**79** I. Time period of a spring-mass system depends on its amplitude.

II. Time period of a spring-mass system depends on its mass.

III. Time period of a spring-mass system depends on spring constant.

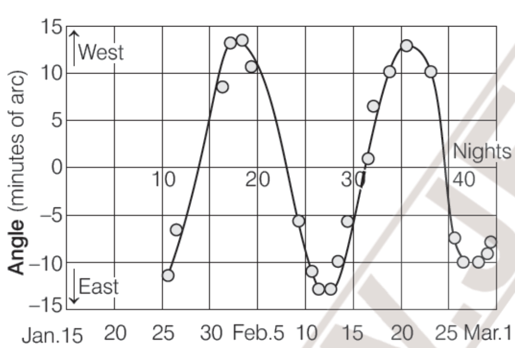
Which of the following statement(s) is/are correct?

(a) Both I and II (b) Both I and III
(c) Both II and III (d) I, II and III

**80** Three simple harmonic motions in the same direction having the same amplitude $a$ and same period are superposed. If each differs in phase from the next by 45°, then choose the correct statement.

(a) The resultant amplitude is $(1+\sqrt{2})a$.
(b) The phase of the resultant motion relative to the first is 90°.
(c) The energy associated with the resulting motion is $(3-2\sqrt{2})$ times the energy associated with any single motion.
(d) The resulting motion is not simple harmonic.

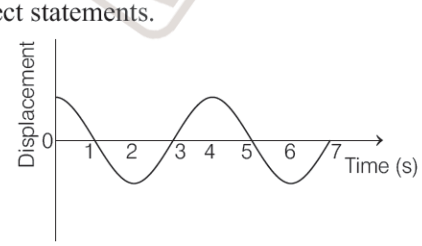
**81** In 1610, Galileo found four moons of planet jupiter. He observe the motion of one moon callisto from earth and his data in graph as shown below suggests that its motion is simple harmonic.




Now, choose the correct statement.

(a) The motion of moon callisto is not simple harmonic in real.
(b) The motion of moon callisto is actually uniform circular motion.
(c) Galileo observed projections of uniform circular motion in a line of plane of motion.
(d) All of the above

**82** Displacement *versus* time curve for a particle executing SHM is shown in figure. Choose the correct statements.



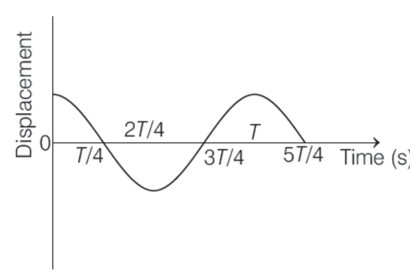

(a) Phase of the oscillator is same at $t = 0$ s and $t = 2$ s.
(b) Phase of the oscillator is same at $t = 2$ s and $t = 6$ s.
(c) Phase of the oscillator is same at $t = 1$ s and $t = 7$ s.
(d) Phase of the oscillator is same at $t = 3$ s and $t = 5$ s.

**83** A particle is in linear simple harmonic motion between two points $A$ and $B$, 10 cm apart (figure). Take the direction from $A$ to $B$ as the positive direction and choose the incorrect statements.


$AO = OB = 5$ cm, $BC = 8$ cm

(a) The sign of velocity, acceleration and force on the particle when it is 3 cm away from $A$ going towards $B$ are positive.
(b) The sign of velocity of the particle at $C$ going towards $B$ is negative.
(c) The sign of velocity, acceleration and force on the particle when it is 4 cm away from $B$ going towards $A$ are negative.
(d) The sign of acceleration and force on the particle when it is at point $B$ is negative.

**84** The displacement-time graph of a particle executing SHM is shown in figure. Which of the following statement is/are incorrect?




(a) The force is zero at $t = \frac{3T}{4}$.
(b) The acceleration is maximum at $t = \frac{4T}{4}$.
(c) The velocity is maximum at $t = \frac{T}{4}$.
(d) The PE is equal to KE of oscillation at $t = \frac{T}{2}$.

**85** A body is performing SHM, then which of the following statement(s) is/are incorrect?

(a) Total energy per cycle is equal to its maximum kinetic energy.
(b) Average kinetic energy per cycle is equal to half of its maximum kinetic energy.
(c) Mean velocity over a complete cycle is equal to $\frac{2}{\pi}$ times of its maximum velocity.
(d) Both (b) and (c)

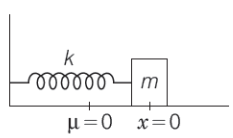
**86** For a SHM, if the maximum potential energy become double, choose the correct statements.

(a) Maximum kinetic energy will become double.
(b) The total mechanical energy will become double.
(c) Both (a) and (b)
(d) Neither (a) nor (b)

**87** A body executes simple harmonic motion. Its Potential Energy (PE), the Kinetic Energy (KE) and Total Energy (TE) were measured as function of displacement $x$. Then, which of the following statement regarding the body is correct?

(a) KE is maximum, when $x = 0$.
(b) TE is zero, when $x = 0$.
(c) KE is maximum, when $x$ is maximum.
(d) PE is maximum, when $x = 0$.

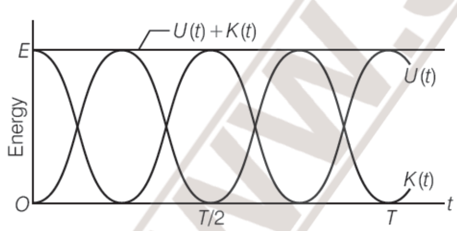
**88** A block is in simple harmonic motion as shown in the figure on a frictionless surface, i.e. $\mu = 0$.




Choose the correct statements.

(a) The kinetic energy varies between a maximum value and zero.
(b) The potential energy varies between a maximum value and zero.
(c) Total energy remains constant.
(d) All of the above

**89** The graph below shows the variation of potential energy $U(t)$, kinetic energy $K(t)$ with time $t$ for a particle executing SHM. Choose the correct statement(s).




(a) The time periods of variation of potential and kinetic energies are same.
(b) The total mechanical energy is maximum at $t = T/2$.
(c) Kinetic energy is negative and potential energy is positive.
(d) Potential energy is negative and kinetic energy is positive.

**90** A vertical spring mass system is taken on moon. It starts oscillating on the moon. Choose the correct statements.

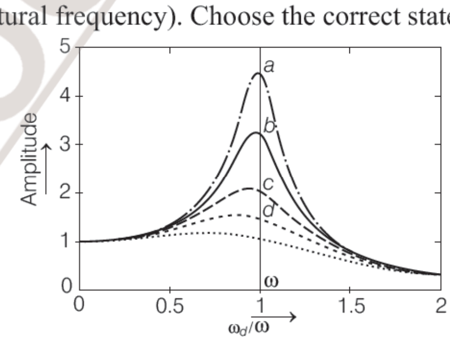
(a) The time period will become $\frac{T_{\text{Earth}}}{\sqrt{6}}$.
(b) The equilibrium position about which spring-mass system oscillates in vertical direction is $\frac{mg}{6k}$ from the unstretched position.
(c) $k$ on moon decreases.
(d) $k$ on moon increases.

**91** Choose the correct statements regarding the expression $x(t) = A\,e^{-bt/2m}\cos(\omega' t + \phi)$.

Here, $\omega' = \sqrt{\frac{k}{m} - \frac{b^2}{4m^2}}$, $\frac{b}{\sqrt{km}}$ is much less than 1.

(a) $x(t)$ is strictly periodic always.
(b) $x(t)$ is approximately periodic.
(c) Amplitude of damped oscillation represented by above expression is constant.
(d) None of the above

**92** Graph below shows variation of amplitude of forced oscillation with respect to frequency of driving force (natural frequency). Choose the correct statement(s).




(a) Amplitude is maximum at $\omega_d = \omega$.
(b) Peak amplitude is maximum for curve $a$ because for curve $a$ damping is minimum.
(c) Both (a) and (b)
(d) None of the above

## III. Matching Type

**93** Match the Column I (examples of different types of motion) with Column II (type of motion) and select the correct answer from the codes given below.

| Column I | Column II |
|---|---|
| A. Circular motion of a rigid body about a common axis | 1. Projectile |
| B. Motion of a pendulum | 2. Rectilinear |
| C. Motion of a car on a straight road | 3. Oscillatory |
| D. Motion of a ball thrown by a boy at an angle with horizontal | 4. Rotational motion |

| | A | B | C | D |
|---|---|---|---|---|
| (a) | 2 | 3 | 2 | 1 |
| (b) | 4 | 3 | 2 | 1 |
| (c) | 3 | 4 | 1 | 2 |
| (d) | 4 | 3 | 1 | 2 |

**94** Match the Column I (quantity) with Column II (value) for an object executing simple harmonic motion in a horizontal plane with displacement given as $x = A\cos\omega t$ and select the correct answer from the codes given below.

| | Column I | | Column II |
|---|---|---|---|
| A. | $v_{max}$ / amplitude is equal to | 1. | $T/8$ |
| B. | $a_{max}$ / amplitude is equal to | 2. | $T/12$ |
| C. | If object starts from $x = +A$, then time to reach at $+A\sqrt{2}$ | 3. | $\omega$ |
| D. | If object starts from $x = 0$ and move towards right, then the time to reach at $+A/2$ | 4. | $\omega^2$ |

| | A | B | C | D |
|---|---|---|---|---|
| (a) | 3 | 1 | 4 | 2 |
| (b) | 3 | 4 | 1 | 2 |
| (c) | 4 | 3 | 2 | 1 |
| (d) | 3 | 4 | 2 | 1 |

**95** For a forced oscillation, match the Column I (quantity) with Column II (expression) and select the correct answer from the codes given below.

| | Column I | | Column II |
|---|---|---|---|
| A. | External periodic force is | 1. | $a(t) = \frac{-k}{m}x(t) - \frac{b}{m}v(t) + \frac{F_0}{m}\cos\omega_d t$ |
| B. | The instantaneous acceleration of the object under forced oscillation is | 2. | $x(t) = A\cos(\omega_d t + \phi)$ |
| C. | The displacement for forced oscillation is | 3. | $A = \frac{F_0}{\{m^2(\omega^2 - \omega_d^2)^2 + \omega_d^2 b^2\}^{1/2}}$ |
| D. | Amplitude for forced oscillation is | 4. | $F = F_0\cos\omega_d t$ |

| | A | B | C | D |
|---|---|---|---|---|
| (a) | 4 | 1 | 3 | 2 |
| (b) | 1 | 2 | 3 | 4 |
| (c) | 4 | 1 | 2 | 3 |
| (d) | 3 | 2 | 1 | 4 |

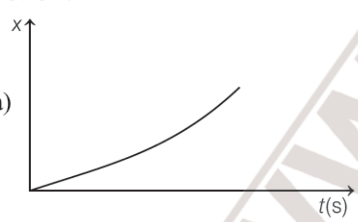
# NCERT & NCERT Exemplar

## MULTIPLE CHOICE QUESTIONS

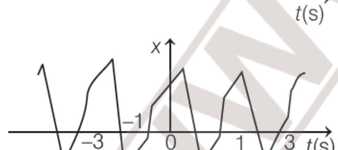
### NCERT

**96** Figures depict four $x$-$t$ plots for linear motion of a particle. Which of the plots represent a non-periodic motion?

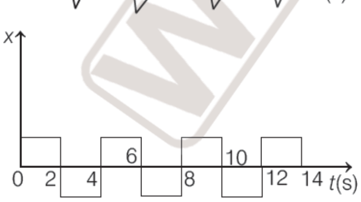
(a)



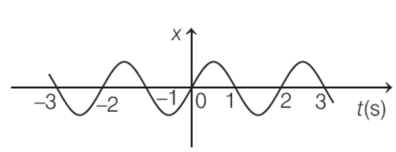

(b)




(c)




(d)




**97** The motion of a particle executing simple harmonic motion is described by the displacement function,

$$x(t) = A\cos(\omega t + \phi)$$

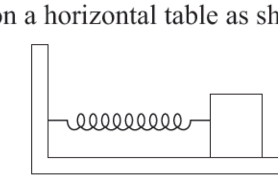
If the initial $(t = 0)$ position of the particle is 1 cm and its initial velocity is $\omega$ cms$^{-1}$, what are its amplitude and initial phase angle?

(a) $\sqrt{2}$ cm, $-\frac{\pi}{4}$ (b) 2 cm, $\frac{\pi}{2}$
(c) $\sqrt{2}$ cm, $-\frac{\pi}{2}$ (d) 2 cm, $\frac{\pi}{4}$

**98** A spring balance has a scale that reads from 0 to 50 kg. The length of the scale is 20 cm. A body suspended from this balance, when displaced and released, oscillates with a period of 0.6 s. What is the weight of the body?

(a) 100 N (b) 200 N
(c) 220 N (d) 250 N

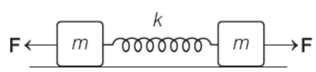
**99** A spring having with a spring constant 1200 Nm$^{-1}$ is mounted on a horizontal table as shown in figure.



A mass of 3 kg is attached to the free end of the spring, then the mass is pulled sideways to a distance of 2.0 cm and released. The frequency of oscillation is

(a) $1.6\ s^{-1}$ (b) $3.2\ s^{-1}$ (c) $4.8\ s^{-1}$ (d) $5\ s^{-1}$

**100** Figure shows the same spring with both ends free and attached to a mass $m$ at either end. Each end of the spring in figure is stretched by the same force $F$.




If masses in figure are released, what is the period of oscillation?

(a) $2\pi\sqrt{\frac{2m}{k}}$ (b) $2\pi\sqrt{\frac{m}{2k}}$ (c) $2\pi\sqrt{\frac{m}{k}}$ (d) $2\pi\sqrt{\frac{2m}{3k}}$

**101** The piston in the cylinder head of a locomotive has a stroke (twice the amplitude) of 1.0 m. If the piston moves with simple harmonic motion with an angular frequency of 200 rad $min^{-1}$, what is its maximum speed?

(a) 25 m $min^{-1}$ (b) 50 m $min^{-1}$
(c) 100 m $min^{-1}$ (d) 200 m $min^{-1}$

**102** The acceleration due to gravity on the surface of moon is 1.7 $ms^{-2}$. What is the time period of a simple pendulum on the surface of moon, if its time period on the surface of earth is 3.5 s?
(Take, $g$ on the surface of earth is 9.8 $ms^{-2}$)

(a) 2.4 s (b) 4.2 s (c) 6.2 s (d) 8.4 s

**103** You are riding in an automobile of mass 3000 kg. Assuming that you are examining the oscillation characteristics of its suspension system. The suspension sags 15 cm when the entire automobile is placed on it. Also, the amplitude of oscillation decreases by 50% during one complete oscillation. What is the value of the damping constant $b$ for the spring and shock absorber system of one wheel? Assuming that each wheel supports 750 kg.

(a) 1252 $kgs^{-1}$ (b) 1352 $kgs^{-1}$
(c) 1562 $kgs^{-1}$ (d) 1632 $kgs^{-1}$

**104** A body describes simple harmonic motion with an amplitude of 5 cm and a period of 0.2 s. What is the acceleration and velocity of the body when the displacement is 5 cm?

(a) $-5\pi^2\ ms^{-2}, 0$ (b) $2\pi^2\ ms^{-2}, 2ms^{-1}$
(c) $1\pi^2\ ms^{-2}, 2ms^{-1}$ (d) $0, 5\ ms^{-1}$

## NCERT Exemplar

**105** The displacement of a particle is represented by the equation $y = 3\cos\left(\frac{\pi}{4} - 2\omega t\right)$. The motion of the particle is

(a) simple harmonic with period $2\pi/\omega$
(b) simple harmonic with period $\pi/\omega$
(c) periodic but not simple harmonic
(d) non- periodic

**106** The displacement of a particle is represented by the equation $y = \sin^3 \omega t$. The motion is

(a) non-periodic
(b) periodic but not simple harmonic
(c) simple harmonic with period $2\pi/\omega$
(d) simple harmonic with period $\pi/\omega$

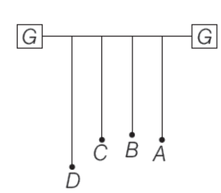
**107** The relation between acceleration and displacement of four particles are given below. Which of the particle is executing SHM?

(a) $a_x = +2x$ (b) $a_x = +2x^2$
(c) $a_x = -2x^2$ (d) $a_x = -2x$

**108** The displacement of a particle varies with time according to the relation $y = a\sin\omega t + b\cos\omega t$.

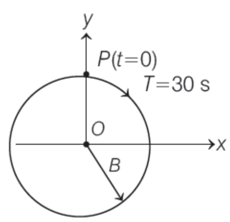
(a) The motion is oscillatory but not SHM
(b) The motion is SHM with amplitude $a + b$
(c) The motion is SHM with amplitude $a^2 + b^2$
(d) The motion is SHM with amplitude $\sqrt{a^2 + b^2}$

**109** Four pendulums $A$, $B$, $C$ and $D$ are suspended from the same elastic support as shown in figure. $A$ and $C$ are of the same length, while $B$ is smaller than $A$ and $D$ is larger than $A$. If $A$ is given a transverse displacement, then




(a) $D$ will vibrate with maximum amplitude
(b) $C$ will vibrate with maximum amplitude
(c) $B$ will vibrate with maximum amplitude
(d) All the four will oscillate with equal amplitude

**110** Figure shows the circular motion of a particle. The radius of the circle, the period, sense of revolution and the initial position are indicated on the figure. The simple harmonic motion of the $x$-projection of the radius vector of the rotating particle $P$ is




(a) $x(t) = B\sin\left(\dfrac{2\pi t}{30}\right)$

(b) $x(t) = B\cos\left(\dfrac{\pi t}{15}\right)$

(c) $x(t) = B\sin\left(\dfrac{\pi t}{15} + \dfrac{\pi}{2}\right)$

(d) $x(t) = B\cos\left(\dfrac{\pi t}{15} + \dfrac{\pi}{2}\right)$

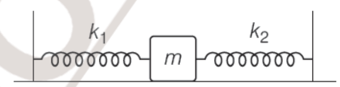
**111** The equation of motion of a particle is $x = a\cos(\alpha t)^2$. The motion is

(a) periodic but not oscillatory
(b) periodic and oscillatory
(c) oscillatory but not periodic
(d) Neither periodic nor oscillatory

**112** A particle executing SHM has a maximum speed of 30 $\text{cms}^{-1}$ and a maximum acceleration of 60 $\text{cms}^{-2}$. The period of oscillation is

(a) $\pi$ s (b) $\dfrac{\pi}{2}$ s (c) $2\pi$ s (d) $\dfrac{\pi}{t}$ s

**113** When a mass $m$ is connected individually to two springs having spring constants $k_1$ and $k_2$, the oscillation frequencies are $\nu_1$ and $\nu_2$. If the same mass is attached to the two springs as shown in figure, the oscillation frequency would be




(a) $\nu_1 + \nu_2$ (b) $\sqrt{\nu_1^2 + \nu_2^2}$

(c) $\left(\dfrac{1}{\nu_1} + \dfrac{1}{\nu_2}\right)^{-1}$ (d) $\sqrt{\nu_1^2 - \nu_2^2}$

# Answers

## > Mastering NCERT with MCQs

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 1 (a) | 2 (c) | 3 (c) | 4 (a) | 5 (c) | 6 (c) | 7 (b) | 8 (b) | 9 (a) | 10 (b) |
| 11 (d) | 12 (b) | 13 (c) | 14 (a) | 15 (b) | 16 (c) | 17 (c) | 18 (d) | 19 (c) | 20 (a) |
| 21 (c) | 22 (b) | 23 (a) | 24 (b) | 25 (c) | 26 (d) | 27 (b) | 28 (d) | 29 (c) | 30 (c) |
| 31 (d) | 32 (d) | 33 (d) | 34 (d) | 35 (c) | 36 (a) | 37 (b) | 38 (d) | 39 (c) | 40 (d) |
| 41 (d) | 42 (c) | 43 (b) | 44 (b) | 45 (c) | 46 (a) | 47 (c) | 48 (a) | 49 (d) | 50 (d) |
| 51 (b) | 52 (d) | 53 (d) | 54 (b) | 55 (b) | 56 (d) | 57 (d) | 58 (c) | 59 (b) | 60 (b) |
| 61 (c) | 62 (a) | | | | | | | | |

## > Special Types Questions

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 63 (d) | 64 (d) | 65 (b) | 66 (c) | 67 (b) | 68 (c) | 69 (a) | 70 (d) | 71 (d) | 72 (d) |
| 73 (c) | 74 (d) | 75 (d) | 76 (d) | 77 (a) | 78 (d) | 79 (c) | 80 (a) | 81 (d) | 82 (b) |
| 83 (b) | 84 (d) | 85 (c) | 86 (c) | 87 (a) | 88 (d) | 89 (a) | 90 (b) | 91 (b) | 92 (c) |
| 93 (b) | 94 (b) | 95 (c) | | | | | | | |

## > NCERT & NCERT Exemplar MCQs

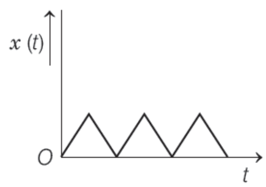
| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 96 (a) | 97 (a) | 98 (c) | 99 (b) | 100 (b) | 101 (c) | 102 (d) | 103 (b) | 104 (a) | 105 (b) |
| 106 (b) | 107 (d) | 108 (d) | 109 (b) | 110 (a) | 111 (c) | 112 (a) | 113 (b) | | |

# Hints & Explanations

**1** ($a$) The rotation of earth about its axis is periodic because it repeats after a regular interval of time.

However, it is obviously not a to and fro type of motion about a fixed point, hence its motion is not simple harmonic motion.

**2** ($c$) The $x$-$t$ graph for an insect climbing up a ramp and falling down with uniform speed is correctly represented by the graph given in option (c).




As we know that, for uniform motion, the $x$-$t$ graph is a straight line. So, for the upward motion the graph is represented by straight line with positive slope (considering motion in upward direction as positive) and the downward motion it is represented by a straight line with negative slope.

**3** ($c$) The beat frequency of heart $= \dfrac{75}{1\text{ min}} = \dfrac{75}{60}\text{s}^{-1}$

$$= 1.25\text{ Hz}$$

Time period, $T = \dfrac{1}{\text{frequency}} = \dfrac{1}{1.25} = 0.8\text{ s}$

The beat frequency and period of human heart are 1.25 Hz and 0.8 s, respectively.

**4** ($a$) A periodic function repeats itself after a time period $T$.

Given, $f(t) = A\sin\omega t$

If the argument of this function $\omega t$, is increased by an integral multiple of $2\pi$ radians. Then, the value of the function remains the same. The given function $f(t)$ is then periodic and repeats itself after every $2\pi$ radians.

$$\Rightarrow \qquad f(t) = f(t+T)$$

**5** ($c$) Given, amplitude, $a = 4$ cm

Time period, $T = 1\text{ s}$,

Displacement, $y = 2\text{ cm}$

Since, particle perform oscillatory motion, hence its displacement equation is

$$y = a\sin\omega t$$

Substituting the given values in the above equation, we get

$$2 = 4\sin\omega t$$

$$\frac{1}{2} = \sin\omega t$$

$$\sin\frac{\pi}{6} = \sin\omega t$$

$$\Rightarrow \qquad \omega t = \frac{\pi}{6}, \frac{2\pi}{T}\cdot t = \frac{\pi}{6} \qquad \left(\because \omega = \frac{2\pi}{T}\right)$$

$$t = \frac{T}{12} = \frac{1}{12}\text{s}$$

**6** ($c$) Given, $x(t) = 20\cos\omega t$

$$x(t) = 20\cos\frac{2\pi}{T}\cdot t \qquad \ldots\text{(i)} \left(\because \omega = \frac{2\pi}{T}\right)$$

Given, $T = 4\text{s}$ and $t = 1\text{ s}$

Substituting the given values in Eq. (i), we get

$$\therefore \qquad x(t) = 20\cos\frac{2\pi}{4}\cdot 1 = 20\cos\frac{\pi}{2} = 20\times 0 = 0$$

**7** ($b$) The function $\log\omega t$ increases monotonically with time $t$. Therefore, it cannot repeat its value and is a non-periodic function. It may be noted that as $t\to\infty$, $\log\omega t$ diverges to $\infty$. Therefore, it cannot represents any kind of physical displacement.

**8** ($b$) **In option** (b),

$$(\sin\omega t + \cos\omega t) = \sqrt{2}\left(\frac{1}{\sqrt{2}}\sin\omega t + \frac{1}{\sqrt{2}}\cos\omega t\right)$$

$$= \sqrt{2}\left(\cos\frac{\pi}{4}\sin\omega t + \sin\frac{\pi}{4}\cos\omega t\right)$$

$$= \sqrt{2}\left[\sin\left\{\omega t + \frac{\pi}{4}\right\}\right]$$

[$\because$ using trigonometric identity, $\sin(A+B) = \sin A\cos B + \cos A\sin B$]

Comparing the above equation with the standard equation of periodic function $f(t) = A\sin(\omega t + \phi)$, we can say that, the above function has amplitude $\sqrt{2}$ with phase constant $\pi/4$. Also, it represents a periodic function with period $2\pi/\omega$.

**In option** (a), $A\sin^3(\omega t)^2$ is not periodic function due to cube of sine value and square of $t$ value.

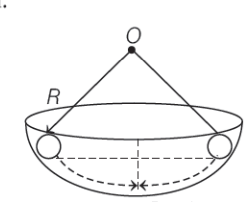
**In option** (c), $\tan(\omega t)^3$ is not periodic function due to cube of $t$ value.

**In option** (d), the function $e^{\omega t}$ is not periodic, as it increases with decreasing time.

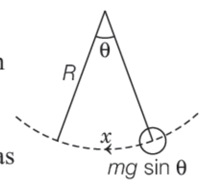
Hence, it never repeats its value.

So, option (b) is correct.

**9** ($a$) Consider the motion of the ball inside a smooth curved bowl.



For small angular displacement or slightly released motion, it can be considered as angular SHM. This can be explained as follows




Let the ball is at an angle θ, the restoring force ($g \sin \theta$) $m$ acts on it as shown.

$\because \qquad ma = mg \sin\theta \Rightarrow a = g \sin\theta$

$\Rightarrow \quad \dfrac{d^2x}{dt^2} = -g \sin\theta = -g \times \dfrac{x}{R} \quad (\because \sin\theta \simeq \theta = x/R)$

$\Rightarrow \quad d^2x/dt^2 \propto (-x)$

$\therefore \qquad a \propto -x$

Hence, the motion of ball bearing inside the bowl is SHM.

As motion is SHM, hence it must be periodic.

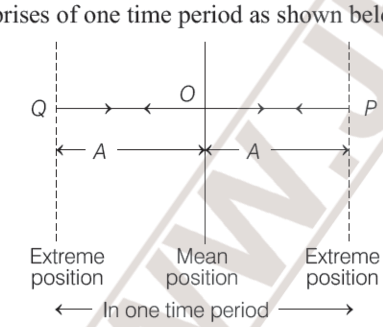
**10** (*b*) From graph, it is clear that time taken to complete one oscillation by SHM represented by curve 1 is equal to time taken to complete one-fourth oscillation by SHM represented by curve 2.

i.e. $\qquad T_1 = \dfrac{T_2}{4} \Rightarrow T_2 = 4T_1$

**11** (*d*) In a simple harmonic motion (SHM), the particle oscillates about its mean position on a straight line.

The particle moves from its mean position $O$ to an extreme position $P$ and then return to its mean position covering same distance of $A$ as shown below.

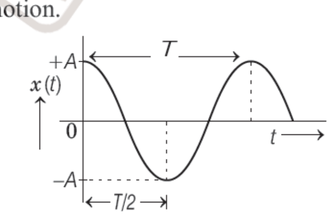
Then by the conservative force, it is moved in opposite direction to a point $Q$ at distance $A$ and then back to mean position covering same distance $A$. This comprises of one time period as shown below




Hence, in one time period it covers a distance of

$$x = OP + PO + OQ + QO$$
$$= A + A + A + A = 4A$$

**12** (*b*) As the particle is moving between $+A$ and $-A$ with varying speed about origin (at $x = 0$) and by observing snapshots we can draw position-time graph for the given motion.




Graph shows a sinusoidal function $x$ with respect to time $t$. From figure,at $t = 0$ particle is at $x = +A$ and crosses mean position at $t = T/4$ and reaches other end in negative direction $(-A)$ at $t = T/2$.

So, $x(t) = A\cos\omega t$

where, ω is the angular frequency $= \dfrac{2\pi}{T}$.

**13** (*c*) Given, $y_1 = a\sin\omega t$

and $\qquad y_2 = b\cos\omega t = b\sin\left(\omega t + \dfrac{\pi}{2}\right)$

The resultant displacement is given by

$$y = y_1 + y_2 = \sqrt{a^2 + b^2}\sin(\omega t + \phi)$$

Hence, the motion of superimposed wave is simple harmonic with amplitude $\sqrt{a^2 + b^2}$.

**14** (*a*) Standard form of the equation of motion of SHM as a linear combination of sine and cosine functions can be given as $y = a\sin\omega t + b\cos\omega t$ ...(i)

Let, $\quad a = d\cos\phi$ and $b = d\sin\phi$

$\Rightarrow \quad y = d\cos\phi\sin\omega t + d\sin\phi\cos\omega t$

$\qquad = d\sin(\omega t + \phi)$, where $d = \sqrt{a^2 + b^2}$

Here, the displacement of given particle is

$$y = A_0 + A\sin\omega t + B\cos\omega t \qquad \text{... (ii)}$$

So, from Eqs. (i) and (ii), we can say that $A_0$ be the value of mean position for the given particle, at which $y = 0$.

Also, $a = A$ and $b = B$

∴ Resultant amplitude of the oscillation in given as

$$= \sqrt{A^2 + b^2}$$

**15** (*b*) Given equation,

$$y_1 = 5(\sin 2\pi t + \sqrt{3}\cos 2\pi t)$$
$$= 10\left(\frac{1}{2}\sin 2\pi t + \frac{\sqrt{3}}{2}\cos 2\pi t\right)$$
$$= 10\left(\cos\frac{\pi}{3}\sin 2\pi t + \sin\frac{\pi}{3}\cos 2\pi t\right)$$
$$= 10\left[\sin\left(2\pi t + \frac{\pi}{3}\right)\right] \Rightarrow A_1 = 10$$

Similarly, $y_2 = 5\sin\left(2\pi t + \dfrac{\pi}{4}\right) \Rightarrow A_2 = 5$

Hence, $\quad \dfrac{A_1}{A_2} = \dfrac{10}{5} = \dfrac{2}{1} = 2:1$

So, the ratio of their amplitudes is 2 : 1.

**16** (*c*) Given function

$x = A\sin^2\omega t + B\cos^2\omega t + C\sin\omega t\cos\omega t$

$x = \dfrac{A}{2}(1 - \cos 2\omega t) + \dfrac{B}{2}(1 + \cos 2\omega t) + \dfrac{C}{2}\sin 2\omega t$

For $A = 0, B = 0; x = \dfrac{C}{2}\sin 2\omega t$

It is also represents SHM.

For $A = -B$ and $C = 2B$

$$x = -\frac{B}{2}(1-\cos 2\omega t) + \frac{B}{2}(1+\cos 2\omega t) + \frac{2B}{2}\sin 2\omega t$$

$= B\cos 2\omega t + B\sin 2\omega t$; Amplitude $\sqrt{B^2 + B^2} = |B\sqrt{2}|$

For $A = B$; $C = 0$;

$$x = \frac{A}{2}(1-\cos 2\omega t) + \frac{A}{2}(1+\cos 2\omega t) = A$$

Hence, option (c) is incorrect.

For $A = B, C = 2B$;

$x = B + B\sin 2\omega t$, it also represents SHM.

**17** (*c*) We know that, $y = a\sin\omega t$ $\quad \left(\because \omega = \frac{2\pi}{T}\right)$

where, $a$ is the amplitude, $y = a\sin\frac{2\pi}{T}t$

Given, $\quad T = 3\text{ s}$

So, when the displacement will be half of its amplitude, i.e.

$$y = \frac{a}{2} \Rightarrow \frac{a}{2} = a\sin\frac{2\pi t}{3}$$

$$\frac{1}{2} = \sin\frac{2\pi t}{3}$$

$$\sin\frac{\pi}{6} = \sin\frac{2\pi t}{3}$$

$$\frac{2\pi t}{3} = \frac{\pi}{6}$$

$$t = \frac{1}{4}\text{ s}$$

**18** (*d*) Equation for a particle executing simple harmonic motion is

$$x = A\sin(\omega t + \phi)$$

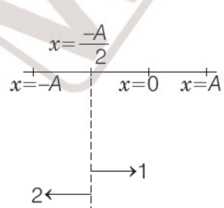
$$A\sin(\omega t + \phi) = \frac{A}{2} \quad \left[\because \text{Given}, x = \frac{A}{2}\right]$$

$$\sin(\omega t + \phi) = \frac{1}{2}$$

$$\sin(\omega t + \phi) = \sin\frac{\pi}{6}$$

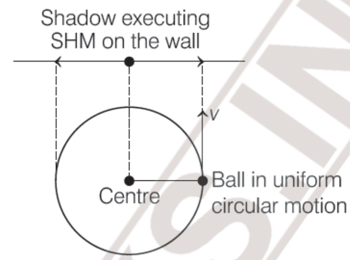
Since, $\quad \delta = \omega t + \phi = \frac{\pi}{6}$ or $\frac{5\pi}{6}$

So, the phase difference of the two particles when they are crossing each other at $x = \frac{A}{2}$ in opposite directions are

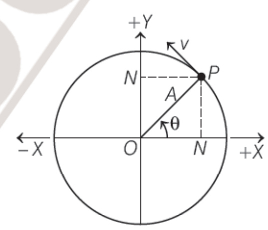



$$\delta = \delta_1 - \delta_2 = \frac{5\pi}{6} - \frac{\pi}{6} = \frac{2\pi}{3}$$

**19** (*c*) The ball will appear to execute to and fro motion along a horizontal line with the point of rotation as the mid-point. Thus, the shadow will execute SHM on the wall as shown below.




**20** (*a*) At $t = 0$, $OP$ makes an angle of $45° = (\pi/4)$ rad with the (positive direction of) $X$-axis. After time $t$, it covers an angle $\frac{2\pi}{T}t$ in the anti-clockwise sense and makes an angle of $\frac{2\pi}{T}t + \frac{\pi}{4}$ with the $X$-axis.




$\therefore$ The projection of $OP$ on the $X$-axis at time $t$ is given by

$$x(t) = A\cos\left(\frac{2\pi}{T}t + \frac{\pi}{4}\right)$$

For $\quad T = 4\text{ s}$,

$$x(t) = A\cos\left(\frac{2\pi}{4}t + \frac{\pi}{4}\right)$$

which is a SHM of amplitude $A$, period 4s and an initial phase $\frac{\pi}{4}$.

**21** (*c*) The equation of a simple harmonic motion is given by

$$y = 3\sin(50t - x) \quad \ldots\text{(i)}$$

By comparing Eq. (i) with general equation of a simple harmonic motion $y = A\sin(\omega t + \phi)$, we get

Amplitude $A = 3\text{ m}$

angular frequency, $\omega = 50\text{ Hz}$

$\therefore$ Maximum particle velocity, $v_{max} = A\omega = 3 \times 50$
$= 150\text{ ms}^{-1}$

**22** (*b*) Given, angular frequency of the piston,

$$\omega = 100\text{ rad min}^{-1}$$

and stroke length = 2m

$\therefore$ Amplitude of SHM, $A = \frac{\text{Stroke length}}{2} = \frac{1}{2} = 1\text{ m}$

Now, $\quad v_{max} = \omega A = 100 \times 1 = 100\text{ m min}^{-1}$

**24** (*b*) Maximum acceleration of object in simple harmonic motion is

$$a_{\max} = \omega^2 A$$

$$\Rightarrow \quad \frac{(a_{\max})_1}{(a_{\max})_2} = \frac{\omega_1^2}{\omega_2^2} \quad \text{(as } A \text{ remains same)}$$

$$\Rightarrow \quad \frac{(a_{\max})_1}{(a_{\max})_2} = \frac{(100)^2}{(1000)^2} = \left(\frac{1}{10}\right)^2 = 1:10^2$$

So, the ratio of their maximum acceleration is $1 : 10^2$.

**25** (*c*) The oscillation of a body on a smooth horizontal surface is represented by the equation, $x(t) = A\cos\omega t$

$$\Rightarrow \quad v(t) = \frac{d}{dt}\{x(t)\} = -\omega A \sin\omega t$$

and $$a(t) = \frac{d}{dt}\{v(t)\} = -\omega^2 A\cos\omega t \quad \ldots\text{(i)}$$

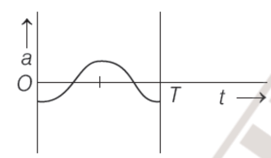
So, for the corresponding *a-t* graph for the given body, let us first calculate the value of *a* at different values of *t*, using Eq. (i), we get

If $t = 0$, then $a = -\omega^2 A$

If $t = \dfrac{T}{4}$, then $a = 0$

If $t = \dfrac{T}{2}$, then $a = +\omega^2 A$

We can see that only graph (c) will satisfy the above results.




Thus, the correct *a-t* graph is represented in option (c).

**26** (*d*) From the given graph, we have

Given, $T = 8\text{s}$,

Angular frequency, $\omega = \dfrac{2\pi}{T} = \dfrac{2\pi}{8} = \left(\dfrac{\pi}{4}\right)\text{rads}^{-1}$

Equation of motion for the particle executing SHM is given as

$$x = A\sin\omega t$$

$$\therefore \text{Acceleration, } a = \frac{d^2x}{dt^2} = -\omega^2 x = -\left(\frac{\pi^2}{16}\right)\sin\left(\frac{\pi}{4}t\right)$$

$$(\because A = 1)$$

On substituting $t = \dfrac{4}{3}$ s, we get

$$a = -\frac{\sqrt{3}}{32}\pi^2 \text{ cms}^{-2}$$

**27** (*b*) The acceleration of particle/body executing SHM at any instant (at position *x*) is given as $a = -\omega^2 x$,

where $\omega$ is the angular frequency of the body.

$$\Rightarrow \quad |a| = \omega^2 x \quad \ldots\text{(i)}$$

Given, $x = 5\text{m}$ and $|a| = 20\text{ms}^{-2}$

Substituting the given values in Eq. (i), we get

$$20 = \omega^2 \times 5$$

$$\Rightarrow \quad \omega^2 = \frac{20}{5} = 4 \quad \text{or} \quad \omega = 2\text{ rad s}^{-1}$$

As we know that, time period, $T = \dfrac{2\pi}{\omega}$ $\ldots$(ii)

$\therefore$ Substituting the value of $\omega$ in Eq. (ii), we get

$$T = \frac{2\pi}{2} = \pi \text{ s}$$

**28** (*d*) For a particle executing SHM, we have maximum acceleration,

$$\alpha = A\omega^2 \quad \ldots\text{(i)}$$

where, *A* is maximum amplitude and $\omega$ is angular velocity of a particle.

Maximum velocity, $\beta = A\omega$ ...(ii)

Dividing Eq. (i) by Eq. (ii), we get

$$\frac{\alpha}{\beta} = \frac{A\omega^2}{A\omega} \Rightarrow \omega = \frac{\alpha}{\beta}$$

But $$\omega = \frac{2\pi}{T}$$

$$\therefore \quad \frac{2\pi}{T} = \frac{\alpha}{\beta} \Rightarrow T = \frac{2\pi\alpha}{\beta}$$

Thus, its time period of vibration, $T = \dfrac{2\pi\beta}{\alpha}$.

**29** (*c*) Magnitude of velocity of particle when it is at displacement *x* from mean position, *v*

$$= \omega\sqrt{A^2 - x^2}$$

Also, magnitude of acceleration of particle in SHM,

$$a = \omega^2 x$$

Given, when $x = 2\text{ cm}$

$$v = a$$

$$\Rightarrow \omega\sqrt{A^2 - x^2} = \omega^2 x$$

$$\Rightarrow \quad \omega = \frac{\sqrt{A^2 - x^2}}{x} = \frac{\sqrt{9-4}}{2} \quad [\because \text{given, } A = 3\text{ cm}]$$

$$\Rightarrow \text{Angular velocity, } \omega = \frac{\sqrt{5}}{2}$$

$$\therefore \text{ Time period of motion, } T = \frac{2\pi}{\omega} = \frac{4\pi}{\sqrt{5}} \text{ s}$$

**33** (*d*) Given, $m = 1\text{kg}$

The given equation of SHM is

$$x = 6.0\cos\left(100t + \frac{\pi}{4}\right)$$

Comparing it with general equation of SHM,

$$x = A\cos(\omega t + \phi),$$

We have, $A = 6.0\text{ cm} = \dfrac{6}{100}\text{ m}$ and $\omega = 100\text{ rad/s}$

Maximum kinetic energy $= \frac{1}{2}m(v_{\max})^2$

$$= \frac{1}{2}m(A\omega)^2 = \frac{1}{2}\times 1\times\left[\frac{6}{100}\times 100\right]^2 = 18\text{ J}$$

**34** (*d*) Potential energy of an object executing SHM is given by

$$U(x) = \frac{1}{2}kx^2$$

Given, $\quad x = A\cos\omega t$

$\Rightarrow \quad U(x) = \frac{1}{2}kA^2\cos^2\omega t$

At $t = \left(\frac{T}{4}\right)$,

$$U(x) = \frac{1}{2}k^2A^2\cos^2\left(\frac{2\pi}{T}\times\frac{T}{4}\right) \quad \left(\because \omega = \frac{2\pi}{T}\right)$$

$$= \frac{1}{2}k^2A^2\cos^2\left(\frac{\pi}{2}\right) = 0$$

So, PE at $t = T/4$ is zero.

**35** (*c*) For SHM,

Displacement, $x(t) = A\cos(\omega t + \phi)$ $\quad$ ...(i)

$\Rightarrow \quad T_2 = 2\pi/\omega$

Potential energy, PE $= \frac{1}{2}kx^2 = \frac{1}{2}kA^2\cos^2(\omega t + \phi)$ ...(ii)

$$\Rightarrow \quad = \frac{1}{2}kA^2\frac{[1+\cos 2(\omega t + \phi)]}{2}$$

$$\therefore \quad T_1 = \frac{2\pi}{2\omega} = \frac{\pi}{\omega}$$

$$\Rightarrow \quad T_1 = \frac{T_2}{2} \Rightarrow 2T_1 = T_2$$

So, the relation between $T_1$ and $T_2$ is $2T_1 = T_2$.

**36** (*a*) Potential energy of particle in SHM,

$$U = \frac{1}{2}m\omega^2x^2$$

$$\Rightarrow \quad U = \frac{1}{2}m(2\pi\nu)^2x^2 = 2\pi^2m\nu^2x^2$$

Kinetic energy of particle in SHM

$$K = \frac{1}{2}m\omega^2(A^2 - x^2)$$

$$\Rightarrow \quad K = 2\pi^2m\nu^2(A^2 - x^2)$$

Hence, total energy

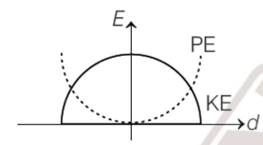
$$E = K + U = 2\pi^2m\nu^2x^2 + 2\pi^2m\nu^2(A^2 - x^2)$$

$$= 2\pi^2m\nu^2A^2 = \frac{2\pi^2mA^2}{T^2} \quad \left(\because T = \frac{1}{\nu}\right)$$

So, total energy depends on amplitude and time period.

**37** (*b*) For a particle executing SHM, during oscillation, its KE is maximum at mean position, where PE is minimum. At extreme position, KE is minimum and PE is maximum.

Thus, correct graph is depicted in option (b).




**38** (*d*) If a force acting on an object is a function of position only, it is said to be conservative force and it can be represented by a potential energy $U$ function which for one-dimensional case satisfies the derivative condition, i.e.

$$F(x) = -\frac{dU}{dx}$$

Given, $\quad U(x) = k[1 - \exp(-x^2)]$

From this we can say that, change in potential energy ($\Delta U$) will depend on initial and final positions only, so particle is under influence of conservative force.

$$\therefore \quad F = -\frac{dU}{dx} = -2kx\exp(-x^2)$$

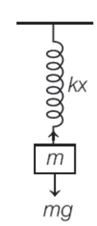
Here, negative sign implies that for any finite non-zero value of $x$, the force is directed towards the origin. This means there is a restoring force acting on the particle. Hence, the motion of the particle is simple harmonic.

At, $\quad x = 0, F = 0.$

Hence, at equilibrium position force exerted on particle is zero.

Also, potential energy of the particle is minimum at $x = 0$ and at $x = \pm\infty$, the potential energy is maximum.

**39** (*c*) At equilibrium the FBD of the block can be shown as




$\Rightarrow \quad mg = kx$

$\Rightarrow \quad$ Stretch, $x = mg/k$

Now, with the extra $\frac{mg}{k}$ stretch, the net stretch become

$$= \frac{mg}{k} + \frac{mg}{k} = \frac{2mg}{k}.$$

**40** (*d*) Total energy of the system in SHM $= \frac{1}{2}kA^2$ ...(i)

Kinetic energy of the system in SHM $= \frac{1}{2}mv^2$ ...(ii)

Since, at equilibrium position, i.e. at $x = 0$, the energy of the system would be kinetic only.

$\therefore$ From Eqs. (i) and (ii), we get

$$\frac{1}{2}kA^2 = \frac{1}{2}mv^2 \Rightarrow v = \sqrt{\left(\frac{k}{m}\right)A^2} = A\sqrt{\frac{k}{m}}$$

**41** (*d*) Let $x_1$ and $x_2$ be the extensions in the spring with spring constants $k_1$ and $k_2$, respectively.

Then, $\quad x_1 + x_2 = A$ $\quad$ ...(i)

and $k_1x_1 = k_2x_2$ or $\frac{x_1}{x_2} = \frac{k_2}{k_1}$ $\quad$ ...(ii)

From Eq. (ii) substitute the value of $x_2$ in Eq. (i), we get

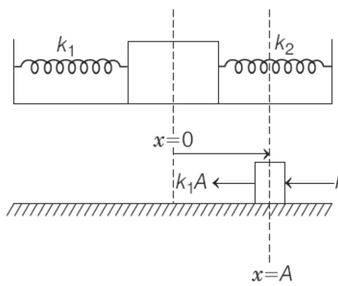
$$x_1 + \frac{k_1}{k_2}x_1 = A \qquad \text{...(iii)}$$

$$k_2x_1 + k_1x_1 = Ak_2$$

On solving these equations, we get

$$x_1 = \left(\frac{k_2}{k_1 + k_2}\right)A$$

**42** (*c*) The situation can be depicted as




As, restoring force of a spring, $F = -kx$

From FBD, $\mathbf{F}_{\text{net}} = -(k_1 + k_2)A$

The magnitude of net force, $|\mathbf{F}_{\text{net}}| = (k_1 + k_2)A$

**43** (*b*) Kinetic energy (KE) of the system in SHM $= \frac{1}{2}mv^2$

Potential energy (PE) of the system in SHM $= \frac{1}{2}kx^2$

Total kinetic energy of the system

$$= \text{KE} + \text{PE} = \frac{1}{2}kA^2 \qquad \text{...(i)}$$

If KE = PE, then Eq. (i) can be written as,

$$\frac{1}{2}kA^2 = 2 \times \text{PE} = 2\left(\frac{1}{2}kx^2\right)$$

or $$\frac{1}{2}kx^2 = \frac{1}{2}\left[\frac{1}{2}kA^2\right] \Rightarrow x = A/\sqrt{2}$$

**44** (*b*) For a block executing SHM, its

Velocity, $v = -A\omega\sin(\omega t + \phi)$ ...(i)

$$\Rightarrow \quad T_1 = \frac{2\pi}{\omega}$$

$$\text{KE} = \frac{1}{2}mv^2 = \frac{1}{2}mA^2\omega^2\sin^2(\omega t + \phi)$$

$$= mA^2\omega^2[1 - \cos(2\omega + 2\phi)] \Rightarrow T_2 = \frac{2\pi}{2\omega} \text{...(ii)}$$

$$\therefore \quad T_2 = \frac{T_1}{2} \quad \text{So, } T_1 = 2T_2$$

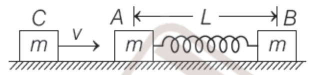
**45** (*c*) Frequency of oscillation of a spring mass system is given as $\quad \nu = 2\pi\sqrt{\frac{m}{k}}$

This means, it is independent of acceleration due to gravity.

Given, $m = 2\text{kg}$, $k = 10\ \text{Nm}^{-1}$

$$\Rightarrow \quad \nu = 2\pi\sqrt{\frac{2}{10}} = 2\pi\sqrt{0.2} = 2.8\ \text{Hz}$$

**46** (*a*) Given condition can be seen in the figure given below



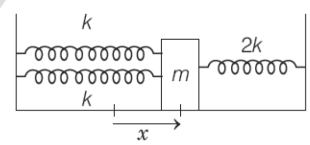

If collision is elastic, $C$ will stop and $A$ will start moving with speed $v$ towards $B$. At maximum compression (say $x$), both $A$ and $B$ will move with same speed $v/2$.

∴ At maximum compression, applying the conservation of energy, we get

$$\frac{1}{2}mv^2 = \frac{1}{2}(2m)\left(\frac{v}{2}\right)^2 + \frac{1}{2}kx^2$$

$$\Rightarrow \quad x = v\sqrt{\frac{m}{2k}}$$

**47** (*c*) Since, the three springs with spring constants $k$, $k$ and $2k$ are attached to the mass $m$ in parallel. So, net spring constant $= k + k + 2k = 4k$




∴ The net restoring force on mass $m$,

$$|\mathbf{F}_{\text{net}}| = -4kx \qquad \text{...(i)}$$

As, force in SHM, $|\mathbf{F}_{\text{SHM}}| = -m\omega^2 x$ ...(ii)

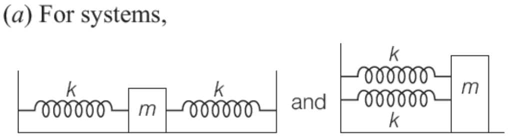
On comparing Eqs. (i) and (ii), we get

$$m\omega^2 = 4k$$

$$\omega = \sqrt{\frac{4k}{m}}$$

$$\therefore \quad T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{4k}}$$

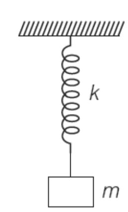
**48** (*a*) For systems,




there are two springs of same spring constants $k$, attached to mass $m$ in parallel. So, both systems have same time periods.

So, the correct option is (a).

**49** (*d*) As we know that, time period for a spring-mass system as shown




$$T = 2\pi\sqrt{\frac{m}{k}}$$

**Case I** $\quad T_1 = 2\pi\sqrt{\frac{m}{k}} = 3\ \text{s}$ ...(i)

**Case II** When the mass $m$ is increased by 1 kg

$$T_2 = 2\pi\sqrt{\frac{m+1}{k}} = 5\ \text{s} \qquad \text{...(ii)}$$

Dividing Eq. (ii) by Eq. (i), we get

$$\frac{T_2}{T_1} = \sqrt{\frac{m+1}{m}}$$

$$\Rightarrow \quad \frac{5}{3} = \sqrt{\frac{m+1}{m}}$$

$$\Rightarrow \quad \frac{25}{9} = \frac{m+1}{m}$$

$$\Rightarrow \quad \frac{25}{9} = 1 + \frac{1}{m}$$

$$\Rightarrow \quad \frac{1}{m} = \frac{16}{9}$$

$$\therefore \quad m = \frac{9}{16}\,\text{kg}$$

**50** (*d*) Since, the given spring with spring constants $k_1$ and $k_2$ are in parallel, so the net spring constant of the system is $k_{\text{net}} = k_1 + k_2$.

Initially frequency of oscillation,

$$f = \frac{1}{2\pi}\sqrt{\frac{k_{\text{net}}}{m}} = \frac{1}{2\pi}\sqrt{\frac{k_1 + k_2}{m}} \quad \ldots\text{(i)}$$

Now, when $k_1$ and $k_2$ are made four times their original value, then

$$f' = \frac{1}{2\pi}\sqrt{\frac{k'_1 + k'_2}{m}} = \frac{1}{2\pi}\sqrt{\frac{4k_1 + 4k_2}{m}} = 2f$$

[$\because$ using Eq. (i)]

Thus, the frequency of oscillation becomes $2f$.

**51** (*b*) Given, $m_1 = 1\,\text{kg}$,

Extension in length, $l_1 = 5\,\text{cm} = 5 \times 10^{-2}\,\text{m}$

$$\therefore \quad m_1 g = k l_1$$

where, $k$ = spring constant of the spring

$$\Rightarrow \quad k = \frac{m_1 g}{l_1} = \frac{1 \times 10}{5 \times 10^{-2}} = 200\,\text{Nm}^{-1}$$

So, the spring constant of the given spring is $200\,\text{Nm}^{-1}$.

Now, if a 2 kg block is suspended to this spring and pulled, then

Time period of the block,

$$T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{2}{200}} = 2\pi \times \frac{1}{10} = \frac{\pi}{5}\,\text{s}$$

As, maximum velocity, $v_{\max} = A\omega$

where, $A$ = amplitude = 10 cm = $10 \times 10^{-2}$ m (given)

$$\Rightarrow \quad v_{\max} = A \times \frac{2\pi}{T} = 10 \times 10^{-2} \times \frac{2\pi}{\pi/5}$$

$$= 10^{-1} \times 2 \times 5 = 1\,\text{ms}^{-1}$$

**52** (*d*) The time period $T$ of a simple pendulum of length $l$ is given by

$$T = 2\pi\sqrt{\frac{l}{g}} = \frac{1}{\text{frequency}\,(n)}$$

where, $g$ is acceleration due to gravity.

$$\therefore \quad \frac{n_1}{n_2} = \sqrt{\frac{l_2}{l_1}} \Rightarrow \frac{l_1}{l_2} = \left(\frac{n_2}{n_1}\right)^2$$

Given, $\quad \dfrac{n_1}{n_2} = \dfrac{2}{3}$ or $\dfrac{n_2}{n_1} = \dfrac{3}{2}$

$$\Rightarrow \quad \frac{l_1}{l_2} = \left(\frac{3}{2}\right)^2 = \frac{9}{4}$$

**53** (*d*) The distance $s$ covered by the mass falling from height $h$ during its time of fall $t$ is given by

$$s = h = ut + \frac{1}{2}gt^2$$

As, $\quad u = 0 \Rightarrow h = \dfrac{1}{2}gt^2 \Rightarrow t = \sqrt{\dfrac{2h}{g}} \quad \ldots\text{(i)}$

The time period of simple pendulum is

$$T = 2\pi\sqrt{\frac{l}{g}} \quad \ldots\text{(ii)}$$

where, $l$ is the length of the pendulum.

From Eqs. (i) and (ii), since $h$ and $l$ are constants, so we can conclude that

$$t \propto \frac{1}{\sqrt{g}} \quad \text{and} \quad T \propto \frac{1}{\sqrt{g}}$$

$$\therefore \quad \frac{t}{T} = 1$$

Thus, the ratio of time of fall and time period of pendulum is independent of value of gravity $g$ or any other parameters like mass and radius of the planet. Thus, the relation between $t'$ and $T'$ on another planet irrespective of its mass or radius will remains same as it was on earth, i.e. $t' = 2T'$.

**54** (*b*) Time period of a simple pendulum which is suspended to the ceiling of the lift, which is initially at rest is given by

$$T = 2\pi\sqrt{\frac{l}{g}} \quad \ldots\text{(i)}$$

When the lift is moving up with an acceleration $a$, then time period becomes

$$T' = 2\pi\sqrt{\frac{l}{g+a}}$$

Given, $\quad T' = \dfrac{T}{2}$

$$\Rightarrow \quad T' = \frac{T}{2} = 2\pi\sqrt{\frac{l}{g+a}} \quad \ldots\text{(ii)}$$

On dividing Eq. (ii) by Eq. (i), we get

$$\frac{1}{2} = \sqrt{\frac{g}{g+a}}$$

$$\Rightarrow \quad g + a = 4g$$

$$\Rightarrow \quad a = 3g$$

**55** (*b*) As we know that, according to force of gravitation, at the surface of the earth for a simple pendulum of mass $m$

$$mg = \frac{GMm}{R^2} \quad \ldots(i)$$

where, $M$ is the mass of earth.

When it is taken to a height $h$, above the earth's surface, then

$$mg' = \frac{GMm}{(h+R)^2} \quad \ldots(ii)$$

From Eqs. (i) and (ii), we get

$$g' = g\left(1+\frac{h}{R}\right)^{-2}$$

$$= g\left(1+\frac{2R}{R}\right)^{-2} \quad \text{(given, } h = 2R)$$

$$= g\,(3)^{-2}$$

The time period of a simple pendulum is given by

$$T = 2\pi\sqrt{\frac{l}{g}}$$

$\therefore$ Ratio of time period $T_1$ of a simple pendulum, when on the earth's surface and $T_2$ when on height $2R$ above the earth's surface is $\frac{T_1}{T_2} = \sqrt{\frac{g'}{g}}$

$$\therefore \quad \frac{T_1}{T_2} = \sqrt{\frac{g(3)^{-2}}{g}} = \sqrt{\frac{1}{3^2}}$$

$$\Rightarrow \quad T_2 = 3T_1 \Rightarrow \frac{T_1}{T_2} = \frac{1}{3}$$

**56** (*d*) Here, the rod is oscillating about an end point $O$. Hence, moment of inertia of rod about the point of oscillation is $I = \frac{1}{3}ml_0^2$

Moreover, length $l$ of the pendulum = distance from the oscillation axis to centre of mass of rod $= l_0/2$

$\therefore$ Time period of oscillation,

$$T = 2\pi\sqrt{\frac{I}{mgl}} = 2\pi\sqrt{\frac{\frac{1}{3}ml_0^2}{mg\left(\frac{l_0}{2}\right)}}$$

$$\Rightarrow \quad T = 2\pi\sqrt{\frac{2l_0}{3g}}$$

**57** (*d*) We know that,

Time period of a pendulum is given by

$$T = 2\pi\sqrt{L/g_{\text{eff}}} \quad \ldots(i)$$

Here, $L$ is the length of the pendulum and $g_{\text{eff}}$ is the effective acceleration due to gravity in the respective medium in which bob is oscillating.

Initially, when bob is oscillating in air, $g_{\text{eff}} = g$.

So, initial time period, $T = 2\pi\sqrt{\frac{L}{g}}$ $\quad \ldots(ii)$

Let $\rho_{\text{bob}}$ be the density of the bob.

When this bob is dipped into a liquid whose density is given as

$$\rho_{\text{liquid}} = \frac{\rho_{\text{bob}}}{16} = \frac{\rho}{16} \quad \text{(given)}$$

$\therefore$ Net force on the bob is

$$F_{\text{net}} = V\rho g - V\cdot\frac{\rho}{16}\cdot g \quad \ldots(iii)$$

(where, $V$ = volume of the bob = volume of displaced liquid by the bob when immersed in it). If effective value of gravitational acceleration on the bob in this liquid is $g_{\text{eff}}$, then net force on the bob can also be written as

$$F_{\text{net}} = V\rho g_{\text{eff}} \quad \ldots(iv)$$

Equating Eqs. (iii) and (iv), we have

$$V\rho g_{\text{eff}} = V\rho g - V\rho g/16$$

$$\Rightarrow \quad g_{\text{eff}} = g - g/16 = \frac{15}{16}g \quad \ldots(v)$$

Substituting the value of $g_{\text{eff}}$ from Eq. (v) in Eq. (i), the new time period of the bob will be

$$T' = 2\pi\sqrt{\frac{L}{g_{\text{eff}}}} = 2\pi\sqrt{\frac{16}{15}\frac{L}{g}}$$

$$\Rightarrow \quad T' = \sqrt{\frac{16}{15}} \times 2\pi\sqrt{\frac{L}{g}}$$

$$= \frac{4}{\sqrt{15}} \times T \quad \text{[using Eq. (ii)]}$$

**59** (*b*) For this case, $\omega_d b << m(\omega^2 - \omega_d^2)$,

So, from amplitude in the case of forced oscillations

$$A = \frac{F_0}{\{m^2(\omega^2-\omega_d^2)^2 + \omega_d^2 b^2\}^{1/2}}$$

we get $\quad A = \frac{F_0}{m(\omega^2 - \omega_d^2)}$

**60** (*b*) The amplitude of forced oscillation,

$$A = \frac{F_0}{\{m^2(\omega^2-\omega_d^2)^2 + \omega_d^2 b^2\}^{1/2}}$$

when driving frequency $\omega_d$ is close to natural frequency $\omega$, so we can take $(\omega_d = \omega)$.

Hence, $A = \frac{F_0}{\omega_d b}$.

**61** (*c*) Amplitude of a damped oscillator is given as

$$A = A_0\, e^{-kt}$$

where, $\quad k = \frac{-b}{2m}$.

When the amplitude decreases to 0.9 times in 5s, then

$$0.9\,A_0 = A_0 e^{-k5}$$

$$0.9 = e^{-k5}$$

$$\ln(0.9) = -k5 \quad \dots (i)$$

In another 10 s, the decrease in amplitude is $\alpha$-times, then

$$\alpha A_0 = A_0 e^{-k(15)} \text{ or } \alpha = e^{-k(15)}$$

$$\Rightarrow \quad \ln(\alpha) = -k(15) = -k(5)(3)$$

Using Eq. (i), we can write

$$\ln(\alpha) = +\ln(0.9)(3)$$

$$\Rightarrow \quad \ln(\alpha) = \ln(0.9)^3$$

$$\alpha = (0.9)^3 = 0.729$$

**62** (*a*) Given, frequency of oscillations is

$$f = 5 \text{osc s}^{-1}$$

$\Rightarrow$ Time period of oscillations is $T = \frac{1}{f} = \frac{1}{5}$s

So, time for 10 oscillations is $= \frac{10}{5} = 2$ s

Now, if $A_0$ = initial amplitude at $t = 0$ and $\gamma$ = damping factor, then for damped oscillations, amplitude after $t$ second is given as

$$A = A_0 e^{-\gamma t}$$

$\therefore$ After 2 s,

$$\frac{A_0}{2} = A_0 e^{-\gamma(2)} \Rightarrow 2 = e^{2\gamma}$$

$$\Rightarrow \quad \gamma = \frac{\log 2}{2} \quad \dots (i)$$

Now, when amplitude is $\frac{1}{1000}$ of initial amplitude, i.e.

$$\frac{A_0}{1000} = A_0 e^{-\gamma t}$$

$$\Rightarrow \quad \log(1000) = \gamma t$$

$$\Rightarrow \quad \log(10^3) = \gamma t$$

$$3 \log 10 = \gamma t$$

$$\Rightarrow \quad t = \frac{2 \times 3 \log 10}{\log 2} \quad \text{[using Eq. (i)]}$$

$$\Rightarrow \quad t = 19.93 \text{ s} \quad \text{or} \quad t \approx 20 \text{ s}$$

**63** (*d*) If the body is given a small displacement from the position, a force comes into play which tries to bring the body back to the equilibrium point, giving rise to oscillations or vibrations.

There is no significant difference between oscillations and vibrations. It seems that, when frequency is small, then it is called oscillations (like, the oscillation of branch of a tree), while when frequency is high, then it is called vibration (like, the vibration of a string of musical instrument).

Thus, vibrations and oscillations are not two different types of motion.

Therefore, Assertion is incorrect but Reason is correct.

**64** (*d*) $x(t) = A \sin \omega t$ is a sinusoidal periodic function that can represent an oscillatory motion.

Also, $\sin \theta$ is a sinusoidal periodic function.

Therefore, Assertion is incorrect but Reason is correct.

**65** (*b*) $x = A \cos \omega t$ and $x = A \sin \omega t$ both represent the displacement of particle undergoing periodic motion.

At $t = 0$,

If $x = A$, we can represents its displacement by

$$x = A \cos \omega t$$

and if $x = 0$, we can represent its displacement by

$$x = A \sin \omega t$$

This implies, both $x = A \cos \omega t$ and $x = A \sin \omega t$, represents the same motion depending on initial position of particle.

Since, $x = A \cos \omega t$ represents a periodic function with time period of $2\pi$ rad. So, if the argument of this function $\omega t$ is increased by an integral multiple of $2\pi$ rad, the value of the function remains the same.

Therefore, Assertion and Reason are correct but Reason is not the correct explanation of Assertion.

**66** (*c*) In $x = A \cos \omega t$, since $\cos \omega t$ varies between $+1$ to $-1$, thus the value of $x$ varies between $+A$ and $-A$.

Amplitude $A$ is a scalar quantity.

Therefore, Assertion is correct but Reason is incorrect.

**67** (*b*) In oscillatory or vibratory motion, an object moves about an equilibrium position due to a restoring force. When the body is at equilibrium position, no net external force acts on it, i.e. $F_{net} = 0$. Therefore, if it is left there at rest, it remains there forever.

If the body is then given then a small displacement from that position, a force comes into play, i.e. restoring force which tries to bring the body back to the equilibrium point giving rise to oscillation or vibrations.

Therefore, Assertion and Reason are correct but Reason is not the correct explanation of Assertion.

**68** (*c*) Time period of oscillation of spring mass system is

$$T = 2\pi \sqrt{\frac{m}{k}}$$

which is independent of the amplitude.

Thus, if the amplitude of the system is increased, then $T$ will remain same.

Therefore, Assertion is correct but Reason is incorrect.

**69** (*a*) A stiff spring has large spring constant $k$ and a soft spring has small $k$.

As, frequency of oscillation, for a spring mass system,

$$\nu = 2\pi\omega = 2\pi \sqrt{\frac{k}{m}} \quad \dots (i)$$

This means, a block of mass $m$ attached to stiff spring have large frequency of oscillation according to Eq. (i).

Therefore, Assertion and Reason are correct and Reason is the correct explanation of Assertion.

**70** (*d*) Time period of simple pendulum when it is falling under acceleration of $a$ is

$$T = 2\pi\sqrt{\frac{l}{g-a}} \quad \ldots(\text{i})$$

In the case of freely falling, $a = g$.

$\therefore$ From Eq. (i),

$$T = 2\pi\sqrt{\frac{l}{g-g}}$$

$$\Rightarrow \quad T = \infty$$

Therefore, Assertion is incorrect but Reason is correct.

**71** (*d*) In damped oscillations, the energy of the system is dissipated continuously although the motion is approximately periodic for small damping but not strictly periodic.

This is because, due to the presence of dissipative forces, such as drag friction, etc. the amplitude of oscillation decreases.

Thus, Assertion is incorrect but Reason is correct.

**72** (*d*) Total mechanical energy of an oscillation executing SHM, $E = \frac{1}{2}kA^2$

But for damped oscillation, $A(t) = Ae^{-bt/2m}$

So, for damped oscillation,

$$E = \frac{1}{2}k\,(Ae^{-bt/2m})^2 = \frac{1}{2}kA^2\,e^{-bt/m}$$

Thus, $E$ decreases with time $t$.

Therefore, Assertion is incorrect but Reason is correct.

**73** (*c*) Air drag and friction at the support oppose the motion of the pendulum and dissipate its energy gradually. Thus, the pendulum is said to be executing damped oscillations.

However, for small damping, the oscillations remain approximately periodic.

Therefore, Assertion is correct but Reason is incorrect.

**74** (*d*) When a system is displaced from its equilibrium position and released, it oscillates with its natural frequency $\omega$ and the oscillations are called free oscillations.

All free oscillations eventually die out because of the ever present damping forces. However, an external agency can maintain these oscillations. These are called forced or driven oscillations.

The most familiar example of forced oscillation is when a child in a garden swing periodically presses his feet against the ground or someone else periodically gives the child a push to maintain the oscillations.

Therefore, Assertion is incorrect but Reason is correct.

**75** (*d*) For forced oscillations, external force can be represented as

$$F_{\text{ext}} = F_0 \cos \omega_d t$$

This means, $F_{\text{ext}}$ varies with time and is not constant. Also, this force helps in sustaining the oscillations.

Thus, these types of oscillations are called forced or driven oscillations.

Thus, Assertion is incorrect but Reason is correct.

**76** (*d*) Resonance is a phenomenon of increase in amplitude, when driving frequency is equal to natural frequency of the system.

At resonance amplitude is maximum, but it can never be infinity due to the ever present dissipative forces in nature.

Therefore, Assertion is incorrect but Reason is correct.

**77** (*a*) The army troops are suggested to break their march because the hanging bridge could collapse, if the frequency of their march and the frequency at which bridge oscillate would match.

As, it this point the condition of the resonance would be satisfied. Thus, the amplitude with which the bridge was oscillating would increase, thereby leading the bridge to collapse.

Therefore, Assertion and Reason are correct and Reason is the correct explanation of Assertion.

**79** (*c*) We know that, for a spring mass system, restoring force, $\mathbf{F}_s = -k\mathbf{x}$ (spring force) $\quad \ldots$(i)

where, $k$ is spring constant.

$$\mathbf{F} = -m\omega^2\,\mathbf{x} \text{ (for SHM condition)} \quad \ldots(\text{ii})$$

On comparing Eqs. (i) and (ii), we get

$$\omega = \sqrt{\frac{k}{m}} \Rightarrow T = 2\pi\sqrt{\frac{m}{k}} \qquad \left(\because T = \frac{2\pi}{\omega}\right)$$

$$\Rightarrow \quad T \propto \sqrt{m} \quad \Rightarrow T \propto \frac{1}{\sqrt{k}}$$

So, time period $T$ does not depends on the amplitude of the oscillation

but depend on $m$ and $k$.

So, statements II and III are correct but I is incorrect.

**80** (*a*) Let simple harmonic motions be represented by

$$y_1 = a\sin\left(\omega t - \frac{\pi}{4}\right);\; y_2 = a\sin\omega t$$

and $y_3 = a\sin\left(\omega t + \frac{\pi}{4}\right)$.

On superimposing, resultant SHM will be

$$y = a\left[\sin\left(\omega t - \frac{\pi}{4}\right) + \sin\omega t + \sin\left(\omega t + \frac{\pi}{4}\right)\right]$$

$$= a\left[2\sin\omega t\cos\frac{\pi}{4} + \sin\omega t\right]$$

$$= a\,[\sqrt{2}\sin\omega t + \sin\omega t] = a\,(1+\sqrt{2})\sin\omega t$$

Thus, this function represents SHM with time period $T = \frac{2\pi}{\omega}$ and resultant amplitude, $A = (1+\sqrt{2})a$.

As, energy in SHM $\propto$ (amplitude)$^2$

$$\therefore \quad \frac{E_{\text{resultant}}}{E_{\text{single}}} = \left(\frac{A}{a}\right)^2 = (\sqrt{2}+1)^2 = (3+2\sqrt{2})$$

$$\Rightarrow \quad E_{\text{resultant}} = (3+2\sqrt{2})E_{\text{single}}$$

Also, the phase of the resultant motion $y$ relative to the first motion $y_1$ is differ by $\frac{\pi}{4}$.

Thus, the statement given in option (a) is correct, rest are incorrect.

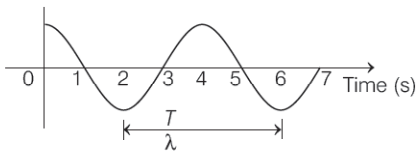
**81** (*d*) Actually, the motion of moon callisto is uniform circular motion.

However, what Galileo observed the projection of that uniform circular motion in a line of plane of motion.

Hence, when it was viewed from earth, it looked like a to and fro motion, i.e. a simple harmonic motion.

Thus, the statements given in options (a), (b) and (c) are all correct.

**82** (*b*) It is clear from the curve that points corresponding to $t = 2$ s and $t = 6$ s are separated by a distance $\lambda$ belongs to one time period. Hence, these points must be in same phase.

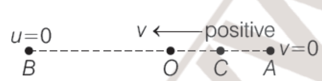



Similarly, points belonging to $t = 0$ s and $t = 2$ s are separated by half the distance that belongs to one time period. Hence, they are not in phase.

However, points belonging to $t = 3$ s and $t = 5$ s or $t = 1$ s and 7 s are at separation of different time period, hence they must not be in phase.

Thus, the statement given in option (b) is correct, rest are incorrect.

**83** (*b*) Consider the diagram,




where, the direction from $A$ to $B$ is taken as positive.

(a) When the particle is 3cm away from $A$ going towards $B$, velocity is towards $AB$, i.e. positive.

As, in SHM, acceleration and force is always towards mean position $O$. So in the given case, it will also be positive in this case.

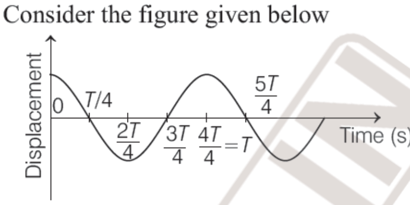
(b) When the particle is at $C$ going towards $B$ , then the velocity is towards $B$. Hence, it will also be positive.

(c) When the particle is 4 cm away from $B$ going towards $A$, i.e. velocity is towards $BA$. Thus, velocity will be negative in this case. Similarly, acceleration and force is also negative.

(d) When the particle is at $B$ acceleration and force are towards $BA$, i.e. negative.

Thus, the statement given in option (b) is incorrect, rest are correct.

**84** (*d*) Consider the figure given below




From this figure, it is clear that

(a) at $t = \frac{3T}{4}$, the displacement of the particle is zero.

Hence, the particle executing SHM will be at mean position, i.e. $x = 0$. So, acceleration is zero and force is also zero.

(b) at $t = \frac{4T}{4} = T$, displacement is maximum, i.e. the particle is at extreme position, so acceleration is maximum.

(c) Similarly, at $t = \frac{T}{4}$, the particle will be at to mean position, so velocity will be maximum at this position.

(d) at $t = \frac{2T}{4} = \frac{T}{2}$, the particle will be at extreme position, so KE $= 0$ and PE = maximum.

Thus, the statement given in option (d) is incorrect, rest are correct.

**85** (*c*) Let the equation of a SHM is represented as $x = a \sin \omega t$

Assume, mass of the body $= m$.

(a) Total mechanical energy of the body at any time $t$ is

$$E = \frac{1}{2} m \omega^2 a^2 \quad \text{...(i)}$$

Kinetic energy at any instant $t$ is

$$K = \frac{1}{2} m v^2 = \frac{1}{2} m \left[\frac{dx}{dt}\right]^2 \quad \left(\because v = \frac{dx}{dt}\right)$$

$$= \frac{1}{2} m\omega^2 (a^2 - x^2)$$

$$\Rightarrow K_{\max} = \frac{1}{2} m\omega^2 a^2 = E \quad \text{...(ii)}$$

Thus, total energy per cycle is equal to its maximum KE.

(b) KE at any instant $t$ can also be written as

$$K = \frac{1}{2} m\omega^2 a^2 \cos^2 \omega t$$

$$K_{\text{av}} \text{ for a cycle} = \frac{1}{2} m\omega^2 a^2 [(\cos^2 \omega t)_{\text{av}}]$$

$$\text{For a cycle } = \frac{1}{2} m\omega^2 a^2 \left(\frac{0+1}{2}\right)$$

$$= \frac{1}{4} m\omega^2 a^2 = \frac{K_{max}}{2} \quad \text{[from Eq. (ii)]}$$

(c) Velocity, $v = \frac{dx}{dt} = a\,\omega\cos\omega t$

$$v_{mean} = \frac{v_{max} + v_{min}}{2}$$

$$= \frac{a\omega + (-a\omega)}{2} = 0 \quad \text{(for a complete cycle)}$$

$$\Rightarrow \quad v_{max} \neq \frac{2}{\pi} v_{mean}$$

Thus, the statement given in option (c) is incorrect, rest are correct.

**86** (*c*) For a particle executing SHM, its kinetic energy,

$$KE = \frac{1}{2} m\omega^2 (A^2 - x^2)$$

$$\Rightarrow \quad (KE)_{max} = \frac{1}{2} m\omega^2 A^2$$

Potential energy, $PE = (PE)_{max} = \frac{1}{2} m\omega^2 A^2$

Total mechanical energy TE = KE + PE

$$= \frac{1}{2} m\omega^2 A^2$$

$$\Rightarrow \quad TE = (KE)_{max} = (PE)_{max}$$

So, if maximum potential energy becomes double, then both total energy and kinetic energy will also become double.

Thus, the statements given in both options (a) and (b) are correct.

**87** (*a*) For a body executing SHM, its kinetic energy,

$$KE(x) = \frac{1}{2} m\omega^2 (A^2 - x^2)$$

where, $A$ is the amplitude.

So, KE at $x = 0$, $KE(0) = \frac{1}{2} m\omega^2 A^2 = (KE)_{max}$

$x$ is maximum at $A$

So, at $x = A$

$$KE = \frac{1}{2} m\omega^2 (A^2 - A^2) = 0$$

$\therefore$ KE is minimum when $x$ is maximum.

Potential Energy (PE) $= \frac{1}{2} m\omega^2 x^2$

So, PE at $x = 0$, $PE(0) = 0$

So, PE is minimum at $x = 0$.

At $x = A$

$$PE = \frac{1}{2} m^0 \omega^2 A^2 = (PE)_{max}$$

$\therefore$ PE is maximum when $x$ is maximum.

Total Energy (TE) $= \frac{1}{2} kA^2 = \frac{1}{2} m\omega^2 A^2$

This means TE remains constant.

Thus, statement given in option (a) is correct, rest are incorrect.

**88** (*d*) Both kinetic and potential energies of a particle, i.e. block in SHM vary between zero and maximum values.

Since, total mechanical (KE + PE) is constant for this system, there will be interconversion of KE and PE during motion.

KE will be maximum at mean position (i.e. $x = 0$) and potential energy will be maximum at extreme (i.e. $x = \pm A$).

Thus, all statements given in options (a), (b) and (c) are correct.

**89** (*a*) Kinetic energy and potential energy of a particle executing SHM are periodic with period $\frac{T}{2}$.

Time periods of variation of potential and kinetic energies are same.

However, for such a particle the total mechanical energy, i.e. $U(t) + K(t)$ remains constant always.

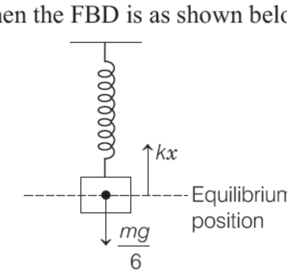
This can also be seen from the graph given in question.

Also, kinetic and potential both energies are positive.

Thus, statement given in option (a) is correct, rest are incorrect.

**90** (*b*) If $g$ is the acceleration due to gravity on earth, then acceleration due to gravity on moon is $\frac{g}{6}$.

So, when the spring-mass system is taken to the moon, then the FBD is as shown below.




In equilibrium, $\quad kx = \frac{mg}{6}$

where, $k$ is spring constant and $x$ is the extension in the spring.

$$\Rightarrow \quad x = \frac{mg}{6k}$$

Time period of oscillation for spring-mass system,

$$T = 2\pi \sqrt{\frac{m}{k}}$$

Since, the value of $k$ is governed by the elastic properties of the spring only. Also, mass $m$ remains same everywhere irrespective of its position.

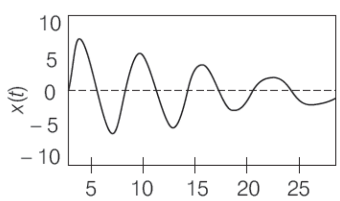
Thus, time period $T$ remains unchanged.

Thus, statement given in option (b) is correct but rest are incorrect.

**91** (*b*) The given expression represents the displacement for a damped oscillator.

In this amplitude for a damped oscillation varies as $Ae^{-bt/2m}$.

But this oscillation is approximately periodic, if the damping is small. So, for the given displacement variation w.r.t. time is shown in the graph below.




From this, we can conclude that the amplitude of the damped oscillator decreases with time.

Thus, statement given in option (b) is correct, rest are incorrect.

**92** (*c*) From the given figure, amplitude is maximum when $\frac{\omega_d}{\omega} = 1$, i.e. $\omega_d = \omega$.

Also the peak of amplitude is maximum for curve *a* which has least damping. With further increase in damping, amplitude decreases.

Thus, the statements given in options (a) and (b) is correct, so option (c) is correct.

**93** (*b*) If a rigid body is moved in such a way that all the particles constituting it undergo circular motion about a common axis, then that type of motion is called rotational motion.

Motion of a pendulum represents to and fro movement about its equilibrium. This represents oscillatory motion.

Motion of car on a straight road represents rectilinear motion.

Motion of a ball thrown by a boy at an angle with horizontal represents projectile motion.

Hence, A → 4, B → 3, C → 2 and D → 1.

**94** (*b*) For an object executing SHM,

A. $v_{max} = A\omega$

$$\Rightarrow \quad \frac{v_{max}}{\text{Amplitude}} = \frac{A\omega}{A} = \omega$$

B. Similarly, $a_{max} = A\omega^2$

$$\text{So,} \quad \frac{a_{max}}{\text{Amplitude}} = \frac{A\omega^2}{A} = \omega^2$$

C. If object starts from $x = +A$, its equation is

$$x = A\cos\omega t$$

$$\text{For} \quad x = +A/\sqrt{2}$$

$$\frac{A}{\sqrt{2}} = A\cos\omega t$$

$$\cos\omega t = \frac{1}{\sqrt{2}} = \cos\left(\frac{\pi}{4}\right)$$

$$\Rightarrow \quad \frac{2\pi}{T} \times t = \frac{\pi}{4} \Rightarrow t = T/8$$

D. If object starts from $x = 0$, its equation is

$$x = A\sin\omega t$$

$$\text{For } x = +A/2 \Rightarrow \frac{A}{2} = A\sin\omega t$$

$$\therefore \quad \frac{1}{2} = \sin\omega t$$

$$\Rightarrow \quad \sin\frac{\pi}{6} = \sin\omega t$$

$$\Rightarrow \quad \omega t = \frac{\pi}{6}$$

$$\frac{2\pi}{T}\cdot t = \frac{\pi}{6}$$

$$t = \frac{T}{12}$$

Hence, A → 3, B → 4, C → 1 and D → 2.

**95** (*c*)

A. Suppose an external force $F(t)$ of amplitude $F_0$ that varies periodically with time is applied to a damped oscillator (a system representing forced oscillation). Such a force can be represented as

$$F(t) = F_0 \cos\omega_d t \quad \ldots\text{(i)}$$

where, $\omega_d$ = driving frequency.

B. The motion of the particle in such a system is under the combined action of a linear restoring force, damping force and a time dependent driving force is represented by

$$F_{net} = -kx(t) - bv(t) + F_0\cos\omega_d t$$

$$\Rightarrow m\,a(t) = -kx(t) - b\,v(t) + F_0\cos\omega_d t$$

$$\Rightarrow a(t) = -\frac{k}{m}x(t) - \frac{b}{m}v(t) + \frac{F_0}{m}\cos\omega_d t \quad \ldots\text{(ii)}$$

C. The oscillator initially oscillates with its natural frequency $\omega$ when we apply the external periodic force, the oscillations with the natural frequency die out, and then the body oscillates with the (angular) frequency of the external periodic force. Its displacement after the natural oscillations die out is given by

$$x(t) = A\cos(\omega_d t + \phi)$$

D. Where, amplitude,

$$A = \frac{F_0}{\{m^2(\omega^2 - \omega_d^2)^2 + \omega_d^2 b^2\}^{1/2}}$$

$$\text{and} \quad \tan\phi = \frac{-v_0}{\omega_d x_0}$$

Here, $v_0$ and $x_0$ are velocity and displacement, respectively.

Hence, A → 4, B → 1, C → 2 and D → 3.

**96** (*a*) In the given $x$-$t$ graph, for

(a) No repetition of motion takes place rather it represents a unidirectional, linear but non-uniform motion of the particle, hence motion is non-periodic.

(b) Motion repeats after every 2 s. Hence, it is periodic with time period 2 s.

(c) Motion repeats after every 4s, hence it is periodic with time period of 4 s.

(d) Clearly, the motion repeats itself after 2 s. Hence, periodic having a time-period of 2 s.

**97** (*a*) Given $x(t) = A\cos(\omega t + \phi)$

At $t = 0$; position, $x(t) = 1\,\text{cm}$, velocity, $v = \omega\ \text{cms}^{-1}$

$\Rightarrow$ For $t = 0$, $\quad 1 = A\cos\phi \quad$ ...(i)

Now, $\quad v(t) = \dfrac{dx(t)}{dt} = \dfrac{d}{dt}[A\cos(\omega t + \phi)]$

$= -A\omega\sin(\omega t + \phi)$

Again at $t = 0$, $\quad v = \omega\ \text{cms}^{-1} \Rightarrow \omega = -A\omega\sin\phi$

$\Rightarrow \quad -1 = A\sin\phi \quad$ ...(ii)

Squaring and adding Eqs. (i) and (ii), we get

$$A^2\cos^2\phi + A^2\sin^2\phi = (1)^2 + (-1)^2$$

$$A^2 = 2$$

$$\Rightarrow \quad A = \pm\sqrt{2}\ \text{cm}$$

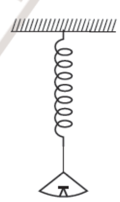
Hence, the amplitude $= \sqrt{2}$ cm

Dividing Eq. (ii) by Eq. (i), we have

$$\frac{A\sin\phi}{A\cos\phi} = \frac{-1}{1} \text{ or } \tan\phi = -1$$

$\Rightarrow$ Initial phase angle, $\phi = -\dfrac{\pi}{4}$

**98** (*c*) As the length of the scale is 20 cm and it can read upto 50 kg. The maximum extension of 20 cm will correspond to maximum weight of $w = mg = 50\ \text{kg} \times 9.8\ \text{ms}^{-2}$.



Using, $\quad \mathbf{F} = -k\mathbf{x}$

$|\mathbf{F}| = F = kx$

As, $\quad F = mg$

$\Rightarrow \quad k = \dfrac{mg}{x}$

Here, substituting the given values, we get

$$k = \frac{50 \times 9.8}{20 \times 10^{-2}} = 2450\ \text{Nm}^{-1}$$

As we know, for a spring mass system time period for oscillation, $T = 2\pi\sqrt{\dfrac{m}{k}}$

or $\quad m = \dfrac{T^2 k}{4\pi^2} = \dfrac{(0.6)^2 \times 2450}{4 \times (3.14)^2} = 22.36\,\text{kg}$

Weight of the body, $w = mg = 22.36 \times 9.8 = 219.17\,\text{N}$

$\simeq 220\,\text{N}$

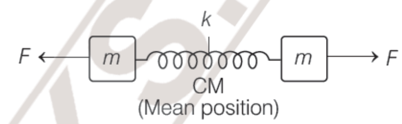
**99** (*b*) Given, spring constant, $k = 1200\ \text{Nm}^{-1}$, mass, $m = 30\,\text{kg}$

Now, frequency of oscillation,

$$\nu = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = \frac{1}{2 \times 3.14}\sqrt{\frac{1200}{3}}$$

$$= \frac{1}{2 \times 3.14} \times 20 = 3.18 \cong 3.2\ \text{s}^{-1}$$

**100** (*b*) The given system of springs can be shown below as




The system is divided into two similar systems with spring divided in two equal halves, $k' = 2k$

Hence, $\quad F = -k'x$

$\Rightarrow \quad F = -2kx$

But $\quad F = ma$

$\therefore \quad ma = -2kx$

$\Rightarrow \quad a = -\left(\dfrac{2k}{m}\right)x \quad$ ...(i)

$\Rightarrow \quad a \propto -x$ (displacement) (as $\dfrac{2k}{m}$ is a constant)

On comparing Eq. (i) with $a = -\omega^2 x$, we get

$$\omega = \sqrt{\frac{2k}{m}}$$

Period of oscillation, $T = \dfrac{2\pi}{\omega} = 2\pi\sqrt{\dfrac{m}{2k}}$

**101** (*c*) Given, angular frequency of the piston, $\omega = 200\ \text{rad min}^{-1}$

Stroke length $= 1\,\text{m}$

$\therefore$ Amplitude of SHM, $A = \dfrac{\text{Stroke length}}{2} = \dfrac{1}{2} = 0.5\,\text{m}$

Now, $\quad v_{\max} = \omega A = 200 \times 0.5 = 100\ \text{m min}^{-1}$

**102** (*d*) On the surface of the earth, time period,

$\therefore \quad T_e = 2\pi\sqrt{\dfrac{l}{g_e}} \quad$ ...(i)

On the surface of the moon, time period,

$T_m = 2\pi\sqrt{\dfrac{l}{g_m}} \quad$ ...(ii)

where, $g_e$ and $g_m$ are acceleration due to gravity on the earth and moon surfaces, respectively.

On dividing Eq. (i) by Eq. (ii), we get

$$\frac{T_e}{T_m} = \frac{2\pi}{2\pi}\sqrt{\frac{l}{l} \times \frac{g_m}{g_e}}$$

$\Rightarrow \quad T_m = \sqrt{\dfrac{g_e}{g_m}} \cdot T_e \quad$ ...(iii)

Given, $g_e = 9.8\ \text{ms}^{-2}$, $g_m = 1.7\text{ms}^{-2}$ and $T_e = 3.5$ s

Putting the given values in Eq. (iii), we get

$$T_m = \sqrt{\frac{9.8}{1.7}} \times 3.5 = 8.4\ \text{s}$$

**103** (*b*) Mass supported by each wheel = 750 kg

For damping factor $b$, the equation of displacement is

$$x = x_0 e^{-bt/2m}$$

As, $x = x_0/2$

we have $\frac{x_0}{2} = x_0 e^{-bt/2m}$

$$\Rightarrow \quad \log_e 2 = \frac{bt}{2m} \text{ or } b = \frac{2m \log_e 2}{t} \quad \ldots\text{(i)}$$

Given, $m = 3000\,\text{kg}$, $g = 9.8\,\text{ms}^{-2}$ and $x = 15\,\text{cm} = 0.15\,\text{m}$

Restoring force of the system,

$$F = -4kx = mg$$

$$\therefore \quad k = \frac{mg}{4x} = \frac{3000 \times 9.8}{4 \times 0.15}$$

(neglecting negative sign, which is only for direction)

$$\approx 5 \times 10^4\ \text{Nm}^{-1}$$

The time taken in 50% damping, $t$ = One time period = $T$

$$T = 2\pi\sqrt{\frac{m}{4k}} = 2\pi\sqrt{\frac{3000}{4 \times 5 \times 10^4}} = 0.769\ \text{s}$$

$\therefore$ Substituting values in Eq. (i), we get

$$b = \frac{2 \times 750 \times 0.693}{0.769}$$

$$= 1351.58\ \text{kgs}^{-1} \simeq 1352\ \text{kgs}^{-1}$$

**104** (*a*) When displacement is $x = 5$ cm = 0.05 m

$$\text{Acceleration, } a = -\omega^2 x = -\left(\frac{2\pi}{T}\right)^2 (x)$$

$$= -\left(\frac{2\pi}{0.2}\right)^2 (0.05) = -5\pi^2\ \text{ms}^{-2}$$

$$\text{Velocity, } v = \omega\sqrt{a^2 - x^2} = \left(\frac{2\pi}{T}\right)\sqrt{(0.05)^2 - (0.05)^2}$$

$$= \left(\frac{2\pi}{T}\right) \times 0 = 0$$

**105** (*b*) Given, $y = 3\cos\left(\frac{\pi}{4} - 2\omega t\right)$

$$\text{Velocity of the particle, } v = \frac{dy}{dt} = \frac{d}{dt}\left[3\cos\left(\frac{\pi}{4} - 2\omega t\right)\right]$$

$$= 6\omega \sin\left(\frac{\pi}{4} - 2\omega t\right)$$

$$\text{Acceleration, } a = \frac{dv}{dt} = \frac{d}{dt}\left[6\omega \sin\left(\frac{\pi}{4} - 2\omega t\right)\right]$$

$$\Rightarrow \quad a = -12\omega^2 y$$

$\Rightarrow$ As acceleration, $a \propto -y$

Hence, motion is SHM.

Clearly, from the equation

$$\omega' = 2\omega$$

[$\because$ Comparing the given equation with standard equation, $y = a\cos(\omega' t + \phi)$]

$$\Rightarrow \quad \frac{2\pi}{T'} = 2\omega \quad \Rightarrow \quad T' = \frac{2\pi}{2\omega} = \frac{\pi}{\omega}$$

So, motion is SHM with period $\frac{\pi}{\omega}$.

**106** (*b*) Given equation of motion is

$$y = \sin^3 \omega t = (3\sin\omega t - 4\sin 3\omega t)/4$$

$$(\because \sin 3\theta = 3\sin\theta - 4\sin^3\theta)$$

$$\Rightarrow \quad \frac{dy}{dt} = \left[\frac{d}{dt}(3\sin\omega t) - \frac{d}{dt}(4\sin 3\omega t)\right]/4$$

$$\Rightarrow \quad 4\frac{dy}{dt} = 3\omega\cos\omega t - 36 \times [3\omega^2 \cos 3\omega t]$$

$$\Rightarrow \quad 4 \times \frac{d^2y}{dt^2} = -3\omega^2 \sin\omega t + 36\omega^2 \sin 3\omega t$$

$$\Rightarrow \quad \frac{d^2y}{dt^2} = -\frac{3\omega^2 \sin\omega t + 36\omega^2 \sin 3\omega t}{4}$$

$\Rightarrow \frac{d^2y}{dt^2}$ is not proportional to $y$.

Hence, motion is not SHM.

As the expression is involving sine function, hence it will be periodic.

**107** (*d*) For a particle executing SHM,

acceleration $a \propto -$ displacement $x$

which is correctly given in option (d) only.

**108** (*d*) According to the question,

displacement, $y = a\sin\omega t + b\cos\omega t \quad \ldots\text{(i)}$

Let $a = A\sin\phi$ and $b = A\cos\phi$

Now, $a^2 + b^2 = A^2\sin^2\phi + A^2\cos^2\phi = A^2$

$$\Rightarrow \quad A = \sqrt{a^2 + b^2}$$

Now, substituting the values of $a, b$ and $A$ in Eq. (i), we get

$$\Rightarrow \quad y = A\sin\phi \cdot \sin\omega t + A\cos\phi \cdot \cos\omega t$$

$$= A\sin(\omega t + \phi)$$

[$\because$ using trigonometric identity $\sin(A + B) = \sin A\cos B + \cos A\sin B$]

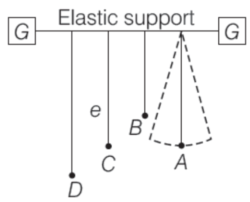
$$\Rightarrow \quad \frac{dy}{dt} = A\omega\cos(\omega t + \phi)$$

$$\Rightarrow \quad \frac{d^2y}{dt^2} = -A\omega^2\sin(\omega t + \phi) = -Ay\omega^2 = (-A\omega^2)y$$

$$\Rightarrow \quad \frac{d^2y}{dt^2} \propto -y$$

Hence, it is an equation of SHM with amplitude, $A = \sqrt{a^2 + b^2}$.

**109** (*b*) According to the question, $A$ is given a transverse displacement as shown below

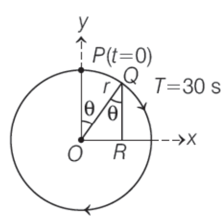



Through the elastic support, the disturbance is transferred to all the pendulums. $A$ and $C$ are having same length, hence they will be in resonance, it is because their time period of oscillation,

$T = 2\pi\sqrt{\dfrac{l}{g}}$ will be same and hence frequency will be same.

So, $C$ will vibrate with maximum amplitude.

**110** (*a*) Let angular velocity of the particle executing circular motion is $\omega$ and when it is at $Q$ it makes an angle $\theta$ as shown in the diagram.




Clearly, $\theta = \omega t$

Now, we can write $OR = OQ\cos(90° - \theta)$

$$= OQ\sin\theta = OQ\sin\omega t$$

$$= r\sin\omega t \qquad (\because OQ = r)$$

$$\Rightarrow \quad x = r\sin\omega t = B\sin\omega t \qquad (\because r = B)$$

$$= B\sin\frac{2\pi}{T}t = B\sin\left(\frac{2\pi}{30}t\right)$$

Clearly, this equation represents SHM.

**111** (*c*) As the given equation is $x = a\cos(\alpha t)^2$ is a cosine function, hence it is an oscillatory motion.

Now, putting $t + T$ in place of $t$

$$x(t+T) = a\cos[\alpha(t+T)]^2$$

$$= a\cos(\alpha t^2 + \alpha T^2 + 2\alpha tT) \neq x(t)$$

where, $T$ is supposed to be the period of the given function.

Hence, it is not periodic.

**112** (*a*) Let equation of the particle executing SHM is represented by

$$y = A\sin\omega t$$

Particle's speed, $v = \dfrac{dy}{dt} = A\,\omega\cos\omega t$

$\Rightarrow$ Maximum speed,

$$(v)_{max} = A\omega = 30\,\text{cms}^{-1} \qquad \text{...(i) (given)}$$

Particle's acceleration, $a = \dfrac{dx^2}{dt^2} = -A\omega^2\sin\omega t$

Maximum acceleration,

$$|a_{max}| = \omega^2 A = 60\,\text{cms}^{-2} \qquad \text{...(ii) (given)}$$

From Eqs. (i) and (ii), we get

$$\omega(\omega A) = 60$$

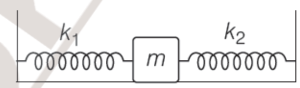
$$\Rightarrow \quad \omega(30) = 60$$

$$\Rightarrow \quad \omega = 2\,\text{rads}^{-1}$$

$$\Rightarrow \quad \frac{2\pi}{T} = 2\,\text{rads}^{-1} \Rightarrow T = \pi\,\text{s}$$

$\therefore$ The period of oscillations in $\pi$ s.

**113** (*b*) In the given figure (as shown below), it can be said that the two springs are connected in parallel.




where, equivalent spring constant $= k_{eq} = k_1 + k_2$.

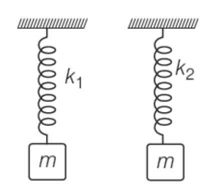
Time period of oscillation of the spring-block system,

$$T = 2\pi\sqrt{\frac{m}{k_{eq}}} = 2\pi\sqrt{\frac{m}{k_1 + k_2}}$$

$$\Rightarrow \quad \nu = \frac{1}{T} = \frac{1}{2\pi}\times\sqrt{\frac{k_1 + k_2}{m}} \qquad \text{...(i)}$$

= equivalent oscillation frequency.

Initially when the mass is connected to the two springs individually as shown below, then




$$\nu_1 = \frac{1}{2\pi}\sqrt{\frac{k_1}{m}} \qquad \text{...(ii)}$$

and
$$\nu_2 = \frac{1}{2\pi}\sqrt{\frac{k_2}{m}} \qquad \text{...(iii)}$$

From Eq. (i), we get

$$\nu = \frac{1}{2\pi}\left[\frac{k_1}{m} + \frac{k_2}{m}\right]^{1/2}$$

$$= \frac{1}{2\pi}\left[\frac{4\pi^2\nu_1^2}{1} + \frac{4\pi^2\nu_2^2}{1}\right]^{1/2}$$

$$\left[\begin{array}{l}\because \text{from Eq. (ii), } \dfrac{k_1}{m} = 4\pi^2\nu_1^2 \\ \text{and from Eq. (iii), } \dfrac{k_2}{m} = 4\pi^2\nu_2^2\end{array}\right]$$

$$= \frac{2\pi}{2\pi}[\nu_1^2 + \nu_2^2]^{1/2}$$

$$\Rightarrow \quad \nu = \sqrt{\nu_1^2 + \nu_2^2}$$

CHAPTER › 15

# Waves

## KEY NOTES

- **Wave** is a pattern which moves without the actual physical transfer or flow of matter as a whole.
- Waves transport energy and the pattern of disturbance has information that propagates from one point to another.
- There are mainly three types of waves, which are as follows
  (i) **Mechanical Waves** These waves require a medium for their propagation, i.e. they cannot propagate through vacuum, e.g. waves on string, water wave, etc.
  They involve oscillations of constituent particles and depend on the elastic properties of the medium.
  (ii) **Electromagnetic Waves** These waves do not necessarily require a medium for their propagation and thus they can travel through vacuum, e.g. light, radio waves, X-rays, etc.
  In vacuum, all electromagnetic waves have the same speed $c$, where $c = 299{,}792{,}458 \text{ ms}^{-1}$.
  (iii) **Matter Waves** These waves are associated with constituents of matter namely, electrons, protons, neutrons, atoms and molecules, e.g. a beam of electron, etc.
- Wave propagates in a medium due to **restoring force** which is proportional to the disturbance.
- The compression or rarefaction moves from one region to another, making the propagation of a disturbance possible in air.

## Transverse and Longitudinal Waves

- In **transverse waves**, the constituents of the medium oscillate about their mean positions in a direction perpendicular to the direction of wave propagation like a harmonic wave travelling along a stretched string.
- In **longitudinal waves**, the constituents of the medium oscillate about their mean positions along the direction of wave propagation like propagation of sound waves.
- Transverse waves can propagate only in those media, which can sustain shearing stress, such as solids not in fluids.
  While fluids as well as solids can sustain compressive strain, therefore longitudinal waves can be propagated in all elastic media.
- The waves on the surface of water are of two types
  (i) **Capiliary Waves** These are ripples of fairly short wavelength (not more than a few centimeter) and the restoring force that produces them is the surface tension of water.
  (ii) **Gravity Waves** These waves have wavelengths typically ranging from several meters to several hundred meters. The restoring force that produces these waves is the pull of gravity, which tends to keep the water surface at its lowest level.
- Transverse and longitudinal waves travel with different speed in the same medium.

## Progressive Waves

- A transverse or longitudinal wave that travel from one point of the medium to another is called a progressive wave.
- The **displacement relation** of a progressive wave is given by
  $$y(x, t) = a \sin(kx - \omega t + \phi)$$
  where, $y(x, t)$ = displacement as a function of position $x$ and time $t$; $a$ = amplitude of a wave; $\omega$ = angular frequency of the wave; $k$ = angular wave number;
  $(kx - \omega t + \phi)$ = phase angle
  and $\phi$ = initial phase (at $x = 0, t = 0$)
- If displacement equation can be considered to be a linear combination of sine and cosine function,
  $$y(x, t) = A \sin(kx - \omega t) + B \cos(kx - \omega t)$$
  Then amplitude, $a = \sqrt{A^2 + B^2}$ and phase, $\phi = \tan^{-1}\left(\frac{B}{A}\right)$.
- In a transverse progressive wave, the **crest** is the point of maximum positive displacement and the **trough** is the point of maximum negative displacement.

Some important definitions related to wave motion are given below

- **Amplitude** The maximum displacement suffered by the particles of the medium from their mean position is called amplitude. It is denoted by $a$.
- **Phase** The position of a point in time on a waveform is known as the phase of a wave. It can also be expressed as relative displacement between two corresponding peaks of a waveform. It is denoted by $\phi$.
- **Wavelength** It is the minimum distance between two points having same phase or it is the distance between two consecutive crests or troughs in a wave. It is denoted by $\lambda$.
- **Wave Number** The number of waves present in a unit distance along the direction of propagation is known as wave number.

  Angular wave number, $k = \frac{2\pi}{\lambda}$ (in rad $m^{-1}$)
- **Period of oscillation** of the wave is the time it takes for a particle to complete one full oscillation. It is given by

$$T = \frac{2\pi}{\omega}$$

  where, $\omega$ = **angular frequency** $= \left(\frac{2\pi}{T}\right)$ (in rad $s^{-1}$)
- **Frequency** is the number of oscillations per second and given by

$$\nu = \frac{1}{T} = \frac{\omega}{2\pi} \text{ (in Hz)}$$

## Speed of a Travelling or Progressive Wave

- Wave speed is a measure of how fast a wave travels. For a progressive wave,

$$\text{Speed, } v = \frac{\omega}{k} = \nu\lambda = \lambda / T$$

- The **speed of transverse wave on a stretched string** is given by $v = \sqrt{\frac{T}{\mu}}$

  where, $T$ = tension in the string and $\mu$ = linear mass denstiy (or mass per unit length).

  So, speed $v$ depends only on the properties of the medium $T$ (due to restoring force) and $\mu$ (due to inertial force). It does not depend on wavelength or frequency of the wave itself.
- The general formula for **speed of longitudinal waves** (sound) in a medium is $v = \sqrt{\frac{B}{\rho}}$

  where, $B$ is the bulk modulus of liquid and $\rho$ is its mass density.
- The speed of longitudinal waves (sound) in a solid bar is

$$v = \sqrt{\frac{Y}{\rho}}$$

  where, $Y$ is Young's modulus of elasticity of the bar and $\rho$ is its mass density.
- Liquids and solids generally have higher speed of sound than gases.
- The speed of a longitudinal wave in an ideal gas (for isothermal change) is given by

$$v = \sqrt{\frac{p}{\rho}}$$

  where $p$ is the pressure of the gas and $\rho$ is its density.

  This relation is known as **Newton's formula**.
- **Laplace's correction** It was pointed out by Laplace that the pressure variations in the propagation of sound waves are so fast that there is short time for the heat flow to maintain constant temperature. So, variations are adiabatic not isothermal.

  Thus, speed of sound is given by

$$v = \sqrt{\frac{\gamma p}{\rho}}$$

  where, $\gamma$ is the ratio of specific heat of the gas at constant pressure $p$ to that at constant volume $V$, i.e. $\gamma = C_p / C_V$.

## Principle of Superposition of Waves

- According to this principle, "when the pulses travelling in different directions overlap, then the resultant displacement is the algebraic sum of the displacement due to each pulse."
- It is basic to the phenomenon of interference.

  For two waves differing only in their phases, the resultant displacement is given by

$$y(x, t) = a \sin(kx - \omega t) + a \sin(kx - \omega t + \phi)$$
$$= 2a \cos\frac{\phi}{2} \sin\left(kx - \omega t + \frac{\phi}{2}\right)$$

  Here, amplitude, $A(\phi) = 2a\cos\frac{\phi}{2}$
- At those points where phase difference $\phi = 0$ or $2n\pi$, i.e. $\phi$ is an integer multiple of $2\pi$ ($n = 0, 1, 2, 3, \ldots$), the resultant amplitude is

$$A(0) = 2a$$

  This is called **constructive interference**, where the amplitudes add up in the resultant wave.
- At those points where phase difference $\phi = (2n-1)\pi$ i.e. $\phi$ is an odd multiple of $\pi$, the resultant amplitude is $A(\pi) = 0$.

  This is called **destructive interference**, where amplitude subtract out in the resultant wave.

## Reflection of Waves

- When a progressive wave travelling through a medium reaches a rigid boundary, it gets reflected and the phenomena is called reflection of waves.

KEY NOTES

If equation of incident travelling wave is

$$y_i(x, t) = a \sin (kx - \omega t)$$

At a rigid boundary, the reflected wave is given by

$$y_r(x, t) = a \sin (kx + \omega t + \pi)$$
$$= -a \sin (kx + \omega t)$$

- A travelling wave or pulse suffers a phase change of $\pi$ (displacement $y = 0$) on reflection at a rigid boundary and no phase change on reflection at an open boundary.
- If the boundary is not completely rigid or is an interface between two different elastic media, then a point of the incident wave is reflected and a part is transmitted into second media.

**Note** If a wave incidents obliquely on the boundary between two different media, the transmitted wave is called the **refracted wave**.

- The incident and refracted waves obey Snell's law of refraction and the incident and reflected waves obey the **usual laws of reflection**.

## Standing or Stationary Waves

- These waves are produced due to the continuous reflection of wave between two ends of a string, until a steady wave pattern is observed.

  The resultant wave is given by

  $$y(x, t) = 2a \sin kx \cos \omega t$$

  where, amplitude $= 2a \sin kx$.

  These waves have following characteristics

  (i) The amplitude varies from point to point, but each element of the string oscillates with the same angular frequency $\omega$ or time period.

  (ii) There is no phase difference between oscillations of different elements of the wave.

  (iii) The wave pattern is neither moving to the right nor to the left. Hence, they are called standing or stationary waves.

- In stationary waves, the system cannot oscillate with any arbitrary frequency, but is characterised by a set of natural frequencies or **normal modes** of oscillation.
- The points at which the amplitude is zero (i.e. where there is no motion at all) are **nodes** and the points at which the amplitude is largest are called **anti-nodes**.
- Position of nodes is given by

  $$x = \frac{n\lambda}{2}; \text{ where } \quad n = 0, 1, 2, 3, \dots .$$

- Position of anti-nodes is given by

  $$x = \left(n + \frac{1}{2}\right)\frac{\lambda}{2}; \text{ where } \quad n = 0, 1, 2, \dots .$$

- The distance between any two successive nodes or anti-nodes is $\lambda / 2$.
- For a stretched string, length $L$ is related by $\lambda$,

  $$L = \frac{n\lambda}{2}; \text{ where } \quad n = 1, 2, 3, \dots .$$

  Possible wavelengths, $\lambda = \frac{2L}{n}$ with corresponding

  frequencies, $\nu = \frac{nv}{2L}$ $\qquad n = 1, 2, 3$

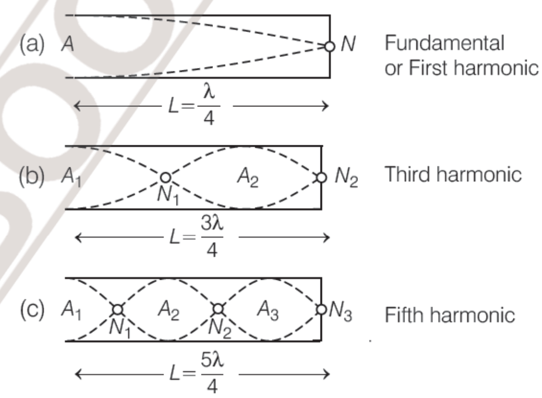
- The lowest possible natural frequency of a system is called its **fundamental mode** or the **first harmonic.**

  $\therefore$ For a stretched string fixed at either end, it is given by

  $$\nu = \frac{v}{2L} \qquad (\text{for } n = 1)$$

### Close Organ Pipe

- In a closed organ pipe, longitudinal stationary waves are formed such a way that open end of pipe behaves as an anti-node and a node is formed at the closed end of pipe as shown below.




Length $L$ of the pipe is related $\lambda$ as

$$L = \left(n + \frac{1}{2}\right)\frac{\lambda}{2}$$

The possible wavelengths, $\lambda = \frac{2L}{(n + 1/2)}$ with

corresponding frequencies, $\nu = \left(n + \frac{1}{2}\right)\frac{v}{2L}$.

- For fundamental mode ($n = 0$), $L = \frac{\lambda}{4}$ or $\lambda = 4L$.

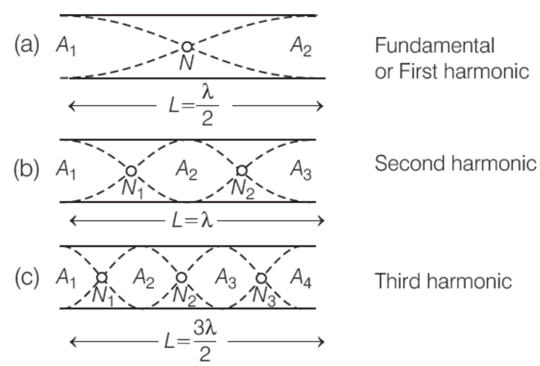
  $\therefore$ Fundamental (1st harmonic) frequency,

  $$\nu = \frac{v}{\lambda} = \frac{v}{4L}$$

- Only **odd harmonics** are present in closed pipe with their frequencies in the ratio 1 : 3: 5 : ... .

### Open Organ Pipe

- In an open organ pipe, stationary waves are formed in such a way that anti-nodes are formed at both the open ends of pipe and one or more nodes are formed in between symmetrically as shown below.




**Note** The number of nodes is 1 less than the number of anti-nodes.

Length of the pipe is $L = \frac{n\lambda}{2}$, where $n = 1, 2, 3, \ldots$ .

Possible wavelengths, $\lambda = \frac{2L}{\lambda}$ with corresponding frequencies, $\nu = \frac{nv}{2L}$.

- In fundamental mode, only one nodes is formed at the centre of pipe, so $L = \frac{\lambda}{2}$ or $\lambda = 2L$.

  $\therefore$ Fundamental (1st harmonic) frequency, $\nu = \frac{v}{\lambda} = \frac{v}{2L}$.
- All harmonics are present in open pipe with their frequencies in the ratio 1 : 2 : 3 : 4 ... .

  At the nodes, the pressure changes are largest, while the diplacement is minimum (zero).

  At the anti-nodes (open end), pressure change is least and the displacement is maximum.
- If the external frequency is close to one of the natural frequencies, the system shows **resonance**.

## Beats

- The alternate rise and fall of sound at a given position, when two sound waves of slightly different frequencies or amplitudes superimpose at a given point is called beats.
- **Beat Frequency** Number of beats formed per second are expressed as $|\nu_1 - \nu_2|$, i.e. either $(\nu_1 - \nu_2)$ or $(\nu_2 - \nu_1)$, where $\nu_1$ and $\nu_2$ are frequencies of two sound waves.
- **Beat Period** It is the reciprocal of beat frequency expressed as

$$T = \frac{1}{|\nu_1 - \nu_2|}$$

- Essential condition for hearing beats is that $|\nu_1 - \nu_2|$ should not exceed 10.

## Doppler's Effect

- If a wave source and a receiver are moving relative to each other, the frequency observed by receiver is different from the actual source frequency. This phenomena is known as doppler's effect.
- **When the source is moving and the observer is stationary**, the apparent or observer frequency is given by

  (i) When source is moving away from observer

  $$\nu = \nu_o\left(\frac{v}{v + v_s}\right)$$

  where, $\nu_o$ is original frequency of sound wave, $v$ is the speed of sound in a medium and $v_s$ is the speed of source.

  (ii) When source is approaching the observer

  $$\nu = \nu_o\left(\frac{v}{v - v_s}\right)$$
- **When the observer is moving and the source is stationary**, the apparent frequency is given by

  (i) When observer is moving away from source

  $$\nu = \left(\frac{v - v_o}{v}\right)\nu_o$$

  (ii) When source is approaching the source

  $$\nu = \left(\frac{v + v_o}{v}\right)\nu_o$$

  where, $v_o$ = velocities of observer relative of medium.
- **When both source and observer are moving**, then the observer frequency is given by

$$\nu = \nu_o\left(\frac{v \pm v_o}{v \mp v_s}\right)$$

### Applications of Doppler's Effect

Doppler effect is a wave phenomenon, it holds not only for sound waves but also for electromagnetic waves.

It has following applications as given below

(i) The change in frequency called **doppler shift** is used at airports to guide aircraft and in the military to detect enemy aircraft.

(ii) Astrophysicists use it to measure the velocities of stars.

(iii) Doctors use it to study heart beats and blood flow in different parts of body using ultrasonic waves. This process is called **sonography**.

In the case of heart, the picture generated is called **echocardiogram**.

# Mastering NCERT

## MULTIPLE CHOICE QUESTIONS

## TOPIC 1 ~ Transverse and Longitudinal Waves

**1** The correct possible order of interconversion of waves when our sound is transferred to the receiver *via* a mobile phone is
(a) sound waves $\rightarrow$ electric current signal $\rightarrow$ electromagnetic waves (radio waves)
(b) sound waves $\rightarrow$ electromagnetic waves $\rightarrow$ electric current signal
(c) sound waves $\rightarrow$ electric current signal $\rightarrow$ sound wave
(d) None of the above

**2** Which of the following waves require a medium for their propagation?
(a) Light waves (b) Radiowaves
(c) X-rays (d) Seismic waves

**3** Matter waves used in electron microscopes are associated with
(a) electrons (b) protons
(c) neutrons (d) atoms

**4** The wave generated from up and down jerk given to the string or by up and down motion of the piston at end of the pipe is
(a) transverse or longitudinal (b) progressive
(c) standing (d) Both (a) and (b)

**5** In which of the following cases, the material medium as a whole does not move?
(a) A stream (b) A wind
(c) Water wave (d) Both (a) and (c)

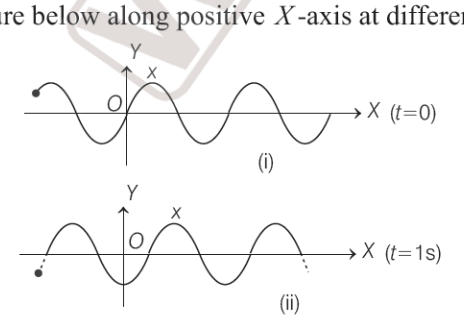
## TOPIC 2 ~ Displacement and Speed of Progressive Waves

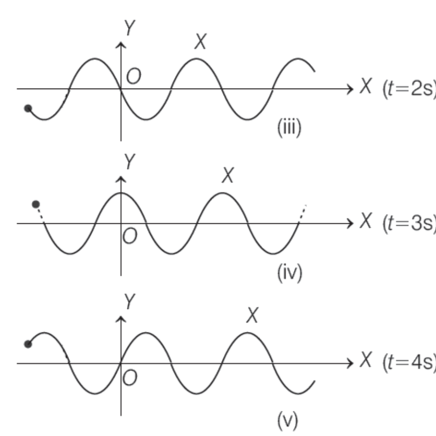
**6** For mathematical description of a travelling wave, we need a function of both position $x$ and time $t$. Such a function describes
(a) shape of the wave
(b) motion of the constituent of the medium
(c) Both (a) and (b)
(d) None of the above

**7** The displacement of an elastic wave is given by the function $y = 3\sin \omega t + 4\cos \omega t$, where $y$ is in cm and $t$ is in second. The resultant amplitude will be
(a) 15 cm (b) 4 cm (c) 5 cm (d) 3 cm

**8** A simple wave motion is represented by $y = 5\,(\sin 4\pi t + \sqrt{3}\cos 4\pi t)$. Its amplitude is
(a) 5 (b) $5\sqrt{3}$ (c) $10\sqrt{3}$ (d) 10

**9** The propagation of a harmonic wave is shown in figure below along positive $X$-axis at different times.




Then, the number of waves or oscillations crossed origin $O$ in 4 s are
(a) 1 (b) 2
(c) 3 (d) 4

**10** The displacement as a function of position $x$ and time $t$ is given by $y\,(x, t) = a\sin\,(kx - \omega t + \phi)$.
The range of possible value of $y(x, t)$, if $a$ is a positive constant, is
(a) $-a \le y\,(x, t) \le a$ (b) $-\frac{a}{2} \le y\,(x, t) \le \frac{a}{2}$
(c) $-1 \le y(x, t) \le +1$ (d) $0 \le y(x, t) \le a$

**11** The equation of a wave is given by $y=10\sin\left(\frac{2\pi}{45}t+\alpha\right)$. If the displacement is 5 cm at $t=0$, then the total phase at $t=7.5$ s is
(a) $\pi$ (b) $\pi/6$ (c) $\pi/2$ (d) $\pi/3$

**12** The displacement of the wave is given by equation $y(x,t)=a\sin(kx-\omega t+\phi)$. Taking $\phi=0$ at point $x$ and $t=0$, the displacement is $y'$. The point at which the displacement is same as that of $x$ is
(a) $x+2n\pi$ (b) $x+\frac{2n\pi}{k}$
(c) $kx+2n\pi$ (d) Both (a) and (b)

**13** A wave equation is given by $y=4\sin\left[\pi\left(\frac{x}{9}-\frac{t}{5}+\frac{1}{6}\right)\right]$ where, $x$ is in cm and $t$ is in second. The wave number of the wave is
(a) $\frac{\pi}{9}$ rad cm$^{-1}$ (b) $\frac{1}{6}$ rad cm$^{-1}$
(c) $\frac{\pi}{5}$ rad cm$^{-1}$ (d) $\frac{2\pi}{9}$ rad cm$^{-1}$

**14** Equation of a progressive wave is
$$y=a\sin\left(10\pi x+11\pi t+\frac{\pi}{3}\right)$$
The wavelength of the wave is
(a) 0.2 unit (b) 0.1 unit
(c) 0.5 unit (d) 1 unit

**15** The equation of a wave is given as
$$y=0.07\sin(12\pi x-300\pi t)$$
where, $x$ and $y$ are in metre and $t$ is in second, then the correct option is
(a) $\lambda=(1/6)$ m (b) $a=0.07$ m
(c) Both (a) and (b) (d) None of these

**16** A plane progressive wave equation is given by
$$y=2\cos 2\pi(330t-x)$$
What is the period of the wave?
(a) $\frac{1}{330}$ s (b) $2\pi\times 330$ s
(c) $(2\pi\times 330)^{-2}$ s (d) $\frac{6.284}{330}$ s

**17** In a sinusoidal wave, the time required for a particular point to move from maximum displacement to zero displacement is 0.14 s. The frequency of the wave is
(a) 0.42 Hz (b) 2.75 Hz (c) 1.79 Hz (d) 0.56 Hz

**18** The equation of a progressive wave can be given by $y=15\cos(660\pi t-0.02\pi x)$ cm. The frequency of the wave is
(a) 330 Hz (b) 342 Hz
(c) 365 Hz (d) 660 Hz

**19** A wave travelling in the positive $x$-direction having displacement along $y$-direction 1 m, wavelength $(2\pi)$ m and frequency of $(1/\pi)$ Hz is represented by **NEET 2013**
(a) $y=\sin(x-2t)$ (b) $y=\sin(2\pi x-2\pi t)$
(c) $y=\sin(10\pi x-20\pi t)$ (d) $y=\sin(2\pi x+2\pi t)$

**20** The angle between particle velocity and wave velocity in a transverse wave is
(a) zero (b) $\pi/4$ (c) $\pi/2$ (d) $\pi$

**21** The displacement $y$ of a particle in a medium can be expressed as $y=10^{-6}\sin\left(100t+20x+\frac{\pi}{4}\right)$ m, where $t$ is in second and $x$ is in metre. The speed of the wave is
(a) 2000 ms$^{-1}$ (b) 5 ms$^{-1}$
(c) 20 ms$^{-1}$ (d) $5\pi$ ms$^{-1}$

**22** The equation of progressive wave is $y=0.2\sin 2\pi\left(\frac{t}{0.01}-\frac{x}{0.3}\right)$, where $x$ and $y$ are in metres and $t$ is in second. The velocity of propagation of the wave is
(a) 30 ms$^{-1}$ (b) 40 ms$^{-1}$ (c) 300 ms$^{-1}$ (d) 400 ms$^{-1}$

**23** The speed of transverse wave on a stretched string depends on
(a) properties of the medium (both $T$ and $\mu$)
(b) wavelength of the wave
(c) frequency of the wave
(d) Both (b) and (c)

**24** A transverse wave travels on a taut steel wire with a velocity of $v$ when tension in it is $2.06\times10^4$ N. When the tension is changed to $T$, the velocity changed to $v/2$. The value of $T$ is close to **JEE Main 2020**
(a) $10.2\times10^2$ N (b) $5.15\times10^3$ N
(c) $2.50\times10^4$ N (d) $30.5\times10^4$ N

**25** A string wave equation is given by $y=0.002\cos(300t-15x)$ and mass density is $\mu=0.1$ kg/m. Then find the tension force in the string. **AIIMS 2019**
(a) 30 N (b) 20 N
(c) 40 N (d) 45 N

**26** Two strings $A$ and $B$, made of same materials are stretched by same tension. The radius of string $A$ is double the radius of $B$. A transverse wave travels on $A$ with speed $v_A$ and on $B$ with speed $v_B$. The ratio of $v_A/v_B$ is
(a) 1/2 (b) 2 (c) 1/4 (d) 4

**27** In a longitudinal wave, the elastic property of the constituents of the medium that determines the stress under compressional strain is
(a) Young's modulus $Y$ (b) Bulk modulus $B$
(c) shear modulus $S$ (d) Either (b) or (c)

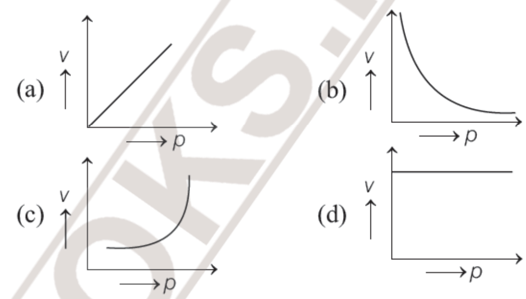
**28** Speed of sound wave in a fluid depends
(a) directly on density of the medium
(b) directly on square of bulk modulus of the medium
(c) inversely on the square root of density
(d) inversely on the square root of bulk modulus of the medium

**29** Two solid bars having Young's modulus $Y_1$ and $Y_2$ is in the ratio of $Y_1/Y_2 = 4$. If the bars are made up of materials with same density, then the ratio of the speeds of longitudinal waves in the solid bars i.e. $v_1/v_2$ is
(a) 2 (b) 1
(c) 3 (d) 4

**30** The correct order of speed of sound in different media is (consider the speed at constant temperature)
(a) $v_{\text{air}} > \underset{\text{(aluminium)}}{v_{\text{solid}}} > \underset{\text{(water)}}{v_{\text{liquid}}}$
(b) $\underset{\text{(aluminium)}}{v_{\text{solid}}} > v_{\text{air}} > \underset{\text{(water)}}{v_{\text{liquid}}}$
(c) $\underset{\text{(aluminium)}}{v_{\text{solid}}} > \underset{\text{(water)}}{v_{\text{liquid}}} > v_{\text{air}}$
(d) None of these

**31** If the mass of 1 mol of air is $29.0 \times 10^{-3}$ kg, then the speed of sound in air at standard temperature and pressure (using Newton's formula) is
(a) 250 $\text{ms}^{-1}$ (b) 280 $\text{ms}^{-1}$ (c) 300 $\text{ms}^{-1}$ (d) 380 $\text{ms}^{-1}$

**32** A student plotted the following four graphs representing the variation of velocity of sound in a gas with the pressure $p$ at constant temperature. Which one is correct?

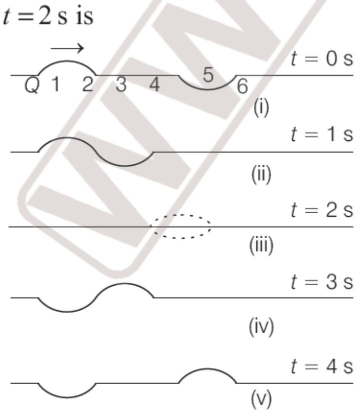



**33** According to Laplace correction, the pressure variations in the medium during propagation of sound waves are
(a) adiabatic (b) isothermal
(c) isobaric (d) isochoric

## TOPIC 3 ~ *Principle of Superposition of Waves*

**34** When two wave pulses travelling in opposite direction overlap, the resultant displacement is
(a) the algebraic sum of the displacement due to each pulse
(b) always zero
(c) the vector in the direction of the displacement of right travelling pulse
(d) Both (a) and (b)

**35** Two pulses having equal and opposite displacements moving in opposite directions overlap at $t = 2$ s as shown in figure. The resultant displacement of the wave at $t = 2$ s is




(a) twice the displacement of each pulse
(b) half the displacement of each pulse
(c) zero
(d) Either (a) or (c)

**36** If there are $n$ number of waves moving in the medium represented by the wave functions

$$y_i = f_i\,(x - vt), \text{ where } i = 1, 2, 3, \cdots, n,$$

then the resultant wave function describing the disturbance in the medium is
(a) $y = \sum_{i=1}^{n} f_i\,(x - vt)$
(b) $y = f_1\,(x - vt) + f_2\,(x - vt) + \cdots + f_n\,(x - vt)$
(c) $y = f_1(x - vt) - f_2\,(x - vt) + f_3\,(x - vt) - f_4\,(x - vt) + \cdots + (-1)^{n+1}\, f_n(x - vt)$
(d) Both (a) and (b)

**37** Two waves of equal amplitude $A$ and equal frequency travel in the same direction in a medium having phase difference of $\phi$. The amplitude of the resultant wave is
(a) 0 (b) $A$
(c) $2A$ (d) between 0 to $2A$

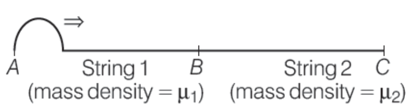
**38** Two sine waves having equal frequency travel in the same direction in a medium. The amplitude of each wave is $A$ and the phase difference between the two waves is 120°. The resultant amplitude will be
(a) $A$ (b) $2A$
(c) $4\,A$ (d) $\sqrt{2}A$

# TOPIC 4 ~ *Stationary Waves and Beats*

**39** Echo is based on which amongst the given phenomenon that sound wave follows?
(a) Refraction (b) Reflection
(c) Interference (d) Resonance

**40** When a wave is reflected normally from a rigid boundary, the reflected wave suffers a phase change of (assuming no absorption)
(a) $\pi/2$ (b) $\pi/4$
(c) $\pi$ (d) $2\pi$

**41** Two strings $AB$ and $BC$ having mass densities $\mu_1$ and $\mu_2$ are tied at $B$ and are stretched between $A$ and $C$. A wave pulse is generated at end $A$ and progresses towards junction $B$. It is given that $(\mu_2 > \mu_1)$.




If the magnitude of velocity of the reflected wave and transmitted wave are $v_r$ and $v_t$ respectively, then the correct relation is
(a) $v_r > v_t$
(b) $v_r < v_t$
(c) $v_r > v_t$ and $v_t = 0$
(d) None of the above

**42** Let a wave $y(x, t) = a \sin (kx - \omega t)$ is reflected from an open boundary and then the incident and reflected waves overlap. Then the amplitude of resultant wave is
(a) $2a \cos kx$ (b) $2a \sin kx$
(c) $2a \sin \left(\frac{kx}{2}\right)$ (d) $a \sin kx$

**43** At nodes in stationary waves, change in
(a) pressure is maximum
(b) pressure is minimum
(c) displacement is maximum
(d) density is minimum

**44** A stationary wave equation is given as $y = 20 \sin 20x \cos 1000\, t$. What will be the speed of stationary wave? ***JIPMER 2019***
(a) 20 m/s (b) 50 m/s
(c) 2 cm/s (d) Cannot be defined

**45** A string is clamped at both the ends and it is vibrating in its 4th harmonic. The equation of the stationary wave is $Y = 0.3 \sin(0.157x) \cos(200\pi t)$. The length of the string is (All quantities are in SI units) ***JEE Main 2019***
(a) 60 m (b) 40 m (c) 80 m (d) 20 m

**46** A wire under tension vibrates with a fundamental frequency of 600 Hz. If the length of the wire is doubled, the radius is halved and the wire is made to vibrate under one-ninth of the tension, then the fundamental frequency will become
(a) 400 Hz (b) 600 Hz
(c) 300 Hz (d) 200 Hz

**47** A string is stretched between fixed points separated by 75 cm. It is observed to have resonant frequencies of 420 Hz and 315 Hz. There are no other resonant frequencies between these two. The lowest resonant frequency for this string is ***CBSE AIPMT 2015***
(a) 155 Hz (b) 205 Hz
(c) 10.5 Hz (d) 105 Hz

**48** A string 2.0 m long and fixed at its ends is driven by a 240 Hz vibrator. The string vibrates in its third harmonic mode. The speed of the wave and its fundamental frequency is ***JEE Main 2019***
(a) 180 m/s, 80 Hz (b) 320 m/s, 80 Hz
(c) 320 m/s, 120 Hz (d) 180 m/s, 120 Hz

**49** A wire of length $L$ and mass per unit length $6.0 \times 10^{-3}\ \text{kgm}^{-1}$ is put under tension of 540 N. Two consecutive frequencies that it resonates at are : 420 Hz and 490 Hz. Then, $L$ in metres is ***JEE Main 2020***
(a) 8.1 m (b) 2.1 m (c) 5.1 m (d) 1.1 m

**50** A person blows into an open end of a long pipe. As a result, a high pressure pulse of air travels down the pipe. When this pulse reaches the other end of the pipe,
(a) a high pressure pulse starts travelling up the pipe, if the other end of the pipe is open
(b) a low pressure pulse starts travelling up the pipe, if the other end of the pipe is open
(c) a low pressure pulse starts travelling down the pipe, if the other end of the pipe is closed
(d) None of the above

**51** An air column, closed at one end and open at the other, resonates with a tuning fork when the smallest length of the column is 50 cm. The next larger length of the column resonating with the same tuning fork is ***NEET 2016***
(a) 100 cm (b) 150 cm (c) 200 cm (d) 66.7 cm

**52** The two nearest harmonics of a tube closed at one end and open at other end are 220 Hz and 260 Hz. What is the fundamental frequency of the system? ***NEET 2017***
(a) 10 Hz (b) 20 Hz
(c) 30 Hz (d) 40 Hz

**53** A pipe of length 85 cm is closed from one end. Find the number of possible natural oscillations of air column in the pipe whose frequencies lie below 1250 Hz. The velocity of sound in air is 340 $ms^{-1}$. ***AIIMS 2018, JEE Main 2014***

(a) 12 (b) 8 (c) 6 (d) 4

**54** If the speed of sound in air is 330 m/s, then find the number of tones present in an open organ pipe of length 1 m whose frequency is $\leq 1000$ Hz. ***AIIMS 2019***

(a) 2 (b) 4 (c) 8 (d) 6

**55** An open tube is in resonance with string. If tube is dipped in water, so that 75% of length of tube is inside water, then the ratio of the frequency $\nu_0$ of tube to string is

(a) 1 (b) 2 (c) 2 / 3 (d) 3 / 2

**56** A tuning fork with frequency 800 Hz produces resonance in a resonance column tube with upper end open and lower end closed by water surface. Successive resonance are observed at lengths 9.75 cm, 31.25 cm and 52.75 cm. The speed of sound in air is ***NEET (Odisha) 2019***

(a) 500 m/s (b) 156 m/s
(c) 344 m/s (d) 172 m/s

**57** An organ pipe open on both ends in the *n*th harmonic is in resonance with a source of 1000 Hz. The length of pipe is 16.6 cm and speed of sound in air is 332 m/s, then find the value of *n*. ***JIPMER 2018***

(a) 3 (b) 2 (c) 1 (d) 4

**58** An pipe 30 cm long is open at both ends. The speed of sound in air is 330 $ms^{-1}$. Which harmonic mode of the pipe resonates at 1.1 kHz source?

(a) First harmonic (b) Third harmonic
(c) Second harmonic (d) Fifth harmonic

**59** The fundamental frequency in an open organ pipe is equal to the third harmonic of a closed organ pipe. If the length of the closed organ pipe is 20 cm, the length of the open organ pipe is ***NEET 2018***

(a) 12.5 cm (b) 8 cm (c) 13.3 cm (d) 16 cm

**60** A tuning fork is used to produce resonance in a glass tube. The length of the air column in this tube can be adjusted by a variable piston. At room temperature of 27°C, two successive resonances are produced at 20 cm and 73 cm of column length. If the frequency of the tuning fork is 320 Hz, the velocity of sound in air at 27°C is ***NEET 2018***

(a) 350 $ms^{-1}$ (b) 339 $ms^{-1}$ (c) 330 $ms^{-1}$ (d) 300 $ms^{-1}$

**61** The systems such as strings and air columns undergo forced oscillation. If the external frequency of oscillation for forced oscillation is close to one of the natural frequencies, then the system is said to be in

(a) resonance
(b) fundamental mode of oscillation
(c) simple harmonic motion
(d) Both (a) and (b)

**62** When two harmonic sound waves of close (but not equal) frequencies are heard at the same time, we hear

(a) a sound of similar frequency
(b) a sound of frequency which is the average of two close frequencies
(c) audibly distinct waxing and waning of the intensity of the sound with a frequency equal to the difference in the two close frequencies
(d) All of the above

**63** If frequencies are $(\nu - 1)$ and $(\nu + 1)$, then find the value of beats. ***JIPMER 2019***

(a) 2 (b) 1
(c) 3 (d) 4

**64** Two sitar strings *A* and *B* playing the note 'Dha' are slightly out of tune and produce beats of frequency 5 Hz. The tension of the string *B* is slightly increased and the beat frequency is found to decrease to 3 Hz. The original frequency of *B* , if the frequency of *A* is 427 Hz, is

(a) 422 Hz (b) 420 Hz
(c) 424 Hz (d) 419 Hz

**65** A source of unknown frequency gives 4 beats/s when sounded with a source of known frequency 250 Hz. The second harmonic of the source of unknown frequency gives 5 beats/s when sounded with a source of frequency 513 Hz. The unknown frequency is ***NEET 2013***

(a) 254 Hz (b) 246 Hz
(c) 240 Hz (d) 260 Hz

**66** When two tuning forks (fork 1 and fork 2) are sounded simultaneously, 4 beats/s are heard. Now, some tape is attached on the prong of the fork 2. When the tuning forks are sounded again, 6 beats/s are heard. If frequency of fork 1 is 200 Hz, then what was the original frequency of fork 2?

(a) 200 Hz
(b) 202 Hz
(c) 196 Hz
(d) 204 Hz

## TOPIC 5 ~ *Doppler Effect*

**67** Doppler effect is a wave phenomenon, which holds for

(a) sound waves only
(b) electromagnetic waves only
(c) light waves only
(d) All of the above

**68** A siren emitting sound of frequency 500 Hz is fitted on a car moving towards a vertical wall at a speed of $36\ \text{kmh}^{-1}$. A stationary person is standing on the road behind the car listens the siren sound. The apparent frequency of the wave coming directly from the siren to the person is (consider speed of sound $= 340\ \text{ms}^{-1}$) ***AIIMS 2019***

(a) 480 Hz (b) 486 Hz (c) 400 Hz (d) 350 Hz

**69** A source of sound $S$ is moving with a velocity of 50 m/s towards a stationary observer. The observer measures the frequency of the source as 1000 Hz. What will be the apparent frequency of the source when it is moving away from the observer after crossing him? (Take, velocity of sound in air is 350 m/s) ***JEE Main 2019***

(a) 807 Hz (b) 1143 Hz
(c) 750 Hz (d) 857 Hz

**70** A siren emitting a sound of frequency 800 Hz moves away from an observer towards a cliff at a speed of $15\ \text{ms}^{-1}$. Then, the frequency of sound that the observer hears in the echo reflected from the cliff is (take, velocity of sound in air $= 300\ \text{ms}^{-1}$) ***NEET 2016***

(a) 800 Hz (b) 838 Hz
(c) 885 Hz (d) 765 Hz

**71** A rocket is moving at a speed of $220\ \text{ms}^{-1}$ towards a stationary target, emits a sound of frequency 1000 Hz. Some of the sound reaching the target gets reflected back to the rocket as an echo. The frequency of the echo as detected by the rocket is (take, velocity of sound $= 330\ \text{ms}^{-1}$)

(a) 3500 Hz (b) 4000 Hz
(c) 5000 Hz (d) 3000 Hz

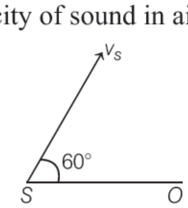
**72** A stationary observer receives sound from two identical tuning forks, one of which approaches and the other one recedes with the same speed (much less than the speed of sound). The observer hears 2 beats/s. The oscillation frequency of each tuning fork is $\nu_0 = 1400$ Hz and the velocity of sound in air is 350 m/s. The speed of each tuning fork is close to ***JEE Main 2020***

(a) $\frac{1}{8}$ m/s (b) $\frac{1}{2}$ m/s
(c) 1 m/s (d) $\frac{1}{4}$ m/s

**73** Two cars moving in opposite directions approach each other with speed of 22 m/s and 16.5 m/s, respectively. The driver of the first car blows a horn having a frequency 400 Hz, then the frequency heard by the driver of the second car is [velocity of sound is 340 m/s] ***NEET 2017, JEE Main 2019***

(a) 350 Hz (b) 361 Hz (c) 411 Hz (d) 448 Hz

**74** A source of sound $S$ emitting waves of frequency 100 Hz and an observer $O$ are located at some distance from each other. The source is moving with a speed of $19.4\ \text{ms}^{-1}$ at an angle of $60°$ with the source observer line as shown in the figure. The observer is at rest. The apparent frequency observed by the observer (velocity of sound in air is $330\ \text{ms}^{-1}$), is




***CBSE AIPMT 2015***

(a) 100 Hz (b) 103 Hz
(c) 106 Hz (d) 97 Hz

# SPECIAL TYPES QUESTIONS

## I. Assertion and Reason

**Direction** (Q. Nos. 75-81) *In the following questions, a statement of Assertion is followed by a corresponding statement of Reason. Of the following statements, choose the correct one.*

(a) Both Assertion and Reason are correct and Reason is the correct explanation of Assertion.
(b) Both Assertion and Reason are correct but Reason is not the correct explanation of Assertion.
(c) Assertion is correct but Reason is incorrect.
(d) Assertion is incorrect but Reason is correct.

**75** **Assertion** The light emitted by stars, which are hundreds of light years away, reaches us through interstellar space even though the interstellar space is practically a vacuum.

**Reason** Light is an electromagnetic wave and do not necessarily require a medium for propagation, so it can even travel in vacuum.

**76** **Assertion** Transverse waves are possible in solids and strings (under tension) but not in fluids.

**Reason** Solids and strings have non-zero shear modulus, but fluids have approximately zero shear modulus.

**77** **Assertion** Longitudinal waves can be propagated through solids and fluids both.

**Reason** Solids as well as fluids have non-zero bulk modulus, i.e. they can sustain compressive stress.

**78** **Assertion** Speed of sound is more in liquids and solids than gases.

**Reason** Liquids and solids have higher densities than gases.

**79** **Assertion** The amplitude $A(\phi)$ of the resultant of the two light travelling waves given by equations

$$y_1(x, t) = A \sin(kx - \omega t) \text{ and}$$
$$y_2(x, t) = A \sin(kx - \omega t + \phi)$$

decreases as $\phi$ increases from 0 to $\pi$.

**Reason** The amplitude of the resultant of the two above mentioned waves is given by $A(\phi) = 2A \cos(\phi/2)$.

**80** **Assertion** Superposition of two harmonic waves, one of frequency 11 Hz and the other of frequency 9 Hz give rise to beats of frequency 2 Hz.

**Reason** Harmonic waves of nearly equal frequencies interfere to give rise to beat having beat frequency, $\nu_{\text{beat}} = |\nu_1 - \nu_2|$.

**81** **Assertion** To hear different beats, difference of the frequencies of two sources should be less than 10.

**Reason** More the number of beats more is the confusion. ***AIIMS 2018***

## II. Statement Based Questions

**82** A pebble is dropped in a pond of still water to disturb the water surface.

I. The disturbance produced does not remain confined to one place but propagates outward along a circle.
II. If a cork piece is put on the disturbed surface, it moves along with the disturbance in the same direction.
III. The water mass does not flow outward with the circles formed but rather a moving disturbance is created.

Which of the given statement(s) is/are correct for the above situation?

(a) Both I and III (b) Both II and III
(c) Both I and II (d) I, II and III

**83** I. Waves are patterns of disturbance which move without the actual physical transfer or flow of matter as a whole.
II. Waves cannot transport energy.
III. The pattern of disturbance in the form of waves carries information that propagates from one point to another.
IV. All our communications essentially depend on transmission of signals through waves.

Which of the following statement(s) is/are correct about waves?

(a) Both I and III (b) Only IV
(c) I, II and III (d) I, III and IV

**84** I. A steel bar possesses both bulk and shear elastic moduli.
II. A steel bar can propagate both longitudinal as well as transverse waves having different speeds.
III. Air can propagate both longitudinal and transverse wave.

Which of the following statement(s) is/are correct?

(a) Both I and III (b) Both II and III
(c) Both I and II (d) I, II and III

**85** A wave is incident obliquely on the boundary between two different media.

I. The incident and the refracted waves obey Snell's law of refraction.

II. The incident and the reflected waves obey the usual laws of reflection.

III. The incident wave only gets reflected but not refracted.

Which of the following statements is/are incorrect?

(a) Only I (b) Both II and III
(c) Only II (d) Only III

**86** I. In a standing wave, the disturbance produced is confined to the region where it is produced.

II. In a standing wave, all the particles cross their mean position together.

III. In a standing wave, energy is transmitted from one region of space to other.

IV. All the particles are oscillating with same amplitude.

Which of the given statement(s) is/are correct about the standing wave?

(a) Both I and II (b) Both II and IV
(c) Only III (d) I, II, III and IV

**87** A standing wave is formed on a string fixed at both the ends. The individual wave, i.e. incident wave and reflected wave are $y_1(x, t) = a \sin(kx - \omega t)$ and $y_2(x, t) = a \sin(kx + \omega t)$, respectively. The two waves have same wavelength $\lambda$.

I. The position of nodes is given as $x = n\lambda$, where $n = 0, 1, 2, 3, \dots$.

II. The position of anti-nodes is given by the equation, $x = \left(n + \frac{1}{2}\right)\frac{\lambda}{2}$, where $n = 0, 1, 2, 3, \dots$.

III. The distance between any two consecutive anti-nodes is $\lambda / 2$.

Which of the following statement(s) is/are incorrect about the standing wave?

(a) Only I (b) Both I and II
(c) Both II and III (d) I, II and III

**88** A standing wave is generated on a stretched string of length $L$ fixed at both the ends. Taking one end of the string at $x = 0$, the boundary conditions are that $x = 0$ and $x = L$ are positions of nodes.

I. For $x = L$ to be the node, the length $L$ is related to $\lambda$ by relation given as $L = \frac{n\lambda}{2}$, where $n = 0, 1, 2, 3, \dots$.

II. The possible wavelengths of the stationary waves are given by $\lambda = \frac{2L}{n}$, where $n = 0, 1, 2, 3, \dots$.

III. The natural frequencies or the normal modes of oscillation of the system are given by $\nu = \frac{nv}{2L}$, where $n = 0, 1, 2, 3, \dots$. (consider $v$ as the wave velocity).

IV. The fundamental mode or the first harmonic is given by $\nu_0 = \frac{v}{4L}$.

Which of the following statement(s) is/are correct?

(a) Both I and II (b) Both II and III
(c) I, II and III (d) I, II, III and IV

**89** I. In a closed organ pipe, longitudinal standing waves can be formed.

II. In a closed organ pipe (closed at one end), only odd harmonics are present.

III. The harmonics which are present in a pipe, open at both ends are odd harmonics only.

Which of the following statements related to organ pipe is/are correct?

(a) Only I (b) Both II and III
(c) Both I and II (d) I, II and III

**90** Two harmonic waves of slightly different frequency interfere to form beats given by equation

$$S = [2a \cos \omega_b t] \cos \omega_a t$$

If the above equation is represented as $S = A(t) \cos \omega_a t$, then which of the given statement(s) is/are correct?

I. The resultant wave is oscillating with the average angular frequency $\omega_a$.

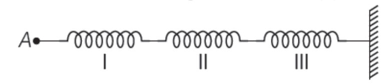
II. The amplitude is largest when the term $\cos \omega_b t$ takes its limit $+1$ or $-1$.

(a) Only I (b) Both I and II
(c) Only II (d) Neither I nor II

**91** Which of the following statement(s) is/are incorrect for mechanical waves?

(a) These waves require a medium for propagation, they cannot propagate through vacuum.
(b) They involve oscillations of constituent particles.
(c) They depend on the elastic properties of the medium.
(d) They have the same speed in all the media.

**92** A collection of springs connected to one another is fixed at one end is as shown in figure below. If the other end $A$ is pulled suddenly and released, then which of the following statement(s) is/are correct?




(a) The disturbance generating at end $A$ will propagate to the other end but each spring will only executes small oscillations about its equilibrium position.
(b) Since, the spring II is connected to the I, it will also be stretched or compressed and so on.
(c) Springs I and III will displace from its equilibrium position while spring II will execute small oscillations about its equilibrium position.
(d) Both (a) and (b)