Also, velocity at any instant say $t = t_1$ is the slope of the tangent at point $P$ which again coincides with $PQ$ or with the graph. Hence, velocity is same as the average velocity at all instants.

Therefore, Assertion and Reason are correct but Reason is not the correct explanation of Assertion.

**74** $(a)$ The $x$ - $t$, $v$ - $t$ and $a$ - $t$ graphs will be smooth,

which means that physically, the values of acceleration and velocity cannot change abruptly as changes are always continuous.
Therefore, Assertion and Reason are correct and Reason is the correct explanation of Assertion.

**75** $(d)$ The uniform motion of a body means that the body is moving with constant velocity.

But if the direction of motion is changing (such as in uniform circular motion), its velocity changes and thus acceleration is produced in a body moving uniformly.
Therefore, Assertion is incorrect but Reason is correct.

**76** $(c)$ When a particle is released from rest position under gravity, then $v = 0$ but $a \neq 0$.
Also, a body is momentarily at rest at the instant, if it reverse the direction.
Therefore, Assertion is correct but Reason is incorrect.

**78** $(b)$ In given case, when objects move in same direction then relative velocity of object $A$ w.r.t. object $B$ is
$v_{AB} = v_A - v_B$.
When objects $A$ and $B$ move in opposite direction then relative velocity of object $B$ w.r.t. object $A$ will be
$v_{BA} = v_B - v_A$.

Therefore, Assertion and Reason are correct but Reason is not the correct explanation of Assertion.

**79** $(c)$ Statements II and III are correct but I is incorrect and it can be corrected as,

Even, when we are sleeping, air and blood flow are treated as objects which are in motion w.r.t. the body. Also, while sleeping human beings are in rest, not in motion.

**80** $(d)$ Statements I and II are correct, but III is incorrect and it can be corrected as,

Object is in motion only when it changes its position with time. So, the object is in motion from point $O$ to point $P$, i.e. from $t = 0$ s to 5 s and object is at rest from $t = 5$ s to $t = 10$ s.

**81** $(c)$ Since, Rahul's initial and final positions coincides.
Thus, his displacement,

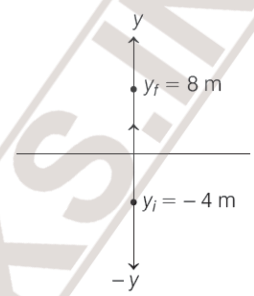
$$\Delta x = x_{\text{final}} - x_{\text{initial}} = 0$$

However, corresponding path length

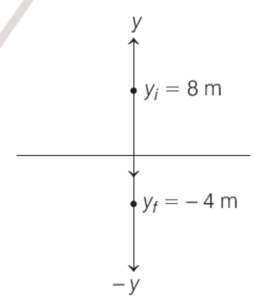
$$= 240 + 240 = 480 \text{ m}$$

Thus, the magnitude of the displacement for the given course of motion is zero but the corresponding path length is 480 m.
So, all statements are correct.

**82** $(c)$ Displacement is defined as the shortest distance between the initial and the final positions of an object or body. It is given as $\Delta x = x_f - x_i$
where, $x_f$ and $x_i$ are final and initial positions of the object/body, respectively.
**Case (i)** $\Delta y_1 = 8 - (-4) = +12$ m




**Case (ii)** $\Delta y_2 = -4 - 8 = -12$ m




Thus, his displacement is negative in Case (ii) and positive in Case (i).
So, statement III is correct but I and II are incorrect.

**83** $(a)$ Only statement I is correct but II is incorrect and it can be corrected as

Coordinate of an object with respect to a rectangular coordinate system is described as $(x, y, z)$. Here, at least one of the coordinates, i.e. either $x$ or $y$ or $z$ must change for an object in motion. If none of the coordinates change, then the object is said to be at rest with respect to the given reference frame.

**84** $(a)$ From the relation of stopping distance, $d_s = -\dfrac{v_0^2}{2a}$

Keeping $a =$ constant, $d_s \propto v_0^2$

When initial velocity is doubled,

$$v_0' = 2v_0$$

$$\Rightarrow \quad d_0' = -\frac{(2v_0)^2}{2a} = -\frac{4v_0^2}{2a} = 4d_s$$

Hence, doubling the initial velocity increases the stopping distance by a factor of 4.
Stopping distance is an important factor considered in setting speed limits because it is the distance travelled by vehicle before stopping, e.g. in school zones.
So, statement I is incorrect but II and III are correct.

**85** (*a*) If $v_A$ and $v_B$ are of opposite signs, relative velocity for two cases will be

$$v_{AB} = v_A - v_B \quad \text{...(i)}$$
$$v_{BA} = v_B - v_A \quad \text{...(ii)}$$

**Case I** $v_A > 0$ and $v_B < 0$

Let $v_A = x\ \text{ms}^{-1}$

and $v_B = -y\ \text{ms}^{-1}$

$$v_{AB} = v_A - v_B = x - (-y) = x + y$$
$$|v_{AB}| = |x + y| = x + y$$
$$v_{BA} = v_B - v_A = -y - x = -(y + x)$$
$$\Rightarrow \quad |v_{BA}| = (y + x)$$

**Case II** If $v_A < 0$ and $v_B > 0$

Let, $v_A = -x\ \text{ms}^{-1}$

and $v_B = +y\ \text{ms}^{-1}$

$$v_{AB} = v_A - v_B = -x - y = -(x + y)$$
$$\Rightarrow \quad |v_{AB}| = x + y$$
$$v_{BA} = v_B - v_A = y - (-x) = y + x$$
$$\Rightarrow \quad |v_{BA}| = x + y$$

So, from Case I and Case II $|v_{AB}| = |v_{BA}| > |v_A|$ or $|v_B|$

Magnitude of $v_{BA}$ or $v_{AB}$ is greater than the magnitude of velocity of $A$ or that of $B$.

Also, they will never meet.

If the objects under consideration are two trains, then for a person sitting in either of the two, the other train seems to go very fast.

So, statement III is incorrect but I and II are correct.

**86** (*d*) Statement given in option (d) is incorrect and it can be corrected as,

Path length is a scalar quantity as it has only magnitude but no direction whereas displacement has magnitude as well as direction, so displacement is vector quantity.

Rest statements are correct.

**87** (*b*) Statement given in option (b) is correct but the rest are incorrect and these can be corrected as,

In general, average speed is not equal to magnitude of average velocity. It can be so if the motion is along a straight line without change in direction.

When acceleration of particle is not constant, then motion is called as non-uniformly accelerated motion.

Displacement is zero, when a particle returns to its starting point.

**88** (*b*) Statement given in option (b) is correct, rest are incorrect, these can be corrected as,

Stopping distance is inversely proportional to deceleration $a$ of the vehicle as

Stopping distance, $d_s = -\dfrac{v_0^2}{2a}$

For constant acceleration, average velocity is $\bar{v} = \dfrac{v + u}{2}$.

When a body thrown vertically upwards, then acceleration due to gravity $g$ will be taken as negative.

**89** (*a*) The instantaneous speed is always positive as it is the magnitude of the velocity at an instant. So, it is positive during $t = 5$ s to $t = 10$ s.

For $t = 0$ s to $t = 5$ s, the motion is uniform and $x$-$t$ graph has positive slope. So, the velocity and average velocity, instantaneous velocity and instantaneous speed are equal and positive.

During $t = 0$ s to $t = 5$ s, the slope of the graph is positive, hence the average velocity and the velocity both are positive.

During $t = 5$ s to $t = 10$ s, the slope of the graph is negative, hence the velocity is negative. Since, there is a change in sign of velocity at $t = 5$ s, so the car changes its direction at this instant.

Hence, option (a) is incorrect, while all other are correct.

**90** (*d*) Here, $AB$ represents uniform motion of a car as $x$-$t$ graph from $t_1$ to $t_2$ varies linearly.

At $t = t_2$ brakes must have been applied such that it stops at $t = t_3$.

After which the $x$-$t$ graph becomes parallel to time axis.

Thus, all given statements are correct.

**91** (*a*) The relation $x = \left(\dfrac{v + v_0}{2}\right) t$ means that the object has undergone displacement $x$ with an average velocity equal to the arithmetic average of the initial and final velocities.

Thus, the statement given in option (a) is correct, rest are incorrect.

**92** (*c*) Displacement of the object = Area under $v$-$t$ curve.

$$x = \frac{1}{2}(v - v_0)t + v_0 t \quad \text{... (i)}$$

Also, $\quad v = v_0 + at \Rightarrow v - v_0 = at \quad$ ...(ii)

Putting the value of $(v - v_0)$ from Eq. (ii) in Eq. (i), we get

$$x = \frac{1}{2} at \times t + v_0 t$$
$$\Rightarrow \quad x = \frac{1}{2} at^2 + v_0 t$$

Thus, the statement given in option (c) is correct, rest are incorrect.

**93** (*c*) For negative acceleration, the $x$-$t$ graph moves downward. But the car is moving in positive direction as the position coordinate is increasing in the positive direction.

Thus, the statement given in option (c) is correct, rest are incorrect.

**94** (*d*) Given, $\quad |v_A| = |v_B|$ (given, speed of $A$ = speed of $B$)

**Case I** $\quad v_{AB} = v_A - v_B = 0$

$$\Rightarrow \quad v_A = v_B \quad \text{...(i)}$$

Also, $\quad v_{BA} = v_B - v_A = 0$

$$\Rightarrow \quad v_B = v_A \quad \text{...(ii)}$$

From Eqs. (i) and (ii), we get

$A$ and $B$ must be moving in same direction as ($v_A = v_B$).

**Case II** If particles are moving in opposite direction,

i.e. $v_A = -v_B$

$v_{AB} = v_A - v_B = -v_B - v_B = -2v_B$

$|v_{AB}| = 2|v_B| = 2|v_A|$

Also, $v_{BA} = v_B - v_A = v_B - (-v_B) = 2v_B$

$|v_{BA}| = 2|v_B| = 2|v_A|$

Hence, for motion in opposite direction, the magnitude of $v_{BA}$ or $v_{AB}$ is twice than the magnitude of velocity of $A$ or that of $B$.

Thus, the statements given in options (a) and (b) are correct, rest is incorrect.

**95** (*a*)

A. Displacement of the car is moving $O$ to $P$ is

$\Delta x = x_2 - x_1 = (+360\text{ m}) - 0\text{ m} = +360\text{ m}$

B. Path length of the car is moving $O$ to $R$ is 120 m, as path length is the total distance traversed by the car from $O$ to $R$.

C. For the motion of the car from $O$ to $P$ and back to $Q$.

Path length $= (+360\text{ m}) + (+120\text{ m}) = +480\text{ m}$

D. However, in the above case displacement

$= (+240\text{ m}) - (0\text{ m})$

$= +240\text{ m}$

Hence, A $\rightarrow$ 2, B $\rightarrow$ 4, C $\rightarrow$ 1 and D $\rightarrow$ 3.

**96** (*d*)

A. Here, average velocity $= \dfrac{\text{displacement}}{\text{time interval}}$

$$= \frac{x_2 - x_1}{t_2 - t_1} = \frac{+20-0}{10\text{ s}} = +2\text{ms}^{-1}$$

Average speed $= \dfrac{\text{Total path length}}{\text{Time interval}}$

$$= \frac{OP}{t_2 - t_1} = \frac{20\text{ m}}{10\text{ s}} = 2\text{ms}^{-1}$$

B. For path travelled from $O$ to $P$ and and back to $R$,

Average velocity $= \dfrac{\text{Displacement}}{\text{Time interval}} = \dfrac{x_2 - x_1}{t_2 - t_1}$

$$= \frac{(-20)-0}{(10+20)} = -\frac{20}{30}\text{ ms}^{-1}$$

$$= -\frac{2}{3}\text{ ms}^{-1}$$

Average speed $= \dfrac{\text{Path length}}{\text{Time interval}}$

$$= \frac{OP + PR}{10+20}$$

$$= \frac{(20+40)\text{ m}}{30\text{ s}} = \frac{60}{30}\text{ ms}^{-1}$$

$$= 2\text{ms}^{-1}$$

C. When object moves from $O$ to $Q$ and back to $R$, then

Average velocity $= \dfrac{x_2 - x_1}{t_2 - t_1} = \dfrac{[(-20)-0]\text{ m}}{40\text{ s}}$

$$= -\frac{20}{40}\text{ms}^{-1} = -0.5\text{ms}^{-1}$$

Average speed $= \dfrac{OQ + QR}{\Delta t}$

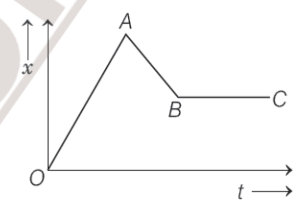
$$= \frac{(50+70)\text{ m}}{40\text{ s}} = \frac{120}{40}\text{ ms}^{-1} = 3\text{ms}^{-1}$$

Hence, A $\rightarrow$ 2, B $\rightarrow$ 3 and C $\rightarrow$ 1.

**97** (*b*) In $x$-$t$ graph, $OA \rightarrow$ Positive slope $\rightarrow$ Positive velocity

$AB \rightarrow$ Negative slope $\rightarrow$ Negative velocity

$BC \rightarrow$ Zero slope $\rightarrow$ Object at rest




At point $A$, there is a change in sign of velocity, hence the direction of motion must have changed at $A$.

Hence, A $\rightarrow$ 1, B $\rightarrow$ 3, C $\rightarrow$ 2 and D $\rightarrow$ 4.

**98** (*c*)

A. Distance of the object between $t = 0$ s to $t = 2$ s.

= Area under $v$-$t$ graph

= Area of triangle of base = 2 and height = 4

$= \dfrac{1}{2} \times \text{base} \times \text{height} = \dfrac{1}{2} \times 2 \times 4 = 4\text{ m}$

B. Displacement of the object between $t = 0$ s to $t = 2$ s

= Distance covered in same interval = + 4 m

C. For $t = 0$ s to $t = 4$ s,

Displacement = Area under $v$-$t$ curve ($t = 0$ s to $t = 2$ s) + Area under $v$-$t$ curve ($t = 2$s to $t = 4$ s)

$$= \frac{1}{2}\times 2\times 4 + \frac{1}{2}\times 2\times(-4) = 4\text{ m} + (-4\text{ m}) = 0$$

D. For $t = 0$ s to $t = 4$ s,

Distance covered = Area under $v$-$t$ curve considering all areas as positive.

= Area under $v$-$t$ curve ($t = 0$ s to $t = 2$ s) + Area under $v$-$t$ curve ($t = 2$ s to $t = 4$ s)

$= 4\text{ m} + 4\text{ m} = 8\text{ m}$

Hence, A $\rightarrow$ 3, B $\rightarrow$ 2, C $\rightarrow$ 4 and D $\rightarrow$ 1.

**99** (*d*)

A. For $t = 0$ s to $t = 2$ s, the motion is in positive direction as the velocity is positive and the acceleration is positive, since the slope of the straight line is positive.

B. For $t = 2\,s$ to $t = 4\,s$, the object is moving in positive direction, till time $t_1 = 2\,s$, and then turns back with the same negative acceleration till $t = 4\,s$.

C. For $t = 4\,s$ to $t = 6\,s$, the object is moving in negative direction, since the velocity is negative. The acceleration is positive, since the slope of the straight line is positive.

D. Displacement for overall journey ($t = 0\,s$ to $t = 6\,s$) = Total area under $v$-$t$ graph considering the area below time axis as negative = [Area under $v$-$t$ curve for $t = 0$ to $t = 3\,s$] + [Area under $v$-$t$ curve for $t = 3\,s$ to $t = 6\,s$] = Area of triangle $(OAB)$ + Area of triangle $(BCD) = \frac{1}{2} \times 3 \times 5 + \frac{1}{2} \times 3 \times (-5) = 0$

Hence, A $\rightarrow$ 4, B $\rightarrow$ 1, C $\rightarrow$ 2 and D $\rightarrow$ 3.

**100** ($b$) Given, $x(t) = a - bt^2$, $a = 8.5\text{ m}$ and $b = 2.5\text{ ms}^{-2}$

$\therefore \quad x(t) = 8.5 - 2.5t^2$

Velocity of object $= \frac{dx}{dt} = -2bt$

A. Velocity at $t = 2.0\,s = \left.\frac{dx}{dt}\right|_{t=2} = -4b$

$= -4 \times 2.5 = -10\text{ ms}^{-1}$

B. Velocity at $t = 0\,s = \left.\frac{dx}{dt}\right|_{t=0} = 0\text{ ms}^{-1}$

C. Instantaneous speed at $t = 2\,s$ = Magnitude of velocity $= |-10\text{ ms}^{-1}| = 10\text{ ms}^{-1}$

D. Average velocity $= \frac{x(t_2) - x(t_1)}{t_2 - t_1} = \frac{x(4) - x(2)}{4-2}$

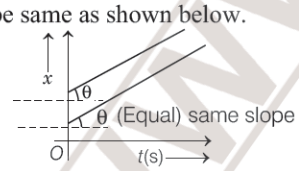
$= \frac{[a - b(4)^2] - [a - b(2)^2]}{2} = \frac{4b - 16b}{2}$

$= -\frac{12b}{2} = -6b = -6 \times 2.5\text{ ms}^{-1} = -15\text{ ms}^{-1}$

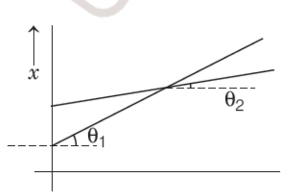
Hence, A $\rightarrow$ 2, B $\rightarrow$ 3, C $\rightarrow$ 4 and D $\rightarrow$ 1.

**101** ($b$)

A. For equal velocities, the slope of the straight lines must be same as shown below.



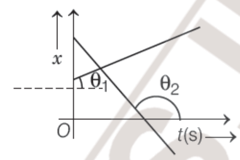

B. For unequal velocity, slope is different, but since, the objects are moving in the same direction, the slope for both the graphs must be of same sign (positive or negative) and they meet at a point as shown below




C. For velocities in opposite direction, slopes must be of opposite sign. Slope = $\tan\theta$, where $\theta$ is the angle of the straight line with horizontal in anti-clockwise direction. As, we know $\tan\theta_1 > 0$, $\tan\theta_2 < 0$

Hence, slopes are of opposite sign.

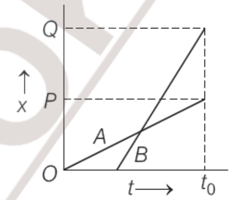
This condition is shown below.




Hence, A $\rightarrow$ 2, B $\rightarrow$ 1 and C $\rightarrow$ 3.

**102** ($c$)

(a) As $OP < OQ$, $A$ lives closer to the school than $B$.




(b) When $x = 0, t = 0$ for $A$; while $B$ has some finite value of $t$. So, $A$ starts from school earlier than $B$.

(c) Speed = Slope of $x$-$t$ graph

Slope for $B$ > Slope for $A$

$\therefore$ $B$ walks faster than $A$.

(d) Corresponding to the positions $P$ and $Q$, time $t_0$ is same on $t$-axis.

$\therefore$ $A$ and $B$ reach home at the same time.

Hence, statement given in option (c) is incorrect.

**103** ($d$) As drunkard takes 5 steps forward and 3 steps backward, therefore he moves 5 m forward and 3 m backward.

Time taken in 8 steps = 8 s

$\therefore$ Distance travelled in 8 s in 8 steps $= 5 - 3 = 2\text{ m}$

Distance travelled in 16 s in 16 steps $= 2 \times 2 = 4\text{ m}$

Distance travelled in 24 s in 24 steps $= 2 \times 3 = 6\text{ m}$

Distance travelled in 32 s in 32 steps $= 2 \times 4 = 8\text{ m}$

Distance travelled in next 5 s in next 5 steps taken in forward direction = 5 m

$\therefore$ Total distance travelled in $(32 + 5 =)$ 37 s in 37 steps $= 8 + 5 = 13\text{ m}$

Distance of the pit from the starting point = 13 m

Therefore, drunkard will fall in the pit in 37 s.

**104** ($a$) Let jet airplane be moving left (+ ve direction) with velocity $v_j$ and ejected gases be moving right (– ve direction) with velocity $v_g$ while observer be at rest on the ground, i.e. $v_0 = 0$

$\therefore \quad v_j = 500\text{ kmh}^{-1}$

$\Rightarrow \quad v_g = -1500\text{ kmh}^{-1} \Rightarrow v_0 = 0$

Relative velocity of plane with respect to the observer,

$v_j - v_0 = 500 - 0 = 500\text{ kmh}^{-1}$ ...(i)

Relative velocity of products of combustion with respect to the jet plane,

$$v_g - v = -1500\,\text{kmh}^{-1} \qquad \text{(given)} \quad \ldots\text{(ii)}$$

(Velocity of ejected gas $v_g$ and velocity of $v_j$ are in opposite directions)

Adding Eqs. (i) and (ii), we get

$$(v_j - v_0) + (v_g - v_j) = 500 - 1500$$

$$\Rightarrow \qquad v_g - v_0 = -1000\,\text{kmh}^{-1}$$

Therefore, relative velocity of the ejected gases with respect to the observer is 1000 kmh$^{-1}$, –ve sign shows that this velocity is in a direction opposite to the motion of the jet airplane.

**105** ($c$) Given, $u = 126\,\text{kmh}^{-1} = 126 \times \dfrac{5}{18} = 35\,\text{ms}^{-1}$,

$$v = 0, s = 200\,\text{m}$$

As, $v^2 - u^2 = 2as$

$$\Rightarrow \qquad 0^2 - 35^2 = 2a \times 200$$

or $$a = -\frac{35 \times 35}{2 \times 200} = -\frac{49}{16} = -3.06\,\text{ms}^{-2}$$

$\therefore$ Retardation, $a = 3.06\,\text{ms}^{-2}$

As, $$v = u + at \quad \Rightarrow \quad 0 = 35 + \frac{49}{16}t$$

As, time cannot be negative

$$\therefore \qquad \text{Time, } t = \frac{35 \times 16}{49} = \frac{80}{7} = 11.43\text{ s}$$

**106** ($c$) Length of each train, $l_A = l_B = 400\,\text{m}$

Initial velocities of both trains,

$$u_A = u_B = 72\,\text{kmh}^{-1} = 72 \times \frac{5}{18}\,\text{ms}^{-1}$$

$$\left(\because 1\,\text{kmh}^{-1} = \frac{5}{18}\,\text{ms}^{-1}\right)$$

$$= 20\,\text{ms}^{-1}$$

Distance travelled by train $A$ in 50 s, $s_A = u_A \times t$

(as for unaccelerated motion, distance = speed × time)

$$s_A = 20 \times 50 = 1000\,\text{m}$$

Distance travelled by train $B$ in 50 s,

$$s_B = u_B t + (1/2) a_B t^2$$

(as motion of train $B$ is an accelerated motion)

$$s_B = 20 \times 50 + (1/2) \times 1 \times (50)^2$$

$$= 1000 + 1250 = 2250\,\text{m}$$

Original distance between the two trains $= s_B - s_A$

$$= 2250 - 1000 = 1250\,\text{m}$$

**107** ($c$) At the instant when $B$ decides to overtake $A$, the speeds of three cars are

$$v_A = 36\,\text{kmh}^{-1} = 36 \times \frac{5}{18} = +10\,\text{ms}^{-1}$$

$$v_B = +54\,\text{kmh}^{-1} = +54 \times \frac{5}{18} = +15\,\text{ms}^{-1}$$

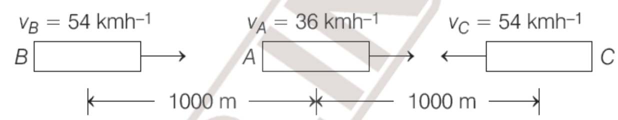
$$v_C = -54\,\text{kmh}^{-1} = -54 \times \frac{5}{18} = -15\,\text{ms}^{-1}$$

Relative velocity of $C$ w.r.t. $A$,

$$v_{CA} = v_C - v_A = -15 - 10 = -25\,\text{ms}^{-1}$$

$\therefore$ Time that $C$ requires to just cross $A$

$$= \frac{1\,\text{km}}{v_{CA}} = \frac{1000\,\text{m}}{25\,\text{ms}^{-1}} = 40\,\text{s}$$




In order to avoid the accident, $B$ must overtake $A$ in a time less than 40 s. So, for car $B$ we have

Relative velocity of car $B$ w.r.t. $A$,

$$v_{BA} = v_B - v_A = 15 - 10 = 5\,\text{ms}^{-1}$$

Here, $s = 1\,\text{km} = 1000\,\text{m}$, $u = 5\,\text{ms}^{-1}$, $t = 40\,\text{s}$

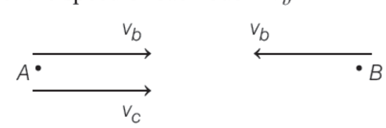
As, $$s = ut + \frac{1}{2}at^2$$

$$\therefore \qquad 1000 = 5 \times 40 + \frac{1}{2}a \times (40)^2$$

or $$1000 = 200 + 800a \quad \text{or} \quad a = 1\,\text{ms}^{-2}$$

Thus, 1 ms$^{-2}$ is the minimum acceleration that car $B$ requires to avoid an accident.

**108** ($a$) Let the speed of each bus $= v_b$ kmh$^{-1}$




Speed of cyclist, $v_c = 20\,\text{kmh}^{-1}$

(i) **In the direction of motion of the cyclist (from $A$ to $B$)**

Relative velocity of bus w.r.t. cyclist $= v_b - v_c$

$$= (v_b - 20)\,\text{kmh}^{-1}$$

The bus goes past the cyclist after every 18 min.

$\therefore$ Distance covered by the bus w.r.t. the cyclist in this time interval $= (v_b - 20) \times \dfrac{18}{60}$ km $\qquad \ldots$(i)

A bus leaves in either direction after every $T$ min.

$\therefore$ Distance travelled by a bus in $T$ min $= v_b \times \dfrac{T}{60}$ km $\qquad \ldots$(ii)

From Eqs. (i) and (ii), we get

$$(v_b - 20) \times \frac{18}{60} = v_b \times \frac{T}{60}$$

or $$v_b - 20 = v_b \times \frac{T}{18} \qquad \ldots\text{(iii)}$$

(ii) **In the opposite direction of the cyclist (from $B$ to $A$)**

Relative velocity of bus w.r.t. cyclist coming from $B$ to $A = (v_b + 20)\,\text{kmh}^{-1}$

The bus goes past the cyclist after every 6 min.

$\therefore$ Distance covered by the bus w.r.t. the cyclist in this time interval $= (v_b + 20) \times \dfrac{6}{60}$ km ...(iv)

Distance travelled by the bus in $T$ min

$$= v_b \times \frac{T}{60}$$

km ...(v)

From Eqs. (iv) and (v), we get

$$(v_b + 20) \times \frac{6}{60} = v_b \times \frac{T}{60}$$

$$v_b + 20 = v_b \times \frac{T}{6}$$

...(vi)

Dividing Eq. (vi) by Eq. (iii), we get

$$\frac{v_b + 20}{v_b - 20} = \frac{18}{6} = 3$$

$\Rightarrow$ $v_b + 20 = 3v_b - 60$

or $2v_b = 80 \Rightarrow v_b = 40\,\text{kmh}^{-1}$

**109** (*a*) For upward motion,

$u = 29.4\,\text{ms}^{-1}, g = -9.8\,\text{ms}^{-2}, v = 0$

If $s$ is the height to which the ball rises, then

$$v^2 - u^2 = 2as$$

or $0^2 - (29.4)^2 = 2 \times -9.8 \times s$

or $$s = \frac{(29.4)^2}{2 \times 9.8} = 44.1\,\text{m}$$

If the ball reaches the highest point in time $t$, then

$$v = u + at \text{ or } 0 = 29.4 - 9.8t$$

or $$t = \frac{29.4}{9.8} = 3\,\text{s}$$

As, time of ascent = time of descent

$\therefore$ Total time taken $= 3 + 3 = 6\,\text{s}$

**110** (*b*) Displacement covered in going to market = 2.5 km

Displacement covered coming back to home = 2.5 km

Net displacement $= 2.5 - 2.5 = 0$

Total distance covered $= 2.5 + 2.5 = 5$ km

(a) Average speed $= \dfrac{\text{Total distance covered}}{\text{Total time taken}}$

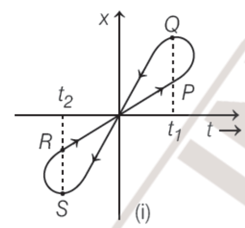
$$= \frac{5\text{km}}{(50/60)\text{h}} = 6\,\text{kmh}^{-1}$$

(b) Average velocity $= \dfrac{\text{Net displacement}}{\text{Time taken}}$

$$= \frac{0}{(50/60)\text{h}} = 0$$

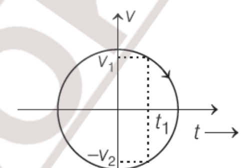
**111** (*d*)

(i) No, graph (i) cannot represents one-dimensional motion of a particle, because graph shows two different positions of the particle at same instant of time. (At time $t_1$ particle is at position $P$ and $Q$ and at time $t_2$ particle is at position $R$ and $S$), which is not possible.



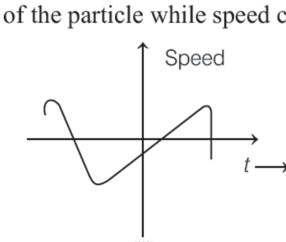
(i)

(ii) No, graph (ii) cannot represents one-dimensional motion of a particle, because graph shows one positive velocity ($v_1$) and another negative velocity ($-v_2$) of the particle at the same instant of time ($t_1$) which is not possible.



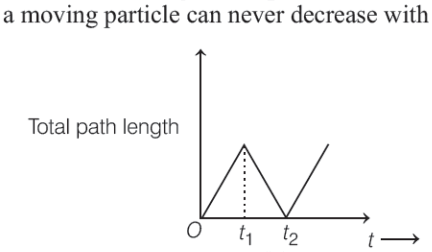
(ii)

(iii) No, graph (iii) cannot represents one-dimensional motion of a particle, because graph shows negative speed of the particle while speed cannot be negative.



(iii)

(iv) No, graph (iv) cannot represents one-dimensional motion of a particle, because graph shows that total path length increases from time $t = 0$ to $t = t_1$, but decreases from $t = t_1$ to $t = t_2$. But total path length of a moving particle can never decrease with time.



(iv)

**112** (*d*) No, it is wrong to say that the particle moves in a straight line for $t < 0$ and on a parabolic path for $t > 0$, because a position-time ($x$-$t$) graph does not represents the trajectory of a moving particle.

This graph can represents the motion of a freely falling particle dropped from a tower, when we take its initial position as $x = 0$ at $t = 0$.

**113** (*b*) Speed of police van, $v_P = 30\,\text{kmh}^{-1}$

$$= 30 \times \frac{5}{18}\,\text{ms}^{-1} \quad (\because 1\text{ kmh}^{-1} = \frac{5}{18}\,\text{ms}^{-1})$$
$$= \frac{25}{3}\,\text{ms}^{-1}$$

Speed of thief's car, $v_T = 192\,\text{kmh}^{-1} = 192 \times \frac{5}{18}\,\text{ms}^{-1}$

$$= \frac{160}{3}\,\text{ms}^{-1}$$

Muzzle speed of bullet, $v_B = 150\,\text{ms}^{-1}$

The bullet is sharing the speed of the police van, therefore effective speed of the bullet,

$$v_B{}' = v_B + v_P = 150 + \frac{25}{3} = \frac{475}{3}\,\text{ms}^{-1}$$

Speed of the bullet with which it hits the thief's car

= Relative speed of the bullet w.r.t. thief's car ($v_{BT}$)

$$v_{BT} = v_B{}' - v_T = \left(\frac{475}{3} - \frac{160}{3}\right)\text{m/s} = \frac{315}{3} = 105\,\text{ms}^{-1}$$

Therefore, bullet will hit the thief's car with a speed $105\text{ ms}^{-1}$.

**114** (*a*) The acceleration-time graph represents the motion of a uniformly moving cricket ball turned back by hitting it with a bat for a very short time interval.

**115** (*a*) In simple harmonic motion, the acceleration is given by

$$a = -\omega^2 x \qquad \text{...(i)}$$

where, $x$ is the displacement, $\omega$ is the angular frequency and negative sign shows that the direction of acceleration is opposite to the direction of displacement.

The velocity is given by

$$v = \frac{dx}{dt} = \text{Slope of } x\text{-}t \text{ graph} \qquad \text{...(ii)}$$

(i) At time $t = 0.3$ s, $x$ is negative and slope of $x$-$t$ graph is negative, therefore position and velocity of the particle is negative but according to Eq. (i) acceleration is positive.

(ii) At time $t = 1.2$ s, $x$ is positive and slope of $x$-$t$ graph is positive, therefore position and velocity of the particle is positive but according to Eq. (i) acceleration is negative.

(iii) At time $t = -1.2$ s, $x$ and $t$ both are negative, therefore position of the particle is negative. As $x$ and $t$ both are negative, therefore from Eq. (ii) velocity is positive and according to Eq. (i) acceleration is positive.

(iv) At $t = -0.3$ s, $x$ is positive but $v$ and $a$ are negative.

Hence, A → 4, B → 3, C → 1 and D → 2.

**116** (*b*) Slope of $x$-$t$ graph in a small interval = Average speed in that interval

As slope for interval 2 > slope for interval 1.

$\therefore \qquad v_2 > v_1$

**117** (*d*) (i) As the change in speed is greatest in interval 2, so magnitude of average acceleration is greatest in interval 2.

(ii) Obviously from the graph, average speed is greatest in interval 3.

**118** (*a*) Distance travelled in $n$th second, $s_n = u + \frac{a}{2}(2n-1)$

As, $\qquad u = 0, a = 1\text{ms}^{-2}$

$\Rightarrow \qquad s_n = \frac{(2n-1)}{2}$

Thus, the distances travelled by the three wheeler at the end of each second are given by

| $n$(s) | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|
| $s_{nth}$ (m) | 0.5 | 1.5 | 2.5 | 3.5 | 4.5 | 5.5 | 6.5 | 7.5 | 8.5 | 9.5 |

Now, velocity of the three wheeler at the end of 10th second is given by

$$v = u + at = 0 + 1 \times 10 = 10\text{ ms}^{-1}$$

Upto $n = 10$ s, the motion is accelerated and the graph between $s_{nth}$ and $n$ is a straight line inclined to time axis as shown in Fig. (a) After 10th second, the three wheeler moves with uniform velocity of $10\text{ ms}^{-1}$, so graph is a straight line parallel to time axis.

**119** (*d*)

(i) **When the lift is stationary** For upward motion of the ball, we have

$$u = 49\text{ms}^{-1}, g = -9.8\text{ms}^{-1}, v = 0, t = ?$$

As, $\qquad v = u + at$

$\therefore \qquad 0 = 49 - 9.8t$ or $t = \frac{49}{9.8} = 5$ s

As, time of ascent = time of descent

$\therefore$ Total time taken = 5 + 5 = 10 s

(ii) **When the lift moves up with uniform** The uniform speed of the lift does not change the relative velocity of the ball w.r.t. the boy, i.e., still remain $49\text{ms}^{-1}$. Hence, total time in which the ball returns is 10 s.

**120** (*b*)

(i) Speed of the child running in the direction of motion of the belt

$$= (9 + 4)\text{ kmh}^{-1} = 13\text{ kmh}^{-1}$$

(ii) Speed of the child running opposite to the direction of the belt

$$= (9 - 4)\text{ kmh}^{-1} = 5\text{ kmh}^{-1}$$

(iii) Speed of the child w.r.t. either parent

$$= 9\text{ kmh}^{-1} = 9 \times \frac{5}{18} = 2.5\text{ ms}^{-1}$$

Distance to be covered = 50 m

$$\text{Time taken} = \frac{50}{2.5} = 20\text{ s}$$

**121** ($c$) Height of the edge of the cliff, $x_0 = 200\,\text{m}$

Acceleration, $a = -g = -10\,\text{m/s}^2$

**For first stone**, $u_1 = 15\,\text{m/s}$

Using equation, $x_1 = x_0 + u_1 t + \frac{1}{2}at^2$

$$= 200 + 15t + \frac{1}{2}(-10)t^2$$

$$x_1 = 200 + 15t - 5t^2 \quad \ldots\text{(i)}$$

**For second stone**, $u_2 = 30\,\text{ms}^{-1}$

Using equation, $x_2 = x_0 + u_2 t + \frac{1}{2}at^2$

$$= 200 + 30t + \frac{1}{2}(-10)\,t^2$$

$$x_2 = 200 + 30t - 5t^2 \quad \ldots\text{(ii)}$$

Now, subtracting Eq. (i) from Eq. (ii), we get

$$x_2 - x_1 = 15t$$

**122** ($c$) Distance travelled by the particle between time interval $t = 0\,\text{s}$ to $t = 10\,\text{s}$

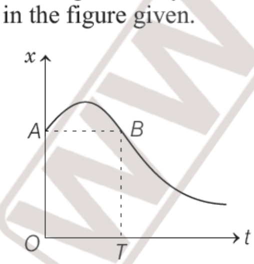
$$= \text{Area of triangle } OAB = \frac{1}{2} \times \text{Base} \times \text{Height}$$

$$= \frac{1}{2} \times OB \times AC$$

$$= \frac{1}{2} \times 10 \times 12 = 60\,\text{m}$$

**123** ($b$) The slope of the given graph over the time interval $t_1$ to $t_2$ is not constant and is not uniform. It means acceleration is not constant and is not uniform, therefore relations (i) and (ii) are not correct which is for uniform accelerated motion.
But relations (iii) and (iv) are correct, because these relations are true for both uniform or non-uniform accelerated motion.

**124** ($b$) In graph (b), for one value of displacement, there are two different points of time. Hence, for one time, the average velocity is positive and for other time, it is negative. As there are opposite velocities in the interval 0 to $T$, hence average velocity can vanish in (b). This can be seen in the figure given.




Here, $OA = BT$ (same displacement) for two different points of time.

**125** ($a$) As the lift is coming in downward direction, displacement will be negative i.e., $x < 0$. When the lift reaches 4th floor, it is about to stop and hence motion is retarding in nature, hence $a > 0$.

As displacement is in negative direction, so velocity will also be negative, i.e. $v < 0$. This can be shown in the graph below.


**126** ($b$) For maximum and minimum displacement, we have to keep in mind the magnitude and direction of maximum velocity.
As maximum velocity in positive direction is $v_0$, maximum velocity in opposite direction is also $v_0$ with negative sign.
Maximum displacement in one direction $= v_0 T$
Maximum displacement in opposite directions $= -v_0 T$
Hence, $-v_0 T < x < v_0 T$.

**127** ($c$) Time taken to travel first half distance,

$$t_1 = \frac{l/2}{v_1} = \frac{l}{2v_1}$$

Time taken to travel second half distance, $t_2 = \frac{l}{2v_2}$

$$\text{Total time} = t_1 + t_2 = \frac{l}{2v_1} + \frac{l}{2v_2} = \frac{l}{2}\left[\frac{1}{v_1} + \frac{1}{v_2}\right]$$

We know that,

$$v_{av} = \text{Average speed} = \frac{\text{Total distance}}{\text{Total time}}$$

$$= \frac{l}{\frac{l}{2}\left[\frac{1}{v_1} + \frac{1}{v_2}\right]} = \frac{2v_1 v_2}{v_1 + v_2}$$

**128** ($b$) Given, $x = (t-2)^2$

$$\text{Velocity, } v = \frac{dx}{dt} = \frac{d}{dt}(t-2)^2 = 2(t-2)\,\text{ms}^{-1}$$

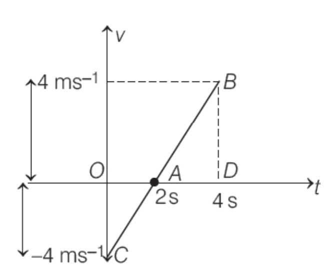
$$\text{Acceleration, } a = \frac{dv}{dt} = \frac{d}{dt}[2(t-2)] = 2[1-0] = 2\,\text{ms}^{-2}$$

When $t = 0$; $v = -4\,\text{ms}^{-1}$

$t = 2\,\text{s}$; $v = 0\,\text{ms}^{-1}$

$t = 4\,\text{s}$; $v = 4\,\text{ms}^{-1}$

$v$-$t$ graph for these values is shown below



Distance travelled = Area of the graph

$= \text{Area } OAC + \text{Area } ABD$

$= \frac{1}{2} \times 4 \times 2 + \frac{1}{2} \times 2 \times 4 = 8\,\text{m}$

**129** (*c*) Let displacement is $L$, then

Velocity of girl, $v_g = \frac{L}{t_1}$

Velocity of escalator, $v_e = \frac{L}{t_2}$

Net velocity of the girl $= v_g + v_e = \frac{L}{t_1} + \frac{L}{t_2}$

If $t$ is total time taken in covering distance $L$, then

$$\frac{L}{t} = \frac{L}{t_1} + \frac{L}{t_2} \Rightarrow t = \frac{t_1 t_2}{t_1 + t_2}$$

**130** (*a*) When we are representing motion by a graph, it may be displacement-time, velocity-time or acceleration-time, hence $B$ may represent time.

For uniform motion, velocity-time graph should be a straight line parallel to time axis and displacement-time graph a straight line inclined to time axis.

Hence, quantity $A$ is displacement. For uniformly accelerated motion, slope will be positive and $A$ will represent velocity.

**131** (*c*) As point $A$ is the starting point. Therefore, particle is starting from rest.

At point $B$, the graph is parallel to time axis, so the velocity is constant here. Thus, acceleration is zero.

Also point $C$, the graph changes slope, hence velocity also changes.

After graph at $C$ is almost parallel to time axis, hence we can say that velocity and acceleration vanishes.

From the graph, it is clear that

$$|\text{slope at } D| > |\text{slope at } E|$$

Hence, speed at $D$ will be more than at $E$.

**132** (*d*) Given, $x = t - \sin t$

Velocity, $v = \frac{dx}{dt} = \frac{d}{dt}[t - \sin t] = 1 - \cos t$

Acceleration, $a = \frac{dv}{dt} = \frac{d}{dt}[1 - \cos t] = \sin t$

As acceleration $a > 0$ for all $t > 0$

Hence, $x(t) > 0$ for all $t > 0$

Velocity, $v = 1 - \cos t$

When $\cos t = 1$, velocity $v = 0$

$v_{\max} = 1 - (\cos t)_{\min} = 1 - (-1) = 2$

$v_{\min} = 1 - (\cos t)_{\max} = 1 - 1 = 0$

Hence, $v$ lies between 0 and 2.

**133** (*a*) Since, the ball is moving with a small speed in the moving train, the direction of motion of the ball is the same as that of the train. The direction of motion of ball does not changes with respect to an observer on ground, i.e. constant for every 10 s.

Compared to velocity of trains ($10\ \text{ms}^{-1}$), speed of ball is less ($1\ \text{ms}^{-1}$). The speed of the ball before collision with side of train is $10 + 1 = 11\,\text{ms}^{-1}$.

Speed after collision with the side of train

$= 10 - 1 = 9\,\text{ms}^{-1}$.

Since, the collision of the ball with side of train is perfectly elastic; the total momentum and kinetic energy are conserved, so average speed of the ball over any 20 s interval is constant or fixed as observed by observer on ground.

Since, the train is moving with constant velocity hence, it will act as inertial frame of reference like Earth and acceleration of ball will be same as from the train.

CHAPTER > 04

# Motion in a Plane

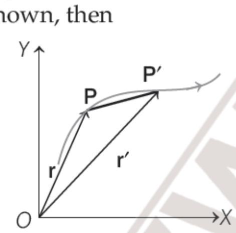
## KEY NOTES

### Scalars and Vectors

- A **scalar** quantity is a quantity that has magnitude only. It is specified completely by a number, along with the proper unit. e.g. Distance, mass of an object, etc.
- A **vector** quantity is a quantity that has both a magnitude and a direction. e.g. Displacement, velocity, acceleration, etc.

### Position and Displacement Vectors

- If $P$ and $P'$ be the positions of the object at time $t$ and $t'$, respectively as shown, then




  (i) **OP** is the **position vector** of the object at time $t$.

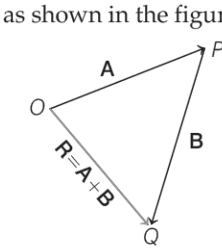
  (ii) If the object moves from $P$ to $P'$, the vector **PP′** is called the **displacement vector** corresponding to motion from point $P$ to point $P'$.
- The magnitude of displacement is either less or equal to the path length of an object between two points.

### Equality of Vectors and Multiplication of Vectors by Real Numbers

- Two vectors **A** and **B** are said to be equal, if and only, if they have the same magnitude and the same direction.
- Multiplying a vector **A** with a positive number $\lambda$ gives a vector whose magnitude is changed by the factor $\lambda$ but the direction is the same as that of **A**, i.e. $|\lambda \mathbf{A}| = \lambda |\mathbf{A}|$, if $\lambda > 0$
- Multiplying a vector **A** by a negative number $-\lambda$ gives another vector whose direction is opposite to the direction of **A** and whose magnitude is $\lambda$ times $|\mathbf{A}|$.

### Addition and Subtraction of Vectors

- Sum of two vectors **A** and **B** can be found, if we place vector **B**, so that its tail is at the head of the vector **A**, then join the tail of **A** to the head of **B**. This line $OQ$ represents a vector **R**, the sum of vectors **A** and **B** as shown in the figure.




Since, in this procedure of vector addition, vectors are arranged head-to-tail, this graphical method is known as **head-to-tail method.** This is also known as **triangle method of vector addition.**

- Following are important points related to vector addition

  (i) Vectors addition is commutative,

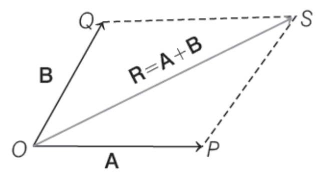
  i.e. $\mathbf{A} + \mathbf{B} = \mathbf{B} + \mathbf{A}$

  (ii) Vector addition is associative,

  i.e. $(\mathbf{A} + \mathbf{B}) + \mathbf{C} = \mathbf{A} + (\mathbf{B} + \mathbf{C})$
- When two equal and opposite vectors are added (says **A** and **–A**), as their magnitudes are the same but the directions are opposite. So, the resultant vector will have zero magnitude and is represented by **0** called a **null vector** or a **zero vector**, i.e.

$$\mathbf{A} - \mathbf{A} = \mathbf{0} \text{ and } |\mathbf{0}| = 0$$

- Since, the magnitude of a null vector is zero, its direction cannot be specified. The main properties of **0** are
  (i) $\mathbf{A} + \mathbf{0} = \mathbf{A}$
  (ii) $\lambda\mathbf{0} = \mathbf{0}$
  (iii) $0\mathbf{A} = \mathbf{0}$
- **Parallelogram method** of vector addition is equivalent to the triangle method. In this vectors to be added, says **A** and **B**, have their tail at common origin *O*, then according to this law, the sum of **A** + **B** is represented by the diagonal (*OS*) of the parallelogram directed from *O* as shown in the figure below

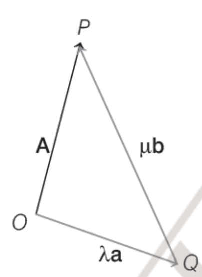



- **Subtraction of vectors** can be defined in terms of addition of vectors. We define the difference of two vectors **A** and **B** as the sum of two vectors **A** and **–B**, i.e.

$$\mathbf{A} - \mathbf{B} = \mathbf{A} + (-\mathbf{B})$$

## Resolution of Vectors

- Vector **A** can be resolved along two coplanar vectors **a** and **b** as shown in figure below as $\mathbf{A} = \lambda\,\mathbf{a} + \mu\mathbf{b}$, where $\lambda$ and $\mu$ are real numbers.




- A **unit vector** is a vector of unit magnitude and points in a particular direction. It has no dimension and unit. It is used to specify a direction only.

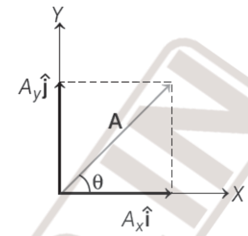
  A unit vector associated with a vector **A** is $\hat{\mathbf{n}} = \dfrac{\mathbf{A}}{|\mathbf{A}|}$,

  where $\hat{\mathbf{n}}$ is along **A**.
- Unit vectors along the *X*, *Y* and *Z*-axes of a rectangular coordinate systems are denoted by $\hat{\mathbf{i}}$, $\hat{\mathbf{j}}$ and $\hat{\mathbf{k}}$, respectively. Since, these are unit vectors, we have

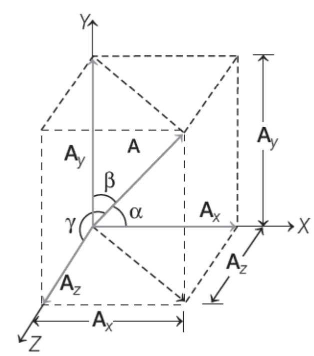
$$|\hat{\mathbf{i}}| = |\hat{\mathbf{j}}| = |\hat{\mathbf{k}}| = 1$$

  These unit vectors are perpendicular to each other.
- If we multiply a unit vector, say $\hat{\mathbf{n}}$ by a scalar $\lambda$, the result is a vector $\lambda = \lambda\hat{\mathbf{n}}$.

- If a vector **A** lies in *xy*-plane such that it is inclined at an angle θ with *X*-axis as shown in the figure, then




  (i) $\mathbf{A} = A_x\hat{\mathbf{i}} + \mathbf{A}_y\hat{\mathbf{j}}$
  (ii) Component along *X*-axis is $A_x = A\cos\theta$.
  (iii) Component along *Y*-axis is $A_y = A\sin\theta$.
  (iv) $A = \sqrt{A_x^2 + A_y^2}$ and $\tan\theta = \dfrac{A_y}{A_x}$.
- If a vector **A** lies in space, such that α, β & γ are the angles between **A** and *X*, *Y* & *Z*-axes respectively, as shown in the figure, then we have




  (i) $A_x = A\cos\alpha;\ A_y = A\cos\beta;\ A_z = A\cos\gamma$
  (ii) $\mathbf{A} = A_x\hat{\mathbf{i}} + A_y\hat{\mathbf{j}} + A_z\hat{\mathbf{k}}$
  (iii) $|\mathbf{A}| = A = \sqrt{A_x{}^2 + A_y{}^2 + A_z{}^2}$
- Vectors can be added by **analytical method**, i.e. adding the vectors by combining their respective components along the coordinate axis.

  If two vectors are $\mathbf{A} = A_x\hat{\mathbf{i}} + A_y\hat{\mathbf{j}} + A_z\hat{\mathbf{k}}$

  and $\mathbf{B} = B_x\hat{\mathbf{i}} + B_y\hat{\mathbf{j}} + B_z\hat{\mathbf{k}}$

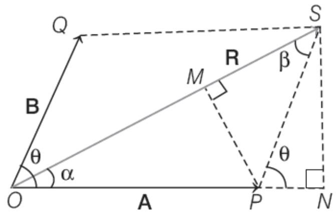
  Then, resultant, $\mathbf{R} = \mathbf{A} + \mathbf{B} = R_x\hat{\mathbf{i}} + R_y\hat{\mathbf{j}} + R_z\hat{\mathbf{k}}$

  where, $R_x = A_x + B_x$, $R_y = A_y + B_y$
  and $R_z = A_z + B_z$.

KEY NOTES

## Law of Cosines and Sines

- If two vectors **A** and **B** inclined at an angle $\theta$, as shown below




Then, from parallelogram method of vector addition,

(i) Resultant vector, $\mathbf{R} = \mathbf{A} + \mathbf{B}$.

(ii) $R = \sqrt{A^2 + B^2 + 2AB\cos\theta}$ is known as **law of cosines.**

(iii) $\dfrac{R}{\sin\theta} = \dfrac{A}{\sin\beta} = \dfrac{B}{\sin\alpha}$ is known as **law of sines.**

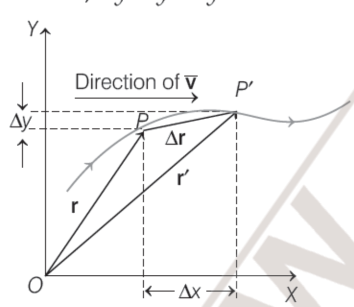
(iv) Angle of **R** from **A**, $\tan\alpha = \dfrac{B\sin\theta}{A + B\cos\theta}$.

## Motion in a Plane

- **Position vector r** of a particle $P$ located in a plane (two dimensions) with reference to the origin of an $xy$-reference frame is given by $\mathbf{r} = x\hat{\mathbf{i}} + y\hat{\mathbf{j}}$, where $x$ and $y$ are components of **r** along $X$ and $Y$-axes.
- If the positions of a particle are $P$ and $P'$ at time $t$ and $t'$, respectively as shown below, then its **displacement**,

$\Delta\mathbf{r} = \mathbf{r}' - \mathbf{r} = (x'\hat{\mathbf{i}} + y'\hat{\mathbf{j}}) - (x\hat{\mathbf{i}} + y\hat{\mathbf{j}}) = \hat{\mathbf{i}}\Delta x + \hat{\mathbf{j}}\Delta y$

where, $\Delta x = x' - x$, $\Delta y = y' - y$.




- The **average velocity** $\overline{\mathbf{v}}$ of an object is the ratio of the displacement $\Delta\mathbf{r}$ and the corresponding time-interval $\Delta t$. Mathematically,

$$\overline{\mathbf{v}} = \frac{\Delta\mathbf{r}}{\Delta t} = \hat{\mathbf{i}}\frac{\Delta x}{\Delta t} + \hat{\mathbf{j}}\frac{\Delta y}{\Delta t} \text{ or } \overline{\mathbf{v}} = \overline{v}_x\hat{\mathbf{i}} + \overline{v}_y\hat{\mathbf{j}}$$

The direction of the average velocity is same as that of $\Delta\mathbf{r}$.

- The velocity **(instantaneous velocity)** is given by the limiting value of the average velocity as the time-interval approaches zero, i.e.

$$\mathbf{v} = \lim_{\Delta t \to 0} \overline{\mathbf{v}} = \lim_{\Delta t \to 0} \frac{\Delta\mathbf{r}}{\Delta t} = \frac{d\mathbf{r}}{dt}$$

- The direction of velocity at any point on the path of an object is tangential to the path at that point and is in the direction of motion.
- **v** in component form can be expressed as

$$\mathbf{v} = \frac{dx}{dt}\hat{\mathbf{i}} + \frac{dy}{dt}\hat{\mathbf{j}} = v_x\hat{\mathbf{i}} + v_y\hat{\mathbf{j}} \Rightarrow |\mathbf{v}| = \sqrt{v_x^2 + v_y^2}$$

and direction of **v** is given by

$$\tan\theta = \frac{v_y}{v_x} \text{ or } \theta = \tan^{-1}\left(\frac{v_y}{v_x}\right)$$

- **Average acceleration** $\overline{\mathbf{a}}$ of an object for a time-interval $\Delta t$ moving in $xy$-plane is the change in velocity divided by the time interval, i.e.

$$\overline{\mathbf{a}} = \frac{\Delta\mathbf{v}}{\Delta t} = \frac{\Delta v_x}{\Delta t}\hat{\mathbf{i}} + \frac{\Delta v_y}{\Delta t}\hat{\mathbf{j}} = a_x\hat{\mathbf{i}} + a_y\hat{\mathbf{j}}$$

- The acceleration **(instantaneous acceleration)** is the limiting value of the average acceleration as the time-interval approaches zero i.e.,

$$\mathbf{a} = \lim_{\Delta t \to 0} \frac{\Delta\mathbf{v}}{\Delta t} = \frac{d\mathbf{v}}{dt}$$

- In one-dimension, the velocity and acceleration of an object are always along the same straight line (either in the same direction or in the opposite direction). However, for motion in two or three dimensions, velocity and acceleration vectors may have any angle 0° and 180° between them.

## Motion in a Plane with Constant Acceleration

- Suppose an object is moving in $xy$-plane with constant acceleration **a**. Let the position and velocity of the object be $\mathbf{r}_0$ and $\mathbf{v}_0$ at time $t = 0$ and **r** and **v** at any time $t$. Then,

(i) $\mathbf{v} = \mathbf{v}_0 + \mathbf{a}t$ (ii) $\overline{\mathbf{v}} = \dfrac{\mathbf{v}_0 + \mathbf{v}}{2}$

(iii) $\mathbf{r} = \mathbf{r}_0 + \mathbf{v}_0 t + \dfrac{1}{2}\mathbf{a}t^2$.

In component form, $x = x_0 + v_{0x}t + \dfrac{1}{2}a_x t^2$

$$y = y_0 + v_{0y}t + \frac{1}{2}a_y t^2$$

- The motion in a plane (two-dimensions) can also be treated as two separate simultaneous one dimensional motions with constant acceleration along two perpendicular directions.

## Relative Velocity in Two-dimensions

- Suppose two objects $A$ and $B$ are moving with velocities $\mathbf{v}_A$ and $\mathbf{v}_B$ respectively (each with respect to some common frame of reference, say ground), then velocity of object $A$ relative to that of $B$ is $\mathbf{v}_{AB} = \mathbf{v}_A - \mathbf{v}_B$.

Similarly, the velocity of object $B$ relative to that of $A$ is

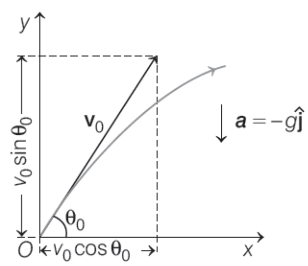
$$\mathbf{v}_{BA} = \mathbf{v}_B - \mathbf{v}_A$$

Therefore, $\mathbf{v}_{AB} = -\mathbf{v}_{BA}$

$\Rightarrow$ $|\mathbf{v}_{AB}| = |\mathbf{v}_{BA}|$

## Projectile Motion

- An object that is in flight after being thrown or projected is called a **projectile**. Such a projectile might be a football, a cricket ball, a baseball, etc.
- If a projectile (particle or body) moves in a horizontal as well as vertical direction simultaneously, the motion of particle is known as projectile motion.
- For an object, after being projected with initial velocity $\mathbf{v}_0$ that makes an angle $\theta_0$ with $X$-axis as shown below




(i) **Acceleration acting on it is that due to gravity** which is directed vertically downward, i.e.

$$\mathbf{a} = -g\hat{\mathbf{j}} \text{ or } a_x = 0 \text{ and } a_y = -g$$

(ii) **Components of its velocity at time $t$** can be given by

$$v_x = v_0 \cos\theta_0$$
$$v_y = v_0 \sin\theta_0 - gt$$

(iii) One of the component of velocity, i.e. ***x*-component** remains constant throughout the motion and only the ***y*-component** changes.

(iv) **At maximum height**, $v_y = 0$ and therefore $\tan^{-1}\dfrac{v_y}{v_x} = 0$.

- The equation of **path of a projectile** given by

$$y = x \tan\theta_0 - \frac{gx^2}{(v_0 \cos\theta_0)^2}$$

This is the equation of a parabola, i.e. the path of the projectile is a **parabola**.

- The shape of trajectory of the motion of an object is not determined by the acceleration alone but also depends on the initial conditions of motion (initial and final velocity).

For example, the trajectory of an object moving under the same acceleration due to gravity can be straight line or a parabola depending on the initial conditions.

- **Time of maximum height** is the time taken by the projectile to reach the maximum height. It is given as

$$t_m = \frac{v_0 \sin\theta_0}{g}$$

- **Time of flight** is the total time $T_f$ during which the projectile is in flight. It is given as $T_f = \dfrac{2v_0 \sin\theta_0}{g}$.

**Note** $T_f = 2t_m$, which is expected because of the symmetry of the parabolic path.

- **Maximum height of a projectile** is the maximum height $h_m$ reached by the projectile. It is given as

$$h_m = \frac{(v_0 \sin\theta_0)^2}{2g}$$

- The horizontal distance travelled by a projectile from its initial position to the position, where it passes $y = 0$ during its fall is called **horizontal range** $R$ of projectile. It is the distance travelled during the time of flight $T_f$. It is given as $R = \dfrac{v_0^2 \sin 2\theta_0}{g}$.

$R$ is maximum when $2\theta$ is maximum, i.e. when $\theta_0 = 45°$, then its maximum value is $R_{\max} = \dfrac{v_0^2}{g}$.

## Uniform Circular Motion

- When an object follows a circular path at a constant speed, the motion of the object is called **uniform circular motion**.
- The acceleration of an object moving with speed $v$ in a circle of radius $R$ has a magnitude $\dfrac{v^2}{R}$ and is always directed towards the centre. This acceleration is called **centripetal acceleration**.
- Since, $v$ and $R$ are constants, the magnitude of centripetal acceleration is also constant. However, the direction changes. Therefore, a centripetal acceleration is not a constant vector.
- The resultant acceleration of an object in circular motion is towards the centre only, if the speed is constant.
- **Angular speed** is defined as the time rate of change of angular displacement. It is given as

$$\omega = \frac{\Delta\theta}{\Delta t}$$

- **Relation between linear speed and angular speed** is given as

$$v = R\omega$$

So, centripetal acceleration, $a_c = \omega^2 R$.

- The time taken by an object to make one revolution is known as its **time period** $T$ and the number of revolution made in one second is called its **frequency** $\nu (= 1 / T)$.
- In term of frequency $\nu$, we have

$$\omega = 2\pi\nu$$
$$a_c = 4\pi^2\nu^2 R$$

**Note** The kinematic equations for uniform acceleration do not apply to the case of uniform circular motion. Since, in this case, the magnitude of acceleration is constant but the direction is changing.

# Mastering NCERT

## MULTIPLE CHOICE QUESTIONS

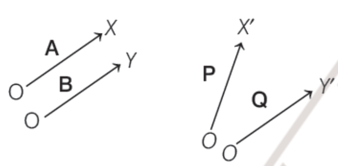
## TOPIC 1 ~ *Scalars and Vectors*

**1** Amongst the following quantities, which is not a vector quantity?
(a) Force (b) Acceleration
(c) Temperature (d) Velocity

**2** In order to describe the motion in two or three dimensions, we use
(a) positive sign
(b) vectors
(c) negative sign
(d) Both (b) and (c)

**3** If length and breadth of a rectangle are 1.0 m and 0.5 m respectively, then its perimeter will be a
(a) free vector (b) scalar quantity
(c) localised vector (d) Neither (a) nor (b)

**4** Set of vectors **A** and **B**, **P** and **Q** are as shown below




Length of **A** and **B** is equal, similarly length of **P** and **Q** is equal. Then, the vectors which are equal, are
(a) **A** and **P** (b) **P** and **Q**
(c) **A** and **B** (d) **B** and **Q**

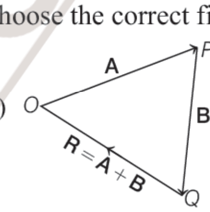
**5** $|\lambda \mathbf{A}| = \lambda |\mathbf{A}|$, if
(a) $\lambda > 0$ (b) $\lambda < 0$
(c) $\lambda = 0$ (d) $\lambda \neq 0$

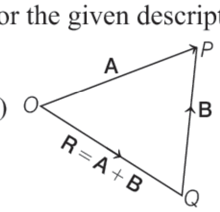
**6** If a vector is multiplied by a negative number, we get a vector whose
(a) magnitude and direction both are changed
(b) only direction is changed
(c) only magnitude is changed
(d) only direction is reversed

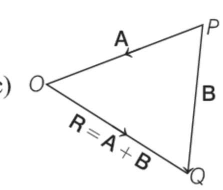
**7** Choose the correct option regarding the given figure.

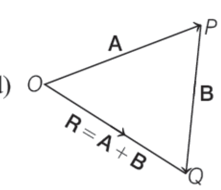

(a) $\mathbf{B} = \mathbf{A}$ (b) $\mathbf{B} = -\mathbf{A}$
(c) $|\mathbf{B}| = |\mathbf{A}|$ (d) $|\mathbf{B}| \neq |\mathbf{A}|$

**8** **A** and **B** are two inclined vectors and **R** is their sum. Choose the correct figure for the given description.

(a)



(b)



(c)



(d)



**9** Among the following properties regarding null vector which is incorrect?
(a) $\mathbf{A} + 0 = \mathbf{A}$ (b) $\lambda \mathbf{0} = \lambda$
(c) $\mathbf{0}\mathbf{A} = \mathbf{0}$ (d) $\mathbf{A} - \mathbf{A} = \mathbf{0}$

**10** Suppose an object is at point $P$ at time $t$ moves to $P'$ and then comes back to $P$. Then, displacement is a
(a) unit vector (b) null vector
(c) scalar (d) None of these

**11** Find the correct option about vector subtraction.
(a) $\mathbf{A} - \mathbf{B} = \mathbf{A} + \mathbf{B}$
(b) $\mathbf{A} + \mathbf{B} = \mathbf{B} - \mathbf{A}$
(c) $\mathbf{A} - \mathbf{B} = \mathbf{A} + (-\mathbf{B})$
(d) None of the above

# TOPIC 2 ~ *Resolution of Vectors*

**12** If **A** is a vector with magnitude $A$, then the unit vector $\hat{\mathbf{a}}$ in the direction of vector **A** is

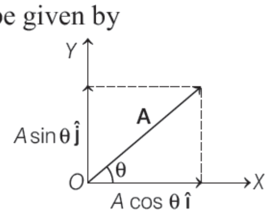
(a) $A\mathbf{A}$ (b) $\mathbf{A}\cdot\mathbf{A}$ (c) $\mathbf{A}\times\mathbf{A}$ (d) $\dfrac{\mathbf{A}}{|\mathbf{A}|}$

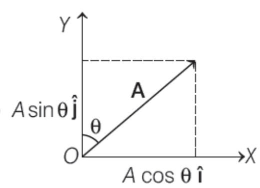
**13** Unit vector of $4\hat{\mathbf{i}}-3\hat{\mathbf{j}}+\hat{\mathbf{k}}$ is

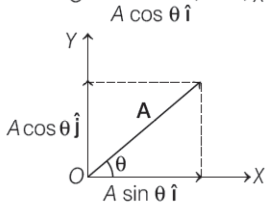
(a) $\hat{\mathbf{i}}-\hat{\mathbf{j}}+\hat{\mathbf{k}}$ (b) $\sqrt{26}\hat{\mathbf{i}}-\sqrt{26}\hat{\mathbf{j}}+\sqrt{26}\hat{\mathbf{k}}$

(c) $\dfrac{4\hat{\mathbf{i}}-3\hat{\mathbf{j}}+\hat{\mathbf{k}}}{\sqrt{26}}$ (d) $5\hat{\mathbf{i}}-4\hat{\mathbf{j}}+5\hat{\mathbf{k}}$

**14** Consider a vector **A** that lies in $xy$-plane. If $A_x$ and $A_y$ are the magnitudes of its $x$ and $y$-components respectively, then the correct representation of **A** can be given by

(a) 


(b) 


(c) 


(d) None of these

**15** Magnitude of a vector **Q** is 5 and magnitude of its $y$-component is 4. So, the magnitude of the $x$-component of this vector is

(a) 8 (b) 3 (c) 6 (d) 8

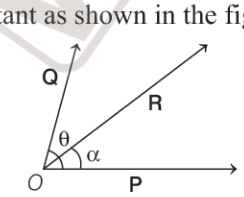
**16** A vector is inclined at an angle 60° to the horizontal. If its rectangular component in the horizontal direction is 50 N, then its magnitude in the vertical direction is

(a) 25 N (b) 75 N (c) 87 N (d) 100 N

**17** Three vectors are given as $\mathbf{P}=3\hat{\mathbf{i}}-4\hat{\mathbf{j}}, \mathbf{Q}=6\hat{\mathbf{i}}-8\hat{\mathbf{j}}$ and $\mathbf{R}=(3/4)\hat{\mathbf{i}}-\hat{\mathbf{j}}$, then which of the following is correct?

(a) $P$, $Q$ and $R$ are equal vectors
(b) $P$ and $Q$ are parallel but $R$ is not parallel
(c) $P$, $Q$ and $R$ are parallel
(d) None of the above

**18** Two vectors **P** and **Q** are inclined at an angle θ and **R** is their resultant as shown in the figure.




Keeping the magnitude and the angle of the vectors same, if the direction of **P** and **Q** is interchanged, then their is a change in which of the following with regard to **R**?

(a) Magnitude
(b) Direction
(c) Both magnitude and direction
(d) None of the above

**19** Consider vectors **a**, **b** and **c** as

$$\mathbf{a}=a_x\hat{\mathbf{i}}+a_y\hat{\mathbf{j}}+a_z\hat{\mathbf{k}}$$
$$\mathbf{b}=b_x\hat{\mathbf{i}}+b_y\hat{\mathbf{j}}+b_z\hat{\mathbf{k}}$$
$$\mathbf{c}=c_x\hat{\mathbf{i}}+c_y\hat{\mathbf{j}}+c_z\hat{\mathbf{k}}$$

Then, for a vector $\mathbf{T}=\mathbf{a}+\mathbf{b}-\mathbf{c}$ has its $y$-component in the form

(a) $a_y+b_y+c_y$ (b) $-a_y+b_y-c_y$
(c) $a_y+b_y-c_y$ (d) $a_y-b_y+c_y$

**20** Unit vector in the direction of the resultant of vectors $\mathbf{A}=-3\hat{\mathbf{i}}-2\hat{\mathbf{j}}-3\hat{\mathbf{k}}$ and $\mathbf{B}=2\hat{\mathbf{i}}+4\hat{\mathbf{j}}+6\hat{\mathbf{k}}$ is

(a) $\dfrac{-3\hat{\mathbf{i}}+2\hat{\mathbf{j}}-3\hat{\mathbf{k}}}{\sqrt{14}}$ (b) $-\hat{\mathbf{i}}+2\hat{\mathbf{j}}+3\hat{\mathbf{k}}$

(c) $\dfrac{-\hat{\mathbf{i}}+2\hat{\mathbf{j}}+3\hat{\mathbf{k}}}{\sqrt{14}}$ (d) $-2\hat{\mathbf{i}}-4\hat{\mathbf{j}}+8\hat{\mathbf{k}}$

**21** Two forces $P$ and $Q$ of magnitude $2F$ and $3F$, respectively,are at an angle θ with each other. If the force $Q$ is doubled, then their resultant also gets doubled. Then, the angle θ is **JEE Main 2019**

(a) 60° (b) 120°
(c) 30° (d) 90°

**22** It is found that $|\mathbf{A}+\mathbf{B}|=|\mathbf{A}|$. This necessarily implies

(a) $|\mathbf{B}|=0$ (b) **A**, **B** are parallel
(c) **A**, **B** are perpendicular (d) **A**; $\mathbf{B}\leq 0$

**23** Find the value of difference of unit vectors **A** and **B** whose angle of intersection is θ.

(a) $2\sin(\theta/2)$ (b) $2\cos(\theta/2)$
(c) $\sin(\theta/2)$ (d) $\cos(\theta/2)$

**24** Given, $|\mathbf{A}+\mathbf{B}|=P, |\mathbf{A}-\mathbf{B}|=Q$. The value of $P^2+Q^2$ is

(a) $2(A^2+B^2)$ (b) $A^2-B^2$
(c) $A^2+B^2$ (d) $2(A^2-B^2)$

**25** For two vectors **A** and **B**, $|\mathbf{A}+\mathbf{B}|=|\mathbf{A}-\mathbf{B}|$ is always true, when

(a) $|\mathbf{A}|=|\mathbf{B}|\neq 0$
(b) $|\mathbf{A}|=|\mathbf{B}|\neq 0$ and **A** and **B** are parallel or anti-parallel
(c) when either $|\mathbf{A}|$ or $|\mathbf{B}|$ is zero
(d) None of the above

**26** Two vectors $\mathbf{A}$ and $\mathbf{B}$ have equal magnitudes. The magnitude of $(\mathbf{A}+\mathbf{B})$ is '$n$' times the magnitude of $(\mathbf{A}-\mathbf{B})$. The angle between $\mathbf{A}$ and $\mathbf{B}$ is

(a) $\sin^{-1}\left(\frac{n^2-1}{n^2+1}\right)$ (b) $\sin^{-1}\left(\frac{n-1}{n+1}\right)$

(c) $\cos^{-1}\left(\frac{n^2-1}{n^2+1}\right)$ (d) $\cos^{-1}\left(\frac{n-1}{n+1}\right)$

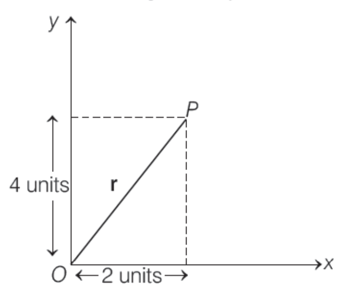
**27** Rain is falling vertically with a speed of $35\ \text{ms}^{-1}$. Winds starts blowing after sometime with a speed of $12\ \text{ms}^{-1}$ in east to west direction. In which direction from vertical should boy waiting at a bus stop hold his umbrella?

(a) $\tan^{-1}(0.45)$, west (b) $\tan^{-1}(0.343)$, west

(c) $\tan^{-1}(0.343)$, east (d) $\tan^{-1}(0.24)$, east

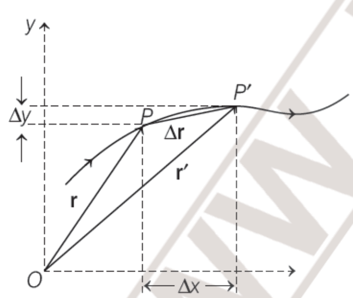
# TOPIC 3 ~ *Motion in a Plane*

**28** Position vector $\mathbf{r}$ of a particle $P$ located in a plane with reference to the origin of an $xy$-plane as shown in the figure below is given by




(a) $2\hat{\mathbf{i}}+4\hat{\mathbf{j}}$ (b) $4\hat{\mathbf{i}}+2\hat{\mathbf{j}}$ (c) $6\hat{\mathbf{k}}$ (d) $\hat{\mathbf{i}}+\hat{\mathbf{j}}+4\hat{\mathbf{k}}$

**29** Suppose a particle moves along a curve shown by the thick line and the positions of particle are represented by $P$ at $t$ and $P'$ at $t'$.




where, coordinates of $P$ is (2, 3) and $P'$ is (5, 6). Net displacement of the particle will be

(a) zero (b) $7\hat{\mathbf{i}}+9\hat{\mathbf{j}}$

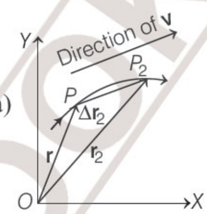
(c) $10\hat{\mathbf{i}}+18\hat{\mathbf{j}}$ (d) $3\hat{\mathbf{i}}+3\hat{\mathbf{j}}$

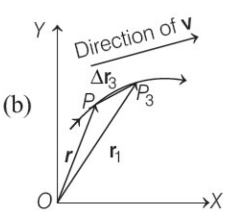
**30** A particle is moving such that its position coordinates $(x, y)$ are $(2m, 3m)$ at $t=0$ s, $(6m, 7m)$ at time 2s and $(13m, 14m)$ at time $t=5$s. Average velocity vector $(\mathbf{v}_{av})$ from $t=0$s to $t=5$ s is

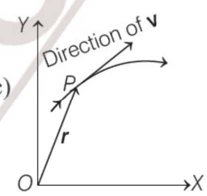
(a) $\frac{1}{5}(13\hat{\mathbf{i}}+14\hat{\mathbf{j}})$ (b) $\frac{11}{5}(\hat{\mathbf{i}}+\hat{\mathbf{j}})$

(c) $2(\hat{\mathbf{i}}+\hat{\mathbf{j}})$ (d) $\frac{7}{3}(\hat{\mathbf{i}}+\hat{\mathbf{j}})$

**31** The direction of instantaneous velocity is shown by

(a)



(b)



(c)



(d) None of these

**32** The position of a particle is given by
$\mathbf{r}=3t\hat{\mathbf{i}}+2t^2\hat{\mathbf{j}}+5\hat{\mathbf{k}}$, then the direction of
$\mathbf{v}(t)$ at $t=1$ s in

(a) 45° with $X$-axis (b) 63° with $Y$-axis

(c) 30° with $Y$-axis (d) 53° with $X$-axis

**33** The $x$ and $y$-coordinates of the particle at any time are $x=5t-2t^2$ and $y=10t$ respectively, where $x$ and $y$ are in metres and $t$ in seconds. The acceleration of the particle at $t=2$ s is **NEET 2017**

(a) 0 (b) $5\ \text{ms}^{-2}$ (c) $-4\ \text{ms}^{-2}$ (d) $-8\ \text{ms}^{-2}$

**34** The position vector of particle changes with time according to the relation $\mathbf{r}(t)=15t^2\hat{\mathbf{i}}+(4-20t^2)\hat{\mathbf{j}}$. What is the magnitude of the acceleration (in $\text{ms}^{-2}$) at $t=1$? **JEE Main 2019**

(a) 50 (b) 100 (c) 25 (d) 40

**35** In three dimensional system, the position coordinates of a particle (in motion) are given below **JEE Main 2019**

$$x=a\cos\omega t$$
$$y=a\sin\omega t$$
$$z=a\omega t$$

The velocity of particle will be

(a) $\sqrt{2}\,a\omega$ (b) $2\,a\omega$ (c) $a\omega$ (d) $\sqrt{3}\,a\omega$

**36** A particle starts from origin at $t = 0$ with a velocity $5.0\,\hat{\mathbf{i}}\ \text{ms}^{-1}$ and moves in $xy$-plane under action of force which produces a constant acceleration of $(3.0\,\hat{\mathbf{i}} + 2.0\,\hat{\mathbf{j}})\ \text{ms}^{-2}$. What is the $y$-coordinate of the particle at the instant, if $x$-coordinate is 84 m?
(a) 36 m (b) 24 m (c) 39 m (d) 18 m

**37** When an object is shot from the bottom of a long smooth inclined plane kept at an angle 60° with horizontal, it can travel a distance $x_1$ along the plane. But when the inclination is decreased to 30° and the same object is shot with the same velocity, it can travel $x_2$ distance. Then $x_1 : x_2$ will be ***NEET 2019***
(a) $\sqrt{2}:1$ (b) $1:\sqrt{3}$ (c) $1:2\sqrt{3}$ (d) $1:\sqrt{2}$

# TOPIC 4 ~ *Relative Velocity in Two-dimensions*

**38** If two objects $P$ and $Q$ move along parallel straight lines in opposite direction with velocities $\mathbf{v}_P$ and $\mathbf{v}_Q$ respectively, then relative velocity of $P$ w.r.t $Q$,
(a) $\mathbf{v}_{PQ} = \mathbf{v}_P = \mathbf{v}_Q$ (b) $\mathbf{v}_P - \mathbf{v}_Q$
(c) $\mathbf{v}_P + \mathbf{v}_Q$ (d) $\mathbf{v}_Q - \mathbf{v}_P$

**39** Buses $A$ and $B$ are moving in the same direction with velocities $20\hat{\mathbf{i}}\ \text{ms}^{-1}$ and $15\hat{\mathbf{i}}\ \text{ms}^{-1}$, respectively. Then, relative velocity of $A$ w.r.t. $B$ is
(a) $35\hat{\mathbf{i}}\ \text{ms}^{-1}$ (b) $5\,\hat{\mathbf{i}}\ \text{ms}^{-1}$ (c) $5\,\hat{\mathbf{j}}\ \text{ms}^{-1}$ (d) $35\,\hat{\mathbf{j}}\ \text{ms}^{-1}$

**40** Rain is falling vertically with a speed of 35 $\text{ms}^{-1}$. A woman rides a bicycle with a speed of 12 $\text{ms}^{-1}$ in east to west direction. The direction in which she should hold her umbrella is
(a) at $\cos^{-1}$ (0.343) with vertical towards east
(b) at $\tan^{-1}$ (0.343) with vertical towards west
(c) at $\cos^{-1}$ (0.343) with vertical towards west
(d) at $\tan^{-1}$ (0.343) with vertical towards east

**41** A car driver is moving towards a fired rocket with a velocity of $8\hat{\mathbf{i}}\ \text{ms}^{-1}$. He observed the rocket to be moving with a speed of 10 $\text{ms}^{-1}$. A stationary observer will see the rocket to be moving with a speed of
(a) 5 $\text{ms}^{-1}$ (b) 6 $\text{ms}^{-1}$
(c) 7 $\text{ms}^{-1}$ (d) 8 $\text{ms}^{-1}$

**42** The stream of a river is flowing with a speed of 2 km/h. A swimmer can swim at a speed of 4 km/h. What should be the direction of the swimmer with respect to the flow of the river to cross the river straight? ***JEE Main 2019***
(a) 60° (b) 120° (c) 90° (d) 150°

**43** A man standing on a road has to hold his umbrella at 30° with the vertical to keep the rain away. He throws the umbrella and starts running at 10 $\text{kmh}^{-1}$. He finds that raindrops are hitting his head vertically. The actual speed of raindrops is
(a) 20 $\text{kmh}^{-1}$ (b) $10\sqrt{3}$ $\text{kmh}^{-1}$
(c) $20\sqrt{3}$ $\text{kmh}^{-1}$ (d) 10 $\text{kmh}^{-1}$

**44** A girl can swim with speed 5$\text{kmh}^{-1}$ in still water. She crosses a river 2 km wide, where the river flows steadily at 2 $\text{kmh}^{-1}$ and she makes strokes normal to the river current. Find how far down the river she go when she reaches the other bank.
(a) 1 km (b) 2 km (c) 800 m (d) 750 m

**45** A girl riding a bicycle with a speed of 5 $\text{ms}^{-1}$ towards east direction sees raindrops falling vertically downwards. On increasing the speed to 15 $\text{ms}^{-1}$, rain appears to fall making an angle of 45° of the vertical. Find the magnitude of velocity of rain.
(a) 5 $\text{ms}^{-1}$ (b) $5\sqrt{5}$ $\text{ms}^{-1}$ (c) 25 $\text{ms}^{-1}$ (d) 10 $\text{ms}^{-1}$

# TOPIC 5 ~ *Projectile Motion*

**46** The motion of an object that is in flight after being projected is a result of two simultaneously occurring components of motion, which are the components in
(a) horizontal direction with constant acceleration
(b) vertical direction with constant acceleration
(c) horizontal direction without acceleration
(d) Both (b) and (c)

**47** At the top most point of the trajectory for an object that has been projected at an angle θ with the horizontal, has acceleration
(a) $> g$ (b) $< g$ (c) zero (d) $= g$

**48** A ball is projected with velocity 10 $\text{ms}^{-1}$ in a direction making an angle 30° with the horizontal, what is the position coordinate (in metres) of the ball after 1s?
(a) (8.66, 0.1) (b) (9.8, 1.0)
(c) (4.26, 5.29) (d) (0.4, 8.66)

**49** When a ball is thrown obliquely from the ground level, then the $x$-component of the velocity
(a) decreases with time
(b) increases with time
(c) remains constant
(d) zero

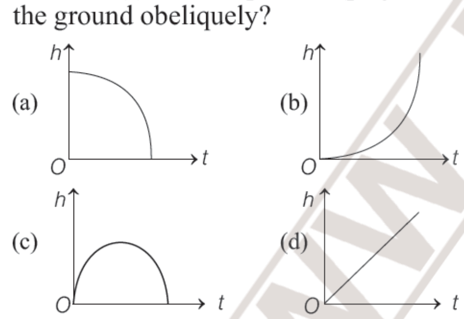
**50** At which point of a projectile motion, acceleration and velocity are perpendicular to each other?
(a) At the point of projection
(b) At the point of drop
(c) At the top most point
(d) Anywhere in between the point of projection and top most points

**51** The equations of motion of a projectile are given by $x = 18t$ and $2y = 54t - 9.8t^2$. The angle $\theta$ of projection is
(a) $\tan^{-1}(3)$ (b) $\tan^{-1}(1.5)$ (c) $\sin^{-1}\frac{2}{3}$ (d) $\cos^{-1}\frac{2}{3}$

**52** A projectile is fired from the surface of the earth with a velocity of 5 $ms^{-1}$ at an angle $\theta$ with the horizontal. Another projectile fired from another planet with a velocity of 3 $ms^{-1}$ at the same angle, follows a trajectory of the projectile fired from the earth. The value of the acceleration due to gravity on the planet is (in $ms^{-2}$) is (given, $g = 9.8\ ms^{-2}$) ***CBSE AIPMT 2014***
(a) 3.5 (b) 5.9 (c) 16.3 (d) 110.8

**53** A projectile is given an initial velocity of $(\hat{\mathbf{i}} + 2\hat{\mathbf{j}})\ ms^{-1}$, where $\hat{\mathbf{i}}$ is along the ground and $\hat{\mathbf{j}}$ is along vertical. If $g = 10\ ms^{-2}$, the equation of its trajectory is
(a) $y = x - 5x^2$ (b) $y = 2x - 5x^2$
(c) $4y = 2x - 5x^2$ (d) $4y = 2x - 25x^2$

**54** Amongst the following graphs, which graph represents the correct relation between the height of projectile ($h$) and time ($t$), when a particle (projectile) is thrown from the ground obeliquely?




**55** Two stones were projected simultaneously in the same vertical plane from same point obliquely, with different speeds and angles with the horizontal. The trajectory of path followed by one, as seen by the other, is
(a) parabola (b) straight line (c) circle (d) hyperbola

**56** Time taken by a stone to reach the maximum height is 5.8 s, then total time taken by the stone during which it was in flight is
(a) 5.8 s (b) 11.6 s (c) 2.9 s (d) 4.2 s

**57** An aircraft flying horizontally with the speed 480 $kmh^{-1}$ releases a parachute at a height of 980 m from the ground. It will strike the ground at (use, $g = 10\ ms^{-2}$)
(a) 1 km (b) 2 km (c) 2.8 km (d) 1.867 km

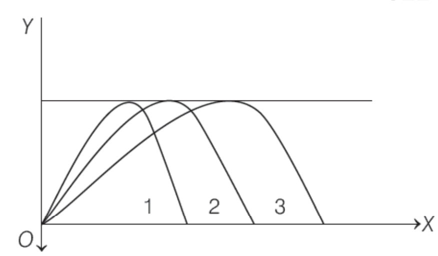
**58** The ceiling of a hall is 30 m high. A ball is thrown with 60 $ms^{-1}$ at an angle $\theta$, so that it could reach the ceiling of the hall and come back to the ground. The angle of projection $\theta$ that the ball was projected is given by
(a) $\sin\theta = \frac{1}{\sqrt{8}}$ (b) $\sin\theta = \frac{1}{\sqrt{6}}$
(c) $\sin\theta = \frac{1}{\sqrt{3}}$ (d) None of these

**59** Two projectiles having different masses $m_1$ and $m_2$ are projected at an angle $\alpha$ and $(90° - \alpha)$ with the same speed from some point. The ratio of their maximum heights is
(a) $\cot\alpha : \sin\alpha$ (b) 1 : 1
(c) $\tan^2\alpha : 1$ (d) $1 : \tan\alpha$

**60** Two projectiles $A$ and $B$ thrown with speeds in the ratio $1 : \sqrt{2}$ acquired the same height. If $A$ is thrown at an angle of 45° with the horizontal, then angle of projection of $B$ will be
(a) 0° (b) 60° (c) 30° (d) 45°

**61** What is the range of a projectile thrown with velocity 98 $ms^{-1}$ with angle 30° from horizontal? ***JIPMER 2018***
(a) $490\sqrt{3}$ m (b) $245\sqrt{3}$ m (c) $980\sqrt{3}$ m (d) 100 m

**62** Given below figure show three paths of a rock with different initial velocities. The correct increasing order for the respective initial horizontal velocity component (ignoring the effect of air resistance) is ***JEE Main 2013***




(a) 1< 2< 3 (b) 3< 2< 1 (c) 2< 1< 3 (d) 3< 1< 2

**63** A man can throw a stone to a maximum distance of 80 m. The maximum height to which it will rise, is
(a) 30 m (b) 20 m (c) 10 m (d) 40 m

**64** If a person can throw a stone to maximum height of $h$ metre vertically, then the maximum distance through which it can be thrown horizontally at an angle $\theta$ by the same person is
(a) $\frac{h}{2}$ (b) $h$ (c) $2h$ (d) $3h$

**65** Find angle of projection with the horizontal in terms of maximum height attained and horizontal range.
(a) $\tan^{-1}\frac{2H}{R}$ (b) $\tan^{-1}\frac{4R}{H}$ (c) $\tan^{-1}\frac{4H}{R}$ (d) $\tan^{-1}\frac{H}{R}$

**66** The speed of a projectile at the maximum height is (1/2) of its initial speed. Find the ratio of range of projectile to the maximum height attained.

(a) $4\sqrt{3}$ (b) $\frac{4}{\sqrt{3}}$

(c) $\frac{\sqrt{3}}{4}$ (d) 6

**67** A body is projected at $t = 0$ with a velocity $10\ ms^{-1}$ at an angle of $60°$ with the horizontal. The radius of curvature of its trajectory at $t = 1$ s is $R$. Neglecting air resistance and taking acceleration due to gravity $g = 10\ ms^{-2}$, the value of $R$ is **JEE Main 2019**

(a) 10.3 m (b) 2.8 m (c) 5.1 m (d) 2.5 m

# TOPIC 6 ~ *Uniform Circular Motion*

**68** In a uniform circular motion, velocity and acceleration vectors are

(a) perpendicular to each other
(b) in same direction
(c) in opposite direction
(d) in arbitrary direction

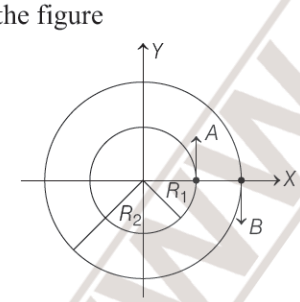
**69** If a car is executing a uniform circular motion, then its centripetal acceleration represents

(a) a scalar quantity
(b) constant vector
(c) not a constant vector
(d) constant direction and changing magnitude

**70** The displacement of a particle moving on a circular path when it makes $60°$ at the centre is

(a) $2r$ (b) $r$
(c) $\sqrt{2}r$ (d) None of these

**71** Two particles $A$ and $B$ are moving on two concentric circles of radii $R_1$ and $R_2$ with equal angular speed $\omega$. At $t = 0$, their positions and direction of motion are shown in the figure




The relative velocity $\mathbf{v}_A - \mathbf{v}_B$ at $t = \frac{\pi}{2\omega}$ is given by **JEE Main 2019**

(a) $\omega(R_1 + R_2)\hat{\mathbf{i}}$ (b) $-\omega(R_1 + R_2)\hat{\mathbf{i}}$

(c) $\omega(R_1 - R_2)\hat{\mathbf{i}}$ (d) $\omega(R_2 - R_1)\hat{\mathbf{i}}$

**72** A particle is moving with $10\ ms^{-1}$ in a circle of radius 5m, find out the magnitude of average velocity if particle is moved by $60°$ in 1 s. **JIPMER 2019**

(a) $5\ ms^{-1}$ (b) $10\ ms^{-1}$
(c) $5\sqrt{3}\ ms^{-1}$ (d) $20\ ms^{-1}$

**73** Two particles $A$ and $B$ are moving in uniform circular motion in concentric circles of radii $r_A$ and $r_B$ with speed $v_A$ and $v_B$, respectively. Their time period of rotation is the same. The ratio of angular speed of $A$ to that of $B$ will be **NEET 2019**

(a) $v_A : v_B$ (b) $r_B : r_A$ (c) $1:1$ (d) $r_A : r_B$

**74** Two cars $A$ and $B$ move along a concentric circular path of radius $r_A$ and $r_B$ with velocities $v_A$ and $v_B$ maintaining constant distance, then $\frac{v_A}{v_B}$ is equal to

(a) $\frac{r_B}{r_A}$ (b) $\frac{r_A}{r_B}$ (c) $\frac{r_A^2}{r_B^2}$ (d) $\frac{r_B^2}{r_A^2}$

**75** If the length of the second's hand in a stop clock is 3 cm, then linear velocity of its tip is

(a) $4.28\ ms^{-1}$ (b) $0.1047\ ms^{-1}$
(c) $0.00314\ ms^{-1}$ (d) $1.424\ ms^{-1}$

**76** What is the centripetal acceleration of a point mass which is moving on a circular path of radius 5m with speed $23\ ms^{-1}$?

(a) $106\ ms^{-2}$ (b) $90\ ms^{-2}$
(c) $60\ ms^{-2}$ (d) None of these

**77** Two bodies of masses $m_1$ and $m_2$, respectively are moving in circles of radii $r_1$ and $r_2$, respectively. Their speeds are such that they make complete circles in the same time $t$. The ratio of their centripetal accelerations is

(a) $m_1 r_1 : m_2 r_2$ (b) $m_1 : m_2$ (c) $r_1 : r_2$ (d) $1:1$

**78** A car revolves uniformly in a circle of diameter 0.80m and completes 100 rev $min^{-1}$. Its angular velocity is

(a) $10.467\ rads^{-1}$ (b) $0.6\ rads^{-1}$
(c) $46.26\ rads^{-1}$ (d) $8\ rads^{-1}$

**79** A car is revolving at the rate of 72 revolutions per minute in a uniformly circular path. Find the velocity of the car which is 25 cm away from its centre.

(a) $1\ ms^{-1}$ (b) $1.5\ ms^{-1}$ (c) $2\ ms^{-1}$ (d) $2.5\ ms^{-1}$

**80** A particle is revolving at 1200 rpm in a circle of radius 30 cm. Then, its acceleration is

(a) $1600\ ms^{-2}$ (b) $4740\ ms^{-2}$
(c) $2370\ ms^{-2}$ (d) $5055\ ms^{-2}$

# SPECIAL TYPES QUESTIONS

## I. Assertion and Reason

■ **Direction** (Q. Nos. 81-86) *In the following questions, a statement of Assertion is followed by a corresponding statement of Reason. Of the following statements, choose the correct one.*

(a) Both Assertion and Reason are correct and Reason is the correct explanation of Assertion.
(b) Both Assertion and Reason are correct but Reason is not the correct explanation of Assertion.
(c) Assertion is correct but Reason is incorrect.
(d) Assertion is incorrect but Reason is correct.

**81** **Assertion** Magnitude of resultant of two vectors may be less than the magnitude of either vector.

**Reason** Vector addition is commutative.

**82** **Assertion** Vector addition of two vectors is always greater than their vector subtraction.

**Reason** At $\theta = 90°$, addition and subtraction of vectors are equal.

**83** **Assertion** The maximum height of projectile is always 25% of the maximum range.

**Reason** For maximum range, projectile should be projected at 90°. *AIIMS 2018*

**84** **Assertion** The range of a projectile is maximum at 45°.

**Reason** At $\theta = 45°$, the value of $\sin\theta$ is maximum.

**85** **Assertion** Sum of maximum height for angles $\alpha$ and $90° - \alpha$ is independent of the angle of projection.

**Reason** For angles $\alpha$ and $90° - \alpha$, the horizontal range $R$ is same.

**86** **Assertion** Uniform circular motion is uniformly accelerated motion.

**Reason** Kinematics equations for uniform acceleration motion cannot be applied in the case of uniform circular motion.

## II. Statement Based Questions

**87** I. Magnitude of a vector is always a scalar.
II. Each component of a vector is always a scalar.
III. Vectors obey triangle law of addition.

Which of the following statement(s) is/are correct?

(a) Both I and II (b) Both II and III
(c) Both III and I (d) I, II and III

**88** Unit vector
I. has dimensions and a unit.
II. has unit magnitude.
III. when multiplied by a scalar results in a scalar.

Which of the following statement(s) is/are incorrect?

(a) Both I and II (b) Only II
(c) Both I and III (d) Both II and III

**89** A motorboat is racing towards north at 25 $kmh^{-1}$ and the water current in that region is 10 $kmh^{-1}$ in the direction of 60° east of south. Then, study the statement regarding the resultant velocity of boat.
I. Its magnitude is 22 $kmh^{-1}$.
II. It makes an angle 23.2° from velocity of boat.
III. Its magnitude is 36 $kmh^{-1}$.
IV. It makes an angle 25° from velocity of boat.

Which of the following statement(s) is/are correct?

(a) Only I (b) Both I and II
(c) Only III (d) Only IV

**90** I. The motion of a projectile may be thought of as the result of two separate, simultaneously occurring components of motions.
II. At the maximum height for a projectile, the $x$-component of the velocity is zero.
III. In projectile motion, one component is along a horizontal direction without any acceleration and the other along the vertical direction with constant acceleration due to the force of gravity.

Which of the following statement(s) is/are correct?

(a) Both I and II (b) Both I and III
(c) Only I (d) Only III

**91** I. When an object follows a circular path at a constant speed, the motion of the object is called uniform circular motion.
II. The acceleration of an object in uniform circular motion is always directed away the centre of the circle.

Which of the following statement(s) is/are correct?

(a) Only I (b) Only II
(c) Both I and II (d) Neither I nor II

**92** Which of the following statement(s) is/are correct?
(a) Displacing a vector parallel to itself leaves the vector unchanged.
(b) Three equal vectors cannot add upto zero.
(c) If $|\mathbf{A}+\mathbf{B}| \neq |\mathbf{A}-\mathbf{B}|$, they are perpendicular vectors.
(d) Both (b) and (c)

**93** For two vectors **A** and **B** which lie in a plane. Which of the following statement is correct?

(a) If magnitude of **A** and **B** vector is 3 and 4 and they add upto give vector having magnitude of 5, then they must be perpendicular vector.

(b) If they add up to give more than 5, then they must be inclined at obtuse angle.

(c) If they add upto give less than 5, then they must be inclined at acute angle.

(d) None of the above

**94** Given, $\mathbf{a}+\mathbf{b}+\mathbf{c}+\mathbf{d}=0$, which of the following statements are incorrect?

(a) **a**, **b**, **c** and **d** must each be a null vector.

(b) The magnitude of $(\mathbf{a}+\mathbf{c})$ equals the magnitude of $(\mathbf{b}+\mathbf{d})$.

(c) The magnitude of **a** can never be greater than the sum of the magnitudes of **b**, **c** and **d**.

(d) $\mathbf{b}+\mathbf{c}$ must lie in the plane of **a** and **d**, if **a** and **d** are not collinear and in the line of **a** and **d**, if they are collinear.

**95** A particle moves in a plane such that its coordinate changes with time as $x=at^2+bt$ and $y=ct$, where $a$, $b$ and $c$ are constants.

Then, which of the following statment is correct?

(a) Acceleration of particle is constant.

(b) Velocity of particle is constant.

(c) Acceleration depends only on its $x$-component.

(d) Acceleration depends only on its $y$-component.

**96** Three particles $A$, $B$ and $C$ are projected from the same point with the same initial speeds making angle 30°, 45° and 60°, respectively with the horizontal. Which of the following statement is correct?

(a) $A$, $B$ and $C$ have unequal ranges.

(b) Ranges of $A$ and $C$ are less than that of $B$.

(c) Ranges of $A$ and $C$ are equal and greater than that of $B$.

(d) $A$, $B$ and $C$ have equal ranges.

**97** A biker stands on the edge of a cliff 490 m above the ground and throws a stone horizontally with an initial speed of 15 $ms^{-1}$. Which one of the following statement is correct?

(a) The time taken by the stone to reach the ground is 30 s.

(b) The time taken by the stone to reach the ground is 20 s.

(c) The speed with which it hits the ground is 99 $ms^{-1}$.

(d) None of the above

**98** An insect trapped in a circular groove of radius 12 cm moves along the groove steadily and completes 7 revolutions in 100 s. Which one of the following statement is correct?

(a) The angular speed of insect is 1.68 rad/s.

(b) The linear speed of the motion is 5.3 cm/s.

(c) The magnitude of acceleration is 6.78 $cm/s^2$.

(d) The acceleration vector is a constant vector.

**99** When the projectile is projected obliquely from the ground level with initial velocity $v_0$ and if the air resistance is not neglected, then choose the correct statement.

(a) The projectile traverses a perfect parabolic path.

(b) It will hit the ground with the same speed with which it was projected from it.

(c) Range of the projectile $< \frac{v_0^2}{g}$.

(d) Maximum height of the projectile $> \left(\frac{v_0 \sin\theta_0}{2g}\right)$.

**100** For a particle performing uniform circular motion, choose the incorrect statement(s) from the following.

(a) Magnitude of particle velocity (speed) remains constant.

(b) Particle velocity remains directed parallel to radius vector.

(c) Direction of acceleration keeps changing as particle moves.

(d) None of the above

## III. Matching Type

**101** Match the Column I (example of motion) with Column II (type of motion) and select the correct answer from the codes given below.

| Column I | Column II |
|---|---|
| A. Free fall | 1. One-dimensional motion |
| B. Projectile motion | 2. Two-dimensional motion |
| C. Circular motion | 3. Three-dimensional motion |
| D. Motion along a straight road | |

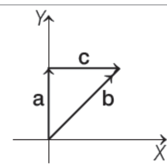
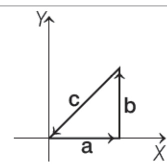
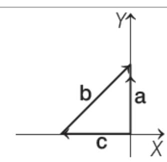
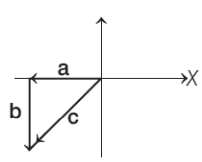
| | A | B | C | D |
|---|---|---|---|---|
| (a) | 2 | 1 | 3 | 1 |
| (b) | 1 | 2 | 3 | 2 |
| (c) | 1 | 2 | 2 | 1 |
| (d) | 1 | 3 | 2 | 1 |

**102** Match the Column I (magnitude of vectors **A** and **B**) with Column II (relationship between **A** and **B**) and select the correct answer from the codes given below.

| | Column I | Column II |
|---|---|---|
| A. | $\xrightarrow{\lvert\mathbf{A}\rvert = l}$ $\xrightarrow{\lvert\mathbf{B}\rvert = 2l}$ | 1. $\mathbf{A}=-\mathbf{B}$ |
| B. | $\xrightarrow{\lvert\mathbf{A}\rvert = l}$ $\xleftarrow{\lvert\mathbf{B}\rvert = l}$ | 2. $\mathbf{A}=\mathbf{B}$ |
| C. | $\xrightarrow{\lvert\mathbf{A}\rvert = 2l}$ $\xleftarrow{\mathbf{B} = l}$ | 3. $2\mathbf{A}=\mathbf{B}$ |
| D. | $\xrightarrow{\lvert\mathbf{A}\rvert = l}$ $\xrightarrow{\lvert\mathbf{B}\rvert = l}$ | 4. $\mathbf{A}=-2\mathbf{B}$ |

| | A | B | C | D |
|---|---|---|---|---|
| (a) | 2 | 4 | 1 | 2 |
| (b) | 3 | 1 | 4 | 2 |
| (c) | 3 | 1 | 2 | 4 |
| (d) | 2 | 3 | 4 | 1 |

**103** Match the Column I (the relations between vectors **a**, **b** and **c**) with Column II (the orientations of **a**, **b** and **c** in the *xy*- plane) and select the correct answer from the codes given below.

| | Column I | | Column II |
|---|---|---|---|
| A. | **a** + **b** = **c** | 1. |   |
| B. | **a** − **c** = **b** | 2. |   |
| C. | **b** − **a** = **c** | 3. |   |
| D. | **a** + **b** + **c** = 0 | 4. |   |

| | A | B | C | D | | A | B | C | D |
|---|---|---|---|---|---|---|---|---|---|
| (a) | 3 | 2 | 1 | 4 | (b) | 4 | 3 | 1 | 2 |
| (c) | 1 | 4 | 3 | 2 | (d) | 3 | 2 | 1 | 4 |

**104** A vector is given by $\mathbf{A} = 4\hat{\mathbf{i}} + 3\hat{\mathbf{j}} + 5\hat{\mathbf{k}}$ (where $\alpha$, $\beta$ and $\gamma$ are the angles made by **A** with coordinate axes, then match the Column I with Column II (respective values) and select the correct answer from the codes given below.

| Column I | Column II |
|---|---|
| A. $\alpha$ | 1. $\cos^{-1}(1/\sqrt{2})$ |
| B. $\beta$ | 2. $\cos^{-1}(4/5\sqrt{2})$ |
| C. $\gamma$ | 3. $\cos^{-1}(3/5\sqrt{2})$ |

| | A | B | C | | A | B | C |
|---|---|---|---|---|---|---|---|
| (a) | 1 | 2 | 3 | (b) | 2 | 3 | 1 |
| (c) | 3 | 2 | 1 | (d) | 1 | 1 | 2 |

**105** Match the Column I (respective change in values of $u_x$, horizontal component of initial velocity of projectile and $u_y$ vertical component of initial velocity) with Column II (respective change in horizontal range $R$ and maximum height $H$) and select the correct answer from the codes given below.

| | Column I | | Column II |
|---|---|---|---|
| A. | Only $u_x$ is doubled | 1. | $R$ will be two times |
| B. | $u_x$ is doubled, $u_y$ is halved | 2. | $H$ will be one-fourth |
| C. | $u_x$ is halved, $u_y$ is doubled | 3. | $H$ will be four times |
| | | 4. | $R$ will be halved |

| | A | B | C | | A | B | C |
|---|---|---|---|---|---|---|---|
| (a) | 2 | 4 | 1 | (b) | 1 | 2 | 3 |
| (c) | 3 | 1 | 2 | (d) | 2 | 4 | 1 |

# NCERT & NCERT Exemplar

## MULTIPLE CHOICE QUESTIONS

### NCERT

**106** Which of the following algebraic operations is not meaningful?
(a) Multiplying any vector by any scalar
(b) Multiplying any two scalars
(c) Adding any two vectors
(d) None of the above

**107** The ceiling of a long hall is 25 m high. What is the maximum horizontal distance that a ball thrown with a speed of 40 $ms^{-1}$ can go without hitting the ceiling of the hall?
(a) 150.5 m (b) 250.5 m
(c) 130.2 m (d) 100.5 m

**108** A cricketer can throw a ball to a maximum horizontal distance of 100 m. How much high above the ground can the cricketer throw the same ball?
(a) 100 m (b) 50 m (c) 25 m (d) 200 m

**109** An aircraft executes a horizontal loop of radius 1 km with a speed of 900 $kmh^{-1}$. What is the ratio of its centripetal acceleration with the acceleration due to gravity?
(a) 6.38 (b) 3.19 (c) 12.76 (d) 5.38

**110** In a harbour, wind is blowing at the speed of 72 $kmh^{-1}$ and the flag on the mast of a boat anchored in the harbour flutters along north-east direction. If the boat starts moving at the speed of 51 $kmh^{-1}$ of the north, what is the direction of the flag on the mast of the boat?
(a) East (b) North (c) South (d) West

**111** A bullet fired at an angle of 30° with the horizontal hits the ground 3 km away. By adjusting its angle of projection, it can (assume, the muzzle speed to be fixed and neglect air resistance.)
(a) hit a target at 5 km
(b) hit a target at 6 km
(c) cannot hit a target at 5 km
(d) None of the above

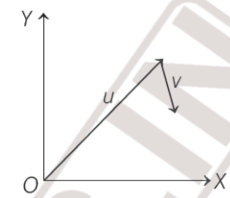
**112** An aircraft is flying at a height of 3400 m above the ground. If the angle subtended at a ground observation point by the aircraft positions 10 s apart is 30°, then what is the speed of the aircraft?
(a) 140 m/s (b) 196.30 m/s
(c) 9.8 m/s (d) 120 m/s

**113** A stone tied to the end of a string 80 cm long is whirled in a horizontal circle with a constant speed. If the stone makes 14 revolutions in 25 s, what is the magnitude of its centripetal acceleration?
(a) 8.2 m/s$^2$ (b) 2 m/s$^2$
(c) 8.8 m/s$^2$ (d) 9.91 m/s$^2$

## NCERT Exemplar

**114** Which one of the following statement is correct?
(a) A scalar quantity is the one that is conserved in a process.
(b) A scalar quantity is the one that can never take negative values.
(c) A scalar quantity is the one that does not vary from one point to another in space.
(d) A scalar quantity has the same value for observers with different orientation of the axes.

**115** Figure shows the orientation of two vectors **u** and **v** in the $xy$-plane. If $\mathbf{u} = a\hat{\mathbf{i}} + b\hat{\mathbf{j}}$ and $\mathbf{v} = p\hat{\mathbf{i}} + q\hat{\mathbf{j}}$, then which of the following option is correct?




(a) $a$ and $p$ are positive, while $b$ and $q$ are negative
(b) $a$, $p$ and $b$ are positive, while $q$ is negative
(c) $a$, $q$ and $b$ are positive, while $p$ is negative
(d) $a$, $b$, $p$ and $q$ are all positive

**116** The component of a vector **r** along $X$-axis will have maximum value, if
(a) **r** is along positive $Y$-axis
(b) **r** is along positive $X$-axis
(c) **r** makes an angle of 45° with the $X$-axis
(d) **r** is along negative $Y$-axis

**117** The horizontal range of a projectile fired at an angle of 15° is 50 m. If it is fired with the same speed at an angle of 45°, its range will be
(a) 60 m (b) 71 m
(c) 100 m (d) 141 m

**118** In a two-dimensional motion, instantaneous speed $v_0$ is a positive constant, then which of the following are necessarily correct?
(a) The average velocity is not zero at any time
(b) Average acceleration must always vanish
(c) Displacements in equal time intervals are equal
(d) Equal path lengths are traversed in equal intervals

# Answers

### > *Mastering NCERT with MCQs*

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 1 (c) | 2 (b) | 3 (b) | 4 (c) | 5 (a) | 6 (a) | 7 (d) | 8 (d) | 9 (b) | 10 (b) |
| 11 (c) | 12 (d) | 13 (c) | 14 (a) | 15 (b) | 16 (c) | 17 (c) | 18 (b) | 19 (c) | 20 (c) |
| 21 (b) | 22 (a) | 23 (a) | 24 (a) | 25 (c) | 26 (c) | 27 (c) | 28 (a) | 29 (d) | 30 (b) |
| 31 (c) | 32 (d) | 33 (c) | 34 (a) | 35 (a) | 36 (a) | 37 (b) | 38 (c) | 39 (b) | 40 (b) |
| 41 (b) | 42 (b) | 43 (a) | 44 (c) | 45 (b) | 46 (d) | 47 (d) | 48 (a) | 49 (c) | 50 (c) |
| 51 (b) | 52 (a) | 53 (b) | 54 (c) | 55 (b) | 56 (b) | 57 (d) | 58 (b) | 59 (c) | 60 (c) |
| 61 (a) | 62 (a) | 63 (b) | 64 (c) | 65 (c) | 66 (b) | 67 (b) | 68 (a) | 69 (c) | 70 (b) |
| 71 (d) | 72 (a) | 73 (c) | 74 (b) | 75 (c) | 76 (a) | 77 (c) | 78 (a) | 79 (c) | 80 (b) |

### > *Special Types Questions*

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 81 (b) | 82 (d) | 83 (c) | 84 (c) | 85 (b) | 86 (d) | 87 (c) | 88 (c) | 89 (b) | 90 (b) |
| 91 (a) | 92 (a) | 93 (a) | 94 (a) | 95 (a) | 96 (b) | 97 (c) | 98 (b) | 99 (c) | 100 (b) |
| 101 (c) | 102 (b) | 103 (b) | 104 (b) | 105 (b) | | | | | |

### > *NCERT & NCERT Exemplar MCQs*

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 106 (c) | 107 (a) | 108 (b) | 109 (a) | 110 (a) | 111 (c) | 112 (b) | 113 (d) | 114 (d) | 115 (b) |
| 116 (b) | 117 (c) | 118 (d) | | | | | | | |

# Hints & Explanations

**1** (*c*) Temperature is not a vector quantity because it has magnitude only.
However, force, acceleration and velocity have both a magnitude and a direction. So, these are vectors in nature.

**2** (*b*) In order to describe two-dimensional or three-dimensional motions, we use vectors.
However, direction of the motion of an object along a straight line is shown by positive and negative signs.

**3** (*b*) The perimeter of the rectangle would be the sum of the lengths of the four sides, i.e. 1.0 m + 0.5 m + 1.0 m + 0.5 m = 3.0 m.
Since, length of each side is a scalar, thus the perimeter is also a scalar.

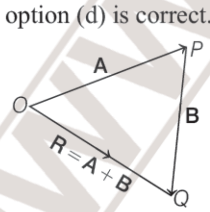
**4** (*c*) Two vectors are said to be equal, if and only if they have the same magnitude and direction.
Among the given vectors **A** and **B** are equal vectors as they have same magnitude (length) and direction.
However, **P** and **Q** are not equal even though they are of same magnitude because their directions are different.

**5** (*a*) $|\lambda\,\mathbf{A}| = \lambda\,|\mathbf{A}|$, if $\lambda > 0$, as multiplication of vector **A** with a positive number $\lambda$ gives a vector whose magnitude is changed by the factor $\lambda$ but the direction is same as that of **A**.

**6** (*a*) Multiplying a vector **A** with a negative number $\lambda$ gives a vector whose magnitude is changed by the factor $\lambda$ but direction is reversed.

**7** (*d*) $|\mathbf{B}| = -1.5\,|\mathbf{A}|$. So when **A** is multiplied by $-1.5$, then its direction gets reversed and magnitude would be 1.5 times $|\mathbf{A}|$.
Thus, $|\mathbf{B}| \neq |\mathbf{A}|$.

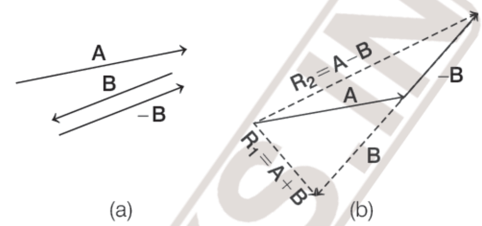
**8** (*d*) Vectors by definition obey the triangle law of addition. According to which, if vector **B** is placed with its tail at the head of vector **A**. Then, when we join the tail of **A** to the head of **B**. The line *OQ* represents a vector **R**, i.e. the sum of the vectors **A** and **B**. Thus, figure given in option (d) is correct.




**9** (*b*) Null vector **0** is a vector whose magnitude is zero and its direction cannot be specified. So, it means, $|\mathbf{0}| = 0$
Thus, $\lambda\,\mathbf{0} = \mathbf{0}$.
Hence, property given in option (b) is incorrect.

**10** (*b*) Since in the given case, the initial and final positions coincides, so the displacement will be zero. Thus, it is a null vector.

**11** (*c*) To subtract **B** from **A**, we can add – **B** and **A**.
So, $\mathbf{A} + (-\mathbf{B}) = \mathbf{A} - \mathbf{B} = \mathbf{R}_2$. This is as shown below




Hence, option (c) is the correct about vector subtraction.

**12** (*d*) In general, a vector **A** can be written as

$$\mathbf{A} = |\mathbf{A}|\,\hat{\mathbf{n}} \qquad \ldots \text{(i)}$$

where, $\hat{\mathbf{n}}$ is a unit vector along **A**.
If $\hat{\mathbf{a}}$ is a unit vector along **A**, then from Eq. (i) we can write, $\hat{\mathbf{a}} = \dfrac{\mathbf{A}}{|\mathbf{A}|}$

**13** (*c*) Given, $\mathbf{A} = 4\hat{\mathbf{i}} - 3\hat{\mathbf{j}} + \hat{\mathbf{k}}$

$$|\mathbf{A}| = \sqrt{A_x^2 + A_y^2 + A_z^2}$$
$$= \sqrt{(4)^2 + (-3)^2 + (1)^2} = \sqrt{26}$$

$\therefore$ Unit vector, $\hat{\mathbf{A}} = \dfrac{\mathbf{A}}{|\mathbf{A}|} = \dfrac{4\hat{\mathbf{i}} - 3\hat{\mathbf{j}} + \hat{\mathbf{k}}}{\sqrt{26}}$

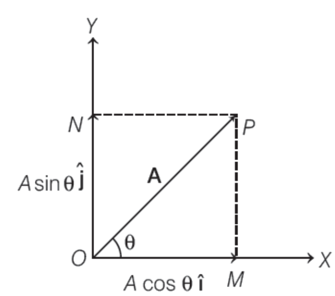
**14** (*a*) Vector along *X*-axis (*x*-component)

$$= A_x\hat{\mathbf{i}} = |\mathbf{A}|\cos\theta\,\hat{\mathbf{i}} = A\cos\theta\,\hat{\mathbf{i}}$$

Vector along *Y*- axis (*y*-component)

$$= A_y\,\hat{\mathbf{j}} = |\mathbf{A}|\sin\theta\,\hat{\mathbf{j}} = A\sin\theta\,\hat{\mathbf{j}}$$

This can be shown as




**15** (*b*) Given, $|\mathbf{Q}| = 5$

$$Q_y = 4$$
$$Q_x = ?$$

As, $\quad |\mathbf{Q}| = \sqrt{Q_x^2 + Q_y^2}$

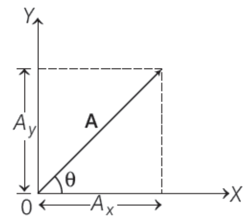
$\Rightarrow \quad |\mathbf{Q}|^2 = Q_x^2 + Q_y^2$

Substituting the given values, we get

$$(5)^2 = Q_x^2 + 4^2$$

$\Rightarrow \quad Q_x = \sqrt{9} = 3$

**16** (*c*) Given, vector can be shown below as




where, $\theta = 60^\circ$

Then, $\tan\theta = \dfrac{A_y}{A_x}$ or $A_y = A_x \tan\theta$

$\Rightarrow \quad A_y = 50\tan 60^\circ = 50\times\sqrt{3} \qquad (\because \sqrt{3} = 1.732)$

$= 86.6 \simeq 87$ N

**17** (*c*) Given, $\mathbf{P} = 3\hat{\mathbf{i}} - 4\hat{\mathbf{j}}$

and $\quad \mathbf{Q} = 6\hat{\mathbf{i}} - 8\hat{\mathbf{j}} = 2(3\hat{\mathbf{i}} - 4\hat{\mathbf{j}}) = 2\mathbf{P}$

Also, $\quad \mathbf{R} = \dfrac{3}{4}\hat{\mathbf{i}} - \hat{\mathbf{j}} = \dfrac{1}{4}(3\hat{\mathbf{i}} - 4\hat{\mathbf{j}}) = \dfrac{\mathbf{P}}{4}$

So, **P**, **Q** and **R** are parallel with unequal magnitude. Thus, they are not equals vectors.

**18** (*b*) Since, the magnitude and angle between the vectors is unchanged, so the magnitude of the resultant **R** will be same. However, the direction of **R** will get changed.

**19** (*c*) $\mathbf{T} = \mathbf{a} + \mathbf{b} - \mathbf{c}$

$= (a_x\hat{\mathbf{i}} + a_y\hat{\mathbf{j}} + a_z\hat{\mathbf{k}}) + (b_x\hat{\mathbf{i}} + b_y\hat{\mathbf{j}} + b_z\hat{\mathbf{k}}) - (c_x\hat{\mathbf{i}} + c_y\hat{\mathbf{j}} + c_z\hat{\mathbf{k}})$

$= (a_x + b_x - c_x)\hat{\mathbf{i}} + (a_y + b_y - c_y)\hat{\mathbf{j}} + (a_z + b_z - c_z)\hat{\mathbf{k}} \quad \ldots(\text{i})$

As, $\mathbf{T} = T_x\hat{\mathbf{i}} + T_y\hat{\mathbf{i}} + T_z\hat{\mathbf{k}} \quad \ldots(\text{ii})$

On comparing Eqs. (i) and (ii), we get

*y*-component of **T**, $T_y = a_y + b_y - c_y$

**20** (*c*) Resultant vector of **A** and **B** is

$$\mathbf{R} = \mathbf{A} + \mathbf{B} = (-3\hat{\mathbf{i}} - 2\hat{\mathbf{j}} - 3\hat{\mathbf{k}}) + (2\hat{\mathbf{i}} + 4\hat{\mathbf{j}} + 6\hat{\mathbf{k}})$$

$$= -\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + 3\hat{\mathbf{k}}$$

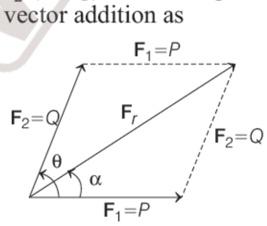
$$|\mathbf{R}| = \sqrt{(-1)^2 + (2)^2 + (3)^2}$$

$$= \sqrt{1+4+9} = \sqrt{14}$$

Unit vector in the direction of **R** is

$$\hat{\mathbf{R}} = \frac{\mathbf{R}}{|\mathbf{R}|} = \frac{-\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + 3\hat{\mathbf{k}}}{\sqrt{14}}$$

**21** (*b*) Resultant force $\mathbf{F}_r$ of any two forces $\mathbf{F}_1$ (i.e. *P*) and $\mathbf{F}_2$ (i.e. *Q*) with an angle θ between them can be given by vector addition as




$$F_r^2 = F_1^2 + F_2^2 + 2F_1F_2\cos\theta \qquad \ldots(\text{i})$$

**In first case** $F_1 = 2F$ and $F_2 = 3F$

$\Rightarrow \quad F_r^2 = 4F^2 + 9F^2 + 2\times 2\times 3F^2\cos\theta$

$\Rightarrow \quad F_r^2 = 13F^2 + 12F^2\cos\theta \qquad \ldots(\text{ii})$

**In second case** $F_1 = 2F$ and $F_2 = 6F$

($\because$ Force *Q* gets doubled)

and $\quad F_r' = 2F_r \qquad$ (Given)

By putting these values in Eq. (i), we get

$$(2F_r)^2 = (2F)^2 + (6F)^2 + 2\times 2\times 6F^2\cos\theta$$

$\Rightarrow \quad 4F_r^2 = 40F^2 + 24F^2\cos\theta \qquad \ldots(\text{iii})$

From Eq. (ii) and Eq. (iii), we get;

$$52F^2 + 48F^2\cos\theta = 40F^2 + 24F^2\cos\theta$$

$\Rightarrow \quad 12 + 24\cos\theta = 0$ or $\cos\theta = -1/2$

or $\quad \theta = 120^\circ \qquad (\because \cos 120^\circ = -1/2)$

**22** (*a*) Given that $|\mathbf{A} + \mathbf{B}| = |\mathbf{A}|$ or $|\mathbf{A} + \mathbf{B}|^2 = |\mathbf{A}|^2$

$\Rightarrow \quad |\mathbf{A}|^2 + |\mathbf{B}|^2 + 2|\mathbf{A}||\mathbf{B}|\cos\theta = |\mathbf{A}|^2$

where, θ is angle between **A** and **B**.

$\Rightarrow \quad |\mathbf{B}|(|\mathbf{B}| + 2|\mathbf{A}|\cos\theta) = 0$

$\Rightarrow \quad |\mathbf{B}| = 0$ or $|\mathbf{B}| + 2|\mathbf{A}|\cos\theta = 0$

$\Rightarrow \quad \cos\theta = -\dfrac{|\mathbf{B}|}{2|\mathbf{A}|} \qquad \ldots(\text{i})$

If **A** and **B** are anti-parallel, then $\theta = 180^\circ$

Hence, from Eq. (i),

$$\cos 180^\circ = -1 = -\frac{|\mathbf{B}|}{2|\mathbf{A}|} \Rightarrow |\mathbf{B}| = 2|\mathbf{A}|$$

Hence, the given condition can only be implied of either $|\mathbf{B}| = 0$ or **A** and **B** are anti-parallel provided $|\mathbf{B}| = 2|\mathbf{A}|$.

**23** (*a*) Difference of vectors **A** and **B** can be given as

$$|(\mathbf{A} - \mathbf{B})|^2 = A^2 + B^2 - 2AB\cos\theta$$

$$= 1 + 1 - 2\cos\theta$$

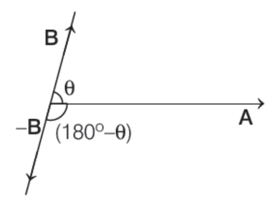
[$\because$ **A** and **B** are unit vectors]

$$= 2 - 2\cos\theta = 2(1 - \cos\theta)$$

$$= 2\times 2\sin^2(\theta/2) = 4\sin^2\left(\frac{\theta}{2}\right)$$

$$|\mathbf{A} - \mathbf{B}| = 2\sin(\theta/2)$$

**24** (*a*) According to the question, representation of vectors **A** and **B** can be shown as follows




Given, $\quad |\mathbf{A} + \mathbf{B}| = P$

$\Rightarrow \quad |\mathbf{A} + \mathbf{B}|^2 = P^2$

$$P^2 = A^2 + B^2 + 2AB\cos\theta \qquad \ldots(\text{i})$$

Also, $|\mathbf{A}-\mathbf{B}| = Q \Rightarrow |\mathbf{A}-\mathbf{B}|^2 = Q^2$

$\Rightarrow \quad A^2 + B^2 + 2AB\cos(180°-\theta) = Q^2$

$\Rightarrow \quad A^2 + B^2 - 2AB\cos\theta = Q^2 \quad \text{...(ii)}$

Adding Eqs. (i) and (ii), we get

$$P^2 + Q^2 = 2(A^2 + B^2)$$

**25** (*c*) Given, $|\mathbf{A}+\mathbf{B}| = |\mathbf{A}-\mathbf{B}|$

$\Rightarrow \quad \sqrt{|\mathbf{A}|^2 + |\mathbf{B}|^2 + 2|\mathbf{A}||\mathbf{B}|\cos\theta}$

$= \sqrt{|\mathbf{A}|^2 + |\mathbf{B}|^2 - 2|\mathbf{A}||\mathbf{B}|\cos\theta}$

$\Rightarrow \quad |\mathbf{A}|^2 + |\mathbf{B}|^2 + 2|\mathbf{A}||\mathbf{B}|\cos\theta$

$= |\mathbf{A}|^2 + |\mathbf{B}|^2 - 2|\mathbf{A}||\mathbf{B}|\cos\theta$

$\Rightarrow 4|\mathbf{A}||\mathbf{B}|\cos\theta = 0 \quad \Rightarrow \quad |\mathbf{A}||\mathbf{B}|\cos\theta = 0$

$\Rightarrow \quad |\mathbf{A}| = 0$ or $|\mathbf{B}| = 0$ or $\cos\theta = 0 \quad \Rightarrow \quad \theta = 90°$

Thus, $|\mathbf{A}+\mathbf{B}| = |\mathbf{A}-\mathbf{B}|$ is always true, when either $|\mathbf{A}|$ or $|\mathbf{B}|$ is zero or $\mathbf{A}$ and $\mathbf{B}$ are perpendicular to each other.

**26** (*c*) Given, $|\mathbf{A}| = |\mathbf{B}|$

or $\quad A = B \quad \text{...(i)}$

Let magnitude of $(\mathbf{A}+\mathbf{B})$ is $R$ and for $(\mathbf{A}-\mathbf{B})$ is $R'$.

Now, $\quad \mathbf{R} = \mathbf{A}+\mathbf{B}$

and $\quad R^2 = A^2 + B^2 + 2AB\cos\theta$

$R^2 = 2A^2 + 2A^2\cos\theta \quad \text{...(ii)}$

[$\because$ using Eq. (i)]

Again, $\quad \mathbf{R}' = \mathbf{A}-\mathbf{B}$

$\Rightarrow \quad R'^2 = A^2 + B^2 - 2AB\cos\theta$

$R'^2 = 2A^2 - 2A^2\cos\theta \quad \text{...(iii)}$

[$\because$ using Eq. (i)]

Given, $R = nR'$ or $\left(\dfrac{R}{R'}\right)^2 = n^2$

Dividing Eq. (ii) with Eq. (iii), we get

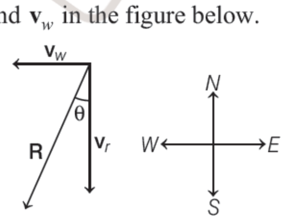
$$\frac{n^2}{1} = \frac{1+\cos\theta}{1-\cos\theta}$$

or $$\frac{n^2-1}{n^2+1} = \frac{(1+\cos\theta)-(1-\cos\theta)}{(1+\cos\theta)+(1-\cos\theta)}$$

$\Rightarrow$ $$\frac{n^2-1}{n^2+1} = \frac{2\cos\theta}{2} = \cos\theta$$

or $$\theta = \cos^{-1}\left(\frac{n^2-1}{n^2+1}\right)$$

**27** (*c*) The velocity of the rain and wind are represented by vectors $\mathbf{v}_r$ and $\mathbf{v}_w$ in the figure below.




Using the rule of vector addition, we see that the resultant of $\mathbf{v}_r$ and $\mathbf{v}_w$ is $\mathbf{R}$ as shown in the figure.

The magnitude of $\mathbf{R}$ is

$$|\mathbf{R}| = \sqrt{\mathbf{v}_r^2 + \mathbf{v}_w^2}$$

$$= \sqrt{35^2 + 12^2} = 37 \text{ ms}^{-1}$$

The direction $\theta$ that $\mathbf{R}$ makes with the vertical is given by

$$\tan\theta = \frac{\mathbf{v}_w}{\mathbf{v}_r} = \frac{12}{35} = 0.343 \quad \text{or} \quad \theta = \tan^{-1}(0.343)$$

Therefore, the boy should hold his umbrella in the vertical plane at an angle of about $\tan^{-1}(0.343)$ with the vertical towards the east.

**28** (*a*) Position vector $\mathbf{r}$ of an object in $xy$-plane at point $P$ with its components along $X$ and $Y$-axes as $x$ and $y$, respectively is given as $\mathbf{r} = x\hat{\mathbf{i}} + y\hat{\mathbf{j}}$.

Given, $x = 2$ units and $y = 4$ units

So, position vector at $P$ will be given as $\mathbf{r} = 2\hat{\mathbf{i}} + 4\hat{\mathbf{j}}$.

**29** (*d*) Position vector of the particle at $P$, $\mathbf{r} = 2\hat{\mathbf{i}} + 3\hat{\mathbf{j}}$

Position vector of the particle at $P'$, $\mathbf{r}' = 5\hat{\mathbf{i}} + 6\hat{\mathbf{j}}$

$\therefore$ Displacement of the particle is $\Delta\mathbf{r} = \mathbf{r}' - \mathbf{r}$

$\Rightarrow \quad \Delta\mathbf{r} = (5\hat{\mathbf{i}} + 6\hat{\mathbf{j}}) - (2\hat{\mathbf{i}} + 3\hat{\mathbf{j}})$

$= (5-2)\hat{\mathbf{i}} + (6-3)\hat{\mathbf{j}} = 3\hat{\mathbf{i}} + 3\hat{\mathbf{j}}$

**30** (*b*) Position vector of the particle at

$t = 0\text{ s}, \ \mathbf{r}_{0s} = 2\hat{\mathbf{i}} + 3\hat{\mathbf{j}}$

$t = 2\text{ s}, \ \mathbf{r}_{2s} = 6\hat{\mathbf{i}} + 7\hat{\mathbf{j}}$

and $t = 5\text{ s}, \quad \mathbf{r}_{5s} = 13\hat{\mathbf{i}} + 14\hat{\mathbf{j}}$

Displacement in $t = 0$ s to $t = 5$ s,

$\Delta\mathbf{r} = \mathbf{r}_{5s} - \mathbf{r}_{0s}$

$= (13-2)\hat{\mathbf{i}} + (14-3)\hat{\mathbf{j}} = 11\hat{\mathbf{i}} + 11\hat{\mathbf{j}}$

Average velocity, $\overline{\mathbf{v}} = \dfrac{\Delta\mathbf{r}}{\Delta t} = \dfrac{11\hat{\mathbf{i}} + 11\hat{\mathbf{j}}}{5} = \dfrac{11}{5}(\hat{\mathbf{i}} + \hat{\mathbf{j}})$

**31** (*c*) The direction of instantaneous velocity at any point on the path of an object is tangential to the path at that point and is in the direction of motion. Also, direction of average velocity is same as that of $\Delta\mathbf{r}$.

So, amongst the given figures we can say that, options (a) and (b) are depicting the direction of averge velocity but option (c) is correctly depicting the direction of instantaneous velocity.

**32** (*d*) Given, $\mathbf{r} = 3t\hat{\mathbf{i}} + 2t^2\hat{\mathbf{j}} + 5\hat{\mathbf{k}}$

$\therefore \quad \mathbf{v}(t) = \dfrac{d\mathbf{r}}{dt} = \dfrac{d}{dt}(3t\hat{\mathbf{i}} + 2t^2\hat{\mathbf{j}} + 5\hat{\mathbf{k}}) = 3\hat{\mathbf{i}} + 4t\hat{\mathbf{j}}$

At $t = 1$ s, $\quad \mathbf{v} = 3\hat{\mathbf{i}} + 4\hat{\mathbf{j}}$

Thus, its direction is $\theta = \tan^{-1}\left(\dfrac{v_y}{v_x}\right) = \tan^{-1}\left(\dfrac{4}{3}\right)$

$\cong 53°$ with $X$-axis

**33** (*c*) Given, $x = 5t - 2t^2$

Velocity of the particle,

$$v_x = \frac{dx}{dt} = \frac{d}{dt}(5t - 2t^2) = 5 - 4t$$

Acceleration, $a_x = \frac{dv_x}{dt} = -4 \text{ ms}^{-2}$

Also, $y = 10t$,

Velocity, $v_y = \frac{dy}{dt} = \frac{d}{dt}(10t) = 10$

$\therefore$ Acceleration, $a_y = \frac{dv_y}{dt} = 0$

$\therefore$ Net acceleration of the particle,

$$\mathbf{a}_{\text{net}} = a_x\hat{\mathbf{i}} + a_y\hat{\mathbf{j}} = (-4 \text{ ms}^{-2})\,\hat{\mathbf{i}} + 0$$

or $\quad \mathbf{a}_{\text{net}} = -4\,\hat{\mathbf{i}} \text{ ms}^{-2}$ or $a_{\text{net}} = -4 \text{ ms}^{-2}$

**34** (*a*) Position vector of particle is given as

$$\mathbf{r} = 15t^2\hat{\mathbf{i}} + (4 - 20t^2)\hat{\mathbf{j}}$$

Velocity of particle is

$$\mathbf{v} = \frac{d\mathbf{r}}{dt} = \frac{d}{dt}\,[15t^2\hat{\mathbf{i}} + (4 - 20t^2)\hat{\mathbf{j}}]$$

$$= 30t\,\hat{\mathbf{i}} - 40\,t\,\hat{\mathbf{j}}$$

Acceleration of particle is

$$\mathbf{a} = \frac{d}{dt}(\mathbf{v}) = \frac{d}{dt}(30t\hat{\mathbf{i}} - 40t\hat{\mathbf{j}})$$

$$= 30\hat{\mathbf{i}} - 40\hat{\mathbf{j}}$$

So, magnitude of acceleration at $t = 1$s is

$$|\mathbf{a}|_{t=1\text{s}} = \sqrt{a_x^2 + a_y^2} = \sqrt{30^2 + 40^2}$$

$$= 50 \text{ ms}^{-2}$$

**35** (*a*) Given that the position coordinates of a particle

$$\left.\begin{aligned} x &= a\cos\omega t \\ y &= a\sin\omega t \\ z &= a\omega t \end{aligned}\right\} \quad \ldots\text{(i)}$$

So, the position vector of the particle is

$$\hat{\mathbf{r}} = x\hat{\mathbf{i}} + y\hat{\mathbf{j}} + z\hat{\mathbf{k}}$$

$\Rightarrow \quad \hat{\mathbf{r}} = a\cos\omega t\,\hat{\mathbf{i}} + a\sin\omega t\,\hat{\mathbf{j}} + a\,\omega t\,\hat{\mathbf{k}}$

$$\hat{\mathbf{r}} = a[\cos\omega t\,\hat{\mathbf{i}} + \sin\omega t\,\hat{\mathbf{j}} + \omega t\,\hat{\mathbf{k}}]$$

therefore, the velocity of the particle is

$\because \quad \hat{\mathbf{v}} = \frac{d\mathbf{r}}{dt} = \frac{d[a][\cos\omega t\;\hat{\mathbf{i}} + \sin\omega t\hat{\mathbf{j}} + \omega t\;\hat{\mathbf{k}}]}{dt}$

$\Rightarrow \quad \hat{\mathbf{v}} = -a\omega\sin\omega t\;\hat{\mathbf{i}} + a\omega\cos\omega t\;\hat{\mathbf{j}} + a\omega\hat{\mathbf{k}})$

The magnitude of velocity is

$$|\mathbf{v}| = \sqrt{v_x^2 + v_y^2 + v_z^2}$$

or $\quad |\mathbf{v}| = \sqrt{(-a\omega\sin\omega t)^2 + (a\omega\cos\omega t)^2 + (a\omega)^2}$

$$= \omega a\sqrt{(-\sin\omega t)^2 + (\cos\omega t)^2 + (1)^2}$$

$$= \sqrt{2}\omega a$$

**36** (*a*) Given, initial velocity of the particle at $t = 0$ s, $\mathbf{v}_0 = 5.0\,\hat{\mathbf{i}} \text{ ms}^{-1}$, acceleration, $\mathbf{a} = (3.0\hat{\mathbf{i}} + 2.0\hat{\mathbf{j}}) \text{ ms}^{-2}$

The position of the particle is given by

$$\mathbf{r}(t) = \mathbf{v}_0 t + \frac{1}{2}\mathbf{a}t^2 = 5.0\hat{\mathbf{i}}\,t + (1/2)(3.0\hat{\mathbf{i}} + 2.0\hat{\mathbf{j}})t^2$$

$$= (5.0t + 1.5t^2)\hat{\mathbf{i}} + 1.0t^2\hat{\mathbf{j}} \quad \ldots\text{(i)}$$

As, $\mathbf{r}(t) = x(t)\hat{\mathbf{i}} + y(t)\hat{\mathbf{j}} \quad \ldots$(ii)

Comparing Eqs. (i) and (ii), we get

$$x(t) = 5.0t + 1.5t^2 \text{ and } y(t) = +1.0t^2$$

Given, $\quad x(t) = 84$m

$\Rightarrow \quad 5.0t + 1.50t^2 = 84$

or $\quad 1.50\,t^2 + 5.0t - 84 = 0$

Solving the above quadratic equation, the value of $t$ is given as,

$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-5 \pm \sqrt{5^2 - 4(1.50)(-84)}}{2(1.50)}$$

$$= \frac{-5 \pm \sqrt{25 + 504}}{3} = \frac{-5 \pm \sqrt{529}}{3} = \frac{-5 \pm 23}{3}$$

$= 6$ or $- 9.33$

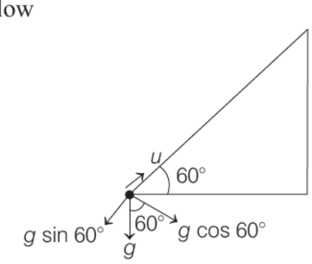
(Neglecting the negative values as $t$ can never be negative)

$\Rightarrow \quad t = 6$ s

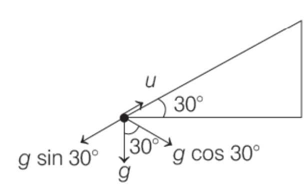
At $t = 6$s, $\quad y = 1.0(6)^2 = 36$ m

**37** (*b*) The motion of object shot in two cases can be depicted as below

**Case I**




**Case II**




Using third equation of motion, $v^2 = u^2 - 2gh \quad \ldots$ (i)

As the object stops finally, so

$$v = 0$$

For inclined motion, $\quad g = g\sin\theta$ and $h = x$

Substituting these values in Eq. (i), we get

$$\Rightarrow u^2 = 2g\sin\theta x \Rightarrow x = \frac{u^2}{2g\sin\theta}$$

**For Case I**, $x_1 = \dfrac{u^2}{2g\sin 60°}$

**For Case II**, $x_2 = \dfrac{u^2}{2g\sin 30°}$

$$\Rightarrow \quad \frac{x_1}{x_2} = \frac{u^2}{2g\sin 60°} \times \frac{2g\sin 30°}{u^2}$$

$$= \frac{1}{2} \times \frac{2}{\sqrt{3}} = \frac{1}{\sqrt{3}} \text{ or } 1 : \sqrt{3}$$

**38** (*c*) Relative velocity of $P$ w.r.t. $Q$ is given by

$$\mathbf{v}_{PQ} = \mathbf{v}_P - (-\mathbf{v}_Q) = \mathbf{v}_P + \mathbf{v}_Q$$

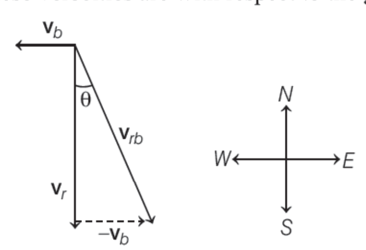
**39** (*b*) Given, $\mathbf{v}_A = 20\hat{\mathbf{i}}\ \text{ms}^{-1}$

$$\mathbf{v}_B = 15\hat{\mathbf{i}}\ \text{ms}^{-1}$$

Relative velocity of $A$ w.r.t. $B$,

$$\mathbf{v}_{AB} = \mathbf{v}_A - \mathbf{v}_B$$
$$= 20\hat{\mathbf{i}} - 15\hat{\mathbf{i}} = 5\hat{\mathbf{i}}\ \text{ms}^{-1}$$

**40** (*b*) In figure below $\mathbf{v}_r$ represents the velocity of rain and $\mathbf{v}_b$ is the velocity of the bicycle, the woman is riding. Both these velocities are with respect to the ground.




Since, the woman is riding a bicycle, the velocity of rain as experienced by her is the velocity of rain relative to the velocity of the bicycle, she is riding, i.e.

$$\mathbf{v}_{rb} = \mathbf{v}_r - \mathbf{v}_b$$

The angle θ made by the relative velocity $\mathbf{v}_{rb}$ with the vertical given by

$$\tan\theta = \frac{v_b}{v_r} = \frac{12}{35} = 0.343$$

or $$\theta = \tan^{-1}(0.343)$$

Therefore, the woman should hold her umbrella at an angle of about $\tan^{-1}(0.343)$ with the vertical towards the west.

**Note** The difference between this question and the Q.27 is that, in Q.40, the boy experiences the resultant (vector sum) of two velocities while in this question, the woman experiences the velocity of rain relative to the bicycle (the vector difference of the two velocities).

**41** (*b*) The velocity of car driver = $8\ \hat{\mathbf{i}}\ \text{ms}^{-1}$

Velocity of rocket = $v_y \hat{\mathbf{j}}\ \text{ms}^{-1}$

Relative velocity of rocket w.r.t. car = $8\hat{\mathbf{i}} - v_y\hat{\mathbf{j}}$

Since, the speed of the rocket observed by the car driver is $10\ \text{ms}^{-1}$.

$$\therefore\ (v_y)^2 + (8)^2 = (10)^2$$
$$v_y^2 = 100 - 64 = 36$$
$$\Rightarrow \quad v_y = 6\,\text{ms}^{-1}$$

Velocity of rocket, $v_y\,\hat{\mathbf{j}} = (6\hat{\mathbf{j}})\,\text{ms}^{-1}$

$\therefore$ Relative speed of rocket w.r.t. a stationary observer

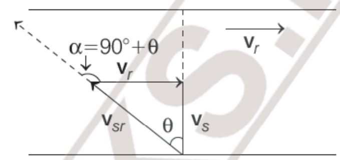
$$= 6 - 0 = 6\,\text{ms}^{-1}$$

**42** (*b*) Let the velocity of the swimmer is

$$v_s = 4\text{ km/h}$$

and velocity of river is $v_r = 2\,\text{km/h}$

Also, angle of swimmer with the flow of the river (down stream) is α as shown in the figure below




From diagram, angle θ is

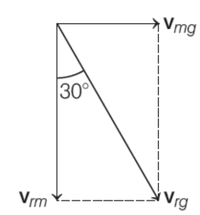
$$\sin\theta = \frac{v_r}{v_{sr}} = \frac{2\text{ km/h}}{4\text{ km/h}} = \frac{1}{2}$$

$$\Rightarrow \quad \theta = 30°$$

Clearly, $\alpha = 90° + 30° = 120°$

**43** (*a*) When the man is at rest with respect to the ground, the rain comes to him at an angle 30° with the vertical. This is the direction of the velocity of raindrops with respect to the ground.

This is as shown below




Here, $\mathbf{v}_{rg}$ = velocity of the rain with respect to the ground,

$\mathbf{v}_{mg}$ = velocity of the man with respect to the ground

and $\mathbf{v}_{rm}$ = velocity of the rain with respect to the man.

Here, when the man throws the umbrella and starts running, then $|\mathbf{v}_{mg}| = 10\ \text{kmh}^{-1}$

From the above figure, we can write

$$v_{rg}\sin 30° = v_{mg}$$

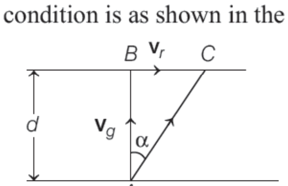
or $$v_{rg} = \frac{10}{\sin 30°} = 20\ \text{kmh}^{-1}$$

**44** (*c*) Given, speed of girl, $v_g = 5\ \text{km h}^{-1}$

Speed of river, $v_r = 2\ \text{km h}^{-1}$

Width of river, $d = 2$ km

The given condition is as shown in the figure below



Since, the girl dive the river normal to the flow of the river, time taken by the girl to cross the river, so

$$t = \frac{d}{v_g} = \frac{2\text{ km}}{5\text{ kmh}^{-1}} = \frac{2}{5}\text{ h}$$

In this time, the girl will go down the river by the distance $AC$ due to river current.

$\therefore$ Distance travelled along the river

$$= v_r \times t = 2 \times \frac{2}{5}$$

$$= \frac{4}{5}\text{ km} = \frac{4000}{5}\text{ m} = 800\text{ m}$$

**45** (*b*) Given, velocity of girl, $v_g = 5\hat{\mathbf{i}}\text{ ms}^{-1}$

Let velocity of rain, $v_r = v_x\hat{\mathbf{i}} + v_y\hat{\mathbf{j}}\text{ ms}^{-1}$

Relative velocity of rain $= v_r - v_g = (v_x - 5)\hat{\mathbf{i}} + v_y\hat{\mathbf{j}}$

Now, it is vertical, so $\tan\theta = \dfrac{v_x - 5}{v_y} = 0$

$$\Rightarrow \quad v_x - 5 = 0 \quad \Rightarrow v_x = 5 \qquad \text{...(i)}$$

On increasing the speed of the girl, relative velocity becomes $(v_x - 15)\hat{\mathbf{i}} + v_y\hat{\mathbf{j}}$

$$\tan\theta = \tan 45^\circ = \frac{v_x - 15}{v_y} = 1$$

$\Rightarrow \; v_x - 15 = v_y \;\Rightarrow\; v_y = -10$ [using Eq. (i)]

$\therefore$ Velocity of rain $= (5\hat{\mathbf{i}} - 10\hat{\mathbf{j}})\text{ ms}^{-1}$

$\Rightarrow$ Magnitude of velocity of rain

$$= \sqrt{(5)^2 + (10)^2}$$

$$= \sqrt{125} = 5\sqrt{5}\text{ ms}^{-1}$$

**46** (*d*) An object that is in flight after being thrown or projected is called a projectile. The motion of projectile may be thought of as the result of two separate, simultaneously occurring components of motions. One component along a horizontal direction without any acceleration

and the other along the vertical direction with constant acceleration due to the force of gravity.

**47** (*d*) When an object projected at an angle $\theta$ with the horizontal, then the acceleration acting on it is that due to gravity which is directed vertically downward and remains constant throughout.

i.e., $\mathbf{a} = -g\hat{\mathbf{j}}$

Thus, at the top most point value of $a = g$.

**48** (*a*) Given, $u = 10\text{ ms}^{-1}, \theta = 30^\circ, t = 1\text{s}$

Horizontal distance, $x = u\cos\theta t = 10\cos 30^\circ \times 1$

$$= \frac{10 \times \sqrt{3}}{2} = 5\sqrt{3}\text{ m} = 8.66\text{ m}$$

Similarly, vertical distance,

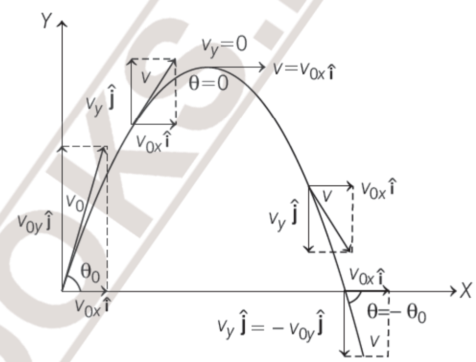
$$y = u\sin\theta t - \frac{1}{2}gt^2$$

$$= 10\sin 30^\circ \times 1 - \frac{1}{2} \times 9.8 \times 1^2$$

$$= 10 \times \frac{1}{2} - \frac{1}{2} \times 9.8 = 5 - 4.9 = 0.1\text{ m}$$

So, the position coordinate of the ball after 1s is (8.66 m, 0.1 m).

**49** (*c*) After the object has been projected, the $x$-component of the velocity remains constant throughout the motion and only the $y$-component changes, like an object in free-fall in vertical direction. This is shown graphically at few instants below




**50** (*c*) At the top most point of the projectile, there is only horizontal component of velocity and acceleration due to the force of gravity in vertically downward direction. So, velocity and acceleration are perpendicular to each other at the top most point.

**51** (*b*) Given, equations of motion are

$$x = 18t,\; 2y = 54t - 9.8t^2$$

General equations of projectile are

$$x = u\cos\theta \cdot t \text{ and } \; y = u\sin\theta \cdot t - \frac{1}{2}gt^2$$

where, $\theta$ is the angle of projection.

Comparing it with given equation, we have

$$u\cos\theta = 18 \text{ and } u\sin\theta = \frac{54}{2}$$

$$\Rightarrow \quad \frac{u\sin\theta}{u\cos\theta} = \frac{54/2}{18}$$

$$\therefore \quad \tan\theta = \frac{54}{2 \times 18} = 1.5 \Rightarrow \theta = \tan^{-1}(1.5)$$

**52** (*a*) The trajectory of a projectile projected at an angle $\theta$ with the horizontal direction from ground is given by

$$y = x\tan\theta - \frac{gx^2}{2u^2\cos^2\theta}$$

For same trajectories with equal angle of projection,

$$\frac{g}{u^2} = \text{constant}$$

$$\Rightarrow \quad \frac{g_{\text{earth}}}{u^2_{\text{earth}}} = \frac{g'_{\text{planet}}}{u^2_{\text{planet}}}$$

Given, $g_{\text{earth}} = g = 9.8\text{ ms}^{-2}$, $u_{\text{earth}} = 5\text{ ms}^{-1}$ and $u_{\text{planet}} = 3\text{ ms}^{-1}$

Let, $g'_{\text{planet}} = g'$

So, substituting these values in Eq. (i), we get

$$\frac{9.8}{5^2} = \frac{g'}{3^2}$$

$$\Rightarrow \quad g' = \frac{9.8 \times 9}{25} = 3.5 \text{ ms}^{-2}$$

**53** (*b*) Given, initial velocity, $\mathbf{u} = (\hat{\mathbf{i}} + 2\hat{\mathbf{j}})\text{ ms}^{-1}$

Magnitude of velocity,

$$u = \sqrt{(1)^2 + (2)^2} = \sqrt{5}\text{ ms}^{-1}$$

Equation of trajectory of projectile,

$$y = x\tan\theta - \frac{gx^2}{2u^2\cos^2\theta}$$

$$= x\tan\theta - \frac{gx^2\sec^2\theta}{2u^2}$$

$$= x\tan\theta - \frac{gx^2}{2u^2}(1+\tan^2\theta) \quad [\because \sec^2\theta = 1 + \tan^2\theta]$$

Substituting the given values, we get

$$\therefore y = x \times 2 - \frac{10(x)^2}{2(\sqrt{5})^2}[1 + (2)^2] \left[\because \tan\theta = \frac{u_y}{u_x} = \frac{2}{1} = 2\right]$$

$$= 2x - \frac{10(x^2)}{2 \times 5}(1+4) = 2x - 5x^2$$

**54** (*c*) When a particle is thrown from the ground obeliquely, i.e. at an angle $\theta$ with the $X$-axis, then firstly height attained by it would increase gradually with time. At a certain instant, it would reach the maximum height. After that, with the increase in the time, the particle's height decreases gradually and then finally comes to zero. This has been correctly depicted in the graph shown in option (c).

**55** (*b*) Velocities of the stones at some instant $t$ can be given as

$$\mathbf{v}_1 = u_1\cos\theta_1\hat{\mathbf{i}} + (u_1\sin\theta_1 - gt)\hat{\mathbf{j}}$$

and $$\mathbf{v}_2 = u_2\cos\theta_2\hat{\mathbf{i}} + (u_2\sin\theta_2 - gt)\hat{\mathbf{j}}$$

$\Rightarrow$ Relative velocity, $\mathbf{v}_1 - \mathbf{v}_2 = (u_1\cos\theta_1 - u_2\cos\theta_2)\hat{\mathbf{i}} + (u_1\sin\theta_1 - u_2\cos\theta_2)\hat{\mathbf{j}}$ = constant

Since, their relative velocity is constant.

So, the trajectory of path followed by one as seen by other will be straight line, making a constant angle with horizontal.

**56** (*b*) Due to symmetry of the parabolic path transversed by a stone during its flight, $2t_m = T_f$

where, $t_m$ is the time-taken by the stone to reach the maximum height and $T_f$ is the total flight of the stone.

Given, $t_m = 5.8\text{s}$

$$\Rightarrow \quad T_f = 2 \times 5.8 = 11.6\text{ s}$$

**57** (*d*) Time taken by the parachute to fall through a height $h$ of 980 m

$$t = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2 \times 980}{10}} = 14\text{ s}$$

Distance at which the parachute strikes the ground
= Horizontal velocity $\times\, t$

$$= 480 \times 14 \times \frac{1}{3600} = \frac{6720}{3600} = 1.867\text{ km}$$

**58** (*b*) Given, $u = 60\text{ ms}^{-1}$

Maximum height $H$ that the ball will achieve
= Height of ceiling of the hall = 30 m

As, maximum height, $H = \dfrac{u^2\sin^2\theta}{2g}$

$$\Rightarrow \quad 30 = \frac{(60)^2\sin^2\theta}{2g}$$

$$\Rightarrow \quad \sin^2\theta = \frac{30 \times 2g}{60 \times 60} = \frac{10}{60} \quad [\because g = 10\text{ ms}^{-2}]$$

$$\Rightarrow \quad \sin\theta = \frac{1}{\sqrt{6}}$$

**59** (*c*) Maximum height, $H = \dfrac{u^2\sin^2\alpha}{2g}$

For same speed of projection,

$$H \propto \sin^2\alpha$$

$$\therefore \quad \frac{H_1}{H_2} = \frac{\sin^2\alpha}{\sin^2(90° - \alpha)} = \frac{\sin^2\alpha}{\cos^2\alpha} = \tan^2\alpha$$

**60** (*c*) Given condition, $h_1 = h_2$

For projectile maximum height attained, $h = \dfrac{u^2\sin^2\theta}{2g}$

$$\Rightarrow \quad u_1^2\sin^2 45° = u_2^2\sin^2\theta$$

$$\Rightarrow \quad \sin^2\theta = \frac{u_1^2}{u_2^2}\sin^2 45° = \frac{1}{2}\cdot\frac{1}{2} = \frac{1}{4} \quad \left[\because \text{given, } \frac{u_1}{u_2} = \frac{1}{\sqrt{2}}\right]$$

$$\Rightarrow \quad \sin\theta = \frac{1}{2}$$

$$\Rightarrow \quad \theta = 30°$$

So, angle of projection of $B$ will be 30°.

**61** (*a*) Given, $u = 98\text{ ms}^{-1}$ and $\theta = 30°$

$\because$ Range of a projectile,

$$R = \frac{u^2\sin 2\theta}{g} = \frac{98 \times 98 \times \sin 60°}{9.8} = 490\sqrt{3}\text{ m}$$

**62** (*a*) Range of a projectile, $R = \dfrac{u^2\sin 2\theta}{g} = \dfrac{2u^2\sin\theta\cos\theta}{g}$

$$= \frac{2(u\sin\theta)(u\cos\theta)}{g} = \frac{2u_xu_y}{g}$$

$\Rightarrow R \propto$ horizontal initial velocity component $(u_x)$

$\therefore$ From the given plot, we can see that for path 3, range is maximum. This implies that the rock has the maximum horizontal velocity component in this path. Thus, the correct order will be $1 < 2 < 3$.

**63** (*b*) Given ,maximum horizontal range, $R_{max} = 80\,m$

As, range of a projectile, $R = \dfrac{u^2 \sin 2\theta}{g}$

and it is maximum $\theta = 45°$

$\therefore \quad \dfrac{u^2}{g} = 80\,m$

Maximum height, $h = \dfrac{u^2 \sin^2\theta}{2g}$

$$= \frac{80}{2}(\sin^2 45°) = 40 \times \frac{1}{2} = 20\,m$$

**64** (*c*) When, stone is thrown vertically upward, then

Maximum height, $h = \dfrac{u^2}{2g} \Rightarrow u^2 = 2gh$ ...(i)

Maximum horizontal distance covered by the stone when it is thrown horizontally at an angle $\theta$ is

$$R_{max} = \frac{u^2}{g} \text{ (when } \theta = 45°\text{)},$$

$\Rightarrow \quad R_{max} = 2h$ [from Eq. (i)]

**65** (*c*) Maximum height, $H = \dfrac{u^2 \sin^2\theta}{2g}$ ... (i)

Horizontal range, $R = \dfrac{u^2 \sin 2\theta}{g}$ ... (ii)

Dividing Eq. (i) by Eq. (ii), we get

$$\frac{H}{R} = \frac{\tan\theta}{4}$$

$$\Rightarrow \quad \theta = \tan^{-1}\frac{4H}{R}$$

**66** (*b*) Let $u$ be the initial speed, so speed at hightest point

$= u\cos\theta = \dfrac{u}{2} \Rightarrow \theta = 60°$

$\because$ Horizontal range, $R = \dfrac{u^2 \sin 2\theta}{g}$

and maximum height, $H = \dfrac{u^2 \sin^2\theta}{2g}$

$$\Rightarrow \quad \frac{R}{H} = \frac{4}{\tan\theta} = \frac{4}{\tan 60°} = \frac{4}{\sqrt{3}}$$

**67** (*b*) Components of velocity at an instant of time $t$ of a body projected at an angle $\theta$ is

$v_x = u\cos\theta + g_x t$ and $v_y = u\sin\theta + g_y t$

Here, components of velocity at $t = 1\,s$, is

$v_x = u\cos 60° + 0$ [as $g_x = 0$]

$= 10 \times \dfrac{1}{2} = 5\,m/s$

and $v_y = u\sin 60° + (-10) \times (1)$

$= 10 \times \dfrac{\sqrt{3}}{2} + (-10) \times (1)$

$= 5\sqrt{3} - 10$

$\Rightarrow \quad |v_y| = |10 - 5\sqrt{3}|\,m/s$

Now, angle made by the velocity vector at time of $t = 1\,s$

$$|\tan\alpha| = \left|\frac{v_y}{v_x}\right| = \frac{|10 - 5\sqrt{3}|}{5}$$

$\Rightarrow \quad \tan\alpha = |2 - \sqrt{3}|$

or $\quad \alpha = 15°$

$\therefore$ Radius of curvature of the trajectory of the projected body $R = v^2 / g\cos\alpha$

$$= \frac{(5)^2 + (10 - 5\sqrt{3})^2}{10 \times 0.97}$$

[$\because v^2 = v_x^2 + v_y^2$ and $\cos 15° = 0.97$]

$\Rightarrow \quad R = 2.77\,m \approx 2.8\,m$

**68** (*a*) In a uniform circular motion, velocity at each point is along the tangent at that point in the direction of motion. However, acceleration is directed towards the centre at each point of the circular path.

$\therefore$ Velocity and acceleration vectors are perpendicular to each other.

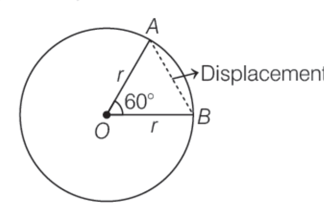
**69** (*c*) For a uniform circular motion,

centripetal acceleration, $a_c = \dfrac{v^2}{R}$

Since, $v$ and $R$ are constants, the magnitude of the centripetal acceleration of the car is also constant. However, the direction changes pointing towards the centre. Therefore, a centripetal acceleration is not a constant vector.

**70** (*b*) In the figure, $AB$ is the required displacement of the particle.

In triangle $OAB$, $OA = OB$ and $\angle AOB = 60°$



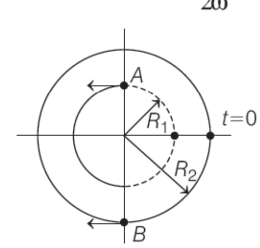

Therefore, $\Delta AOB$ is an equilateral triangle, so

$OA = OB = r = AB$.

**71** (*d*) Angle covered by each particle in time duration 0 to $\dfrac{\pi}{2\omega}$ is

$$\theta = \omega \times t = \omega \times \frac{\pi}{2\omega} = \frac{\pi}{2}\text{ rad}$$

So, positions of particles at $t = \dfrac{\pi}{2\omega}$ is as shown below



Velocities of particles at $t = \dfrac{\pi}{2\omega}$ are

$$\mathbf{v}_A = -\omega R_1 \hat{\mathbf{i}} \quad \text{and} \quad \mathbf{v}_B = -\omega R_2 \hat{\mathbf{i}}$$

The relative velocity of particles is

$$\mathbf{v}_A - \mathbf{v}_B = -\omega R_1 \hat{\mathbf{i}} - (-\omega R_2 \hat{\mathbf{i}})$$
$$= -\omega(R_1 - R_2)\hat{\mathbf{i}} = \omega(R_2 - R_1)\hat{\mathbf{i}}$$

**72** (*a*) Given, velocity of particle, $v = 10\,\text{ms}^{-1}$

Radius, $r = 5\text{m}$

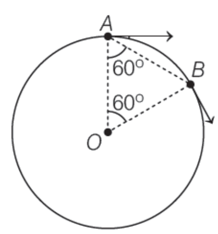
Angular displacement, $\theta = 60°$

$$\therefore \quad \theta = 60° \times \frac{\pi}{180°}\,\text{rad}$$
$$\Rightarrow \quad \theta = \frac{\pi}{3}\,\text{rad}$$

When particle moved by an angle of 60°, i.e. from $A$ to $B$ on the circular path, then according to figure, displacement $AB$ is equal to radius $r$ because it forms an equilateral triangle.

So, displacement $(AB) = 5\,\text{m}$

Given, $t = 1\,\text{s}$




$$\therefore \text{Average velocity} = \frac{\text{Displacement }(AB)}{\text{Time}}$$
$$= \frac{r}{1} = 5\,\text{ms}^{-1}$$

**73** (*c*) Angular speed, $\omega = \dfrac{2\pi}{T}$, where $T$ is the time period of rotation.

For particle $A$, $\omega_A = \dfrac{2\pi}{T_A}$

For particle $B$, $\omega_B = \dfrac{2\pi}{T_B}$

$$\therefore \frac{\omega_A}{\omega_B} = \frac{2\pi}{T_A} \times \frac{T_B}{2\pi} = \frac{T_B}{T_A} = \frac{1}{1} \text{ or } 1:1$$

$[\because T_A = T_B \text{ (given)}]$

**74** (*b*) We know that, linear speed $v = \omega r$, where angular velocity $\omega$ is constant.

$$\therefore \quad v \propto r \text{ or } \frac{v_A}{v_B} = \frac{r_A}{r_B}$$

**75** (*c*) Given, $r = 3\,\text{cm} = 3 \times 10^{-2}\,\text{m}$

Time period, $T = 60\,\text{s}$

Angular velocity, $\omega = \dfrac{2\pi}{T} = \dfrac{2\pi}{60} = 0.1047\,\text{rads}^{-1}$

Linear velocity, $v = \omega r = 0.1047 \times 3 \times 10^{-2}$
$= 0.00314\,\text{ms}^{-1}$

**76** (*a*) Given, speed, $v = 23\,\text{ms}^{-1}$

and radius, $r = 5\,\text{m}$

Centripetal acceleration, $a_c = \dfrac{v^2}{r} = \dfrac{23 \times 23}{5}$
$= 105.8 \approx 106\,\text{ms}^{-2}$

**77** (*c*) As, centripetal acceleration is given as $a_c = \dfrac{v^2}{r}$

For the first body of mass $m_1$, $a_{c_1} = \dfrac{v_1^2}{r_1}$

For the second body of mass $m_2$, $a_{c_2} = \dfrac{v_2^2}{r_2}$

Also time taken to complete one revolution by both the bodies is same.

Hence, $T_1 = T_2 \Rightarrow \dfrac{2\pi r_1}{v_1} = \dfrac{2\pi r_2}{v_2} \Rightarrow \dfrac{v_1}{v_2} = \dfrac{r_1}{r_2}$ ...(i)

i.e. $a_{c_1} : a_{c_2} = \dfrac{v_1^2}{r_1} \times \dfrac{r_2}{v_2^2}$

$= \dfrac{r_1^2}{r_2^2} \times \dfrac{r_2}{r_1} = \dfrac{r_1}{r_2} = r_1 : r_2$ [from Eq. (i)]

**78** (*a*) Radius, $r = \dfrac{\text{Diameter}}{2} = \dfrac{0.80\,\text{m}}{2} = 0.40\,\text{m}$

Frequency, $\nu = 100\,\text{revmin}^{-1} = \dfrac{100}{60}\,\text{revs}^{-1}$

Time period, $T = \dfrac{1}{\nu} = \dfrac{60}{100} = 0.60$

$\therefore$ Angular velocity, $\omega = \dfrac{2\pi}{T} = \dfrac{2 \times 3.14}{0.60} = 10.467\,\text{rads}^{-1}$

**79** (*c*) Given, frequency, $\nu = 72\,\text{revmin}^{-1} = \dfrac{72}{60}\,\text{revs}^{-1}$,

radius $r = 25\,\text{cm} = 25 \times 10^{-2}\,\text{m}$,

Linear velocity, $v = \omega r = (2\pi\nu)r$

$$= 2 \times \frac{22}{7} \times \frac{72}{60} \times \frac{25}{100}$$
$$= \frac{44}{7} \times \frac{6}{5} \times \frac{25}{100} = \frac{66}{35}\,\text{ms}^{-1}$$
$$= 2\,\text{ms}^{-1} \text{ (approx)}$$

**80** (*b*) Given, $\nu = 1200\,\text{rpm} = \dfrac{1200}{60}\,\text{rps}$,

$$r = 30\,\text{cm} = \frac{30}{100}\,\text{m}$$

Acceleration of the particle

$= \text{Centripetal acceleration} = \omega^2 r = (2\pi\nu)^2 r$

$$= \left(2 \times \frac{22}{7} \times \frac{1200}{60}\right)^2 \times \frac{30}{100} \approx 4740\,\text{ms}^{-2}$$

**81** (*b*) Resultant of two vectors **A** and **B** is given as

$$R = \sqrt{A^2 + B^2 + 2AB\cos\theta}$$

∴ We can say that

(i) If $\theta$ is an obtuse angle, then magnitude of **R** will be less than magnitude of the either vectors **A** or **B**.
e.g. if $|\mathbf{A}| = 4, |\mathbf{B}| = 3$ and $\theta = 120°$, then

$$|\mathbf{R}| = \sqrt{4^2 + 3^2 + 2 \times 4 \times 3\cos(120°)}$$
$$= \sqrt{25 - 12} = \sqrt{13} \qquad \left(\because \cos 120° = -\frac{1}{2}\right)$$
$$\therefore \qquad |\mathbf{R}| < |\mathbf{A}|$$

(ii) If the vectors are in opposite direction and are equal in magnitude, then also the magnitude of **R** will be less than the magnitude of either vectors **A** or **B**.
e.g., if $|\mathbf{A}| = |\mathbf{B}| = a$ (say) and $\theta = 180°$

then, $$|\mathbf{R}| = \sqrt{a^2 + a^2 - 2a^2\cos(180°)}$$
$$= \sqrt{2a^2 - 2a^2} \qquad [\because \cos 180° = -1]$$
$$\therefore \qquad |\mathbf{R}| < |\mathbf{A}| \text{ or } |\mathbf{B}|$$

Also, vector addition is commutative in nature.
$$\mathbf{A} + \mathbf{B} = \mathbf{B} + \mathbf{A}$$

Therefore, Assertion and Reason are correct but Reason is not the correct explanation of Assertion.

**82** (*d*) $|\mathbf{A} + \mathbf{B}| = \sqrt{A^2 + B^2 + 2AB\cos\theta}$

$$|\mathbf{A} - \mathbf{B}| = \sqrt{A^2 + B^2 - 2AB\cos\theta}$$
$$|\mathbf{A} + \mathbf{B}| = |\mathbf{A} - \mathbf{B}| = \sqrt{A^2 + B^2} \qquad [\text{At } \theta = 90°]$$

Therefore, Assertion is incorrect but Reason is correct.

**83** (*c*) To obtain maximum range, angle of projection must be 45°, i.e., $\theta = 45°$.

So, $$R_{\max} = \frac{u^2 \sin 2 \times 45°}{g} = \frac{u^2 \sin 90°}{g} = \frac{u^2}{g} \qquad \ldots(\text{i})$$
$$\therefore \qquad H_{\max} = \frac{u^2 \sin^2 45°}{2g} = \frac{u^2}{2g}\left(\frac{1}{\sqrt{2}}\right)^2 = \frac{u^2}{4g} = \frac{R_{\max}}{4}$$
[from Eq. (i)]

So, $H_{\max}$ is 25% of $R_{\max}$.
Therefore, Assertion is correct but Reason is incorrect.

**84** (*c*) Horizontal range, $R = \dfrac{u^2 \sin 2\theta}{g}$

At, $\theta = 45°, \quad \sin 2\theta = 1$

$$\therefore \qquad R_{\max} = \frac{u^2}{g} = \text{maximum range}$$
$$\because \qquad \sin\theta = 1 (\text{maximum}), \text{ at } \theta = 90°$$

Therefore, Assertion is correct but Reason is incorrect.

**85** (*b*) Maximum height, $H_1 = \dfrac{u^2 \sin^2 \alpha}{2g}$

and $$H_2 = \frac{u^2 \sin^2(90° - \alpha)}{2g} = \frac{u^2 \cos^2 \alpha}{2g}$$
$$\Rightarrow H_1 + H_2 = \frac{u^2}{2g}(\sin^2\alpha + \cos^2\alpha) = \frac{u^2}{2g}$$

Thus, the sum of height for angle of projections $\alpha$ and $90° - \alpha$ is independent of the angle of projection.

As, horizontal range, $R = \dfrac{u^2 \sin 2\theta}{g}$

So, for same value of initial velocity, horizontal range of projectile is same for complementary angles.

Therefore, Assertion and Reason are correct but Reason is not the correct explanation of Assertion.

**86** (*d*) In uniform circular motion the velocity of the object is changing continuously in direction, the object undergoes uniform acceleration which is not a constant vector. However, for a uniformly accelerated motions, the acceleration of the object should be constant. Hence, it is not an example of uniformly accelerated motion.

Kinematic equations for constant acceleration is not applicable for uniform circular motion. Since, in this case the magnitude of acceleration is constant but its direction is changing.

Therefore, Assertion is incorrect and Reason is correct.

**87** (*c*) Statements I and III are correct but II is incorrect and it can be corrected as
Each component of a vector is always a vector.

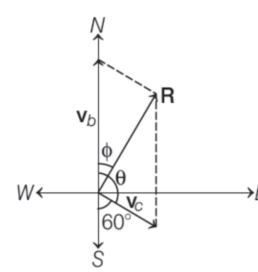
**88** (*c*) A unit vector is a vector of unit magnitude and points in a particular direction.

It has no unit and dimensions. It is just used to specify a direction only.

If we multiply a unit vector, say $\hat{\mathbf{n}}$ by a scalar $\lambda$, then the result is a vector $= \lambda\hat{\mathbf{n}}$.

So, statements I and III are incorrect but II is correct.

**89** (*b*) The vector $\mathbf{v}_b$ representing the velocity of the motorboat and the vector $\mathbf{v}_c$ representing the water current are shown in figure in direction specified by the problem. Using the parallelogram method of addition, the resultant velocity of boat **R** is obtained in the direction shown in the figure.




where, $\theta = 90° + 30° = 120°$

We can obtain the magnitude of **R** using the law of cosine

$$R = \sqrt{v_b^2 + v_c^2 + 2v_b v_c \cos 120°}$$
$$= \sqrt{25^2 + 10^2 + 2 \times 25 \times 10(-1/2)}$$
$$\cong 22 \text{ kmh}^{-1}$$

To obtain the direction, we apply the law of sines

$$\frac{R}{\sin\theta} = \frac{v_c}{\sin\phi}$$

or $$\sin\phi = \frac{v_c}{R}\sin\theta = \frac{10 \times \sin 120°}{22}$$
$$= \frac{10\sqrt{3}}{2 \times 22} = 0.394 \Rightarrow \phi = 23.2°$$

So, statements I and II are correct but III and IV are incorrect.