

# Component Analysis V. Sobol Analysis

(What I've been working on in Matlab)

# System Notation

$$\dot{\vec{x}} = M\vec{x} \quad (1)$$

$$\begin{bmatrix} \dot{\vec{x}}_1 \\ \dot{\vec{x}}_2 \\ \dot{\vec{x}}_3 \\ \vdots \end{bmatrix} = \begin{bmatrix} A_1 & B_{1,2} & B_{1,3} & \dots \\ B_{2,1} & A_2 & B_{2,3} & \dots \\ B_{3,1} & B_{3,2} & A_3 & \dots \\ \vdots & \vdots & \vdots & \ddots \end{bmatrix} \begin{bmatrix} \vec{x}_1 \\ \vec{x}_2 \\ \vec{x}_3 \\ \vdots \end{bmatrix} \quad (2)$$

If  $B$  matrices are all 0 we have a series of completely independent linear systems.

If  $B$  matrices are sparse, we have “compartments” that are connected to form the overall system.

# Parameter Notation

zoom in on one of those compartments and define parameters

$$\dot{\vec{x}}_i = A_i(\vec{\theta})\vec{x}_i \quad (3)$$

$$\begin{bmatrix} \dot{y}_1 \\ \dot{y}_2 \\ \dot{y}_3 \\ \vdots \end{bmatrix} = \begin{bmatrix} \theta_1 a_{1,1} & a_{1,2} & \theta_2 a_{1,3} & \dots \\ a_{2,1} & a_{2,2} & a_{2,3} & \dots \\ \theta_3 a_{3,1} & a_{3,2} & \theta_2 a_{3,3} & \dots \\ \vdots & \vdots & \vdots & \ddots \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ \vdots \end{bmatrix} \quad (4)$$

locations of  $\theta_i$  would depend upon actual model

# Two Approaches for Model Reduction

## Sobol

- Assume  $\theta_i \sim U[0.9, 1.1]$
- Pick number of samples to evaluate,  $N_{Sobol}$
- Compute Sobol index ( $S_i$ ) and total Sobol index ( $ST_i$ ) of each parameter

## Component

- Run model normally:  $\dot{\vec{x}} = M(\vec{\theta})\vec{x}$
- Run model without one component:  $\dot{\vec{x}} = M(\vec{\theta})\vec{x}, \theta_i = 0$
- Quantify how much these two runs differ ( $C_i$ )
- Repeat for all components  $i = 1, 2, \dots, N_\theta$

Want to show that small  $S_i$  or  $ST_i \nRightarrow C_i$  small

# Building the system

inputs:  $[N_1, N_2, \dots], I_{max}, I_{min}, N_{connections}, x_{total}$

outputs:  $\begin{bmatrix} A_1 & B_{1,2} & B_{1,3} & \dots \\ B_{2,1} & A_2 & B_{2,3} & \dots \\ B_{3,1} & B_{3,2} & A_3 & \dots \\ \vdots & \vdots & \vdots & \ddots \end{bmatrix}, \vec{x}_0$

with

$$A_i \in \mathbb{R}^{N_i \times N_i}$$

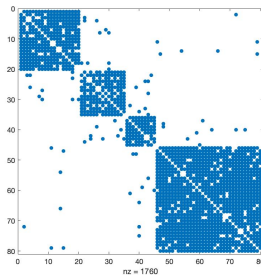
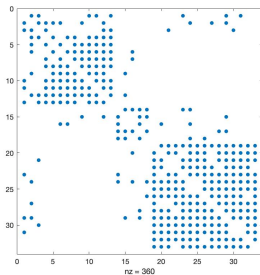
$$a_{i,j}, b_{i,j} \in \mathbb{Z} \cup [I_{min}, I_{max}] \quad a_{i,j} = -a_{j,i} \quad B_{i,j} = -B_{j,i}^T$$

$$\sum_{i,j} \|B_{i,j}\|_0 = N_{connections}$$

$$\|\vec{x}_0\|_1 = x_{total} \quad |\vec{x}_0| = \vec{x}_0$$

# Building the system

Example matrices:



# Selecting Parameters

inputs:  $N_{\theta in}, N_{\theta out}$                        $N_{\theta} = N_{\theta in} + N_{\theta out}$

outputs: list of indexes  $[i, j]$  to be treated as parameters  
with

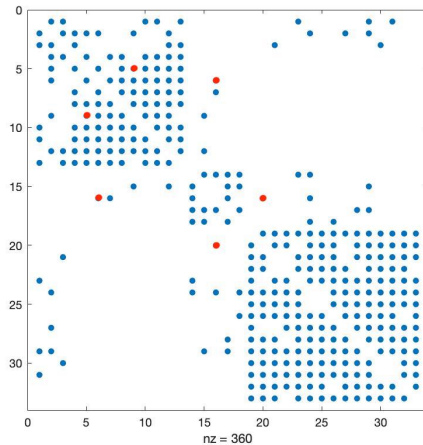
$N_{\theta in}$  parameters chosen “inside” compartments

$N_{\theta out}$  parameters chosen “outside” compartments

All parameters non-zero elements of  $M$

\*currently parameters continue skew symmetry of  $M$

# Selecting Parameters





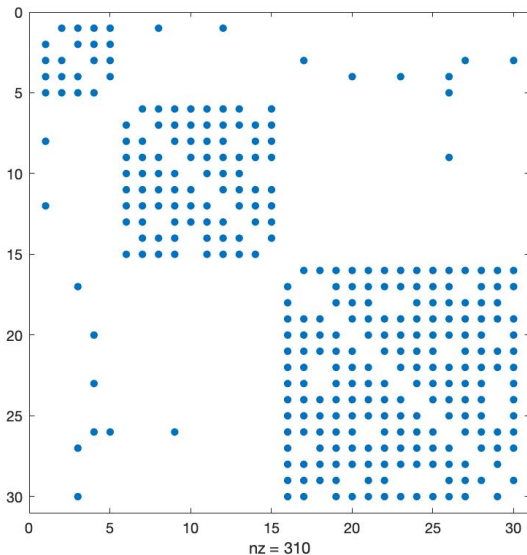
# Running the Analysis 1

Qol = final value of final state variable

Component Analysis:  $C_i = \frac{|\text{nominal Qol} - \text{altered Qol}|}{\text{nominal Qol}}$

rank parameters by increasing  $C_i$  and increasing  $ST_i$ , compare lists

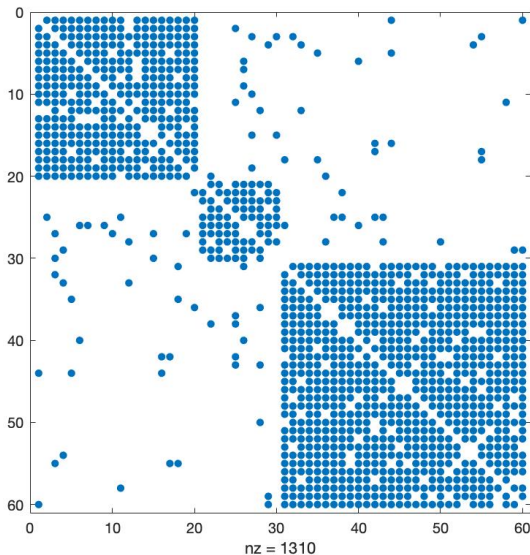
# Running the Analysis 1



# Running the Analysis 1

Parameter	Ranking by C	Ranking by S	Ranking by ST
inside 1	7	10	10
inside 2	5	5	4
inside 3	8	7	9
inside 4	2	3	3
inside 5	9	6	5
outside 1	1	8	6
outside 2	6	9	8
outside 3	4	2	2
outside 4	10	4	7
outside 5	3	1	1

# Running the Analysis 2



# Running the Analysis 1

Parameter	Ranking by C	Ranking by S	Ranking by ST
inside 1	3	7	7
inside 2	1	2	3
inside 3	8	8	8
outside 1	7	10	9
outside 2	10	6	6
outside 3	4	3	2
outside 4	2	9	10
outside 5	6	1	1
outside 6	9	5	5
outside 7	5	4	4

# Questions

- Keep parameters mirrored?
- Keep M skew-symmetric? ( $\dot{\vec{x}} = M e^{-t} \vec{x}$ )
- $C_i$  computed using all state variables (inner-outer norm)
- How to compare different systems? (Distributions)
- Different QoI?