^{SMC-EC}$	Membrane potential of the EC
69	J_{PLC}	IP_3 concentration of the EC
70	$-J_{degrad_j}$	IP_3 concentration of the EC
71	$-J_{IP_3-coupling_j}^{SMC-EC}$	IP_3 concentration of the EC

Table 2

Rctn no.	Reaction name	1-norm	2-norm	inf-norm	(location of max)	equation
0	N/A	0	0	0	E_t	
1	potassium (K^+) channel	53.4665	12.0407	11.5731	v_i	
2	K^+ sodium (Na^+)-ATPase pump	NaN	NaN	NaN	NaN	
3	Na^+, K^+ and Cl flux through the NKCC1 channel	1.8031	1.17673	1.14719	w_k	
4	Cl and K^+ flux through the KCC1 channel J_{KCC1_k}	0.0213536	0.00855816	0.00916892	w_k	$G_{KCC1_k}\phi \ln\left(\frac{K_s Cl_s}{K_k Cl_k}\right)$
5	flux of K^+ into the SC based on the extracellular K^+	0.232818	0.921282	0.912761	w_k	
6	Na^+ flux through the Na^+ channel	6.69095	2.28405	5.53287	v_k	
7	Na^+ and HCO3 flux through the NBC channel J_{NBC_k}	3.04E-01	0.56255	0.501727	w_k	$G_{NBC_k}(v_k - E_{NBC_k})$
8	flux of K^+ into the SC based on the extracellular K^+ = JKNEtoSC	2.37E-02	1.17E-01	1.77E-01	w_k	
9	flux of Ca^{2+} through the IP3R channel	5.63E+00	2.09388	1.00846	h_k	

10	flux of Ca^{2+} through the uptake pump	3179.47	2068.65	260.773	ee_t	
11	flux of Ca^{2+} through the leak channel J_{ERleak_k}	0.098875	0.0412828	0.0276192	w_k	$P_L\left(1-\frac{Ca_k}{s_k}\right)$
12	flux of Ca^{2+} through TRPV4 channel	1.34165	0.590297	0.539168	w_k	
13	flux of Ca^{2+} through CICR into astrocytic cytosol J_{CICR_k}	0.0422956	0.0752756	0.111209	ee_t	$C_k\frac{s_k^4}{sc_k^4+s_k^4}\frac{Ca_k^4}{cc_k^4+Ca_k^4}$
14	rate of IP3 production in astrocyte due to glutamate receptors	6.62065	2.3194	1.00846 h_k		
15	Rate constant for IP3 degradation in astrocyte	357.233	391.514	129.627	I_k	
16	production of astrocytic epoxyeicosatrienoic acid (EET) concentration	1.25478	1.10137	1	ee_t	
17	degredation of astrocytic epoxyeicosatrienoic acid (EET) concentration	2444.74	2033.68	420.738	ee_t	
18	K^+ flux through the big potassium (BK) channel (μM s^{-1}):J_{BK_k}	0.246612	0.194939	0.669853	K_p	$G_{BK_k}w_k\left(v_k-E_{BK_k}\right)$
19	chlorine (Cl^-) flux through the Cl^- channel (μM s^{-1}):	4.60284	1.91901	1.15891	Cl_k	
20	The time constant associated with the opening of the BK channel (s^{-1})	1.24139	1.01878	1	w_k	
21	The time constant associated with the opening of the BK channel (s^{-1})	1616.18	506.339	37.5492	w_k	
22	inactivation variable hk of the astrocytic IP3R channel	6.03146	2.24241	1	h_k	
23	inactivation variable hk of the astrocytic IP3R channel	3175.03	1886.23	269.894	h_k	
24	TRPV4 channel open probability	2.38878	1.16172	0.999886	m_k	
25	TRPV4 channel open probability	6643.8	5033.35	927.663	ee_t	

26	concentration of arachidonic acid in the astrocyte AAk.	0.0316276	0.0429046	0.150997	AA_k	$\frac{(AA_i - AA_k)}{\tau_{AA}}$
27	concentration of arachidonic acid in the astrocyte AA	0.0457825	0.138873	0.654171	AA_k	$\frac{AA_m AA_{max}}{(AA_m + max(Ca_k - Ca_0))^2 \frac{dCa_k}{dt}}$
28	K^+ concentration in the perivascular space (PVS)	0.248199	0.195333	0.669853	K_p	possible repetition of reaction 18
29	K^+ concentration in the PVS	0.00463886	0.0034176	0.00140806	K_p	this is a repetition of reaction 48 see below in SMC
30	K^+ concentration in the PVS	0.374649	0.270297	0.111932	K_p	$K_{decay_p}(K_p - K_{min_p})$
31	calcium (Ca^{2+}) concentration in the PVS	0.0609743	0.0530334	0.0578325	Ca_p	repetition of reaction 12 see above
32	Ca^{2+} concentration in the PVS	0.0211558	0.0183736	0.0235179	Ca_p	repetition of reaction 41 see below in SMC
33	Ca^{2+} concentration in the PVS	NaN	NaN	NaN	NaN	
34	N/A	0	0	0	E_t	
35	N/A	0	0	0	E_t	
36	Cytosolic Ca^{2+} in the smooth muscle cell (SMC)	2.03695	0.619121	0.243314	AMp	
37	Cytosolic Ca^{2+} in the SMC	1.41071	1.76962	9.76799	AMp	
38	Cytosolic Ca^{2+} in the SMC	1.95459	1.43214	1.65995	s_i	
39	Cytosolic Ca^{2+} in the SMC	29.1442	16.4226	10.3864	AMp	
40	Cytosolic Ca^{2+} in the SMC	5.34223	4.79812	10.7775	AMp	
41	Cytosolic Ca^{2+} in the SMC	3.84721	1.15592	0.385505	AM	
42	Cytosolic Ca^{2+} in the SMC	1.37241	0.42607	0.328498	AMp	
43	Cytosolic Ca^{2+} in the SMC	4.73275	1.4221	0.477013	w_i	
44	Cytosolic Ca^{2+} in the SMC	4.20672	1.16217	0.383345	AM	
45	Membrane potential of the SMC	NaN	NaN	NaN	NaN	
46	Membrane potential of the SMC	2.50098	0.781827	0.302132	w_i	

47	Membrane potential of the SMC	30.6122	15.183	10.0965	AMp	
48	Membrane potential of the SMC J_{KIR_i}	0.334658	0.108702	0.124455	w_i	$G_{KIR_i}(v_i - v_{KIR_i})$
49	Membrane potential of the SMC$V_{coupling_i}^{SMC-EC}$	0.794953	0.258294	0.123328	w_i	$-G_{coup}(v_i - v_j)$
50	Open state probability of Ca^{2+}-activated K^+ channels	26.051	14.1635	10.102	AMp	
51	Open state probability of Ca^{2+}-activated K^+ channels	70.4464	68.3471	61.955	w_i	
52	inositol trisphosphate (IP3) concentration in the SMC	20.9808	10.6533	10.1006	AMp	
53	IP3 concentration in the SMC	2.92977	1.12559	1	I_i	
54	Arachidonic acid in the SMC	3.40E+00	1.70348	1	AA_i	
55	Arachidonic acid in the SMC	2.03E+14	6.22E+14	3.16E+15	AA_i	
56	20-HETE in the SMC	5.07E+01	7.1214	1	E_t	
57	20-HETE in the SMC	0.998665	0.675777	0.599416	H_i	$\frac{1}{1+exp(\frac{(NO_i - NO_{rest})}{R_{NO}})} \frac{V_a AA_i}{K_a + AA_i}$
58	20-HETE in the SMC	536.714	589.716	725.883	H_i	
59	Cytosolic Ca^{2+} concentration in the endothelial cell (EC)	4.31153	1.22434	0.725525	s_j	
60	Cytosolic Ca^{2+} concentration in the EC	1.01901	0.990576	0.918791	s_j	
61	Cytosolic Ca^{2+} concentration in the EC	12.5428	12.495	13.2761	s_j	
62	Cytosolic Ca^{2+} concentration in the EC	37.3107	19.2012	10.1376	AMp	
63	Cytosolic Ca^{2+} concentration in the EC$J_{ER_{leak_j}}$	0.0380883	0.0370073	0.0406557	s_j	$L_j s_j$
64	Cytosolic Ca^{2+} concentration in the EC	1.63543	0.466454	0.223622	Ca_j	
65	Cytosolic Ca^{2+} concentration in the EC	1.11102	0.321257	0.147914	Ca_j	
66	Cytosolic Ca^{2+} concentration in the EC	3.01928	0.848642	0.446334	s_j	
67	Membrane potential of the EC	2.35771	0.703111	0.512982	v_j	
68	N/A	0	0	0	E_t	

69	IP3 concentration of the EC	6.9521	1.98954	1	I_i
70	IP3 concentration of the EC	49.3683	7.14973	10.1178	AMp

Table 3 lists the reaction along with the error associated with the removal of that specific reaction when compared to the experiments of Berwick et al. Table 3

Rctn no.	Reaction name	normalized HBO Error; 2-norm	normalized HBR Error; 2-norm
0	N/A	0.000468269	0.000871442
1	potassium (K+) channel	NaN	NaN
2	K+ sodium (Na+)-ATPase pump	NaN	NaN
3	Na+, K+ and Cl flux through the NKCC1 channel	0.00195538	0.000500168
4	Cl and K+ flux through the KCC1 channel	0.000466595	0.000875713
5	flux of K+ into the SC based on the extracellular K+	0.0135393	0.00639256
6	Na+ flux through the Na+ channel	0.017721	0.0073648
7	Na+ and HCO3 flux through the NBC channel	0.0014388	0.000539501
8	flux of K+ into the SC based on the extracellular K+ = JKNEtoSC	NaN	NaN
9	flux of Ca2+ through the IP3R channel	0.029407	0.0101798
10	flux of Ca2+ through the uptake pump	0.00780579	0.00267052
11	flux of Ca2+ through the leak channel	0.000475598	0.000864279
12	flux of Ca2+ through TRPV4 channel	0.00196372	0.00117208
13	flux of Ca2+ through CICR into astrocytic cytosol	0.000512237	0.000949758
14	rate of IP3 production in astrocyte due to glutamate receptors	0.029407	0.0101798
15	Rate constant for IP3 degradation in astrocyte	0.0158477	0.00536754
16	production of astrocytic epoxyeicosatrienoic acid (EET) concentration	0.00566864	0.00263931
17	degredation of astrocytic epoxyeicosatrienoic acid (EET) concentration	0.00201507	0.000941013
18		0.0191554	0.0077376

19		0.0125768	0.00589543
20		0.0191554	0.00773761
21		NaN	NaN
22	inactivation variable hk of the astrocytic IP3R channel	0.029407	0.0101798
23	inactivation variable hk of the astrocytic IP3R channel	0.00777447	0.00266655
24	TRPV4 channel open probability	0.00196378	0.00117213
25	TRPV4 channel open probability	0.0082669	0.00272957
26	concentration of arachidonic acid in the astrocyte AAk.	0.00261581	0.00123312
27	concentration of arachidonic acid in the astrocyte AA	0.00696495	0.00326088
28	K+ concentration in the perivascular space (PVS)	0.0191541 0.00773728	
29	K+ concentration in the PVS 0.00463886	0.000469189	0.000871822
30	K+ concentration in the PVS	0.00242898	0.00132313
31	calcium (Ca2+) concentration in the PVS	NaN	NaN
32	Ca2+ concentration in the PVS	0.000468217	0.000871653
33	Ca2+ concentration in the PVS	NaN	NaN
34	N/A	0.000468269	0.000871442
35	N/A	0.000468269	0.000871442
36	Cytosolic Ca2+ in the smooth muscle cell (SMC)	0.00212447	0.00102861
37	Cytosolic Ca2+ in the SMC	0.00268971	0.00203337
38	Cytosolic Ca2+ in the SMC	0.00875328	0.00455686
39	Cytosolic Ca2+ in the SMC	0.0144208	0.00701218
40	Cytosolic Ca2+ in the SMC	NaN	NaN
41	Cytosolic Ca2+ in the SMC	0.0116207	0.00582014
42	Cytosolic Ca2+ in the SMC	0.00583438	0.00390169
43	Cytosolic Ca2+ in the SMC	0.00364038	0.002412
44	Cytosolic Ca2+ in the SMC	0.00611134	0.00166663
45	Membrane potential of the SMC	NaN	NaN
46	Membrane potential of the SMC	0.00287015	0.00153477
47	Membrane potential of the SMC	0.0149869	0.00710705
48	Membrane potential of the SMC	0.0199942	0.00790603
49	Membrane potential of the SMC	0.00193546	0.000823211
50	Open state probability of Ca2+-activated K+ channels	0.0149869	0.00710705
51	Open state probability of Ca2+-activated K+ channels	0.0190485	0.00847316
52	inositol trisphosphate (IP3) concentration in the SMC	NaN	NaN
53	IP3 concentration in the SMC	0.00120016	0.000903448

54	Arachidonic acid in the SMC	0.00429406	0.00277598
55	Arachidonic acid in the SMC	5.60641	0.82813
56	20-HETE in the SMC	0.00560948	0.00282501
57	20-HETE in the SMC	0.00240002	0.00195237
58	20-HETE in the SMC	0.0149869	0.00710705
59	Cytosolic Ca2+ concentration in the endothelial cell (EC)	0.00410624	0.001356
60	Cytosolic Ca2+ concentration in the EC	0.000479328	0.000871541
61	Cytosolic Ca2+ concentration in the EC	0.00047282	0.000860244
62	Cytosolic Ca2+ concentration in the EC	0.0155095	0.00716025
63	Cytosolic Ca2+ concentration in the EC	0.000465781	0.000871406
64	Cytosolic Ca2+ concentration in the EC	0.00147519	0.00094885
65	Cytosolic Ca2+ concentration in the EC	0.00104536	0.000894999
66	Cytosolic Ca2+ concentration in the EC	0.00268227	0.00112258
67	Membrane potential of the EC	0.00348108	0.00112305
68	N/A	0.000468269 0.000871442	
69	IP3 concentration of the EC	0.00757592	0.00210779
70	IP3 concentration of the EC	NaN	NaN

3 Notes on Table 3 above

We should note that removing reaction 7 given by the Na^+ and HCO_3^- flux through the NBC channel (μM s^{-1}):

$$J_{NBC_k} = G_{NBC_k}\left(v_k - E_{NBC_k}\right) \tag{1}$$

means the NBC flux is zero. HCO_3^- concentration in the synaptic cleft (SC) (μM):

$$\frac{dHCO_{3_s}}{dt} = \frac{1}{VR_{sk}}\left(-2J_{NBC_k}\right) \tag{2}$$

HCO_3^- concentration in the astrocyte (μM):

$$\frac{dHCO_{3_k}}{dt} = 2J_{NBC_k} \tag{3}$$

Hence on the basis of the two equations defined above we can eliminate HCO_3^- from both the SC and Astrocyte. This removes 2 equations from the ode set. However this must be accompanied by the elimination of the definition of the Nernst potential for the NBC channel (mV):

$$E_{NBC_k} = \frac{\phi}{z_{NBC}} \ln\left(\frac{Na_s {HCO_{3_s}}^2}{Na_k {HCO_{3_k}}^2}\right) \tag{4}$$

since HCO_3^- is now effectively zero in both the synaptic cleft and the astrocyte.

Reaction 49 is a membrane voltage coupling equation between SMC and EC given by heterocellular electrical coupling between SMCs and ECs :

$$V_{coupling_i}^{SMC-EC} = -G_{coup}(v_i - v_j) \tag{5}$$

This does not seem to have any effect on the profiles for HbO and HbR.

Reaction 61 defines the Ca^{2+} induced Ca^{2+} release (CICR) mechanism in the EC given by:

$$J_{CICR_j} = C_j \frac{s_j^2}{s_{cj}^2 + s_j^2} \frac{Ca_j^4}{c_{cj}^4 + Ca_j^4} \tag{6}$$

Table 4 shows the reactions ranked from smallest to largest for the error between the numerical output and the experiment for HbR.
Table 4.

Rctn no	normalized HBO Error; 2-norm	normalized HBR Error; 2-norm
3	0.00195538	0.000500168
7	0.0014388	0.000539501
49	0.00193546	0.000823211
61	0.00047282	0.000860244
11	0.000475598	0.000864279
63	0.000465781	0.000871406
0	0.000468269	0.000871442
34	0.000468269	0.000871442
35	0.000468269	0.000871442
68	0.000468269	0.000871442
60	0.000479328	0.000871541
32	0.000468217	0.000871653
29	0.000469189	0.000871822
4	0.000466595	0.000875713
65	0.00104536	0.000894999
53	0.00120016	0.000903448
17	0.00201507	0.000941013
64	0.00147519	0.00094885
13	0.000512237	0.000949758
36	0.00212447	0.00102861
66	0.00268227	0.00112258
67	0.00348108	0.00112305
12	0.00196372	0.00117208
24	0.00196378	0.00117213
26	0.00261581	0.00123312
30	0.00242898	0.00132313
59	0.00410624	0.001356
46	0.00287015	0.00153477
44	0.00611134	0.00166663
57	0.00240002	0.00195237
37	0.00268971	0.00203337
69	0.00757592	0.00210779

43	0.00364038	0.002412
16	0.00566864	0.00263931
23	0.00777447	0.00266655
10	0.00780579	0.00267052
25	0.0082669	0.00272957
54	0.00429406	0.00277598
56	0.00560948	0.00282501
27	0.00696495	0.00326088
42	0.00583438	0.00390169
38	0.00875328	0.00455686
15	0.0158477	0.00536754

41 Cytosolic Ca2+ in the SMC 0.0116207 0.00582014
19 0.0125768 0.00589543
5 flux of K+ into the SC based on the extracellular K+ 0.0135393 0.00639256
39 Cytosolic Ca2+ in the SMC 0.0144208 0.00701218
47 Membrane potential of the SMC 0.0149869 0.00710705
50 Open state probabil-