By question, $\frac{1}{2}(mv'')^2 = \frac{1}{2}\left[\frac{1}{2}m(v')^2\right]$

$$\frac{1}{2}m(v'')^2 = \frac{1}{4}m(v')^2 = \frac{1}{4}m[v^2 - 2gh]$$

[from Eq. (i)]

$$\Rightarrow \quad (v'')^2 = \frac{v^2 - 2gh}{2} = \frac{v^2}{2} - gh$$

$$\Rightarrow \quad v'' = \sqrt{\frac{v^2}{2} - gh} \quad \text{...(ii)}$$

On dividing Eq. (i) by Eq. (ii), we get

$$\frac{v'}{v''} = \frac{\sqrt{v^2 - 2gh}}{\frac{\sqrt{v^2 - 2gh}}{\sqrt{2}}} = \sqrt{2}$$

$$\Rightarrow \quad v'' = \frac{v'}{\sqrt{2}} = \sqrt{2}\left(\frac{v'}{2}\right)$$

$$\Rightarrow \quad \frac{v''}{v'/2} = \sqrt{2} = 1.414 > 1$$

$$\Rightarrow \quad v'' > \frac{v'}{2}$$

Hence, after emerging from the target velocity of the bullet (v'') is more than half of its earlier velocity v' (velocity before emerging into the target).

As the velocity of the bullet changes to v'' which is less than v', hence path followed will change and the bullet reaches at point B instead of A' as shown in the figure but motion is still parabolic.

So, only statement given in option (b) is correct.

116 (*b*) Work done by an agent is given by

$$W = \mathbf{F} \cdot \mathbf{s} = Fs\cos\theta$$

where, F is the applied force, s is the displacement and θ is the smaller angle between F & s.

(A) If $\theta < 90°$, i.e. acute angle, then work done is positive, as in case of coolie lifting luggage.

(B) If $\theta = 90°$, i.e. right angle, then work done is zero, as in case of satellite rotation around the earth.

(C) If $\theta > 90°$, i.e. obtuse angle, work done is negative, as in case of friction.

Hence, A $\rightarrow$ 3, B $\rightarrow$ 2 and C $\rightarrow$ 1.

118 (*b*) Elastic collision keeps kinetic energy constant.

In non-elastic collision, there is loss of kinetic energy due to deformation at the point of contact.

Scattering is also a type of collision without bringing bodies in contact. So, momentum remains conserved.

Hence, A $\rightarrow$ 2, B $\rightarrow$ 1 and C $\rightarrow$ 3.

119 (*b*) Work done, $W = \mathbf{F} \cdot \mathbf{d}$

Force, $\mathbf{F} = (-\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + 3\hat{\mathbf{k}})$N

Displacement, $\mathbf{d} = (4\hat{\mathbf{k}})$ m $= (0\hat{\mathbf{i}} + 0\hat{\mathbf{j}} + 4\hat{\mathbf{k}})$m

$\therefore$ Work done by the force is given by

$$W = \mathbf{F} \cdot \mathbf{d} = (-\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + 3\hat{\mathbf{k}}) \cdot (0\hat{\mathbf{i}} + 0\hat{\mathbf{j}} + 4\hat{\mathbf{k}})$$

$$= (-1 \times 0) + (2 \times 0) + (3 \times 4) = 0 + 12$$

$$[\because \hat{\mathbf{i}} \cdot \hat{\mathbf{i}} = \hat{\mathbf{j}} \cdot \hat{\mathbf{j}} = \hat{\mathbf{k}} \cdot \hat{\mathbf{k}} = 1 \text{ and } \hat{\mathbf{i}} \cdot \hat{\mathbf{j}} = \hat{\mathbf{j}} \cdot \hat{\mathbf{k}} = \hat{\mathbf{k}} \cdot \hat{\mathbf{i}} = 0]$$

$$= 12 \text{ J}$$

120 (*b*) In first case, the man applies a force on the mass 15 kg in vertically upward direction against its weight and walks 2 m in horizontal direction. So, the angle between the applied force and displacement is 90°.

$\therefore$ Work done, $W = Fd \cos 90° = 0 \quad (\because \cos 90° = 0)$

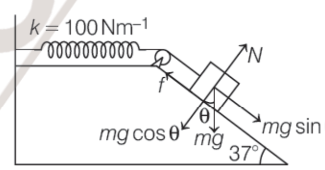
In second case, the man applies a force in horizontal direction and moves also in horizontal direction. So, angle between the applied force and displacement is 0°.

$\therefore$ Work done $= Fd \cos 0° = Fd = mg \times d$

$$= 15 \times 9.8 \times 2 = 294 \text{ J}$$

Difference in work done $= 294 - 0 = 294$ J

121 (*d*) Given, spring constant, $k = 100\,\text{Nm}^{-1}$, mass of block, $m = 1\,\text{kg}$, $\theta = 37°$ and distance moved by block, $x = 10\,\text{cm} = 0.1$ m.



As shown in figure, the net accelerating force acting on block is

$$F = mg \sin\theta - f = mg \sin\theta - \mu N$$

$$= mg \sin\theta - \mu\, mg \cos\theta$$

$\therefore$ Work done by the force F for motion of block,

$$W = Fx = mg\,(\sin\theta - \mu\cos\theta)\,x$$

When the block stops, the work done is stored in the spring in the form of its potential energy, $U = \frac{1}{2}kx^2$

$$\therefore \quad mg\,(\sin\theta - \mu\cos\theta)\,x = \frac{1}{2}kx^2$$

$$\Rightarrow \quad \mu = \frac{1}{\cos\theta}\left(\sin\theta - \frac{kx}{2\,mg}\right)$$

Substituting the values, we get

$$\mu = \frac{1}{\cos 37°}\left(\sin 37° - \frac{100 \times 0.1}{2 \times 1 \times 10}\right)$$

$$\Rightarrow \quad \mu = \frac{1}{0.8}[0.6 - 0.5] = 0.125$$

122 (*b*) Mass of the bolt, $m = 0.3$ kg

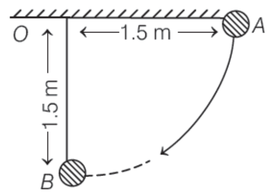
Length of the elevator, $h = 3$ m

As the bolt does not rebound, therefore its total PE is converted into heat.

$\therefore$ Heat produced = PE of the bolt

$$= mgh = 0.3 \times 9.8 \times 3 = 8.82 \text{ J}$$

123 (c) Length of the pendulum = 1.5 m



Potential energy of the bob at position $A = mgh$

As bob moves from position A towards position B, its potential energy converted into kinetic energy. 5% of its potential energy is dissipated against air resistance.

KE at position B = 95% of its PE at position A

$$\frac{1}{2}mv^2 = \frac{95}{100} \times mgh$$

or $$v = \sqrt{\frac{2 \times 95 \times gh}{100}} = \sqrt{\frac{19}{10} \times 9.8 \times 1.5}$$

$$= 5.28\,\text{ms}^{-1}$$

124 (c) Total energy of a particle executing linear simple harmonic motion at any instant is given by

$$E = \text{PE} + \text{KE} \quad \ldots\text{(i)}$$

Given, potential energy, $V(x) = \frac{1}{2}kx^2$

Total energy, $E = 1\,\text{J}$ and $k = 0.5\,\text{Nm}^{-1}$

When particle is at extreme position (the position from which the particle starts to come back to its mean position), then speed of the particle is zero and hence

$$\text{KE} = \frac{1}{2}mv^2 = 0 \quad (\because v = 0)$$

Substituting values in Eq. (i), we get

$$1 = \frac{1}{2}kx^2 + 0 \Rightarrow 1 = \frac{1}{2} \times 0.5 \times x^2$$

$$\Rightarrow \quad x^2 = \frac{2}{0.5} = 4 \text{ or } x = \pm 2\,\text{m}$$

125 (b) Let a body of mass m which is initially at rest undergoes one-dimensional motion under a constant force F with a constant acceleration a.

Acceleration, $a = \frac{F}{m}$ $\quad \ldots$(i)

Using equation of motion, $v = u + at$

$$\Rightarrow \quad v = 0 + \frac{F}{m} \cdot t \quad (\because u = 0)$$

$$\Rightarrow \quad v = \frac{F}{m}t \quad \ldots\text{(ii)}$$

Power delivered, $P = Fv$

Substituting the value from Eq. (ii), we get

$$\Rightarrow \quad P = F \times \frac{F}{m} \times t \Rightarrow P = \frac{F^2}{m}t$$

Dividing and multiplying by m in RHS,

$$P = \frac{F^2}{m^2} \times mt = a^2\,mt \quad \text{[using Eq. (i)]}$$

As, mass m and acceleration a are constants.

$$\therefore \quad P \propto t$$

126 (c) Velocity attained by the body in time t can be obtained using equation of motion, $v = u + at$

$$v = 0 + at \quad [\because u = 0]$$

or $$v = at \quad \ldots\text{(i)}$$

Power delivered, $P = Fv$

But, $F = ma$

$\therefore$ $P = ma \times at$ [using Eq. (i)]

$$P = ma^2t$$

or $$a = \sqrt{\frac{P}{mt}} \quad \ldots\text{(ii)}$$

Using equation of motion, we get

$$s = ut + \frac{1}{2}at^2 \Rightarrow s = \frac{1}{2}\left(\sqrt{\frac{P}{mt}}\right) \times t^2$$

$$s \propto t^{3/2}$$

127 (c) Time taken to fill the tank = 15 min

$$= 15 \times 60 = 900\,\text{s}$$

Work done $= mgh = (V \times d)gh$

[$\because$ mass = volume $\times$ density]

$$= (30 \times 1000) \times 9.8 \times 40$$

$$= 1.176 \times 10^7\,\text{J}$$

$\therefore$ Power required, $P = \frac{\text{Work done}}{\text{Time taken}}$

$$\Rightarrow \quad P = \frac{1.176 \times 10^7}{900} = 13.07 \times 10^3\,\text{W}$$

$$= 13.07\,\text{kW}$$

Efficiency of the pump, $\eta = \frac{\text{Output power}}{\text{Input power}} \times 100$

$\therefore$ Input power $= \frac{\text{Output power}}{\eta} \times 100 = \frac{13.07}{30} \times 100$

$$= \frac{130.7}{3}\,\text{kW} = 43.56\,\text{kW}$$

The pump consumes, 43.6 kW electric power.

128 (c) Power used by a family, $P = 8\,\text{kW}$

Solar energy incident on horizontal surface per square metre = 200 W

Electrical energy obtained from solar energy per unit area $= 200 \times \frac{20}{100}\,\text{W} = 40\,\text{W}$

$\therefore$ Area needed to supply 8 kW $= \frac{8\,\text{kW}}{40\,\text{W}} = \frac{8000}{40}$

$$= 200\,\text{m}^2$$

129 (b) When two bodies of equal masses collides elastically, their velocities are interchanged.

When ball 1 collides with ball 2, then velocity of ball 1, v_1 becomes zero and velocity of ball 2 becomes v.

Similarly when ball 2 collides with ball 3, then velocity of ball 2 becomes zero and velocity of ball 3 becomes v. Thus, option (b) denotes this condition.

130 (*d*) When two billiard balls collide, then distance between their centres is $2R$. Due to impact of collision, there is small temporary deformation of balls. In this process, KE of ball is gradually reduced to zero and converted into elastic potential energy $U(r)$ of balls.

This phenomenon can be successfully explained only by potential energy curve in option (d), because here as $r < 2R$, the potential energy function $U(r)$ is increasing gradually on decreasing value of r and become maximum at $r = 0$.

131 (*c*) Mass, $m = 10\,\text{kg}$, height, $h = 0.5\,\text{m}$

Number of times the mass lifted, $n = 1000$

Work done against gravitational force

$$= n \times mgh = 1000 \times 10 \times 9.8 \times 0.5$$
$$= 49000\,\text{J}$$

132 (*a*) Energy supplied by fat per kilogram $= 3.8 \times 10^7$ J

Mechanical energy supplied by fat per kilogram

$$= 20\% \text{ of total energy supplied by fat}$$
$$= \frac{20}{100} \times 3.8 \times 10^7 = 0.76 \times 10^7\ \text{Jkg}^{-1}$$

$$\text{Fat used up by the dieter} = \frac{1}{0.76 \times 10^7} \times 49000$$
$$= 6.45 \times 10^{-3}\ \text{kg}$$

133 (*a*) The ratio of kinetic energy of electron and proton is

$$\frac{K_e}{K_p} = \frac{\frac{1}{2}m_e v_e^2}{\frac{1}{2}m_p v_p^2} = \frac{10}{100} \Rightarrow \left(\frac{v_e}{v_p}\right)^2 = \frac{10 \times m_p}{100\, m_e}$$

$$\Rightarrow \left(\frac{v_e}{v_p}\right)^2 = \frac{1}{10} \times \frac{1.67 \times 10^{-27}}{9.1 \times 10^{-31}} = \frac{183.5}{1}$$

$$\frac{v_e}{v_p} = \frac{13.5}{1}$$

134 (*b*) When an electron and a proton are moving under influence of their mutual forces, the magnetic forces will be perpendicular to their motion, hence no work is done by these forces.

135 (*c*) Force between two protons is same as that of between proton and a positron.

As positron is much lighter than proton, it moves away through much larger distance compared to proton.

We know that, work done = force × distance. As forces are same in case of proton and positron but distance moved by positron is larger, hence work done will be more in case of positron.

136 (*c*) Here, work is done by the frictional force on the cycle and is equal to $-200 \times 10 = -2000\,\text{J}$.

As the road is not moving, hence work done by the cycle on the road = zero.

137 (*c*) As the body is falling freely under gravity, the potential energy decreases and kinetic energy increases but total mechanical energy (PE + KE) of the body and earth system will be constant as external force on the system is zero.

138 (*c*) When we are considering the two bodies as system, the total external force on the system will be zero.

Hence, total linear momentum of the system remains conserved.

139 (*c*) As the given tracks are frictionless, hence mechanical energy will be conserved. As both the tracks having common height h.

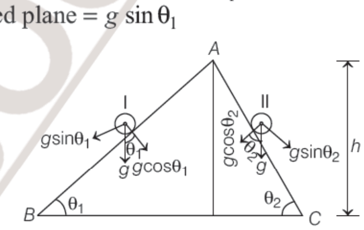
From conservation of mechanical energy,

$$\frac{1}{2}mv^2 = mgh \qquad \text{(for both tracks I and II)}$$

$$v = \sqrt{2gh}$$

Hence, speed is same for both stones.

For stone as shown below, a_1 = acceleration along inclined plane $= g \sin \theta_1$



Similarly, for stone II, $a_2 = g \sin \theta_2$ as $\theta_2 > \theta_1$, hence $a_2 > a_1$.

Also, lengths for track II are also less, hence stone II reaches earlier than stone I.

Thus, the statement given in option (c) is correct, rest are incorrect.

140 (*b*) Total energy, $E = \text{PE} + \text{KE}$ ...(i)

When particle is at $x = x_m$, i.e. at extreme position, it returns back. Hence, at $x = x_m$, $v = 0 \Rightarrow \text{KE} = 0$

From Eq. (i),

$$E = \text{PE} + 0 = \text{PE} = U(x_m) = \frac{1}{2}kx_m^2$$

$$\Rightarrow \quad E = U$$

Thus, option (b) is correct.

141 (*b*) Given, $v = ax^{3/2}$, $m = 0.5\,\text{kg}$, $a = 5\ \text{m}^{-1/2}\text{s}^{-1}$

Work done, $W = ?$

We know that, acceleration,

$$a_0 = \frac{dv}{dt} = v\frac{dv}{dx} = ax^{3/2}\frac{d}{dx}(ax^{3/2})$$
$$= ax^{3/2} \times a \times \frac{3}{2} \times x^{1/2}$$
$$= \frac{3}{2}a^2x^2$$

Now, force $= ma_0 = \frac{3}{2}ma^2x^2$

$$\text{Work done} = \int_{x=0}^{x=2} F dx = \int_0^2 \frac{3}{2} ma^2x^2 dx$$
$$= \frac{3}{2} ma^2 \times (x^3/3)_0^2$$
$$= \frac{1}{2} ma^2 \times 8 = \frac{1}{2} \times (0.5) \times (5)^2 \times 8 = 50 \text{J}$$

142 (*b*) Given, power = constant

We know that, power, $P = \dfrac{dW}{dt} = \dfrac{\mathbf{F} \cdot d\mathbf{s}}{dt}$

As, body is moving unidirectionally.

Hence, $\mathbf{F} \cdot d\mathbf{s} = Fds \cos 0°$

$$P = \frac{Fds}{dt} = Fv = \text{constant}$$

($\because P$ = constant by question)

Now, writing dimensions, [F] [v] = constant

$\Rightarrow \quad [\text{MLT}^{-2}][\text{LT}^{-1}] = \text{constant}$

$\Rightarrow \quad \text{L}^2\text{T}^{-3} = \text{constant} \qquad (\because \text{mass is constant})$

$\Rightarrow \quad L \propto T^{3/2}$ or displacement $(d) \propto t^{3/2}$

Thus, only graph (b) shows this condition.

143 (*d*) When the earth is closest to the sun, speed of the earth is maximum, hence KE is maximum. When the earth is farthest from the sun, speed is minimum, hence KE is minimum but never zero and negative.

Therefore, option (d) is correct.

144 (*c*) When a pendulum oscillates in air, it loses energy continuously in overcoming resistance due to air. Therefore, total mechanical energy of the pendulum decreases continuously with time which is shown in graph (c).

145 (*a*) Given, mass, $m = 5$ kg

Radius, $R = 1$ m

Revolutions per minute, $\omega = 300 \text{ rev min}^{-1}$

$$= (300 \times 2\pi) \text{ rad min}^{-1}$$
$$= (300 \times 2 \times \pi) \text{ rad}/60 \text{ s}$$
$$= \frac{300 \times 2 \times \pi}{60} \text{ rads}^{-1} = 10\pi \text{ rads}^{-1}$$

Linear speed, $v = \omega R = (10\pi \times 1) = 10\pi \text{ ms}^{-1}$

$$\text{KE} = \frac{1}{2} mv^2 = \frac{1}{2} \times 5 \times (10\pi)^2$$
$$= 250\pi^2 \text{ J}$$

146 (*b*) When raindrop falls first velocity increases, hence first KE also increases. After sometime speed (velocity) becomes constant, this is called terminal velocity, hence KE also becomes constant. PE decreases continuously as the drop is falling continuously.

Hence, only graph (b) shows this condition correctly.

147 (*b*) First velocity of the iron sphere increases and after sometime becomes constant. Hence, accordingly first KE increases and then becomes constant which is best represented by (b).

148 (*d*) Given, $h = 1.5$ m, $v = 1 \text{ ms}^{-1}$,

$$m = 10 \text{ kg}, g = 10 \text{ ms}^{-2}$$

From conservation of mechanical energy,

$(\text{PE})_i + (\text{KE})_i = (\text{PE})_f + (\text{KE})_f$

$\Rightarrow \quad mgh + \dfrac{1}{2} mv^2 = 0 + (\text{KE})_f$

$\Rightarrow \quad (\text{KE})_f = mgh + \dfrac{1}{2} mv^2$

$\Rightarrow \quad (\text{KE})_f = 10 \times 10 \times 1.5 + \dfrac{1}{2} \times 10 \times (1)^2$

$$= 150 + 5 = 155 \text{ J}$$

149 (*c*) Given, $m = 150 \text{ g} = \dfrac{150}{1000} \text{ kg} = \dfrac{3}{20} \text{ kg}$

Δt = time of contact = 0.001 s

$$u = 126 \text{ kmh}^{-1} = \frac{126 \times 1000}{60 \times 60} \text{ ms}^{-1}$$
$$= 35 \text{ ms}^{-1}$$

As collision is completely elastic, so

$$v = -126 \text{ kmh}^{-1} = -35 \text{ ms}^{-1}$$

Change in momentum of the ball,

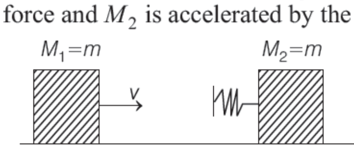
$$\Delta p = m(v - u)$$
$$= \frac{3}{20}(-35 - 35)$$
$$= \frac{3}{20}(-70)$$
$$= -\frac{21}{2} \text{ kg-ms}^{-1}$$

We know that, force, $F = \dfrac{\Delta p}{\Delta t} = \dfrac{-21/2}{0.001}$ N

$$= -1.05 \times 10^4 \text{ N}$$

Here, negative sign shows that force will be opposite to the direction of motion of the ball before hitting.

150 (*c*) Consider the following diagram below when M_1 comes in contact with the spring, M_1 is retarded by the spring force and M_2 is accelerated by the spring force.



Then,

(a) The spring will continue to compress until the two blocks acquire common velocity.

(b) As surfaces are frictionless, so momentum of the system will be conserved.

(c) If spring is massless, whole energy of M_1 will be imparted to M_2 and M_1 will be at rest.

(d) Collision is inelastic, even if friction is not involved. This is because energy is stored as PE during collision.

Hence, only statement given in option (c) is correct.

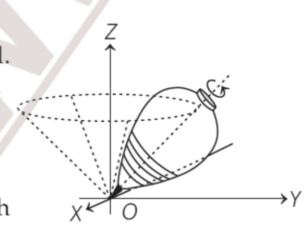
CHAPTER > 07

System of Particles and Rotational Motion

KEY NOTES

Rigid Body

- Ideally a rigid body is a body with a perfectly definite and fixed shape. The distances between all pairs of particles of such a body do not change on the application of a force.
- In **pure translational motion** at any instant of time, all particles of the body have the same velocity.
- The most common way to constrain a rigid body so that it does not have translational motion is, to fix it along a straight line. The only possible motion of such a rigid body is **rotation**. The line or fixed axis about which the body is rotating is its **axis of rotation**.
- In **rotation of a rigid body** about a fixed axis, every particle of the body moves in a circle, which lies in a plane perpendicular to the axis and has its centre on the axis.
- In some examples of rotation, however the axis may not be fixed. A prominent example of this kind of rotation is a top spinning in place (as shown in figure). The axis of such a spinning top moves around the vertical through its point of contact with the ground, sweeping out a cone. This movement of the axis of the top around the vertical is termed **precession**.



Note The point of contact of the top with ground is fixed.

- The motion of a rigid body which is not pivoted or fixed in some way is either a pure translation or a combination of translation and rotation.

The motion of a rigid body which is pivoted or fixed in some way is rotation.

Centre of Mass and Its Motion

- Centre of mass of a system of particles is the point that behaves as, if the entire mass of the system is concentrated on it.
- Centre of mass
  (i) of n-particles system is

$$x_{CM} = \frac{m_1x_1 + m_2x_2 + \dots + m_nx_n}{m_1 + m_2 + \dots + m_n}$$

$$y_{CM} = \frac{m_1y_1 + m_2y_2 + \dots + m_ny_n}{m_1 + m_2 + \dots + m_n}$$

$$z_{CM} = \frac{m_1z_1 + m_2z_2 + \dots + m_nz_n}{m_1 + m_2 + \dots + m_n}$$

  (ii) of rigid bodies is

$$X = \frac{1}{M}\int xdm,\ Y = \frac{1}{M}\int ydm \text{ and } Z = \frac{1}{M}\int zdm$$

- Centre of mass of symmetrical bodies, e.g. for uniform rod, circular plate, etc lies at their centre.
- The motion of centre of mass of a system of particles is considered to be moving as if all the mass of the system was concentrated at the centre of mass and all the external forces were applied at that point.
- Velocity and acceleration of centre of mass respectively are as follows

$$\mathbf{v} = \frac{m_1\mathbf{v}_1 + m_2\mathbf{v}_2 + \dots + m_n\mathbf{v}_n}{m_1 + m_2 + \dots + m_n}$$

$$\mathbf{a} = \frac{m_1\mathbf{a}_1 + m_2\mathbf{a}_2 + \ldots + m_n\mathbf{a}_n}{m_1 + m_2 + \ldots + m_n}$$

- The **total linear momentum of a system of particles** is equal to the product of the total mass of the system and the velocity of its centre of mass.

$$\mathbf{p} = m\mathbf{v}$$
$$\frac{d\mathbf{p}}{dt} = m\frac{d\mathbf{v}}{dt} = m\mathbf{a} = \mathbf{F}_{\text{ext}}$$

This is the statement of **Newton's second law of motion** extended to a system of particles.

- When the total external force acting on a system of particles is zero, the total linear momentum of the system is constant.

i.e. $\frac{d\mathbf{p}}{dt} = 0$

or $\mathbf{p} = \text{constant}$.

This is the **law of conservation of the total linear momentum** of a system of particles.

- If the total external force acting on the system is zero, the centre of mass moves with a constant velocity, i.e. moves uniformly in a straight line like a free particle.

Vector Product of Two Vectors

- A vector product of two vectors $\mathbf{a}$ and $\mathbf{b}$ is a vector $\mathbf{c}$ such that
  - (i) magnitude of $\mathbf{c} = c = ab\sin\theta$, where a & b are the magnitudes of $\mathbf{a}$ & $\mathbf{b}$ and θ is the angle between the two vectors.
  - (ii) $\mathbf{c}$ is perpendicular to the plane containing $\mathbf{a}$ and $\mathbf{b}$.
- Vector product of two vectors $\mathbf{a}$ and $\mathbf{b}$ is not commutative.

i.e. $\mathbf{a}\times\mathbf{b} = -(\mathbf{b}\times\mathbf{a})$

- Vector products are distributive with respect to vector addition, i.e. $\mathbf{a}\times(\mathbf{b}+\mathbf{c}) = \mathbf{a}\times\mathbf{b} + \mathbf{a}\times\mathbf{c}$
- The vector product among cartesian unit vectors $\hat{\mathbf{i}}, \hat{\mathbf{j}}$ and $\hat{\mathbf{k}}$ are given as

$$\hat{\mathbf{i}}\times\hat{\mathbf{i}} = 0, \quad \hat{\mathbf{j}}\times\hat{\mathbf{j}} = 0, \quad \hat{\mathbf{k}}\times\hat{\mathbf{k}} = 0,$$
$$\hat{\mathbf{i}}\times\hat{\mathbf{j}} = \hat{\mathbf{k}}, \quad \hat{\mathbf{j}}\times\hat{\mathbf{k}} = \hat{\mathbf{i}}, \quad \hat{\mathbf{k}}\times\hat{\mathbf{i}} = \hat{\mathbf{j}}$$

- If $\mathbf{a} = a_x\hat{\mathbf{i}} + a_y\hat{\mathbf{j}} + a_z\hat{\mathbf{k}}$ and $\mathbf{b} = b_x\hat{\mathbf{i}} + b_y\hat{\mathbf{j}} + b_z\hat{\mathbf{k}}$,

then $\mathbf{a}\times\mathbf{b} = \begin{bmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ a_x & a_y & a_z \\ b_x & b_y & b_z \end{bmatrix}$.

Angular Velocity and Acceleration

- The **average angular velocity** of the particle over the interval Δt is $\frac{\Delta\theta}{\Delta t}$, i.e. $\omega = \frac{\Delta\theta}{\Delta t}$.

Instantaneous angular velocity, $\omega = \lim_{\Delta t\to 0}\frac{\Delta\theta}{\Delta t} = \frac{d\theta}{dt}$

- At any given instant, the relation $v = \omega r$ applies to all particles of the rotating rigid body.

Thus, for a particle at a perpendicular distance r_i from the fixed axis, the linear velocity v_i at a given instant is given by

$$v_i = \omega r_i$$

- As pure translational motion of a body is characterised by all parts of the body having the same velocity at any instant of time.

Similarly, pure rotation can be characterised by all parts of the body having the same angular velocity at any instant of time.

- For rotation about a fixed axis, the direction of the vector ω does not change with time. Its magnitude may, however change from instant to instant.

For the more general rotation, both the magnitude and the direction of ω may change from instant to instant.

- **Angular acceleration** α is defined as the rate of change of angular velocity, i.e. $\alpha = \frac{d\omega}{dt}$.

If the axis of rotation is fixed, the direction of ω and hence that of α is also fixed.

Torque and Angular Momentum

- The turning effect of the force about the axis of rotation is called **torque** or **moment of force**.
- If a force $\mathbf{F}$ acts on a single particle at a point whose position vector with respect to the origin is given by vector $\mathbf{r}$, then moment of force is given by

$$\tau = \mathbf{r}\times\mathbf{F}$$

- The magnitude of τ is given as $\tau = rF\sin\theta$
  - (a) When $\theta = 0°$ or $180°$, then $\tau = 0$ (minimum).
  - (b) When $\theta = 90°$, then $\tau = rF$ (maximum).
- The moment of force is a vector quantity.
- The quantity **angular momentum** can be referred to as moment of (linear) momentum.

Consider a particle of mass m and linear momentum $\mathbf{p}$ at a position $\mathbf{r}$ relative to origin.

The angular momentum $\mathbf{L}$ is given as $\mathbf{L} = \mathbf{r}\times\mathbf{p}$.

- The magnitude of the angular momentum vector is $L = rp\sin\theta$.
- The time rate of the total angular momentum of a system of particles about a point is equal to the sum of the external torques acting on the system taken about the same point.

i.e. $\tau = \tau_{\text{ext}} = \frac{d\mathbf{L}}{dt}$

KEY NOTES

Equilibrium of a Rigid Body

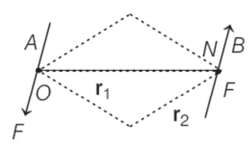
- A rigid body is said to be in equilibrium, if both of its linear momentum and angular momentum remain same with time.
- If total force or vector sum of all forces acting on the body is zero, then linear momentum of the body remains constant, so the body is in **translatory equilibrium**, i.e.

$$\mathbf{F}_1 + \mathbf{F}_2 + \dots\dots + \mathbf{F}_n = \sum_{i=1}^{n} \mathbf{F}_i = 0$$

- If the total torque, i.e. the vector sum of the torques on the rigid body is zero, then angular momentum of the body remains constant, so the body is in **rotational equilibrium**.

i.e. $$\tau_1 + \tau_2 \dots\dots + \tau_n = \sum_{i=1}^{n} \tau_i = 0$$

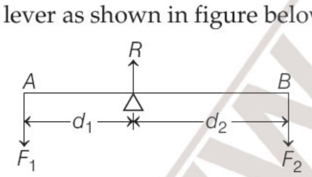
- A pair of equal and opposite forces with parallel lines of action is known as a **couple**. A couple produces rotation without translation.
- For a couple with forces $-\mathbf{F}$ and $\mathbf{F}$ acting at A and B points with position vectors $\mathbf{r}_1$ and $\mathbf{r}_2$ with respect to some origin O as shown in figure.



The moment of couple = total torque

$$= (\mathbf{r}_2 - \mathbf{r}_1) \times \mathbf{F}$$

- According to **principle of moment**, when an object is in rotational equilibrium, then algebraic sum of all torques acting on it is zero. Clockwise torques are taken **negative** and anti-clockwise torques are taken **positive**.
- In case of the lever as shown in figure below



(i) Force F_1 is usually some weight to be lifted. It is called the **load** and its distance from fulcrum d_1 is called the **load arm**.

(ii) Force F_2 is the **effort** applied to lift the load and distance d_2 of the effort from the fulcrum is the **effort arm**.

At equilibrium, load arm × load = effor × effort arm

$$\Rightarrow \quad F_1 d_1 = F_2 d_2$$

The above equation expresses the principle of moments for a lever.

- The ratio $\frac{F_1}{F_2}$ is called the **mechanical advantage** (MA).

$$\text{MA} = \frac{F_1}{F_2} = \frac{d_2}{d_1}$$

Centre of Gravity

If a body is supported on a point such that the total gravitational torque about this point is zero, then this point is called centre of gravity of the body.

$$\Sigma m_i r_i = 0$$

Moment of Inertia and Radius of Gyration

- Moment of inertia is the analogue of mass in rotational motion.

The **moment of inertia** (I) of a body about an axis is defined as the sum of the products of the mass of the particles of the body and the square of the respective distance of the particles from the axis of rotation.

$$I = m_1 r_1^2 + m_2 r_2^2 + \dots + m_n r_n^2 = \sum_{i=1}^{n} m_i r_i^2$$

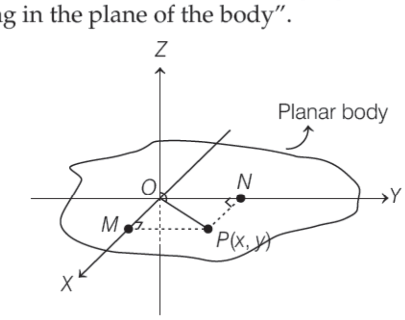
where, $m_1, m_2 \dots, m_n$ are the masses of n-particles and $r_1, r_2, \dots, r_n$ be their distances from axis of rotation.

- The **radius of gyration** of a body about an axis is defined as the distance from the axis of a mass point whose mass is equal to the mass of whole body and whose moment of inertia is equal to the moment of inertia of the body about the axis of rotation. It is denoted by k.

$$k = \sqrt{\frac{I}{M}}$$

Theorem of Perpendicular Axes

- It states that "the moment of inertia of a planar body (lamina) about an axis perpendicular to its plane, is equal to the sum of its moment of inertia about two perpendicular axes concurrent with perpendicular axis and lying in the plane of the body".



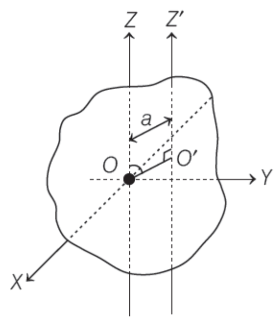
$$\therefore \quad I_Z = I_X + I_Y$$

where, I_X, I_Y and I_Z are the moments of inertia about the X, Y and Z-axes, respectively.

Theorem of Parallel Axes

- It states that " the moment of inertia of a body about any axis is equal to the moment of inertia of the body about a parallel axis passing through the centre of mass plus the product of the mass of the body and the square of the distance between the two parallel axes."

- Two such axes are shown in figure for a body of mass M. If a is the distance between the axes and I_{CM} and I_0 are the respective moments of inertia about these axes, then



$$I_0 = I_{CM} + Ma^2$$

Moment of Inertia of Some Regular Shaped Bodies

- **Moment of inertia of the rod** about an axis passing through its centre and perpendicular to rod, $I = \frac{Ml^2}{12}$

  Moment of inertia of the rod about an axis perpendicular to the length of the rod and passing through its end,

  $I' = \frac{Ml^2}{3}$

  where, l is the length of the rod.
- **Moment of inertia of a thin circular ring** about an axis through its centre and perpendicular to its plane,

  $$I_c = MR^2$$

  Moment of inertia about any diameter of the ring,

  $$I_d = \frac{1}{2} MR^2$$

  where, R is the radius of the circular disc.
- **Moment of inertia of a circular disc** about an axis passing through its centre and perpendicular to its plane is given by $$I_c = \frac{1}{2} MR^2$$

  About its diameter, $I_d = \frac{1}{4} MR^2$
- **Moment of inertia of a solid cylinder** about an axis passing through its centre of mass and parallel to its length, $I = \frac{1}{2} MR^2$
- **Moment of inertia of a solid sphere** about a diameter,

  $$I = \frac{2}{5} MR^2$$

  Moment of inertia about any tangent,

  $$I_{\tan} = \frac{7}{5} MR^2.$$

Kinematic Equations of Rotational Motion

- The kinematic equations for rotational motion with uniform angular acceleration are given below

  (*i*) $\omega = \omega_0 + \alpha t$ (*ii*) $\theta - \theta_0 = \omega_0 t + \frac{1}{2}\alpha t^2$

  (*iii*) $\omega^2 - \omega_0^2 = 2\alpha(\theta - \theta_0)$

  where, θ_0 & ω_0 are initial angular displacement & angular velocity, θ & ω are final angular displacement & angular velocity and α is angular acceleration.

Conservation of Angular Momentum

If the external torque on the rotating body is zero, then angular momentum on the body is conserved. This is law of conservation of angular momentum.

As, $\frac{dL}{dt} = 0$

$$L = \text{constant}, \; I\omega = \text{constant or } I_1\omega_1 = I_2\omega_2$$

Rolling Motion

- When a body performs translation motion as well as rotational motion, then this type of motion is known as rolling motion.
- **Kinetic Energy of Rolling Motion** A body rotating about its axis with angular velocity ω and its centre of mass moving with velocity v_{CM}, then kinetic energy of rolling is given as

$$\text{KE}_{\text{rolling}} = \text{KE}_{\text{translational}} + \text{KE}_{\text{rotational}}$$
$$= \frac{1}{2}Mv_{CM}^2 + \frac{1}{2}I\omega^2 = \frac{1}{2}Mv_{CM}^2\left(1 + \frac{k^2}{R^2}\right)$$

- Various physical quantities of a body, rolling on an inclined plane without slipping are listed below

  (*i*) Acceleration of the body, $a = \dfrac{g\sin\theta}{1 + \dfrac{k^2}{R^2}}$

  where, θ = angle of inclination.

  (ii) Velocity of the body when it reaches the bottom,

  $$v = \sqrt{\frac{2gh}{1 + \dfrac{k^2}{R^2}}}$$

  (*iii*) Time taken by a rolling body to reach the bottom,

  $$t = \frac{1}{\sin\theta}\sqrt{\frac{2h(1 + k^2/R^2)}{g}}$$

- A body with smaller value of k^2/R^2 will take less time to reach the bottom.
- Change in kinetic energy due to rolling $(v_2 > v_1)$

  $$= \frac{1}{2}m\left(1 + \frac{k^2}{R^2}\right)(v_2^2 - v_1^2)$$

KEY NOTES

Mastering NCERT

MULTIPLE CHOICE QUESTIONS

TOPIC 1 ~ Rigid Body, Centre of Mass and Its Motion

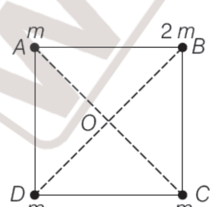
1 A system of particles is called a rigid body, when
(a) any two particles of system may have displacements in opposite directions under action of a force
(b) any two particles of system may have velocities in opposite directions under action of a force
(c) any two particles of system may have a zero relative velocity
(d) any two particles of system may have displacements in same direction under action of a force

2 In pure rotation,
(a) all particles of the body move in a straight line
(b) all particles of body move in concentric circles
(c) all particles of body move in non-concentric circles
(d) all particles of body have same speed

3 In precession of a body,
(a) axis of rotation is fixed
(b) axis of rotation translates on a curved path
(c) both ends of axis of rotation move around circular paths
(d) one end of rotation axis is fixed

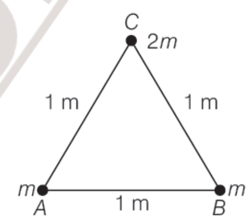
4 The centre of mass of a system of two particles divides, the distance between them
(a) in inverse ratio of square of masses of particles
(b) in direct ratio of square of masses of particles
(c) in inverse ratio of masses of particles
(d) in direct ratio of masses of particles

5 Centre of Mass (CM) of the given system of particles will be at



(a) OD (b) OC
(c) OB (d) AO

6 Two balls each of mass m are placed on the vertices A and B of an equilateral ΔABC of side 1 m. A ball of mass $2m$ is placed at vertex C. The centre of mass of this system from vertex A (located at origin) is



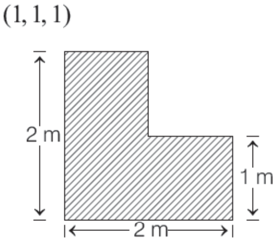
(a) $\left(\frac{1}{2}\text{ m}, \frac{1}{2}\text{ m}\right)$ (b) $\left(\frac{1}{2}\text{ m}, \sqrt{3}\text{ m}\right)$
(c) $\left(\frac{1}{2}\text{ m}, \frac{\sqrt{3}}{4}\text{m}\right)$ (d) $\left(\frac{\sqrt{3}}{4}\text{ m}, \frac{\sqrt{3}}{4}\text{ m}\right)$

7 Three identical spheres of mass M each are placed at the corners of an equilateral triangle of side 2 m. Taking one of the corner as the origin, the position vector of the centre of mass is

(a) $\sqrt{3}\,(\hat{\mathbf{i}} - \hat{\mathbf{j}})$ (b) $\frac{\hat{\mathbf{i}}}{\sqrt{3}} + \hat{\mathbf{j}}$ (c) $\frac{\hat{\mathbf{i}} + \hat{\mathbf{j}}}{3}$ (d) $\hat{\mathbf{i}} + \frac{\hat{\mathbf{j}}}{\sqrt{3}}$

8 The centre of mass of three particles of masses 1 kg, 2 kg and 3 kg is at (3, 3, 3) with reference to a fixed coordinate system. Where should a fourth particle of mass 4 kg be placed, so that the centre of mass of the system of all particles shifts to a point (1, 1, 1)?
(a) $(-1, -1, -1)$ (b) $(-2, -2, -2)$
(c) $(2, 2, 2)$ (d) $(1, 1, 1)$

9 Find the centre of mass of a uniform L-shaped lamina (a thin flat plate) with dimensions as shown in the figure alongside. The mass of the lamina is 3 kg.



(a) $(5/6)$ m, $(5/6)$ m
(b) $(3/4)$ m, $(3/4)$ m
(c) $(5/8)$ m, $(5/8)$ m
(d) $(3/5)$ m, $(3/5)$ m

10 Distance of the centre of mass of a solid uniform cone from its vertex is z_0. If the radius of its base is R and its height is h, then z_0 is equal to

(a) $\frac{h^2}{4R}$ (b) $\frac{3h}{4}$ (c) $\frac{5h}{8}$ (d) $\frac{3h^2}{8R}$

11 Three bodies having masses 5 kg, 4 kg and 2 kg is moving at the speed of 5 m/s, 4 m/s and 2 m/s, respectively along X-axis. The magnitude of velocity of centre of mass is ***AIIMS 2018***

(a) 1.0 m/s (b) 4 m/s (c) 0.9 m/s (d) 1.3 m/s

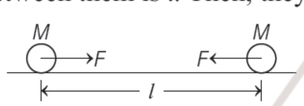
12 Two particles of equal masses have velocities $\mathbf{v}_1 = 4\hat{\mathbf{i}}\ \text{ms}^{-1}$ and $\mathbf{v}_2 = 4\hat{\mathbf{j}}\ \text{ms}^{-1}$. First particle has an acceleration $\mathbf{a}_1 = (2\hat{\mathbf{i}} + 2\hat{\mathbf{j}})\ \text{ms}^{-2}$, while the acceleration of the other particle is zero. The centre of mass of the two particles moves in a path of

(a) straight line (b) parabola
(c) circle (d) ellipse

13 Two persons of masses 55 kg and 65 kg respectively, are at the opposite ends of a boat. The length of the boat is 3 m and weighs 100 kg. The 55 kg man walks upto the 65 kg man and sits with him. If the boat is in still water, the centre of mass of the system shifts by ***CBSE AIPMT 2012***

(a) 3 m (b) 2.3 m (c) zero (d) 0.75 m

14 Two balls of same masses start moving towards each other due to gravitational attraction, if the initial distance between them is l. Then, they meet at

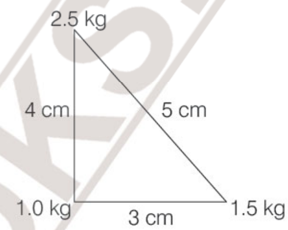


(a) $\frac{l}{2}$ (b) l (c) $\frac{l}{3}$ (d) $\frac{l}{4}$

15 Two particles A and B initially at rest move towards each other under a mutual force of attraction. At the instant, when the speed of A is v and the speed of B is $2v$, the speed of centre of mass of the system is

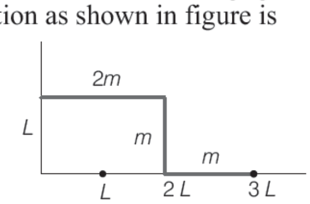
(a) zero (b) v (c) $1.5v$ (d) $3v$

16 Three point particles of masses 1.0 kg, 1.5 kg and 2.5 kg are placed at three corners of a right angle triangle of sides 4.0 cm, 3.0 cm and 5.0 cm as shown in the figure. The centre of mass of the system is at a point ***JEE Main 2020***



(a) 2.0 cm right and 0.9 cm above 1 kg mass
(b) 0.6 cm right and 2.0 cm above 1 kg mass
(c) 1.5 cm right and 1.2 cm above 1 kg mass
(d) 0.9 cm right and 2.0 cm above 1 kg mass

17 The position vector of the centre of mass $\mathbf{r}_{cm}$ of an asymmetric uniform bar of negligible area of cross-section as shown in figure is ***JEE Main 2019***



(a) $\mathbf{r} = \frac{13}{8}L\,\hat{\mathbf{x}} + \frac{5}{8}L\,\hat{\mathbf{y}}$ (b) $\mathbf{r} = \frac{11}{8}L\,\hat{\mathbf{x}} + \frac{3}{8}L\,\hat{\mathbf{y}}$

(c) $\mathbf{r} = \frac{3}{8}L\,\hat{\mathbf{x}} + \frac{11}{8}L\,\hat{\mathbf{y}}$ (d) $\mathbf{r} = \frac{5}{8}L\,\hat{\mathbf{x}} + \frac{13}{8}L\,\hat{\mathbf{y}}$

TOPIC 2 ~ *Vector Product, Angular Velocity, Torque and Angular Momentum*

18 The vector product of two vectors $\mathbf{A} = 2\hat{\mathbf{i}} + \hat{\mathbf{j}} - \hat{\mathbf{k}}$ and $\mathbf{B} = -\hat{\mathbf{i}} - \hat{\mathbf{j}} + 2\hat{\mathbf{k}}$ is

(a) $2\hat{\mathbf{i}} + 3\hat{\mathbf{j}} - \hat{\mathbf{k}}$ (b) $\hat{\mathbf{i}} + 3\hat{\mathbf{j}} + \hat{\mathbf{k}}$

(c) $-\hat{\mathbf{i}} + 3\hat{\mathbf{j}} + 2\hat{\mathbf{k}}$ (d) $\hat{\mathbf{i}} - 3\hat{\mathbf{j}} - \hat{\mathbf{k}}$

19 Two vectors $\mathbf{P}$ and $\mathbf{Q}$ are given in space as $\mathbf{P} = 2\hat{\mathbf{i}} - 3\hat{\mathbf{j}} + \hat{\mathbf{k}}$ and $\mathbf{Q} = -4\hat{\mathbf{i}} + 6\hat{\mathbf{j}} - 2\hat{\mathbf{k}}$. The angle between $\mathbf{P}$ and $\mathbf{Q}$ is

(a) 90° (b) 0°
(c) 30° (d) 60°

20 The unit vector perpendicular to vectors $\mathbf{A} = 2\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}$ and $\mathbf{B} = \hat{\mathbf{i}} - \hat{\mathbf{j}} + 2\hat{\mathbf{k}}$ is

(a) $\frac{1}{\sqrt{3}}(\hat{\mathbf{i}} - \hat{\mathbf{j}} - \hat{\mathbf{k}})$ (b) $\frac{1}{\sqrt{3}}(\hat{\mathbf{i}} + \hat{\mathbf{j}} - \hat{\mathbf{k}})$

(c) $\frac{1}{3}(\hat{\mathbf{i}} - \hat{\mathbf{j}} - \hat{\mathbf{k}})$ (d) $\frac{1}{3}(\hat{\mathbf{i}} + \hat{\mathbf{j}} - \hat{\mathbf{k}})$

21 Angular velocity vector is directed along

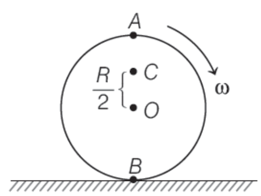
(a) the tangent to the circular path
(b) the inward radius
(c) the outward radius
(d) the axis of rotation

22 A body is rotating with angular velocity $\omega = (3\hat{\mathbf{i}} - 4\hat{\mathbf{j}} - \hat{\mathbf{k}})$. The linear velocity of a point having position vector $\mathbf{r} = (5\hat{\mathbf{i}} - 6\hat{\mathbf{j}} + 6\hat{\mathbf{k}})$ is

(a) $6\hat{\mathbf{i}} + 2\hat{\mathbf{j}} - 3\hat{\mathbf{k}}$ (b) $-18\hat{\mathbf{i}} - 23\hat{\mathbf{j}} + 2\hat{\mathbf{k}}$

(c) $-30\hat{\mathbf{i}} - 23\hat{\mathbf{j}} + 2\hat{\mathbf{k}}$ (d) $6\hat{\mathbf{i}} - 2\hat{\mathbf{j}} + 8\hat{\mathbf{k}}$

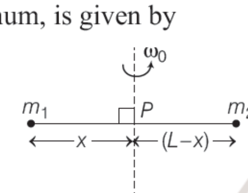
23 A circular plate rotating about its axis with angular speed ω is placed lightly (without any translational push) on a perfectly frictionless table. The radius of the disc is R. Let v_A, v_B and v_C be the magnitudes of linear velocities of the points A, B and C on the disc as shown below. Then,



(a) $v_A > v_B > v_C$ (b) $v_A < v_B < v_C$

(c) $v_A = v_B > v_C$ (d) $v_A = v_B < v_C$

24 Point masses m_1 and m_2 are placed at the opposite ends of a rigid rod of length L and negligible mass. The rod is to be set rotating about an axis perpendicular to it. The position of point P on this rod through which the axis should pass, so that the work required to set the rod rotating with angular velocity ω_0 is minimum, is given by **CBSE AIPMT 2015**



(a) $x = \dfrac{m_1 L}{m_1 + m_2}$ (b) $x = \dfrac{m_1}{m_2} L$

(c) $x = \dfrac{m_2}{m_1} L$ (d) $x = \dfrac{m_2 L}{m_1 + m_2}$

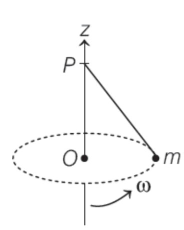
25 The angular momentum $\mathbf{L}$ of a single particle can be represented as

(a) $\mathbf{r} \times \mathbf{p}$ (b) $rp \sin\theta\, \hat{\mathbf{n}}$

(c) $rp \perp \hat{\mathbf{n}}$ (d) Both (a) and (b)

($\hat{\mathbf{n}}$ = unit vector perpendicular to plane of r, so that r, p and $\hat{\mathbf{n}}$ make right handed system)

26 A point mass m is attached to a massless string whose other end is fixed at P as shown in figure. The mass is undergoing circular motion in xy-plane with centre O and constant angular speed ω. If the angular momentum of the system, calculated about O and P be $\mathbf{L}_O$ and $\mathbf{L}_P$ respectively, then



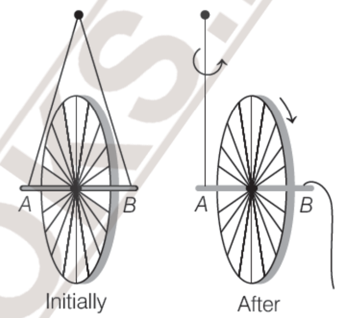
(a) $\mathbf{L}_O$ and $\mathbf{L}_P$ do not vary with time

(b) $\mathbf{L}_O$ varies with time while $\mathbf{L}_P$ remains constant

(c) $\mathbf{L}_O$ remains constant while $\mathbf{L}_P$ varies with time

(d) $\mathbf{L}_O$ and $\mathbf{L}_P$ both vary with time

27 A cycle rim is rotated over its axle in a vertical plane by holding ends of axle using 2-strings A and B.



Such that the rim is vertical. If you leave one string, the rim will tilt. Now, keeping the rim in vertical position with both the strings in one hand, put the wheel in fast rotation around the axle with the other hand. Then, leave one string, say B, from your hand. What will happen, if we leave string B?

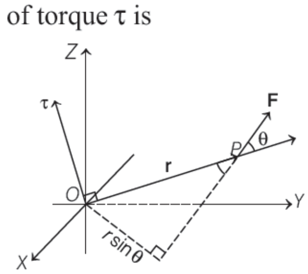
(a) The rim will stop rotating

(b) The rim will rotate in a vertical plane and the plane of rotation will precesses about string A

(c) The rim will rotate in a horizontal plane

(d) String at A is twisted

28 A force $\mathbf{F}$ is applied on a single particle P as shown in the figure. Here, $\mathbf{r}$ is the position vector of the particle. The value of torque τ is



(a) $\mathbf{F} \times \mathbf{r}$ (b) $\mathbf{r} \times \mathbf{F}$

(c) $\mathbf{r} \cdot \mathbf{F}$ (d) $\mathbf{F} \cdot \mathbf{r}$

29 What is the torque of a force $7\hat{\mathbf{i}} + 3\hat{\mathbf{j}} - 5\hat{\mathbf{k}}$ about the origin? The force acts on a particle whose position vector is $\hat{\mathbf{i}} - \hat{\mathbf{j}} + \hat{\mathbf{k}}$.

(a) $2\hat{\mathbf{i}} + 12\hat{\mathbf{j}} - 10\hat{\mathbf{k}}$ (b) $\hat{\mathbf{i}} + 12\hat{\mathbf{j}} + 10\hat{\mathbf{k}}$

(c) $\hat{\mathbf{i}} + 10\hat{\mathbf{j}} + 10\hat{\mathbf{k}}$ (d) $2\hat{\mathbf{i}} + 12\hat{\mathbf{j}} + 10\hat{\mathbf{k}}$

30 Newton's second law for rotational motion of a system of particle can be represented as ($\mathbf{L}$ for a system of particles)

(a) $\frac{d\mathbf{p}}{dt} = \tau_{ext}$ (b) $\frac{d\mathbf{L}}{dt} = \tau_{int}$

(c) $\frac{d\mathbf{L}}{dt} = \tau_{ext}$ (d) $\frac{d\mathbf{L}}{dt} = \tau_{int} + \tau_{ext}$

31 If $\tau_{ext} = 0$, means $\mathbf{L} \rightarrow$ constant, it is

(a) rotational analogue of conservation of linear momentum
(b) rotational analogue of force
(c) rotational analogue of linear momentum
(d) None of the above

32 A man spining in free space, changes the shape of his body, e.g. by spreading his arms or by curling up. By doing this, he cannot change his

(a) moment of inertia (b) angular momentum
(c) angular velocity (d) rotational kinetic energy

33 A force $\mathbf{F} = \alpha\hat{\mathbf{i}} + 3\hat{\mathbf{j}} + 6\hat{\mathbf{k}}$ is acting at a point $\mathbf{r} = 2\hat{\mathbf{i}} - 6\hat{\mathbf{j}} - 12\hat{\mathbf{k}}$. The value of α, for which angular momentum about origin is conserved, is **CBSE AIPMT 2015**

(a) -1 (b) 2
(c) zero (d) 1

34 A bob of mass m attached to an inextensible string of length l is suspended from a vertical support. The bob rotates in a horizontal circle with an angular speed ω rads^{-1} about the vertical support. About the point of suspension,

(a) angular momentum is conserved
(b) angular momentum changes in magnitude but not in direction
(c) angular momentum changes in direction but not in magnitude
(d) angular momentum changes both in direction and magnitude

TOPIC 3 ~ Equilibrium of a Rigid Body

35 A rigid body is said to be in partial equilibrium only, if

(a) it is in rotational equilibrium
(b) it is in translational equilibrium
(c) Either (a) or (b)
(d) None of the above

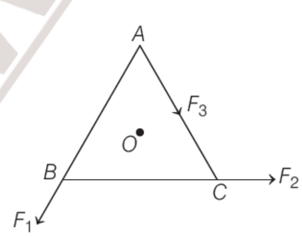
36 For rotational equilibrium,

(a) $\sum_{i=1}^{n} \mathbf{F}_{i\,net} = 0$

(b) $\sum_{i=1}^{n} \tau_{i\,net} = 0$

(c) Both (a) and (b) are the necessary conditions for the rotational equilibrium
(d) Both (a) and (b) are not necessary for rotational equilibrium

37 ABC is an equilateral triangle with O as its centre. $\mathbf{F}_1$, $\mathbf{F}_2$ and $\mathbf{F}_3$ represent three forces acting along the sides AB, BC and AC, respectively. If the total torque about O is zero, then the magnitude of $\mathbf{F}_3$ is **CBSE AIPMT 2012**



(a) $F_1 + F_2$ (b) $F_1 - F_2$

(c) $\frac{F_1 + F_2}{2}$ (d) $2(F_1 + F_2)$

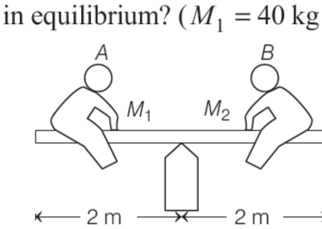
38 Different relations are given below. Which of the following is correct?

(a) Mechanical advantage = $\frac{\text{Effort}}{\text{Load}}$
(b) Load arm $\times$ Effort = Effort arm $\times$ Load
(c) Load arm $\times$ Load = Effort arm $\times$ Effort
(d) None of the above

39 In a lever system, the effort arm is larger than the load arm, then the value of mechanical advantage is

(a) equal to 1
(b) less than 1
(c) greater than 1
(d) None of the above

40 In the game of see-saw, what should be the displacement of boy B from right edge to keep the see-saw in equilibrium? ($M_1 = 40$ kg and $M_2 = 60$ kg)



(a) $\frac{4}{3}$ m (b) 1 m

(c) $\frac{2}{3}$ m (d) Zero

41 A rod of weight w is supported by two parallel knife edges A and B; and is in equilibrium in a horizontal position. The knives are at a distance d from each other. The centre of mass of the rod is at distance x from A. The normal reaction on A is ***CBSE AIPMT 2015***

(a) $\frac{wx}{d}$ (b) $\frac{wd}{x}$

(c) $\frac{w(d-x)}{x}$ (d) $\frac{w(d-x)}{d}$

42 In the given figure, balancing of a cardboard on the tip of a pencil is done. The point of support, G is the centre of gravity.

Choose the correct option. ***JEE Main 2014***

(a) τ_{Mg} about CG = 0

(b) τ_R about CG = 0

(c) Net τ due to $m_1 g, m_2 g......m_n g$ about CG = 0

(d) All of the above

43 Two point objects of masses 1.5 g and 2.5 g respectively, are at a distance of 16 cm apart. The centre of gravity is at a distance x from the object of mass 1.5 g, where x is

(a) 10 cm (b) 6 cm

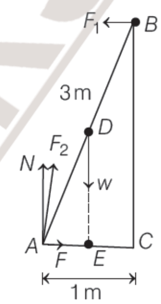
(c) 13 cm (d) 3 cm

44 A metal bar 70 cm long and 4 kg in mass supported on two knife edges placed 10 cm from each end. A 6 kg weight is suspended at 30 cm from one end. What are the reactions at the knife edges? (Assume the bar to be of uniform cross-section and homogeneous)

(a) 45 N and 43 N (b) 50 N and 35 N

(c) 55 N and 43 N (d) 54 N and 30 N

45 A 3 m long ladder weighing 20 kg leans on a frictionless wall. Its feet rest on the floor 1m from the wall as shown in figure. Find the reaction forces of the wall and the floor.



(a) 34.6 N and 199 N (b) 25 N and 175 N

(c) 30 N and 180 N (d) 35 N and 160 N

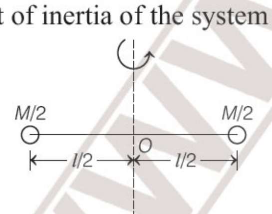
TOPIC 4 ~ *Moment of Inertia*

46 Moment of inertia in rotational motion is analogous to

(a) radius of gyration (b) angular momentum

(c) mass (d) torque

47 Two masses are joined with a light rod and the system is rotating about the fixed axis as shown in the figure. The moment of inertia of the system about the axis is



(a) $Ml^2/2$ (b) $Ml^2/4$

(c) Ml^2 (d) $Ml^2/6$

48 A light rod of length l has two masses m_1 and m_2 are attached to its two ends. The moment of inertia of the system about an axis perpendicular to the rod and passing through the centre of mass is ***NEET 2016***

(a) $\sqrt{m_1 m_2}\, l^2$ (b) $\frac{m_1 m_2}{(m_1 + m_2)} l^2$

(c) $\frac{m_1 m_2}{(m_1 - m_2)} l^2$ (d) $(m_1 + m_2)\, l^2$

49 One solid sphere A and another hollow sphere B are of same mass and same outer radius. Their moments of inertia about their diameters are I_A and I_B respectively, such that

(a) $I_A = I_B$ (b) $I_A > I_B$

(c) $I_A < I_B$ (d) None of these

50 Two discs having mass ratio (1/2) and diameter ratio (2/1), then find ratio of moment of inertia.

(a) 2 : 1 (b) 1 : 1 ***JIPMER 2019***

(c) 1 : 2 (d) 2 : 3

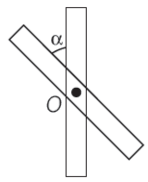
51 A thin wire of mass M and length L is bent to form circular ring. The moment of inertia of this ring about its axis is

(a) $\frac{1}{4\pi^2} ML^2$ (b) $\frac{1}{12} ML^2$ (c) $\frac{1}{3\pi^2} ML^2$ (d) $\frac{1}{\pi^2} ML^2$

52 Two solid spheres A and B are made of metals of different densities ρ_A and ρ_B, respectively. If their masses are equal,then the ratio of their moments of inertia (I_B/I_A) about their respective diameters is

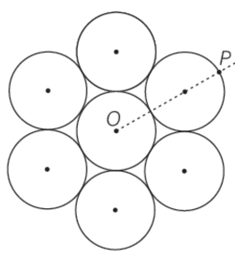
(a) $\left(\frac{\rho_B}{\rho_A}\right)^{2/3}$ (b) $\left(\frac{\rho_A}{\rho_B}\right)^{2/3}$ (c) $\frac{\rho_A}{\rho_B}$ (d) $\frac{\rho_B}{\rho_A}$

53 Two identical rods of mass M and length l are lying in a horizontal plane at an angle α. The moment of inertia of the system of two rods about an axis passing through O and perpendicular to the plane of the rods is



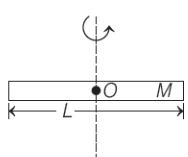
(a) $Ml^2/3$ (b) $Ml^2/12$ (c) $Ml^2/4$ (d) $Ml^2/6$

54 Seven identical circular planar discs, each of mass M and radius R are welded symmetrically as shown in the figure. The moment of inertia of the arrangement about the axis normal to the plane and passing through the point P is **JEE Main 2018**



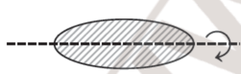
(a) $\frac{19}{2}MR^2$ (b) $\frac{55}{2}MR^2$ (c) $\frac{73}{2}MR^2$ (d) $\frac{181}{2}MR^2$

55 A rod is rotating about an axis passing through its centre and perpendicular to its length. The radius of gyration for the rod is



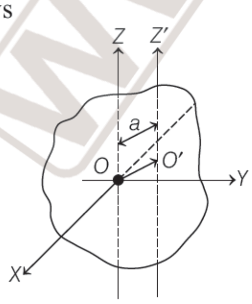
(a) $L/12$ (b) $L/\sqrt{12}$ (c) $L/6$ (d) $L/\sqrt{6}$

56 A disc of mass M and radius R is rotating about one of its diameter. The value of radius of gyration for the disc is **JEE Main 2013**



(a) $R/4$ (b) $R/2$
(c) $R/6$ (d) None of these

57 For the given figure, if we apply theorem of parallel axes, it shows



(a) $I_Z = I_{Z'} + Ma^2$ (b) $I_{Z'} = I_Z + Ma^2$
(c) $I_Z = I_{Z'} + 2Ma^2$ (d) $I_{Z'} = I_Z + 2Ma^2$

58 What is the moment of inertia of a ring about a tangent to the periphery of the ring?

(a) $\frac{1}{2}MR^2$ (b) $\frac{3}{2}MR^2$
(c) MR^2 (d) $MR^2/9$

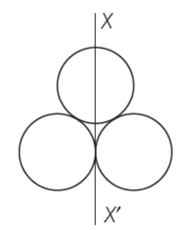
59 The moment of inertia of a thin uniform rod of length L and mass M about an axis passing through a point at a distance of $1/3$ from one of its ends and perpendicular to the rod is

(a) $\frac{ML^2}{12}$ (b) $\frac{ML^2}{9}$ (c) $\frac{7ML^2}{48}$ (d) $\frac{ML^2}{48}$

60 Consider a uniform square plate of side a and mass m. The moment of inertia of this plate about an axis perpendicular to its plane and passing through one of its corner is

(a) $\frac{5}{6}ma^2$ (b) $\frac{1}{12}ma^2$
(c) $\frac{7}{12}ma^2$ (d) $\frac{2}{3}ma^2$

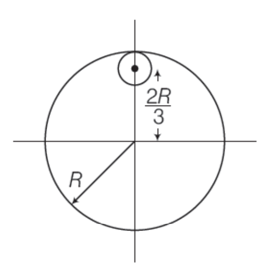
61 Three identical spherical shells, each of mass m and radius r are placed as shown in figure. Consider an axis XX' which is touching two shells and passing through diameter of third shell.



Moment of inertia of the system consisting of these three spherical shells about XX' axis is **CBSE AIPMT 2015**

(a) $4\,mr^2$ (b) $\frac{11}{5}mr^2$
(c) $3\,mr^2$ (d) $\frac{16}{5}mr^2$

62 From a uniform circular disc of radius R and mass $9\,M$, a small disc of radius $\frac{R}{3}$ is removed as shown in the figure. The moment of inertia of the remaining disc about an axis perpendicular to the plane of the disc and passing through centre of disc is **JEE Main 2018**



(a) $4MR^2$ (b) $\frac{40}{9}MR^2$
(c) $10MR^2$ (d) $\frac{37}{9}MR^2$

63 The moment of inertia of a uniform cylinder of length l and radius R about its perpendicular bisector is I. What is the ratio l/R such that the moment of inertia is minimum? ***JEE Main 2017***

(a) $\frac{\sqrt{3}}{2}$ (b) 1 (c) $\frac{3}{\sqrt{2}}$ (d) $\sqrt{\frac{3}{2}}$

64 Let the moment of inertia of a hollow cylinder of length 30 cm (inner radius 10 cm and outer radius 20 cm) about its axis be I. The radius of a thin cylinder of the same mass such that its moment of inertia about its axis is also I, is ***JEE Main 2019***

(a) 16 cm (b) 14 cm
(c) 12 cm (d) 18 cm

65 Find ratio of radius of gyration of a disc and ring of same radii at their tangential axis in plane. ***JIPMER 2017***

(a) $\sqrt{\frac{5}{6}}$ (b) $\sqrt{\frac{5}{3}}$ (c) 1 (d) $\frac{2}{3}$

TOPIC 5 ~ *Kinematics and Dynamics of Rotational Motion About a Fixed Axis*

66 A wheel is rotating at 900 rpm about its axis. When the power is cut-off, it comes to rest in 1 min. The angular retardation (in rad s^{-2}) is

(a) $-\frac{\pi}{2}$ (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{6}$ (d) $\frac{\pi}{8}$

67 The wheel of a car is rotating at the rate of 1200 rpm. On pressing the accelerator for 10 s, it starts rotating at 4500 rpm. The angular acceleration of the wheel is

(a) 30 rads^{-2} (b) 1880 degs^{-2}
(c) 40 rads^{-2} (d) 1980 degs^{-2}

68 When a ceiling fan is switched OFF, its angular velocity fall to half while it makes 36 rotations. How many more rotations will it make before coming to rest? (Assume uniform angular retardation)

(a) 36 (b) 24 (c) 18 (d) 12

69 The angular speed of a motor wheel is increased from 1200 rpm to 3120 rpm in 16 s. (i) What is its angular acceleration, assuming the acceleration to be uniform? (ii) How many revolutions does the motor make during this time?

(a) 4π rads^{-2} and 200 (b) 2π rads^{-2} and 470
(c) 4π rads^{-2} and 576 (d) 3π rads^{-2} and 390

70 The angular velocity of a wheel increases from 100 rps to 300 rps in 10 s. The number of revolutions made during that time is

(a) 600 (b) 1500 (c) 1000 (d) 2000

71 To maintain a rotor at a uniform angular speed of 100 rads^{-1}, an engine needs to transmit torque of 100 N-m. The power of the engine is

(a) 10 kW (b) 100 kW
(c) 10 MW (d) 100 MW

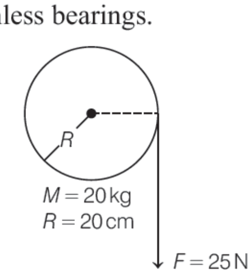
72 Three objects, A :(a solid sphere), B :(a thin circular disc) and C :(a circular ring), each have the same mass M and radius R. They all spin with the same angular speed ω about their own symmetry axes. The amounts of work (W) required to bring them to rest, would satisfy the relation ***NEET 2018***

(a) $W_B > W_A > W_C$ (b) $W_A > W_B > W_C$
(c) $W_C > W_B > W_A$ (d) $W_A > W_C > W_B$

73 A solid sphere of mass m and radius R is rotating about its diameter. A solid cylinder of the same mass and same radius is also rotating about its geometrical axis with an angular speed twice that of the sphere. The ratio of their kinetic energies of rotation ($\text{KE}_{\text{sphere}}/\text{KE}_{\text{cylinder}}$) will be ***NEET 2016***

(a) 3 : 1 (b) 2 : 3
(c) 1 : 5 (d) 1 : 4

74 A cord of negligible mass is wound round the rim of a flywheel (disc) of mass 20 kg and radius 20 cm. A steady pull of 25 N is applied on the cord as shown in figure. The flywheel is mounted on a horizontal axle with frictionless bearings.



Compute the angular acceleration of the flywheel.

(a) 12.50 s^{-2} (b) 6 s^{-2}
(c) 10 s^{-2} (d) 8 s^{-2}

75 A flywheel of moment of inertia 0.4 kg-m^2 and radius 0.2 m is free to rotate about a central axis. If a string is wrapped around it and it is pulled with a force of 10N, then its angular velocity after 4 s will be

(a) 10 rads^{-1} (b) 5 rads^{-1}
(c) 20 rads^{-1} (d) None of these

76 A hollow cylinder and solid sphere of mass M and radius r are rotating about an axis passing through its centre. If torques of equal magnitude are applied to them, then the ratio of angular accelerations produced is

(a) $\frac{2}{5}$ (b) $\frac{5}{2}$ (c) $\frac{5}{4}$ (d) $\frac{4}{5}$

77 A rope is wound around a hollow cylinder of mass 3 kg and radius 40 cm. What is the angular acceleration of the cylinder, if the rope is pulled with a force of 30 N? ***NEET 2017***

(a) 25 m/s^2 (b) 0.25 rad/s^2 (c) 25 rad/s^2 (d) 5 m/s^2

78 A solid cylinder of mass 50 kg and radius 0.5 m is free to rotate about the horizontal axis. A massless string is wound round the cylinder with one end attached to it and other hanging freely. Tension in the string required to produce an angular acceleration of 2 revs^{-2} is ***CBSE AIPMT 2014***

(a) 25 N (b) 50 N (c) 78.5 N (d) 157 N

79 Two rotating bodies A and B of masses m and $2m$ with moments of inertia I_A and I_B ($I_B > I_A$) have equal kinetic energy of rotation. If L_A and L_B be their angular momenta respectively, then ***NEET 2016***

(a) $L_A > L_B$ (b) $L_A = \frac{L_B}{2}$
(c) $L_A = 2L_B$ (d) $L_B > L_A$

80 A solid sphere is rotating freely about its symmetry axis in free space. The radius of the sphere is increased keeping its mass same. Which of the following physical quantities would remain constant for the sphere? ***NEET 2018***

(a) Rotational kinetic energy
(b) Moment of inertia
(c) Angular velocity
(d) Angular momentum

81 A thin horizontal circular disc is rotating about a vertical axis passing through its centre. An insect is at rest at a point near the rim of disc. The insect now moves along a diameter of the disc to reach its other end. During the journey of the insect, the angular speed of the disc ***AIIMS 2018***

(a) continuously decreases
(b) continuously increases
(c) first increases and then decreases
(d) remains unchanged

82 An ice skater spins at 3π rads^{-1} with her arms extended. If her moment of inertia with arms folded is 75% of that with arms extended, her angular velocity when she folds her arms is

(a) π rad s^{-1} (b) 2π rad s^{-1} (c) 3π rad s^{-1} (d) 4π rad s^{-1}

83 A disc of moment of inertia 2 kg-m^2 revolving with 8 rad/s is placed on another disc of moment of inertia 4 kg-m^2 revolving with 4 rad/s. What is the angular frequency of composite disc? ***JIPMER 2018***

(a) 4 rad/s (b) $\frac{3}{16}$ rad/s (c) $\frac{16}{3}$ rad/s (d) $\frac{16}{5}$ rad/s

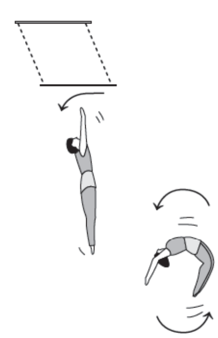
84 Two discs of same moment of inertia rotating about their regular axis passing through centre and perpendicular to the plane of disc with angular velocities ω_1 and ω_2. They are brought into contact face to face coinciding the axis of rotation. The expression for loss of energy during this process is ***NEET 2017***

(a) $\frac{1}{2}I(\omega_1+\omega_2)^2$ (b) $\frac{1}{4}I(\omega_1-\omega_2)^2$
(c) $I(\omega_1-\omega_2)^2$ (d) $\frac{I}{8}(\omega_1-\omega_2)^2$

85 If frictional force is neglected and girl bends her hand, then (initially girl is rotating on chair) ***JEE Main 2012***

(a) I_{girl} will reduce (b) I_{girl} will increase
(c) ω_{girl} will reduce (d) None of the above

86 When acrobat bends his body (assume no external torque)



(a) I_{acrobat} decreases (b) I_{acrobat} increases
(c) ω_{acrobat} increases (d) Both (a) and (c)

TOPIC 6 ~ Rolling Motion

87 A drum of radius R and mass M rolls down without slipping along an inclined plane of angle θ. The frictional force

(a) converts translational energy into rotational energy
(b) dissipates energy as heat
(c) decreases the rotational motion
(d) decreases the rotational and translational motion

88 Kinetic energy of a rolling body will be

(a) $\frac{1}{2}mv_{CM}^2\left(1+\frac{k^2}{R^2}\right)$ (b) $\frac{1}{2}I\omega^2$
(c) $\frac{1}{2}mv_{CM}^2$ (d) None of these

89 A body is rolling down an inclined plane. Its translational and rotational kinetic energies are equal. The body is a

(a) solid sphere (b) hollow sphere
(c) solid cylinder (d) hollow cylinder

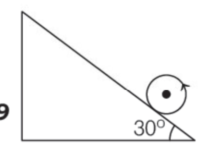
90 A solid sphere of mass 1 kg and radius 10 cm rolls down an inclined plane of height 7 m. The velocity of its centre as it reaches the ground level is

(a) 7 ms^{-1} (b) 10 ms^{-1} (c) 15 ms^{-1} (d) 20 ms^{-1}

91 A round uniform body of radius R, mass M and moment of inertia I, rolls down (without slipping) an inclined plane making an angle θ with the horizontal. Then, its acceleration is

(a) $\frac{g\sin\theta}{1+(I/MR^2)}$ (b) $\frac{g\sin\theta}{1+(MR^2/I)}$
(c) $\frac{g\sin\theta}{1-(I/MR^2)}$ (d) $\frac{g\sin\theta}{1-(MR^2/I)}$

92 A sphere pure rolls on a rough inclined plane with initial velocity 2.8 m/s. Find the maximum distance on the inclined plane. ***AIIMS 2019***



(a) 2.74 m (b) 5.48 m
(c) 1.38 m (d) 3.2 m

93 Three bodies, a ring, a solid cylinder and a solid sphere roll down the same inclined plane without slipping. They start from rest. The radii of the bodies are identical. Which of the bodies reaches the ground with maximum velocity?

(a) Solid sphere
(b) Ring
(c) Solid cylinder
(d) All will have same velocity

94 A solid sphere is in rolling motion. In rolling motion, a body possesses translational kinetic energy K_t as well as rotational kinetic energy K_r simultaneously. The ratio $K_t:(K_t+K_r)$ for the sphere is ***NEET 2018***

(a) 10 : 7 (b) 5 : 7 (c) 7 : 10 (d) 2 : 5

SPECIAL TYPES QUESTIONS

I. Assertion and Reason

■ **Direction** (Q. Nos. 95-102) *In the following questions, a statement of Assertion is followed by a corresponding statement of Reason. Of the following statements, choose the correct one.*

(a) Both Assertion and Reason are correct and Reason is the correct explanation of Assertion.
(b) Both Assertion and Reason are correct but Reason is not the correct explanation of Assertion.
(c) Assertion is correct but Reason is incorrect.
(d) Assertion is incorrect but Reason is correct.

95 **Assertion** The centre of mass of a body must lie on the body.

Reason The centre of mass of a body lie at the geometric centre of body.

96 **Assertion** The motion of the centre of mass describes the translational part of the motion.

Reason Translational motion always means straight line motion.

97 **Assertion** For a system of particles under central force field, the total angular momentum is conserved.

Reason The torque acting on such a system is zero.

98 **Assertion** When a particle is moving in a straight line with a uniform velocity, its angular momentum is constant.

Reason The angular momentum is zero when particle moves with a uniform velocity.

99 **Assertion** Inertia and moment of inertia are same quantities.

Reason Inertia represents the capacity of a body to oppose its state of motion or rest.

100 **Assertion** Moment of inertia of a particle is same, whatever be the axis of rotation.

Reason Moment of inertia depends on mass and distance of the particle from the axis of rotation.

101 **Assertion** If bodies slide down an inclined plane without rolling, then all bodies reach the bottom simultaneously is not necessary.

Reason Acceleration of all bodies are equal and independent of the shape.

102 **Assertion** The work done against force of friction in the case of a disc rolling without slipping down an inclined plane is zero.

Reason When the disc rolls without slipping, friction is required because for rolling condition velocity of point of contact is zero.

II. Statement Based Questions

103 An external force is applied on a rigid body so that it comes in motion. Which of these statements is incorrect?

I. If the total force on the body is zero, then the total linear momentum of the body does not change with time.

II. If the total torque on the rigid body is zero, the total angular momentum of the body does not change with time.

III. Volume of rigid body remains constant.

IV. Shape of rigid body can change easily.

(a) Only I (b) Only II
(c) Only III (d) Only IV

104 Which of these statement(s) is/are incorrect?

I. Moment of force is a vector quantity.

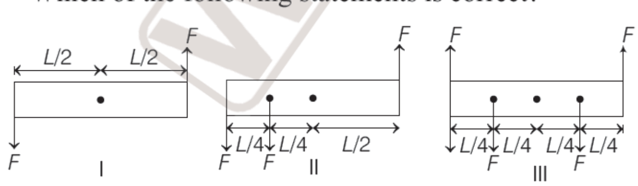
II. Angular momentum may not be parallel to angular velocity.

III. Angular momentum of a particle moving in a straight line is always constant with respect to fixed point.

IV. Moment of inertia of a body remains same irrespective of position of axis of rotation.

(a) Only I (b) Only II
(c) Only III (d) Only IV

105 A rigid rod of length L is acted upon by some forces. All forces labelled F have the same magnitude. Which of the following statements is correct?



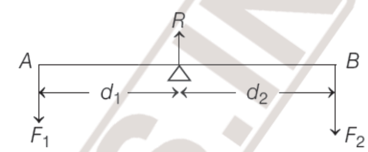
(a) Both I and II cases will have a non-zero net torque acting on the rod about its centre

(b) Both II and III cases will have a non-zero net torque acting on the rod about its centre

(c) Both I and III cases will have a non-zero net torque acting on the rod about its centre

(d) The net torque is zero in all cases

106 A lever system is shown in the figure below.



F_1 is usually some weight to be lifted and force F_2 is effort applied to lift the load. Choose the correct statements.

I. The ratio of $\frac{F_1}{F_2}$ is equal to mechanical advantage, i.e. $\text{MA} = \frac{F_1}{F_2} = \frac{d_2}{d_1}$.

II. If the effort arm d_2 is larger than the load arm, then a small effort can be used to lift a large load.

III. At equilibrium, load arm × effort arm = load × effort.

(a) Both I and III (b) Both III and II
(c) Both I and II (d) All of these

107 A body rolls down on an inclined plane without slipping, choose the correct statements.

I. KE of body is conserved.

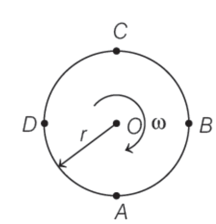
II. Angular momentum of body about its own centre of mass is not conserved.

III. Velocity at any instant of time does not depend on mass.

IV. At bottom all objects (cylinder, sphere, ring) possess equal rotational KE.

(a) Both I and III (b) I, II and III
(c) I, II and IV (d) Only IV

108 For a rolling motion (without slipping) of a disc on a level surface is shown in figure, choose the correct statements.



I. $v_A = 0$

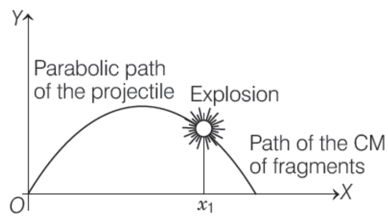
II. $v_B = v_D > v_C$

III. $v_C = 2r\omega$

IV. Velocity of centre of mass of body in pure rolling is zero.

(a) Both I and II (b) Both II and III
(c) I, II and IV (d) Both I and III

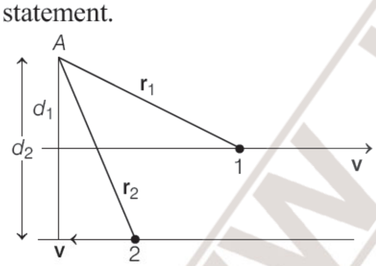
109 A projectile is fired at an angle and it was following a parabolic path. Suddenly, it explodes into fragments. Choose the correct statement regarding this situation.



(a) Due to explosion CM shifts upwards.
(b) Due to explosion CM shifts downwards.
(c) Due to explosion CM traces its path back to origin.
(d) CM continues to move along same parabolic path.

110 Choose the correct statements.
(a) For a general rotational motion, angular momentum **L** and angular velocity ω need not be parallel.
(b) For a rotational motion about a fixed axis, angular momentum **L** and angular velocity ω are always parallel.
(c) For a general translational motion, momentum **p** and velocity **v** are always perpendicular.
(d) For a general translational motion, acceleration **a** and velocity **v** are always parallel.

111 Figure shows two identical particles 1 and 2, each of mass m, moving in opposite directions with same speed **v** along parallel lines. At a particular instant, $\mathbf{r}_1$ and $\mathbf{r}_2$ are their respective position vectors drawn from point A which is in the plane of the parallel lines. Choose the correct statement.

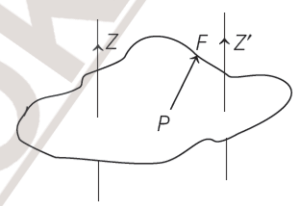


(a) Angular momentum L_1 of particle 1 about A is $mv(d_1)$ ⊗.
(b) Angular momentum L_2 of particle 2 about A is $mv\mathbf{r}_2$ ⊙.
(c) Total angular momentum of the system about A is $L = mv(\mathbf{r}_1 + \mathbf{r}_2)$ ⊙.
(d) Total angular momentum of the system about A is $L = mv(d_2 - d_1)$ ⊗.

where, ⊙ represents a unit vector coming out of the page and ⊗ represents a unit vector going into the page.

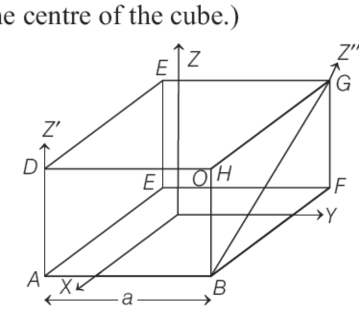
112 The net external torque on a system of particles about an axis is zero. Which of the following statement is incorrect?
(a) The forces may be acting radially from a point on the axis.
(b) The forces may be acting on the axis of rotation.
(c) The forces may be acting perpendicular to the axis of rotation.
(d) The torque caused by some forces may be equal and opposite to that caused by other forces.

113 Figure shows a lamina in xy-plane. Two axes Z and Z' pass perpendicular to its plane. A force **F** acts in the plane of lamina at point P as shown. Which of the following statement is correct? (The point P is closer to Z'-axis than the Z-axis.)



(a) Torque τ caused by **F** about Z-axis is along $-\hat{\mathbf{k}}$.
(b) Torque τ' caused by **F** about Z'-axis is along $-\hat{\mathbf{k}}$.
(c) Torque τ caused by **F** about Z-axis is less in magnitude than that about Z'-axis.
(d) Total torque is given by $\tau = \tau + \tau'$.

114 With reference to figure of a cube of edge a and mass m, state whether the following statements are correct. (O is the centre of the cube.)



(a) The moment of inertia of cube about Z-axis is $I_Z = I_X + I_Y$
(b) The moment of inertia of cube about Z'-axis is $I_{Z'} = I_Z + \frac{ma^2}{2}$
(c) The moment of inertia of cube about Z''-axis is $= I_Z + \frac{ma^2}{2}$
(d) $I_X \neq I_Y$

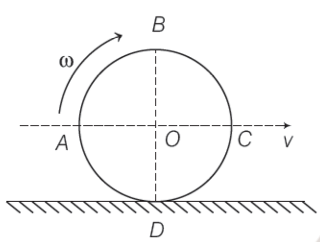
115 A man standing on a platform holds weights in his outstretched arms. The system rotates freely about a central vertical axis. If he now draws the weights inwards close to his body, then choose the incorrect statement.

(a) The angular velocity of the system will increase.
(b) The angular momentum of the system will decrease.
(c) The kinetic energy of the system will increase.
(d) He will have to expand some energy to draw the weights.

116 A uniform rod kept vertically on the ground falls from rest. Its foot does not slip on the ground, then choose the incorrect statement.

(a) No part of the rod can have acceleration greater than g in any position.
(b) At any position of the rod, different points on it have different accelerations.
(c) Any particular point on the rod has different accelerations for different positions of the rod
(d) The maximum acceleration of any point on the rod, at any position, is 1.5 g.

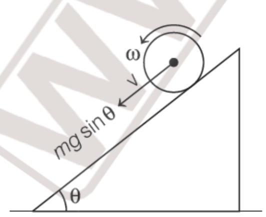
117 A ring rolls without slipping on a horizontal surface. At any instant, its position is as shown in the figure.



Which of the following statement is incorrect?

(a) Section ABC has greater kinetic energy than section ADC.
(b) Section BC has greater kinetic energy than section CD.
(c) Section BC has the same kinetic energy as section DA.
(d) None of the above

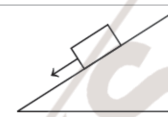
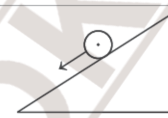
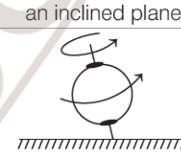
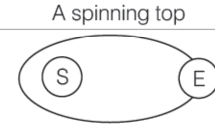
118 Sphere is in pure accelerated rolling motion in the figure shown, choose the correct statement.



(a) The direction of f_s is upwards.
(b) The direction of f_s is downwards.
(c) The direction of gravitational force is upwards.
(d) The direction of normal reaction is downwards.

III. Matching Type

119 Match the examples given in Column I with the type of motion they are executing in Column II and select the correct answer from the codes given blow. There is no information about nature of surfaces of bodies as given.

	Column I		Column II
A.	 A block over an inclined plane	1.	Rolling
B.	 A cylinder over an inclined plane	2.	Translation
C.	 A spinning top	3.	Rotation
D.	 Earth-Sun system	4.	Precession

	A	B	C	D		A	B	C	D
(a)	2	1	3	4	(b)	2	3	1	2
(c)	3	1	2	1	(d)	3	1	2	4

120 Match the Column I (rotation of different bodies) with Column II (their moment of inertia) and select the correct answer from the codes given below.

	Column I		Column II
A.	Thin circular ring of radius R having axis perpendicular to the plane and passing through centre	1.	$MR^2/2$
B.	Thin circular ring of radius R having axis passing through its diameter	2.	$ML^2/12$
C.	Thin rod of length L about an axis perpendicular to the rod and passing through mid-point	3.	MR^2
D.	Circular disc of radius R about an axis perpendicular to the disc and passing through the centre.	4.	$MR^2/4$

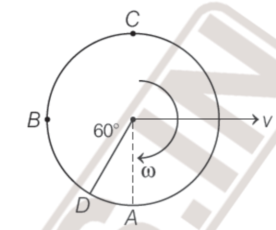
	A	B	C	D		A	B	C	D
(a)	3	2	2	1	(b)	2	3	1	2
(c)	3	1	2	1	(d)	3	1	2	4

121 Match the Column I (rotation of different bodies) with Column II (their moment of inertia) and select the correct answer from the codes given below.

	Column I		Column II
A.	Circular disc of radius R about an axis passing through the diameter	1.	$MR^2/4$
B.	Hollow cylinder of radius R about an axis passing through the axis of cylinder	2.	MR^2
C.	Solid cylinder of radius R about an axis passing through the axis of cylinder.	3.	$MR^2/2$
D.	Solid sphere of radius R about an axis passing through its diameter.	4.	$(2/5)\ MR^2$

	A	B	C	D		A	B	C	D
(a)	4	3	2	1	(b)	1	2	3	4
(c)	1	3	2	4	(d)	2	1	3	4

122 A rigid body is rolling without slipping on the horizontal surface, then match the Column I with Column II and select the correct answer from the codes given below.



	Column I		Column II
A.	Velocity at point A, i.e. v_A	1.	$v\sqrt{2}$
B.	Velocity at point B, i.e. v_B	2.	zero
C.	Velocity at point C, i.e. v_C	3.	v
D.	Velocity at point D, i.e. v_D	4.	$2v$

	A	B	C	D		A	B	C	D
(a)	2	1	4	3	(b)	1	3	4	2
(c)	4	3	2	1	(d)	2	3	4	1

NCERT & NCERT Exemplar

MULTIPLE CHOICE QUESTIONS

NCERT

123 In the HCl molecule, the separation between the nuclei of the two atoms is about 1.27 Å (1 Å $=10^{-10}$ m).

What is the approximate location of the centre of mass of the molecule, given that a chlorine atom is about 35.5 times as massive as a hydrogen atom and nearly all the mass of an atom is concentrated in its nucleus?

(a) 1.24 Å from H-atom (b) 1.1 Å from H-atom
(c) 1 Å from H-atom (d) None of the above

124 A child sits stationary at one end of a long trolley moving uniformly with a speed v on a smooth horizontal floor. If the child gets up and runs about on the trolley in any manner, what is the speed of the centre of mass of the (trolley + child) system?

(a) Increase (b) Remains constant
(c) Decrease (d) None of the above

125 A particle has momentum $\mathbf{p}$ with components p_x, p_y and p_z, has a position vector $\mathbf{r}$ with components x, y and z. If particle moves in the xoy-plane, then

(a) angular momentum of particle is zero
(b) angular momentum of particle has only x-component
(c) angular momentum of particle has only y-component
(d) angular momentum of particle has only z-component

126 Two particles, each of mass m and speed v, travel in opposite directions along parallel lines separated by a distance d. Then, choose the correct statement.

(a) Angular momentum of system about a point depends on choice of position of point.
(b) Angular momentum of system is zero.
(c) Angular momentum of system about any point in space is constant.
(d) Angular momentum keeps on increasing.

127 A car has the weight of 1800 kg. The distance between its front and back axles is 1.8 m. Its centre of gravity is 1.05 m behind the front axle. What are the forces exerted by the level ground on each front wheel and each back wheel?

(a) 3275 N and 5000 N
(b) 3675 N and 5145 N
(c) 3675 N and 4565 N
(d) 3000 N and 5000 N

128 Torques of equal magnitude are applied to a hollow cylinder and a solid sphere, both having the same mass and radius. The cylinder is free to rotate about its standard axis of symmetry and the sphere is free to rotate about an axis passing through its centre.

Which of the two will acquire a greater angular speed after a given time?

(a) Sphere
(b) Hollow cylinder
(c) Both acquire same speed in same time
(d) Data is insufficient to reach any conclusion

129 A child stands at the centre of a turntable with his two arms outstretched. The turntable is set rotating with an angular speed of 40 rev $\min^{-1}$. How much is the angular speed of the child, if he folds his hands back and thereby reduces his moment of inertia to (2/5) times the initial value? Assume that the turntable rotates without friction.

(a) 40 rpm (b) 45 rpm
(c) 55 rpm (d) 100 rpm

130 To maintain a rotor at a uniform angular speed of 200 rads^{-1}, an engine needs to transmit a torque of 180 N-m. What is the power required by the engine?

Note Uniform angular velocity in the absence of friction implies zero torque. In practice, applied torque is needed to counter frictional torque). Assume that the engine is 100% efficient.

(a) 50 kW (b) 20 kW
(c) 16 kW (d) 36 kW

131 A metre stick is balanced on a knife edge at its centre. When two coins, each of mass 5 g are put one on top of the other at the 12 cm mark, the stick is found to be balanced at 45 cm. What is the mass of the metre stick?

(a) 66 g (b) 70 g
(c) 50 g (d) 55 g

132 A hoop of radius 2 m weighs 100 kg. It rolls along a horizontal floor so that its centre of mass has a speed of 20 cms^{-1}. How much work has to be done to stop it?

(a) 10 J (b) 12 J
(c) 4 J (d) 3 J

133 A cylinder rolls up an inclined plane of angle of inclination 30°. At the bottom of the inclined plane, the centre of mass of the cylinder has speed of 5 ms^{-1}. How far will the cylinder go up the plane?

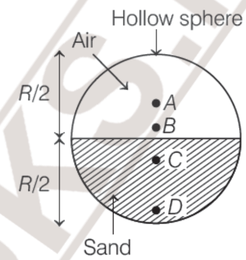
(a) 2.5 m (b) 3.83 m
(c) 4 m (d) 4.9 m

NCERT Exemplar

134 For which of the following does the centre of mass lie outside the body?

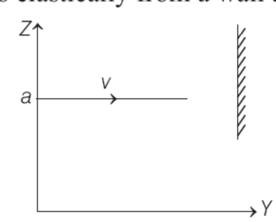
(a) A pencil (b) A shot put
(c) A dice (d) A bangle

135 Which of the following points is the likely position of the centre of mass of the system shown in figure?



(a) A (b) B (c) C (d) D

136 A particle of mass m is moving in yz-plane with a uniform velocity v with its trajectory running parallel to positive Y-axis and intersecting Z-axis at $z = a$ in figure. The change in its angular momentum about the origin as it bounces elastically from a wall at y constant is

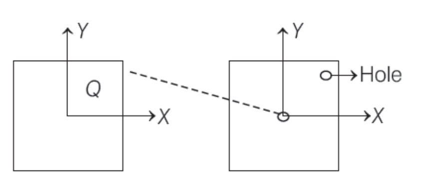


(a) $mva\,\hat{\mathbf{e}}_x$ (b) $2\,mva\,\hat{\mathbf{e}}_x$ (c) $ymv\,\hat{\mathbf{e}}_x$ (d) $2\,ymv\,\hat{\mathbf{e}}_x$

137 When a disc rotates with uniform angular velocity, which of the following statement is not correct?

(a) The sense of rotation remains same.
(b) The orientation of the axis of rotation remains same.
(c) The speed of rotation is non-zero and remains same.
(d) The angular acceleration is non-zero and remains same.

138 A uniform square plate has a small piece Q of an irregular shape removed and glued to the centre of the plate leaving a hole behind as shown in figure below. The moment of inertia about the Z-axis is



(a) increases
(b) decreases
(c) the same
(d) changed in unpredicted manner

139 The density of a non-uniform rod of length 1m is given by $\rho(x) = a(1 + bx^2)$, where a and b are constants and $0 \leq x \leq 1$. The centre of mass of the rod will be at

(a) $\frac{3(2+b)}{4(3+b)}$ (b) $\frac{4(2+b)}{3(3+b)}$

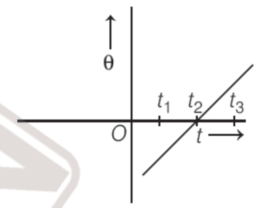
(c) $\frac{3(3+b)}{4(2+b)}$ (d) $\frac{4(3+b)}{3(2+b)}$

140 A merry-go-round, made of a ring like platform of radius R and mass M, is revolving with angular speed ω. A person of mass M is standing on it. At one instant, the person jumps off the round, radially away from the centre of the round (as seen from the round). The speed of the round afterwards is

(a) 2ω (b) ω

(c) $\frac{\omega}{2}$ (d) 0

141 The variation of angular position θ of a point on a rotating rigid body with time t is shown in figure.



In which direction, the body is rotating?

(a) Clockwise
(b) Anti-clockwise
(c) May be clockwise or anti-clockwise
(d) None of the above

142 A disc of radius R is rotating with an angular speed ω_0 about a horizontal axis. It is placed on a horizontal table. The coefficient of kinetic friction is μ_k. What was the velocity of its centre of mass before being brought in contact with the table?

(a) $\omega_0 R$ (b) Zero (c) $\frac{\omega_0 R}{2}$ (d) $2\omega_0 R$

Answers

> Mastering NCERT with MCQs

1 (c)	2 (b)	3 (d)	4 (c)	5 (c)	6 (c)	7 (d)	8 (b)	9 (a)	10 (b)
11 (b)	12 (a)	13 (c)	14 (a)	15 (a)	16 (d)	17 (a)	18 (d)	19 (b)	20 (a)
21 (d)	22 (c)	23 (c)	24 (d)	25 (d)	26 (c)	27 (b)	28 (b)	29 (d)	30 (c)
31 (a)	32 (b)	33 (a)	34 (c)	35 (c)	36 (b)	37 (a)	38 (c)	39 (c)	40 (c)
41 (d)	42 (d)	43 (a)	44 (c)	45 (a)	46 (c)	47 (b)	48 (b)	49 (c)	50 (a)
51 (a)	52 (b)	53 (d)	54 (d)	55 (b)	56 (b)	57 (b)	58 (b)	59 (b)	60 (d)
61 (a)	62 (a)	63 (d)	64 (a)	65 (a)	66 (a)	67 (d)	68 (d)	69 (c)	70 (d)
71 (a)	72 (c)	73 (c)	74 (a)	75 (c)	76 (a)	77 (c)	78 (d)	79 (d)	80 (d)
81 (c)	82 (d)	83 (c)	84 (b)	85 (a)	86 (d)	87 (a)	88 (a)	89 (d)	90 (b)
91 (a)	92 (c)	93 (a)	94 (b)						

> Special Types Questions

95 (d)	96 (c)	97 (a)	98 (c)	99 (d)	100 (d)	101 (c)	102 (a)	103 (d)	104 (d)
105 (a)	106 (c)	107 (d)	108 (d)	109 (d)	110 (a)	111 (d)	112 (c)	113 (b)	114 (b)
115 (b)	116 (a)	117 (c)	118 (a)	119 (a)	120 (c)	121 (b)	122 (a)		

> NCERT & NCERT Exemplar MCQs

123 (a)	124 (b)	125 (d)	126 (c)	127 (b)	128 (a)	129 (d)	130 (d)	131 (a)	132 (c)
133 (b)	134 (d)	135 (c)	136 (b)	137 (d)	138 (b)	139 (a)	140 (a)	141 (b)	142 (b)

Hints & Explanations

1 (*c*) A rigid body does not deform under action of applied force and there is no relative motion of any two particles constituting that rigid body. So, it means that a system of particles is called a rigid body, when any two particles of system have a zero relative velocity.

4 (*c*) Centre of mass of, system of two particles is

$$r_{CM} = \frac{m_1 r_1 + m_2 r_2}{m_1 + m_2}$$

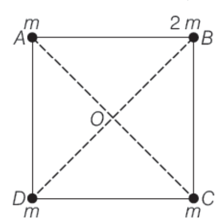
If $m_1 + m_2 = M$ = total mass of the particles

$$r_{CM} = \frac{m_1 r_1 + m_2 r_2}{M}$$

$$\therefore \qquad r_{CM} \propto 1/M$$

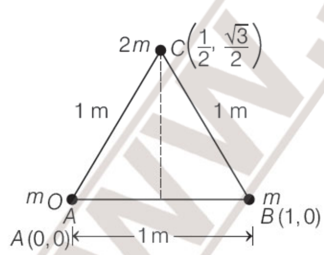
So, the above relation clearly shows that the centre of mass of a system of two particles divide the distance between them in inverse ratio of masses of particles.

5 (*c*) If all the masses were same, the CM was at *O*.



But as the mass at *B* is 2*m*, so the CM of the system will shift towards *B*. So, CM will be on line *OB*.

6 (*c*) The centre of mass is given by



$$x_{CM} = \frac{m_1 x_1 + m_2 x_2 + m_3 x_3}{m_1 + m_2 + m_3}$$

$$x_{CM} = \frac{m \times 0 + m \times 1 + 2m \times \left(\frac{1}{2}\right)}{m + m + 2m}$$

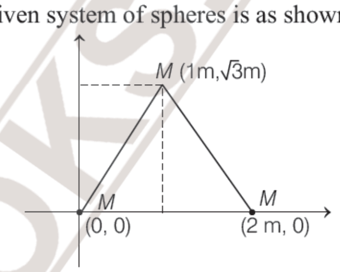
$$x_{CM} = \frac{2m}{4m} = \frac{1}{2}\,\text{m}$$

$$y_{CM} = \frac{m_1 y_1 + m_2 y_2 + m_3 y_3}{m_1 + m_2 + m_3}$$

$$y_{CM} = \frac{m \times 0 + m \times 0 + 2m \times \frac{\sqrt{3}}{2}}{m + m + 2m} = \frac{\sqrt{3}}{4}\,\text{m}$$

Hence, the centre of mass is $\left(\frac{1}{2}\,\text{m}, \frac{\sqrt{3}}{4}\,\text{m}\right)$.

7 (*d*) The given system of spheres is as shown below



The *x* and *y*-coordinates of centre of mass is

$$x = \frac{\Sigma m_i x_i}{\Sigma m_i} = \frac{M \times 0 + M \times 1 + M \times 2}{M + M + M} = 1$$

$$y = \frac{\Sigma m_i y_i}{\Sigma m_i} = \frac{M \times 0 + M\,(\sqrt{3}) + M \times 0}{M + M + M}$$

$$y = \frac{\sqrt{3}M}{3M} = \frac{1}{\sqrt{3}}$$

So, position vector of the centre of mass is $\left(\hat{\mathbf{i}} + \frac{\hat{\mathbf{j}}}{\sqrt{3}}\right)$.

8 (*b*) Centre of mass of a system of particles is given by

$$x_{CM} = \frac{1 \times x_1 + 2 \times x_2 + 3 \times x_3}{1 + 2 + 3} = 3$$

$$[\because x_{CM} = y_{CM} = z_{CM} = 3]$$

$$\Rightarrow \quad x_1 + 2x_2 + 3x_3 = (1 + 2 + 3)3 = 18 \qquad \ldots \text{(i)}$$

When fourth particle is placed, then

$$x_{CM} = y_{CM} = z_{CM} = 1 \qquad \text{(given)}$$

$$\Rightarrow x_{CM} = \frac{1 \times x_1 + 2 \times x_2 + 3 \times x_3 + 4 \times x_4}{(1 + 2 + 3 + 4)}$$

$$\Rightarrow x_1 + 2x_2 + 3x_3 + 4x_4 = 1\,(1 + 2 + 3 + 4) = 10 \quad \ldots \text{(ii)}$$

On solving Eqs. (i) and (ii), we get

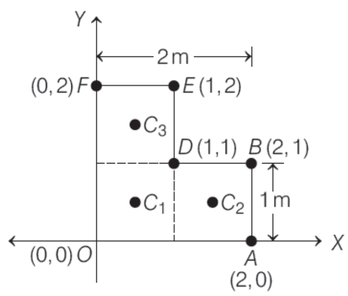
$$4x_4 = 10 - 18 \Rightarrow x_4 = -2$$

Similarly, $\quad y_4 = -2,\ z_4 = -2$

$\therefore$ The fourth particle must be placed at the point $(-2, -2, -2)$.

9 (*a*) We can think of the *L*-shape to consist of 3 squares each of length 1 m as shown in figure.
The mass of each square is 1 kg as the lamina is uniform.
The centres of masses C_1, C_2 and C_3 of the squares are (by symmetry) their geometric centres and have coordinates (1/2, 1/2), (3/2, 1/2) and (1/2, 3/2), respectively.

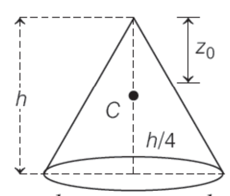
We take the masses of the squares to be concentrated at these points. The centre of mass of the whole L-shape (X, Y) is the centre of mass of these mass points.



Hence, $X = \dfrac{[1(1/2) + 1(3/2) + 1\,(1/2)]\ \text{kg-m}}{(1+1+1)\ \text{kg}} = \dfrac{5}{6}\text{m}$

$$Y = \frac{[1(1/2) + 1(1/2) + 1(3/2)]\ \text{kg - m}}{(1+1+1)\ \text{kg}} = \frac{5}{6}\ \text{m}$$

10 (*b*) We know that, centre of mass of a uniform solid cone of height h is at height $\dfrac{h}{4}$ from base as shown in figure, therefore



$$h - z_0 = \frac{h}{4} \text{ or } z_0 = h - \frac{h}{4} = \frac{3h}{4}$$

11 (*b*) As, velocity of centre of mass is

$$v_{CM} = \frac{m_1 v_1 + m_2 v_2 + m_3 v_3}{m_1 + m_2 + m_3}$$
$$= \frac{5 \times 5 + 4 \times 4 + 2 \times 2}{5 + 4 + 2}$$
$$= \frac{25 + 16 + 4}{11} = \frac{45}{11} = 4.09 \approx 4\ \text{m/s}$$

12 (*a*) Given, $\mathbf{v}_1 = 4\hat{\mathbf{i}}\ \text{ms}^{-1}$, $\mathbf{v}_2 = 4\hat{\mathbf{j}}\ \text{ms}^{-1}$

$$\mathbf{a}_1 = (2\hat{\mathbf{i}} + 2\hat{\mathbf{j}})\ \text{ms}^{-2}, \mathbf{a}_2 = 0\ \text{ms}^{-2},$$

$\therefore$ Velocity of centre of mass,

$$\mathbf{v}_{CM} = \frac{m_1 \mathbf{v}_1 + m_2 \mathbf{v}_2}{m_1 + m_2} = \frac{(\mathbf{v}_1 + \mathbf{v}_2)m}{2m} \quad [\because m_1 = m_2 = m]$$
$$= \frac{4\hat{\mathbf{i}} + 4\hat{\mathbf{j}}}{2} = 2(\hat{\mathbf{i}} + \hat{\mathbf{j}})\ \text{ms}^{-1}$$

Similarly, acceleration of centre of mass,

$$\mathbf{a}_{CM} = \frac{\mathbf{a}_1 + \mathbf{a}_2}{2} = \frac{2\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + 0}{2} = (\hat{\mathbf{i}} + \hat{\mathbf{j}})\ \text{ms}^{-2}$$

Since, from above values, it can be seen that $\mathbf{v}_{CM}$ is parallel to $\mathbf{a}_{CM}$, so the path will be a straight line.

13 (*c*) Here on the entire system, net external force is zero, hence the centre of mass remains unchanged.

14 (*a*) As the balls were initially at rest and the forces of attraction are internal, then their centre of mass (CM) will always remain at rest.

So, $\quad v_{CM} = 0$

As CM is at rest, they will meet at CM. Hence, they will meet at $l/2$ from any initial positions.

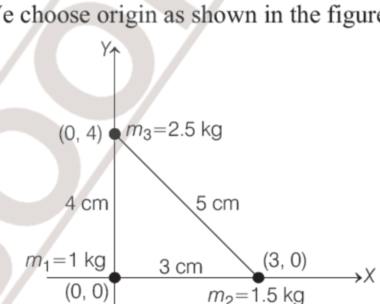
15 (*a*) As per the question, two particles A and B are initially at rest, move towards each other under a mutual force of attraction. It means that, no external force is applied on the system. Therefore, $F_{ext} = 0$

So, there is no acceleration of CM. This means velocity of the CM remain constant.

As, initial velocity of CM, $v_{i\,CM} = 0 \Rightarrow$ final velocity of CM, $v_{f\,CM} = 0$

So, the speed of centre of mass of system will be zero.

16 (*d*) We choose origin as shown in the figure.



Using $x_{CM} = \dfrac{m_1 x_1 + m_2 x_2 + m_3 x_3}{m_1 + m_2 + m_3}$, we have

$$x_{CM} = \frac{1 \times 0 + 1.5 \times 3 + 2.5 \times 0}{1 + 1.5 + 2.5} = 0.9\ \text{cm}$$

Similarly,

$$y_{CM} = \frac{1 \times 0 + 1.5 \times 0 + 2.5 \times 4}{1 + 1.5 + 2.5} = 2.0\ \text{cm}$$

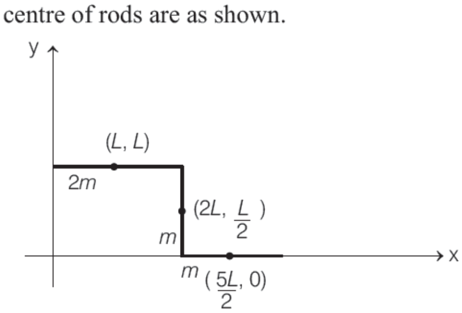
So, centre of mass (CM) is 0.9 cm right and 2.0 cm above 1 kg mass.

17 (*a*) Coordinates of centre of mass (COM) are given by

$$X_{COM} = \frac{m_1 x_1 + m_2 x_2 + m_3 x_3}{m_1 + m_2 + m_3}$$

and $\quad Y_{COM} = \dfrac{m_1 y_1 + m_2 y_2 + m_3 y_3}{m_1 + m_2 + m_3}$

For given system of rods, masses and coordinates of centre of rods are as shown.



Hints & Explanations

So, $X_{\text{COM}} = \left(\dfrac{2mL + m2L + m\dfrac{5L}{2}}{4\,m}\right) = \dfrac{13}{8}L$

and $Y_{\text{COM}} = \dfrac{2mL + m \times \dfrac{L}{2} + m \times 0}{4\,m} = \dfrac{5L}{8}$

So, position vector of COM is

$$\mathbf{r}_{\text{COM}} = X_{\text{COM}}\hat{\mathbf{x}} + Y_{\text{COM}}\hat{\mathbf{y}} = \frac{13}{8}L\hat{\mathbf{x}} + \frac{5}{8}L\hat{\mathbf{y}}$$

18 (*d*) Given, $\mathbf{A} = 2\hat{\mathbf{i}} + \hat{\mathbf{j}} - \hat{\mathbf{k}}$, $\mathbf{B} = -\hat{\mathbf{i}} - \hat{\mathbf{j}} + 2\hat{\mathbf{k}}$

$\therefore$ Vector product, $\mathbf{A} \times \mathbf{B} = \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ 2 & 1 & -1 \\ -1 & -1 & 2 \end{vmatrix}$

$= \hat{\mathbf{i}}(2-1) - \hat{\mathbf{j}}(4-1) + \hat{\mathbf{k}}(-2+1) = \hat{\mathbf{i}} - 3\hat{\mathbf{j}} - \hat{\mathbf{k}}$

19 (*b*) Given, $\mathbf{P} = 2\hat{\mathbf{i}} - 3\hat{\mathbf{j}} + \hat{\mathbf{k}}$,

$\mathbf{Q} = -4\hat{\mathbf{i}} + 6\hat{\mathbf{j}} - 2\hat{\mathbf{k}}$

$\therefore \quad \mathbf{P} \times \mathbf{Q} = \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ 2 & -3 & 1 \\ -4 & 6 & -2 \end{vmatrix}$

$\therefore PQ \sin\theta = \hat{\mathbf{i}}(6-6) - \hat{\mathbf{j}}(-4+4) + \hat{\mathbf{k}}(12-12)$

$(\because \ \mathbf{P} \times \mathbf{Q} = PQ\sin\theta)$

$\Rightarrow \quad PQ\sin\theta = 0$

$\Rightarrow \quad \sin\theta = 0$

$\therefore \quad \theta = 0°$

20 (*a*) Given, $\mathbf{A} = 2\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}$ and $\mathbf{B} = \hat{\mathbf{i}} - \hat{\mathbf{j}} + 2\hat{\mathbf{k}}$

$\therefore \quad \mathbf{A} \times \mathbf{B} = \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ 2 & 1 & 1 \\ 1 & -1 & 2 \end{vmatrix}$

$\Rightarrow \quad \mathbf{A} \times \mathbf{B} = (2+1)\hat{\mathbf{i}} - (4-1)\hat{\mathbf{j}} + (-2-1)\hat{\mathbf{k}}$

$= 3\hat{\mathbf{i}} - 3\hat{\mathbf{j}} - 3\hat{\mathbf{k}}$

$\therefore \quad |\mathbf{A} \times \mathbf{B}| = \sqrt{3^2 + (-3)^2 + (-3)^2} = 3\sqrt{3}$

$\therefore$ Unit vector perpendicular to vector $\mathbf{A}$ and $\mathbf{B}$ is given as

$$\hat{\mathbf{n}} = \frac{\mathbf{A} \times \mathbf{B}}{|\mathbf{A} \times \mathbf{B}|} = \frac{3\hat{\mathbf{i}} - 3\hat{\mathbf{j}} - 3\hat{\mathbf{k}}}{3\sqrt{3}} = \frac{1}{\sqrt{3}}(\hat{\mathbf{i}} - \hat{\mathbf{j}} - \hat{\mathbf{k}})$$

22 (*c*) Given, $\omega = 3\hat{\mathbf{i}} - 4\hat{\mathbf{j}} - \hat{\mathbf{k}}$ and $\mathbf{r} = 5\hat{\mathbf{i}} - 6\hat{\mathbf{j}} + 6\hat{\mathbf{k}}$

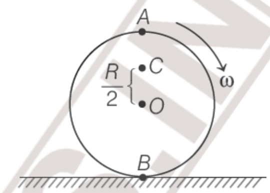
As, linear velocity, $\mathbf{v} = \omega \times \mathbf{r}$

$= \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ 3 & -4 & -1 \\ 5 & -6 & 6 \end{vmatrix}$

$= \hat{\mathbf{i}}(-24-6) - \hat{\mathbf{j}}(18+5) + \hat{\mathbf{k}}(-18+20)$

$= -30\hat{\mathbf{i}} - 23\hat{\mathbf{j}} + 2\hat{\mathbf{k}}$

23 (*c*) Velocity at a point on the circular plate (disc) $v = R\omega$, where r is the distance of point from O.



Since, $\quad R_A = R_B$

$v_A = v_B$

Also, $\quad R_C < R_A$ or R_B

$\Rightarrow \quad v_A = v_B > v_C$

24 (*d*) The position of point P on this rod through which the axis should pass, so that the work required to set the rod rotating with minimum angular velocity ω_0, is their centre of mass, we have,

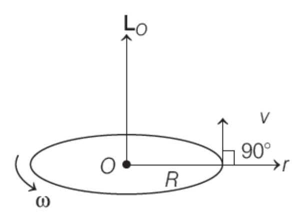
$m_1 x = m_2(L - x)$

$\Rightarrow \quad m_1 x = m_2 L - m_2 x$

$\Rightarrow \quad (m_1 + m_2)x = m_2 L \Rightarrow x = \dfrac{m_2 L}{m_1 + m_2}$

26 (*c*) Angular momentum of a particle about a point is given by $\quad \mathbf{L} = \mathbf{r} \times \mathbf{p} = m(\mathbf{r} \times \mathbf{v})$

For $\mathbf{L}_O$,



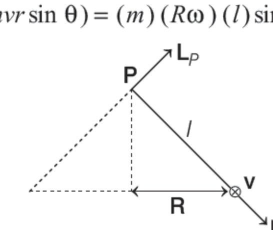
$|\mathbf{L}| = (mvr\sin\theta) = m(R\omega)(R)\sin 90° = mR^2\omega$

$=$ constant

Direction of $\mathbf{L}_O$ is always upwards, therefore $\mathbf{L}_O$ is constant, both in magnitude as well as direction.

For $\mathbf{L}_P$,

$|\mathbf{L}_P| = (mvr\sin\theta) = (m)(R\omega)(l)\sin 90° = (mRl\omega)$.



Magnitude of $\mathbf{L}_P$ will remains constant but direction of $\mathbf{L}_P$ keeps on changing, i.e. it varies with time.

27 (*b*) The rim keeps rotating in a vertical plane and the plane of rotation turns around the string A, i.e. the axis of rotation of the rim or its angular momentum precesses about the string A.

28 (*b*) If a force acts on a single particle at a point P whose position with respect to origin O is given by the position vector $\mathbf{r}$ as shown in given figure, the moment of the force acting on the particle with respect to the origin O is defined as the vector product.

$$\tau = \mathbf{r} \times \mathbf{F} \Rightarrow |\tau| = rF\sin\theta$$

29 (*d*) Given, $\mathbf{r} = \hat{\mathbf{i}} - \hat{\mathbf{j}} + \hat{\mathbf{k}}$

and $\mathbf{F} = 7\hat{\mathbf{i}} + 3\hat{\mathbf{j}} - 5\hat{\mathbf{k}}$

$$\therefore \quad \tau = \mathbf{r} \times \mathbf{F} = \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ 1 & -1 & 1 \\ 7 & 3 & -5 \end{vmatrix}$$

$$= (5-3)\hat{\mathbf{i}} - (-5-7)\hat{\mathbf{j}} + [3-(-7)]\hat{\mathbf{k}}$$

$$\Rightarrow \quad \tau = 2\hat{\mathbf{i}} + 12\hat{\mathbf{j}} + 10\hat{\mathbf{k}}$$

33 (*a*) Given, force, $\mathbf{F} = \alpha\hat{\mathbf{i}} + 3\hat{\mathbf{j}} + 6\hat{\mathbf{k}}$

and $\mathbf{r} = 2\hat{\mathbf{i}} - 6\hat{\mathbf{j}} - 12\hat{\mathbf{k}}$

As, angular momentum about origin is conserved.

i.e. $\tau = \text{constant}$

Torque, $\tau = 0 \Rightarrow \mathbf{r} \times \mathbf{F} = 0$

$$\Rightarrow \quad \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ 2 & -6 & -12 \\ \alpha & 3 & 6 \end{vmatrix} = 0$$

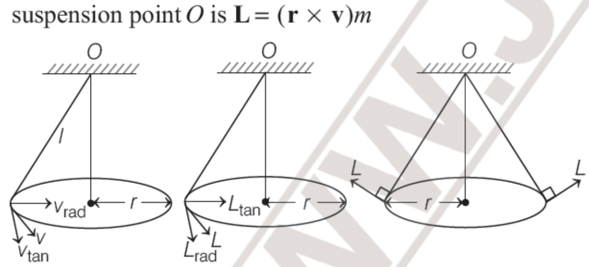
$$\Rightarrow \quad (-36+36)\hat{\mathbf{i}} - (12+12\alpha)\hat{\mathbf{j}} + (6+6\alpha)\hat{\mathbf{k}} = 0$$

$$\Rightarrow \quad 0\hat{\mathbf{i}} - 12(1+\alpha)\hat{\mathbf{j}} + 6(1+\alpha)\hat{\mathbf{k}} = 0$$

$$\Rightarrow \quad (1+\alpha) = 0 \Rightarrow \alpha = -1$$

So, value of α for which angular momentum about origin is conserved is -1.

34 (*c*) Angular momentum of the pendulum about the suspension point O is $\mathbf{L} = (\mathbf{r} \times \mathbf{v})m$



Then, $\mathbf{v}$ can be resolved into two components, radial component v_{rad} and tangential component v_{tan}. Due to v_{rad}, L will be tangential and due to v_{tan}, L will be radially outwards as shown in the figure.

So, net angular momentum will be as shown in figure, whose magnitude will be constant ($|L| = mvr$) but its direction will change as shown in the figure.

37 (*a*) If we take clockwise torque, then magnitude of total torque is $\tau_{net} = \tau_{F_1} + \tau_{F_2} + \tau_{F_3}$

$$0 = -F_1 r - F_2 r + F_3 r$$

$$\Rightarrow \quad F_3 = F_1 + F_2$$

39 (*c*) Mechanical advantage of a lever system is given as

$$\text{MA} = \frac{d_2(\text{effort arm})}{d_1(\text{load arm})}$$

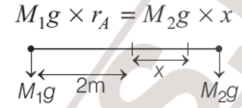
$\because \quad d_2 > d_1$

So, $\quad \text{MA} > 1$

40 (*c*) Let x be the distance from centre, then

For rotational equilibrium,

$$M_1 g \times r_A = M_2 g \times x$$

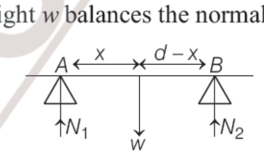


$$(40 \times 10) \times 2 = (60 \times 10)x$$

$$\Rightarrow \quad x = \frac{8}{6} = \frac{4}{3}\text{ m}$$

So, 60 kg boy has to be displaced $= 2 - \frac{4}{3} = \frac{2}{3}$ m.

41 (*d*) As the weight w balances the normal reactions.



So, $\quad w = N_1 + N_2 \quad \ldots$(i)

Now, balancing torque about the CM,

i.e. anti-clockwise momentum = clockwise momentum

$$\Rightarrow \quad N_1 x = N_2(d-x)$$

Putting the value of N_2 from Eq. (i), we get

$$N_1 x = (w - N_1)(d-x)$$

$$\Rightarrow \quad N_1 x = wd - wx - N_1 d + N_1 x$$

$$\Rightarrow \quad N_1 d = w(d-x) \Rightarrow N_1 = \frac{w(d-x)}{d}$$

So, the normal reaction on A is $\frac{w(d-x)}{d}$.

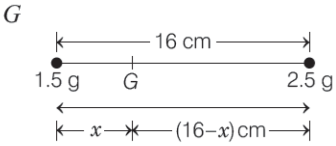
42 (*d*) Point G is the centre of gravity of the cardboard and it is so located that the total torque on it due to forces $m_1\mathbf{g}, m_2\mathbf{g} \cdots m_n\mathbf{g}$ is zero.

It means $\tau_{Mg} = \Sigma\, \tau_i = \Sigma\, \mathbf{r}_i \times m_i\mathbf{g} = 0$.

τ of reaction $\mathbf{R}$, i.e τ_R about CG is also zero as it is at CG.

$\therefore$ CG could be defined as the point, where the total gravitational torque on the body is zero.

43 (*a*) Taking the moment of forces about centre of gravity G

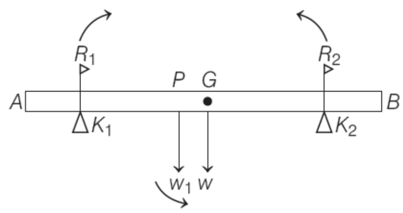


i.e. $\quad 1.5\text{g}\, x = 2.5\text{g}\,(16-x) \Rightarrow 3x = 80 - 5x$

$$\Rightarrow \quad 8x = 80$$

$$\therefore \quad x = 10\text{ cm}$$

44 (*c*) Figure below shows the rod AB, the positions of the knife edges K_1 and K_2, the centre of gravity of the rod is at G and the suspended weight is at P.



The rod is uniform in cross-section and homogeneous. Hence, G is at the centre of rod.

Given, $AB = 70$ cm, $AG = 35$ cm,
$AP = 30$ cm, $PG = 5$ cm,
$AK_1 = BK_2 = 10$ cm and $K_1G = K_2G = 25$ cm.

Also, m = mass of the rod = 4 kg and m_1 = suspended weight of mass = 6 kg; R_1 and R_2 are the normal reactions of the support at the knife edges. For translational equilibrium of the rod,

$$R_1 + R_2 - w_1 - w = 0$$

or $$R_1 + R_2 - m_1g - mg = 0 \quad \ldots\text{(i)}$$

The moments of R_2 and w_1 are anti-clockwise (+ve), whereas the moment of R_1 is clockwise (– ve).

For rotational equilibrium,

$$-R_1\,(K_1G) + w_1\,(PG) + R_2\,(K_2G) = 0 \quad \ldots\text{(ii)}$$

we have, $w = 4.0\,g$ N and $w_1 = 6.0\,g$ N, where g = acceleration due to gravity. Taking, $g = 9.8\ \text{ms}^{-2}$.

With numerical values inserted in Eq. (i), we get

$$R_1 + R_2 - 4.0\,g - 6.0g = 0$$

$$\Rightarrow \quad R_1 + R_2 = 10.00\,g\ \text{N} \quad \ldots\text{(iii)}$$
$$= 98.00\ \text{N}$$

From Eq. (ii), we get

$$-0.25\,R_1 + 0.05\,w_1 + 0.25\,R_2 = 0$$

or $$R_1 - R_2 = \frac{0.05 \times 6.0g}{0.25} = 1.2g\ \text{N} = 11.76\ \text{N} \quad \ldots\text{(iv)}$$

From Eqs. (iii) and (iv), $R_1 = 54.88\ \text{N} \simeq 55\ \text{N}$

$$\Rightarrow \quad R_2 = 43.12\ \text{N}$$

Thus, the reactions of the supports are about 55 N at K_1 and 43 N at K_2.

45 (*a*) The ladder AB is 3 m long and its foot A is at distance $AC = 1$ m from the wall.

From Pythagoras theorem, $BC = \sqrt{AB^2 - AC^2} = \sqrt{8}$
$$= 2\sqrt{2}\text{m}$$

For translational equilibrium, taking the forces in the vertical direction.

$$N - w = 0 \quad \ldots\text{(i)}$$

Taking the forces in the horizontal direction,

$$F - F_1 = 0 \quad \ldots\text{(ii)}$$

For rotational equilibrium, taking the moments of the forces about A, $2\sqrt{2}F_1 - (1/2)w = 0 \quad \ldots$(iii)

Now, $\quad w = 20 \times g = 20 \times 9.8\ \text{N} = 196\ \text{N}$

From Eq. (i), $\quad N = 196.0\ \text{N}$

From Eq. (iii), reaction force of wall,

$$F_1 = w/4\sqrt{2} = 196.0/4\sqrt{2} = 34.6\ \text{N}$$

From Eq. (ii), $\quad F = F_1 = 34.6\ \text{N}$

$\therefore$ Reaction force of floor, $F_2 = \sqrt{F^2 + N^2}$

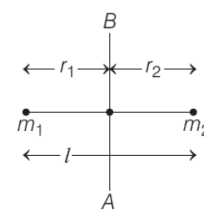
$$= \sqrt{(34.6)^2 + (196)^2} = 199\ \text{N}$$

47 (*b*) Two masses are joined with a light rod and the entire system is rotating about the fixed axis.

Therefore, total moment of inertia is

$$I = \frac{M}{2}\left(\frac{l}{2}\right)^2 + \frac{M}{2}\left(\frac{l}{2}\right)^2 = \frac{Ml^2}{8} + \frac{Ml^2}{8} = \frac{Ml^2}{4}$$

48 (*b*) Consider the situation shown below



Moment of inertia about AB, $I_{AB} = m_1r_1^2 + m_2r_2^2$

$$= m_1\left(\frac{m_2}{m_1 + m_2}l\right)^2 + m_2\left(\frac{m_1}{m_1 + m_2}l\right)^2$$

$$= \frac{m_1m_2(m_1 + m_2)}{(m_1 + m_2)^2}l^2 = \frac{m_1m_2}{(m_1 + m_2)}l^2$$

49 (*c*) Let mass and outer radii of solid sphere and hollow sphere be M and R, respectively. The moment of inertia of solid sphere A about its diameter,

$$I_A = \frac{2}{5}MR^2 \quad \ldots\text{(i)}$$

The moment of inertia of hollow sphere (spherical shell) B about its diameter,

$$I_B = \frac{2}{3}MR^2 \quad \ldots\text{(ii)}$$

It is clear from Eqs. (i) and (ii), that

$$I_A < I_B$$

50 (*a*) Given, mass ratio of two discs,

$$m_1 : m_2 = 1:2, \text{ i.e. } \frac{m_1}{m_2} = \frac{1}{2}$$

and diameter ratio, $\dfrac{d_1}{d_2} = \dfrac{2}{1}$

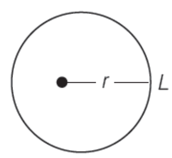
$$\Rightarrow \quad \frac{r_1}{r_2} = \frac{d_1/2}{d_2/2} = \frac{d_1}{d_2} = \frac{2}{1}$$

$\therefore$ Ratio of their moment of inertia,

$$\frac{I_1}{I_2}=\frac{\frac{m_1r_1^2}{2}}{\frac{m_2r_2^2}{2}}=\frac{m_1}{m_2}\cdot\left(\frac{r_1}{r_2}\right)^2=\frac{1}{2}\left(\frac{2}{1}\right)^2=\frac{2}{1}$$

$\therefore \quad I_1 : I_2 = 2 : 1$

51 (*a*) Here, a thin wire of length L is bent to form a circular ring as shown in figure, then $2\pi r = L$ (r is the radius of the ring)



$\Rightarrow \quad r=\frac{L}{2\pi} \quad \ldots(\text{i})$

Hence, the moment of inertia of the ring about its axis,

$$I = Mr^2$$

$\Rightarrow \quad I = M\left(\frac{L}{2\pi}\right)^2 \quad$ [from Eq. (i)]

$\Rightarrow \quad I=\frac{ML^2}{4\pi^2}$

52 (*b*) As two solid spheres are equal in masses, so

$$m_A = m_B$$

$\Rightarrow \quad \frac{4}{3}\pi R_A^3\rho_A=\frac{4}{3}\pi R_B^3\rho_B \Rightarrow \frac{R_A}{R_B}=\left(\frac{\rho_B}{\rho_A}\right)^{1/3}$

The moment of inertia of a solid sphere about diameter,

$$I=\frac{2}{5}mR^2 \Rightarrow \frac{I_A}{I_B}=\left(\frac{R_A}{R_B}\right)^2 \quad (\text{as } m_A = m_B)$$

$\therefore \quad \frac{I_A}{I_B}=\left(\frac{\rho_B}{\rho_A}\right)^{2/3} \Rightarrow \frac{I_B}{I_A}=\left(\frac{\rho_A}{\rho_B}\right)^{2/3}$

53 (*d*) The angle between the rods will not make any difference.

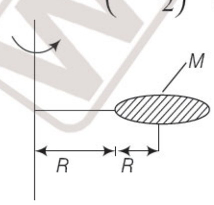
$\therefore$ Net moment of inertia, $I = I_1 + I_2$

$$=\frac{Ml^2}{12}+\frac{Ml^2}{12}=\frac{Ml^2}{6}$$

54 (*d*) Moment of inertia of an outer disc about the axis through centre is

$$=\frac{MR^2}{2}+M(2R)^2$$

$$=MR^2\left(4+\frac{1}{2}\right)=\frac{9}{2}MR^2$$



For 6 such discs,

Moment of inertia $= 6\times\frac{9}{2}MR^2 = 27MR^2$

So, moment of inertia of system

$$=\frac{MR^2}{2}+27MR^2=\frac{55}{2}MR^2$$

Hence, $I_P=\frac{55}{2}MR^2+(7M\times 9R^2)$

$\Rightarrow \quad I_P=\frac{181}{2}MR^2 \quad I_{\text{system}}=\frac{181}{2}MR^2$

55 (*b*) As, moment of inertia of rod,

$$I_{\text{rod}}=\frac{ML^2}{12} \quad \ldots(\text{i})$$

Using radius of gyration, $I = Mk^2 \quad \ldots(\text{ii})$

Comparing Eqs. (i) and (ii), we get

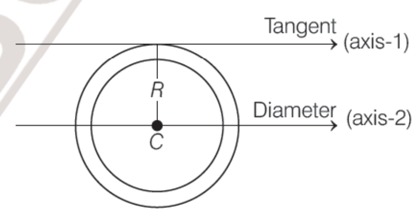
Radius of gyration, $k = L/\sqrt{12}$

56 (*b*) I_{disc} about the axis along its diameter $=\frac{MR^2}{4} \quad \ldots(\text{i})$

Using radius of gyration, $I = Mk^2 \quad \ldots(\text{ii})$

Comparing Eqs. (i) and (ii), we get $k=\frac{R}{2}$.

58 (*b*) Using the parallel axes theorem,

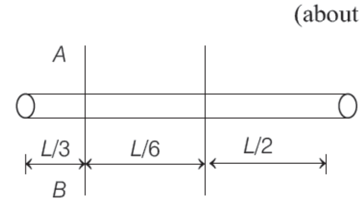


$$I_{\tan}=I_{\text{dia}}+MR^2=\frac{MR^2}{2}+MR^2$$

$$=\frac{3}{2}MR^2$$

59 (*b*) For thin uniform rod, $I_{\text{CM}}=\frac{ML^2}{12}$

(about middle point)



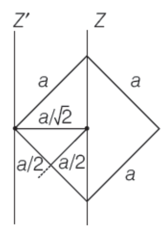
Applying parallel axes theorem, moment of inertia about an axis AB at a distance $\frac{L}{3}$ from one end is given as

$$I = I_{\text{CM}} + Mx^2$$

$$=\frac{ML^2}{12}+M\left(\frac{L}{6}\right)^2 \quad \left(\because x=\frac{L}{2}-\frac{L}{3}=\frac{L}{6}\right)$$

$\Rightarrow \quad I=\frac{ML^2}{9}$

60 (*d*) Moment of inertia of square plate of mass m about Z-axis is $\frac{ma^2}{6}$ and moment of inertia about Z' can be computed using parallel axes theorem,



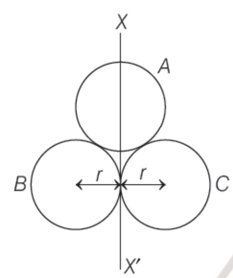
$$I_{Z'} = I_Z + m\left(\frac{a}{\sqrt{2}}\right)^2 = \frac{ma^2}{6} + \frac{ma^2}{2} = \frac{2}{3}ma^2$$

61 (*a*) A is a spherical shell whose mass is m and radius is r.

Its moment of inertia about the XX'-axis is $I_A = \frac{2}{3}mr^2$

B is a spherical shell whose mass is m and radius is r.

Its moment of inertia about its own axis is $I_B = \frac{2}{3}mr^2$



Its moment of inertia about XX'-axis is

$$I_{B'} = I_B + mr^2 = \frac{2}{3}mr^2 + mr^2 = \frac{5}{3}mr^2$$

Similarly, the moment of inertia of the spherical shell C about the XX'-axis is

$$I_{C'} = \frac{5}{3}mr^2$$

Total moment of inertia is, $I = I_A + I_{B'} + I_{C'}$

$$= \frac{2}{3}mr^2 + \frac{5}{3}mr^2 + \frac{5}{3}mr^2 = 4\,mr^2$$

62 (*a*) Moment of inertia of remaining solid

= Moment of inertia of complete solid

− Moment of inertia of removed portion

$$\therefore\ I = \frac{9MR^2}{2} - \left[\frac{M(R/3)^2}{2} + M\left(\frac{2R}{3}\right)^2\right] \Rightarrow I = 4MR^2$$

63 (*d*) MI of a solid cylinder about its perpendicular bisector of length is

$$I = m\left(\frac{l^2}{12} + \frac{R^2}{4}\right)$$

$$\Rightarrow\ I = \frac{mR^2}{4} + \frac{ml^2}{12} = \frac{m^2}{4\pi\rho l} + \frac{ml^2}{12} \qquad [\because \rho\pi R^2 l = m]$$

For I to be maximum,

$$\frac{dI}{dl} = -\frac{m^2}{4\pi\rho}\left(\frac{1}{l^2}\right) + \frac{ml}{6} = 0$$

$$\Rightarrow\ \frac{m^2}{4\pi\rho} = \frac{ml^3}{6}$$

Now, putting $m = \rho\pi R^2 l$

$$\therefore\ l^3 = \frac{3}{2\pi\rho}\cdot\rho\pi R^2 l \ \Rightarrow\ \frac{l^2}{R^2} = \frac{3}{2}$$

$$\therefore\ \frac{l}{R} = \sqrt{\frac{3}{2}}$$

64 (*a*) Moment of inertia of hollow cylinder about its axis is

$$I_1 = \frac{M}{2}(R_1^2 + R_2^2)$$

where, R_1 = inner radius and

R_2 = outer radius.

Moment of inertia of thin hollow cylinder of radius R about its axis is

$$I_2 = MR^2$$

Given, $I_1 = I_2$ and both cylinders have same mass (M).

So, we have

$$\frac{M}{2}(R_1^2 + R_2^2) = MR^2$$

$$(10^2 + 20^2)/2 = R^2$$

$$R^2 = 250 = 15.8$$

$$R \approx 16\,\text{cm}$$

65 (*a*) For disc, moment of inertia about the diameter,

$I_d = \frac{mr^2}{4}$ and moment of inertia about the tangential axis,

$$I_{\text{disc}} = I_d + mr^2 \ \Rightarrow I_{\text{disc}} = \frac{mr^2}{4} + mr^2$$

$$I_{\text{disc}} = \frac{5mr^2}{4}$$

Let the radius of gyration of disc is k_{disc}.

$$\Rightarrow\ I_{\text{disc}} = mk_{\text{disc}}^2 \ \Rightarrow \frac{5mr^2}{4} = mk_{\text{disc}}^2$$

$$\Rightarrow\ k_{\text{disc}} = \sqrt{\frac{5}{4}}r \qquad \text{...(i)}$$

For ring, moment of inertia about the diameter,

$$I'_a = mr^2/2$$

and moment of inertia about the tangential axis,

$$I_{\text{ring}} = I'_a + mr^2 = \frac{mr^2}{2} + mr^2 = \frac{3}{2}mr^2$$

Let the radius of gyration of ring, k_{ring}.

$$\Rightarrow\ I_{\text{ring}} = m\,k^2{}_{\text{ring}} \ \Rightarrow \frac{3}{2}mr^2 = mk^2{}_{\text{ring}}$$

$$\Rightarrow\ k_{\text{ring}} = \sqrt{\frac{3}{2}}r \qquad \text{...(ii)}$$

From Eqs. (i) and (ii), we get

$$\therefore \quad \frac{k_{\text{disc}}}{k_{\text{ring}}} = \frac{\sqrt{\frac{5}{4}}r}{\sqrt{\frac{3}{2}}r} = \sqrt{\frac{5}{6}}$$

66 (*a*) Angular retardation,

$$\alpha = \frac{\omega_f - \omega_i}{\Delta t} = \frac{0 - 900 \times \frac{2\pi}{60}}{60} \text{ rad s}^{-2}$$
$$= -\frac{900 \times 2 \times \pi}{3600} = -\frac{\pi}{2} \text{ rad s}^{-2}$$

67 (*d*) $\therefore$ Angular acceleration,

$$\alpha = \frac{\omega - \omega_0}{t} = \frac{2\pi(\nu - \nu_0)}{t}$$

Given, $\nu = 4500 \text{ rpm} = \frac{4500}{60} \text{s}^{-1}$

$\nu_0 = 1200 \text{ rpm} = \frac{1200}{60} \text{ s}^{-1}$

$t = 10 \text{ s}$

Substituting the given values,we get

$$\alpha = \frac{\omega - \omega_0}{t} = \frac{\frac{2\pi}{60}(4500 - 1200)}{10} = \frac{2\pi\left(\frac{3300}{60}\right)}{10}$$
$$= 11\pi \text{ rads}^{-2} = \frac{11\pi \times 180}{\pi}$$
$$= 1980 \text{ degs}^{-2}$$

68 (*d*) Total angular displacement in 36 rotation,

$\theta = 36 \times 2\pi$

Using $\omega_2^2 - \omega_1^2 = 2\alpha\theta$, we get

$(\omega/2)^2 - \omega^2 = 2\alpha(36 \times 2\pi)$ ...(i)

Similarly, $0^2 - (\omega/2)^2 = 2\alpha\,(n \times 2\pi)$ ...(ii)

Dividing Eq. (i) by Eq. (ii), we get

$$\frac{-\frac{3}{4}\omega^2}{-\omega^2/4} = \frac{36}{n} \Rightarrow n = 12$$

Hence, ceiling fan will make 12 more rotations before coming to rest.

69 (*c*) (i) We shall use $\omega = \omega_0 + \alpha t$

ω_0 = Initial angular speed in rads^{-1}

$= 2\pi \times$ Angular speed in revs^{-1}

$$= \frac{2\pi \times \text{Angular speed in rev/min}}{60 \text{ s/min}}$$
$$= \frac{2\pi \times 1200}{60} \text{ rads}^{-1} = 40\,\pi \text{ rads}^{-1}$$

Similarly, ω = Final angular speed is rads^{-1}

$$= \frac{2\pi \times 3120}{60} = 2\pi \times 52 = 104\pi \text{ rads}^{-1}$$

$\therefore$ Angular acceleration,

$$\alpha = \frac{\omega - \omega_0}{t} = \frac{104\pi - 40\pi}{16} = 4\pi \text{ rads}^{-2}$$

The angular acceleration of the motor is 4π rads^{-2}.

(ii) The angular displacement in time t is given by

$$\theta = \omega_0 t + \frac{1}{2}\alpha t^2 = 40\,\pi \times 16 + \frac{1}{2} \times 4\pi \times 16^2$$
$$= 640\pi + 512\pi = 1152\pi \text{ rad}$$

Number of revolutions $= \frac{1152\pi}{2\pi} = 576$

70 (*d*) Initial angular velocity,

$\omega_0 = 2\pi f_0 = 2\pi \times 100 = 200\pi$ rad/s

Final angular velocity, $\omega = 2\pi f = 2\pi \times 300 = 600\pi$ rad/s,

$t = 10 \text{ s}$

From first equation of rotational motion,

$\omega = \omega_0 + \alpha t$

$$\Rightarrow \quad \alpha = \frac{\omega - \omega_0}{t} = \frac{600\pi - 200\pi}{10} = 40\pi \text{ rad/s}^2$$

If θ be the total angular displacement, then from equation of rotational motion, $\omega^2 = \omega_0^2 + 2\theta\alpha$

$$\Rightarrow \quad \theta = \frac{\omega^2 - \omega_0^2}{2\alpha} = \frac{(600\pi)^2 - (200\pi)^2}{2 \times 40\pi}$$
$$= \frac{(200\pi)^2[3^2 - 1]}{80\pi} = \frac{200\pi \times 200\pi \times 8}{80\pi}$$
$$\theta = 4000\pi$$

$\therefore$ Number of revolution, $n = \frac{\theta}{2\pi} = \frac{4000\pi}{2\pi} = 2000$

71 (*a*) Given, $\omega = 100 \text{ rad s}^{-1}$ and $\tau = 100$ N-m

$\therefore$ Power of the engine, $P = \tau\omega = 100 \times 100$

$= 10 \times 10^3 \text{ W} = 10\text{kW}$

72 (*c*) Work done required to bring an object to rest is given as

$$W = \frac{1}{2}I\omega^2$$

where, I is the moment of inertia and ω is the angular velocity.

Since, here all the objects spin with the same ω, this means, $W \propto I$

As, I_A (for a solid sphere) $= \frac{2}{5}MR^2$

I_B (for a thin circular disc) $= \frac{1}{2}MR^2$

I_C (for a circular ring) $= MR^2$

$$\therefore \quad W_A : W_B : W_C = I_A : I_B : I_C$$
$$= \frac{2}{5}MR^2 : \frac{1}{2}MR^2 : MR^2$$
$$= \frac{2}{5} : \frac{1}{2} : 1 = 4 : 5 : 10$$
$$\Rightarrow \quad W_A < W_B < W_C$$

73 (*c*) Kinetic energy (KE) of sphere

$$= \frac{1}{2} I\omega^2 = \frac{1}{2}\left(\frac{2}{5} mR^2\right)\omega^2 = \frac{1}{5} mR^2\omega^2$$

Kinetic energy (KE) of cylinder

$$= \frac{1}{2}\left(\frac{mR^2}{2}\right)(2\omega)^2 = mR^2\omega^2$$

So, $\dfrac{KE_{sphere}}{KE_{cylinder}} = \dfrac{1}{5}$

74 (*a*) Given, $M = 20\,kg$, $R = 20\,cm = 0.20\,m$

$F = 25\,N$

Torque, $\tau = FR$

$= 25 \times 0.20 = 5.0$ N-m

Moment of inertia of flywheel about its axis,

$$I = \frac{MR^2}{2} = \frac{20 \times (0.2)^2}{2} = 0.4 \text{ kg-m}^2$$

We use, $I\alpha = \tau$

$\therefore$ Angular acceleration, $\alpha = \dfrac{\tau}{I} = 5.0/0.4$

$= 12.50\ s^{-2}$

75 (*c*) Given, moment of inertia,

$I = 0.4$ kg-m^2

Radius, $r = 0.2\,m$

Force, $F = 10\,N$

$\because \quad F \times r = I\alpha = I\dfrac{(\omega_2 - \omega_1)}{t}$

[from $\tau = F \times r$ and $\tau = I\alpha$]

$$\Rightarrow \quad \omega_2 - \omega_1 = \frac{F \times r \times t}{I}$$

$$= \frac{10 \times 0.2 \times 4}{0.4} = 20 \text{ rads}^{-1}$$

76 (*a*) Moment of inertia of hollow cylinder about its axis of symmetry, $I_1 = MR^2$

Moment of inertia of solid sphere about an axis passing its centre, $I_2 = \dfrac{2}{5} MR^2$

Let α_1 and α_2 be angular accelerations produced in the cylinder and the sphere respectively, on applying same torque τ in each case. Then,

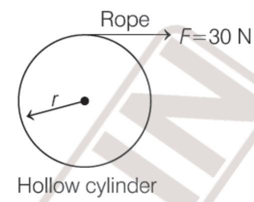
$\alpha_1 = \dfrac{\tau}{I_1}$ (as $\tau = I\alpha$)

and $\alpha_2 = \dfrac{\tau}{I_2}$

Their corresponding ratio is

$$\frac{\alpha_1}{\alpha_2} = \frac{I_2}{I_1} = \frac{\frac{2}{5} MR^2}{MR^2} = \frac{2}{5}$$

77 (*c*) Consider a hollow cylinder, around which a rope is wounded as shown in the figure.



Torque acting on the cylinder due to the force F is

$\tau = Fr$

Now, we have $\tau = I\alpha$

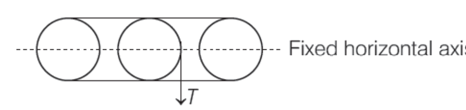
where, I = moment of inertia of the cylinder about the axis through the centre = mr^2

and α = angular acceleration.

$$\Rightarrow \quad \alpha = \frac{\tau}{I} = \frac{Fr}{mr^2} = \frac{F}{mr} = \frac{30}{3 \times 40 \times 10^{-2}}$$

$$= \frac{100}{4} = 25 \text{ rad/s}^2$$

78 (*d*) Given, $m = 50\,kg$, $R = 0.5\,m$, $\alpha = 2$ revs^{-2}



Torque produced by the tension in the string,

$\tau = T \times r = T \times 0.5$

$\tau = \dfrac{T}{2}$ N-m ...(i)

We know that, $\tau = I\alpha$ ...(ii)

From Eqs. (i) and (ii), we get

$\dfrac{T}{2} = I\alpha$

As, $I\alpha = \left(\dfrac{mR^2}{2}\right) \times (2 \times 2\pi)$ rads^{-2}

$\left(\because I_{solid\ cylinder} = \dfrac{mR^2}{2}\right)$

So, $\dfrac{T}{2} = \dfrac{50 \times (0.5)^2}{2} \times 4\pi$

$T = 50 \times \dfrac{1}{4} \times 4\pi = 50\,\pi = 157\,N$

79 (*d*) Given, $(KE)_A = (KE)_B$

$\therefore \quad \dfrac{1}{2} I_A\omega_A^2 = \dfrac{1}{2} I_B\omega_B^2$

Since, $I_B > I_A$, so $\omega_B < \omega_A$

$\Rightarrow \quad \dfrac{1}{2} L_A\omega_A = \dfrac{1}{2} L_B\omega_B$ [$\because L = I\omega$]

$\Rightarrow \quad L_B > L_A$

80 (*d*) Moment of inertia of a rotating solid sphere about its symmetrical (diametric) axis is given as $I=\frac{2}{5}mR^2$

Rotational kinetic energy of solid sphere is

$$K_r=\frac{1}{2}I\omega^2=\frac{1}{2}\times\frac{2}{5}mR^2\omega^2=\frac{1}{5}mR^2\omega^2$$

Angular velocity, $\omega=v_{CM}R$

As, we know that external torque,

$$\tau_{ext}=\frac{dL}{dt}$$

where, L is the angular momentum.

Since, in the given condition, $\tau_{ext}=0$

$$\Rightarrow \quad \frac{dL}{dt}=0 \text{ or } L=\text{constant}$$

Hence, when the radius of the sphere is increased keeping its mass same, only the angular momentum remains constant. But other quantities like moment of inertia, rotational kinetic energy and angular velocity changes as they are related to R which is increasing with time.

81 (*c*) Moment of inertia of the insect-disc system,

$$\text{MI}=\frac{1}{2}MR^2+mx^2$$

where, m = mass of insect

and x = distance of insect from centre.

Clearly, as the insect moves along the diameter of the disc. Moment of inertia first decreases and then increases.

By conservation of angular momentum, i.e. $I\omega=$ constant, i.e. when moment of inertia increases, then angular velocity decreases and *vice-versa*. Therefore, angular speed first increases and then decreases.

82 (*d*) Given, $\omega_1=3\pi$ rad s^{-1}, $I_1=I$

$$I_2=\frac{75}{100}I_1=\frac{3}{4}I, \omega_2=?$$

As, $\quad I_2\omega_2=I_1\omega_1$

$$\therefore \quad \omega_2=\frac{I_1}{I_2}\times\omega_1=\frac{I_1}{\frac{75I_1}{100}}\times 3\pi$$

$$=\frac{4}{3}\times 3\pi=4\pi \text{ rad s}^{-1}$$

83 (*c*) Given, moment of inertia of discs,

$$I_1=2\text{ kg-m}^2 \text{ and } I_2=4\text{ kg-m}^2$$

Angular velocity of discs, $\omega_1=8$ rad/s and $\omega_2=4$ rad/s

From angular momentum conservation principle,

$$I_1\omega_1+I_2\omega_2=(I_1+I_2)\omega$$

or $\quad 2\times 8+4\times 4=(2+4)\omega$

or $\quad \omega=\frac{16+16}{6}=\frac{32}{6}=\frac{16}{3}$ rad/s

84 (*b*) I is the moment of inertia of each discs.

Total angular momentum before contact,

$L_i=I\omega_1+I\omega_2=I(\omega_1+\omega_2)$

Total angular momentum after contact, $L_f=I\omega+I\omega$

$L_f=2I\omega$, where ω is the final angular velocity of the combined system.

According to conservation of angular momentum,

$$L_f=L_i \Rightarrow 2I\omega=I(\omega_1+\omega_2)$$

$$\Rightarrow \quad \omega=\frac{\omega_1+\omega_2}{2} \quad \ldots\text{(i)}$$

Now, loss in energy

= total rotational kinetic energy before contact − total rotational kinetic energy after contact

$$=\frac{1}{2}I\omega_1^2+\frac{1}{2}I\omega_2^2-\frac{1}{2}(I_1+I_2)\cdot\omega^2$$

$$=\frac{1}{2}I\omega_1^2+\frac{1}{2}I\omega_2^2-\frac{1}{2}\cdot 2I\left(\frac{\omega_1+\omega_2}{2}\right)^2$$

$$=\frac{1}{2}I\omega_1^2+\frac{1}{2}I\omega_2^2-\frac{I}{4}\cdot(\omega_1^2+\omega_2^2+2\omega_1\omega_2)$$

$$=\frac{1}{4}I[2\omega_1^2+2\omega_2^2-\omega_1^2-\omega_2^2-2\omega_1\omega_2]$$

$$=\frac{I}{4}[\omega_1^2+\omega_2^2-2\omega_1\omega_2]=\frac{I}{4}(\omega_1-\omega_2)^2$$

85 (*a*) As there is no external torque, so if the girl bends her hands, her moment of inertia about the rotational axis will decrease. By conservation of angular momentum,

$L=I\omega=$ constant. So in order to keep L constant, if I is decreasing, then ω will increase.

86 (*d*) By the conservation of angular momentum, we know that $I\omega=$ constant.

As the acrobat bends his body, then moment of inertia I will decrease

and hence ω of acrobat will increase as no external torque is acting on the acrobat.

87 (*a*) When a body rolls down without slipping along an inclined plane of inclination θ, it rotates about a horizontal axis through its centre of mass and also its centre of mass moves.

As it rolls down, it suffers loss in gravitational potential energy which provides translational energy and due to frictional force, it gets converted into rotational energy.

88 (*a*) Kinetic energy of a rolling body = Rotational kinetic energy + Translational kinetic energy

$$=\frac{1}{2}I\omega^2+\frac{1}{2}mv_{CM}^2=\frac{1}{2}\frac{mk^2v_{CM}^2}{R^2}+\frac{1}{2}mv_{CM}^2$$

where, k is the corresponding radius of gyration of the body.

$$=\frac{1}{2}mv_{CM}^2\left(1+\frac{k^2}{R^2}\right) \quad \left(\because I=mk^2 \text{ and } v_{CM}=R\omega\right)$$

It applies for any rolling body.