

$$\therefore \Delta U = 22.3 \text{ J}$$

So, from first law of thermodynamics,

$$\Delta U = \Delta Q - W \text{ or } W = \Delta Q - \Delta U$$

$$W = 39.18 - 22.3 = 16.88 \text{ J} \approx 16.9 \text{ J}$$

109 (c) Work done by a thermodynamic system,

= Area under p - V curve and V -axis; work done is positive, if volume increases and negative, if volume decreases.

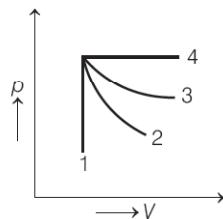
$$\begin{aligned} \text{Work done in process } DE &= + \text{Area of } DEHGD \\ &= + (\text{Area of } \triangle DEF + \text{Area of rectangle } EHGF) \\ &= + \left[\frac{1}{2} \times (600 - 300) \times (5 - 2) + 300 \times (5 - 2) \right] \\ &= 450 + 900 = 1350 \text{ J} \end{aligned}$$

Work done in process, EF

$$\begin{aligned} &= -(\text{Area of rectangle } EHGF) \\ &= -[300 \times (5 - 2)] = -900 \text{ J} \end{aligned}$$

$$\therefore \text{Total work done by the gas from } D \text{ to } E \text{ to } F, \\ W = 1350 - 900 = 450 \text{ J}$$

110 (c) For the curve 4, pressure is constant, so this is an isobaric process.



For the curve 1, volume is constant, so it is isochoric process. Between curves 3 and 2, curve 2 is steeper, so it is adiabatic and 3 is isothermal.

111 (c) According to the question, the equation of the gas is $pV = \text{constant}$.

Hence, we can say that the gas is going through an isothermal process.

From given graph p - V , it is also seen that the gas is expanding as with increasing volume, the pressure of gas decreasing, i.e. $p_2 < p_1$ at constant temperature. This situation is correctly shown in T - p diagram of option (iii).

112 (b) Consider the p - V diagram given in the question.

Work done in the process $ABCDA$

= Area under the curve $ABCDA$

$$W_{\text{total}} = W_{AB} + W_{BC} + W_{CD} + W_{DA} \quad \dots(i)$$

$$\text{Here, } W_{BC} = W_{DA} = 0 \quad (\because \Delta V = 0)$$

$$W_{AB} = p_0(3V_0 - V_0) = 2p_0V_0$$

$$W_{CD} = 2p_0(V_0 - 3V_0) = -4p_0V_0$$

Putting the values in Eq. (i), we get

$$W_{\text{total}} = 2p_0V_0 + 0 - 4p_0V_0 + 0 = -2p_0V_0$$

So, work done by the gas is $-2p_0V_0$.

113 (a) According to the given question, both the processes involves compression of the gas.

For isothermal process (gas A), $p_1V_1 = p_2V_2$

$$\Rightarrow p_0(2V_0) = p_2(V_0) \Rightarrow p_2 = 2p_0$$

For adiabatic process, (gas B)

$$p_1V_1^\gamma = p_2V_2^\gamma \Rightarrow p_0(2V_0)^\gamma = p_2(V_0)^\gamma$$

$$\Rightarrow p_2 = \left(\frac{2V_0}{V_0} \right)^\gamma p_0 \text{ or } p_2 = (2)^\gamma p_0$$

$$\text{Hence, } \frac{(p_2)_B}{(p_2)_A} = \text{Ratio of final pressures}$$

$$= \frac{(2)^\gamma p_0}{2p_0} = 2^{\gamma - 1}$$

where, γ is ratio of specific heat capacities for the gas.

114 (d) Given, temperature of the source, $T_1 = 500 \text{ K}$

Temperature of the sink, $T_2 = 300 \text{ K}$

Work done per cycle, $W = 1 \text{ kJ} = 1000 \text{ J}$

Heat transferred to the engine per cycle, $Q_1 = ?$

Efficiency of a Carnot engine,

$$\eta = 1 - \frac{T_2}{T_1} = 1 - \frac{300}{500} = \frac{2}{5}$$

$$\text{Also, } \eta = \frac{W}{Q_1}$$

$$\text{or } Q_1 = \frac{W}{\eta} = \frac{1000}{(2/5)} = 2500 \text{ J}$$

115 (b) Given, temperature of source,

$$T_1 = (27 + 273) \text{ K} = 300 \text{ K}$$

Temperature of sink, $T_2 = (-3 + 273) \text{ K} = 270 \text{ K}$

Efficiency of a perfect heat engine is given by

$$\eta = 1 - \frac{T_2}{T_1} = 1 - \frac{270}{300} = \frac{1}{10}$$

Efficiency of refrigerator is 50% of a perfect engine,

$$\eta' = 0.5 \times \eta = \frac{1}{2} \eta = \frac{1}{20}$$

. Coefficient of performance of the refrigerator,

$$\begin{aligned} \alpha &= \frac{Q_2}{W} = \frac{1 - \eta'}{\eta'} \\ &= \frac{1 - (1/20)}{(1/20)} = \frac{19/20}{1/20} = 19 \end{aligned}$$

$$\Rightarrow Q_2 = \alpha W = 19 \text{ W} \\ = 19 \times (1 \text{ kW}) = 19 \text{ kW}$$

Therefore, heat is taken out of the refrigerator at a rate of 19 kJ per second.

116 (a) Given, coefficient of performance, $\alpha = 5$

$$T_1 = (27 + 273) \text{ K} = 300 \text{ K}, T_2 = ?$$

$$\begin{aligned} \text{Coefficient of performance, } \alpha &= \frac{T_2}{T_1 - T_2} \\ 5 &= \frac{T_2}{300 - T_2} \end{aligned}$$

$$\Rightarrow 1500 - 5T_2 = T_2$$

$$\Rightarrow 6T_2 = 1500 \Rightarrow T_2 = 250 \text{ K}$$

$$\therefore T_2 = (250 - 273)^\circ \text{C} = -23^\circ \text{C}$$

CHAPTER > 13

Kinetic Theory

KEY NOTES

- Kinetic theory explains the behaviour of gases based on the idea that the gas consists of rapidly moving atoms or molecules.

Molecular Nature of Matter

- According to the molecular theory of matter,
 - (i) The atoms are much free in gases and can travel long distances without colliding. If they are not enclosed, gases disperse away.
 - (ii) In solids and liquids, the closeness makes the interatomic force important. The force has a long range attraction and a short range repulsion. The atoms attract when they are at a few angstroms but repel when they come closer.
 - (iii) In dynamic equilibrium, molecules collide with each other and change their speeds during the collision. However, only the average properties remain constant.

Behaviour of Gases

- Behaviour of gases can be stated as follows
 - (i) Gases at low pressures and high temperatures much above that at which they liquefy (or solidify) approximately satisfy a simple relation between their pressure, temperature and volume given by
$$pV = KT$$
where, T is the temperature in kelvin or absolute scale and K is constant for a given sample but varies with the volume of gas.
 - (ii) With respect to atoms or molecules, K is proportional to the number of molecules (N) in a sample of gas.
i.e. $K = Nk$

It has been observed that, the value of K is same for all gases and it is called Boltzmann constant and is denoted by k_B which is given by

$$k_B = \frac{pV}{NT}$$

- (iii) According to **Avogadro's hypothesis**, the number of molecules per unit volume is the same for all gases at fixed temperature and pressure.
The number of molecules in 22.4 L of any gas is 6.02×10^{23} . This is known as **Avogadro number** and is denoted by N_A .
- (iv) The mass of 22.4 L of any gas is equal to its molecular weight in grams at STP (Standard Temperature 273 K and Pressure 1 atm). This amount of substance is called a **mole**.
- (v) The perfect gas equation can be written as

$$pV = \mu RT$$

where, μ is the number of moles, $R = N_A k_B$ which is a universal constant, T is absolute temperature in kelvin and $R = 8.314 \text{ J mol}^{-1} \text{ K}^{-1}$.

Another useful form of above perfect gas equation is

$$p = \frac{\rho RT}{M_0}$$

where, ρ is the mass density of the gas.

- (vi) If a gas that satisfies the relation, $pV = \mu RT$ exactly at all pressures and temperatures, then it is defined to be an **ideal gas**.
- (vii) Keeping temperature constant, pressure of a given mass of gas varies inversely with volume, i.e. $pV = \text{constant}$ or $p \propto 1/V$.
This is called **Boyle's law**.

- (viii) For a fixed pressure, the volume of a gas is proportional to its absolute temperature T .

i.e. $V \propto T$

This is called **Charles' law**.

- (ix) The total pressure of a mixture of ideal gases is equal to the sum of the partial pressures of the individual gases.

i.e. $p = p_1 + p_2 + \dots = \frac{\mu_1 RT}{V} + \frac{\mu_2 RT}{V} + \dots$

This is called **Dalton's law of partial pressure**.

Kinetic Theory of an Ideal Gas

- It is based on the molecular picture of matter. It correlates the macroscopic properties like pressure and temperature of gases to microscopic properties like speed and kinetic energy of gas molecules.
- At ordinary pressure and temperature, the average distance between molecules is a factor of 10 or more than the typical size of a molecule (2\AA). Thus, interaction between molecules is negligible.
- When the molecules come close to each other, they experience intermolecular forces and their velocities change. These interactions are called **collisions**.
- All collisions between molecules among themselves or between molecules and the walls of the container are elastic, i.e. total kinetic energy and momentum remain conserved.

- Total pressure exerted by a gas is $p = \frac{1}{3}nm\bar{v}^2$

where, \bar{v}^2 denotes the mean of the squared speed.

Also, $pV = \frac{2}{3}E$

- The internal energy E of an ideal gas can be given as

$$E = N \times \frac{1}{2}m\bar{v}^2$$

where, $N (= nV)$ is the number of molecules in the sample.

- The average kinetic energy of one mole of an ideal gas is

$$E = \frac{3}{2}k_BNT \quad \text{or} \quad \frac{E}{N} = \frac{1}{2}m\bar{v}^2 = \frac{3}{2}k_B T$$

- In equilibrium, the average kinetic energy of the molecules of different gases will be equal.

i.e. $\frac{1}{2}m_1\bar{v}_1^2 = \frac{1}{2}m_2\bar{v}_2^2 = \frac{3}{2}k_B T \Rightarrow p = (n_1 + n_2 + \dots)k_B T$
(Dalton's law of partial pressure)

- The mean square speed of a molecule is $\bar{v}^2 = \frac{3k_B T}{m}$.

The square root of \bar{v}^2 is known as **root mean square (rms) speed** and is denoted by v_{rms} .

- When gases diffuse, their rate of diffusion is inversely proportional to square root of the masses.

Law of Equipartition of Energy

- It states that for any system in thermal equilibrium, the total energy is equally distributed in all possible energy modes, with each mode having an average energy equal to $\frac{1}{2}k_B T$.

- The kinetic energy of a single molecule is

$$\epsilon_t = \frac{1}{2}mv_x^2 + \frac{1}{2}mv_y^2 + \frac{1}{2}mv_z^2$$

- For a gas in thermal equilibrium at temperature T , the average value of energy denoted by $\langle \epsilon_t \rangle$ is $\frac{3}{2}k_B T$.

Since there is no preferred direction, hence

$$\langle \frac{1}{2}mv_x^2 \rangle = \langle \frac{1}{2}mv_y^2 \rangle = \langle \frac{1}{2}mv_z^2 \rangle = \frac{1}{2}k_B T.$$

- A molecule has one degree of freedom for motion in a line, two for motion in a plane and three for motion in space.
- Motion of a body as a whole from one point to another is called **translation**. Thus, a molecule free to move in space has three translational degrees of freedom.
- Molecules of a monoatomic gas have only translational degrees of freedom.
- Molecules of diatomic gas are made up of two atoms joined rigidly to one another through a bond. Thus, diatomic molecule has 5 degree of freedom, i.e. 3 translational and 2 rotational. Hence, total energy consists of translational energy ϵ_t and rotational energy ϵ_r .

i.e. $\epsilon_t + \epsilon_r = \frac{1}{2}mv_x^2 + \frac{1}{2}mv_y^2 + \frac{1}{2}mv_z^2 + \frac{1}{2}I_1\omega_1^2 + \frac{1}{2}I_2\omega_2^2$

- Diatomeric molecule like CO, even at moderate temperature have a mode of vibration, i.e. its atom oscillate along the interatomic axis and contribute vibrational energy terms ϵ_v to the total energy,

$$\epsilon_v = \frac{1}{2}m \left(\frac{dy}{dt} \right)^2 + \frac{1}{2}ky^2$$

$$\Rightarrow \epsilon = \epsilon_t + \epsilon_r + \epsilon_v$$

where, k is the force constant of the oscillator and y is the vibrational coordinate.

- Each translational and rotational degrees of freedom of a molecule contributes $\frac{1}{2}k_B T$ to the energy, while each vibrational frequency contributes $2 \times \frac{1}{2}k_B T = k_B T$, since a vibrational mode has both kinetic and potential energy modes.

Specific Heat Capacity

(i) For monoatomic gas

Degree of freedom, $f = 3$ (only translational)

The total internal energy of a mole of such a gas is

$$U = \frac{3}{2} k_B T N_A = \frac{3}{2} RT$$

The molar specific heat at constant volume,

$$C_V = \frac{dU}{dT} = \frac{3}{2} R$$

The molar specific heat at constant pressure,

$$C_p = C_V + R = \frac{5}{2} R \quad [\because C_p - C_V = R]$$

$$\therefore \text{Ratio of specific heats, } \gamma = \frac{C_p}{C_V} = \frac{5}{3}$$

(ii) For diatomic gas (for rigid diatomic)

$f = 5$ (3 translational + 2 rotational)

$$\text{Similarly, } U = \frac{5}{2} k_B T \times N_A = \frac{5}{2} RT$$

The molar specific heats are

$$C_V = \frac{5}{2} R, C_p = \frac{7}{2} R$$

$$\Rightarrow \gamma = \frac{C_p}{C_V} = \frac{7}{5}$$

If the diatomic molecule is not rigid but in addition has a vibrational mode, then

$$U = \left(\frac{5}{2} k_B T + k_B T \right) N_A = \frac{7}{2} RT$$

$$\Rightarrow C_V = \frac{7}{2} R, C_p = \frac{9}{2} R \text{ and } \gamma = \frac{9}{7}$$

(iii) For polyatomic gas

Generally, degree of freedom = $(6+f)$
(3 translational + 3 rotational + f vibrational)

Total internal energy,

$$U = \left(\frac{3}{2} k_B T + \frac{3}{2} k_B T + f k_B T \right) N_A$$

$$\Rightarrow C_V = (3+f) R$$

$$\text{and } C_p = (4+f) R$$

where, f = number of vibrational modes.

$$\gamma = \frac{(4+f)}{(3+f)}$$

- Specific heat capacity of solid is

$$C = \frac{\Delta Q}{\Delta T} = \frac{\Delta U}{\Delta T} = 3R$$

Hence, total energy, $U = 3k_B TN_A = 3RT$

- Specific heat capacity of water is

$$C = \frac{\Delta Q}{\Delta T} = \frac{\Delta U}{\Delta T} = 9R$$

Hence, total energy, $U = 3(3k_B TN_A) = 9RT$

Mean Free Path

- The average distance travelled by a molecule between two successive collisions is known as the mean free path (l) of the molecule.

$$\text{Mean free path, } l = \frac{1}{\sqrt{2} \pi d^2 n}$$

where, d = diameter of each molecule
and n = number of molecules per unit volume.

- The rate of collision of gas molecule is $n\pi d^2 \langle v \rangle$ or the time between two successive collisions is on the average,

$$\tau = \frac{1}{n\pi \langle v \rangle d^2}$$

- The mean free path, in gases, is of the order of thousands of angstroms.

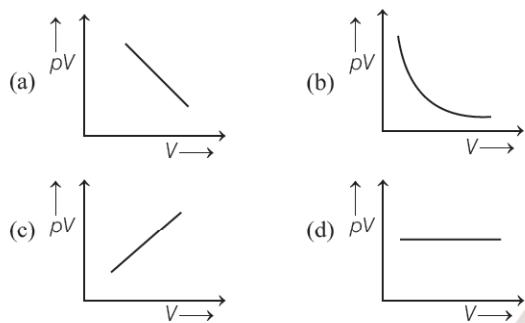
Mastering NCERT

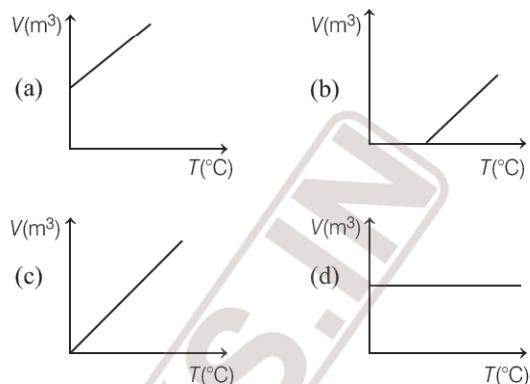
MULTIPLE CHOICE QUESTIONS

TOPIC 1 ~ Molecular Nature of Matter and Behaviour of Gases

- 13** Two balloons are filled, one with pure He gas and the other by air, respectively. If the pressure and temperature of these balloons are same, then the number of molecules per unit volume is
(a) more in the He filled balloon
(b) same in both balloons
(c) more in air filled balloon
(d) in the ratio of 1 : 4

14 A gas is filled in a vessel. Its temperature is increased by 25% on Kelvin scale and volume is decreased by 10%. How much percentage of the gas has to leak for pressure to remain constant?
(a) 25%
(b) 20%
(c) 28%
(d) 30%



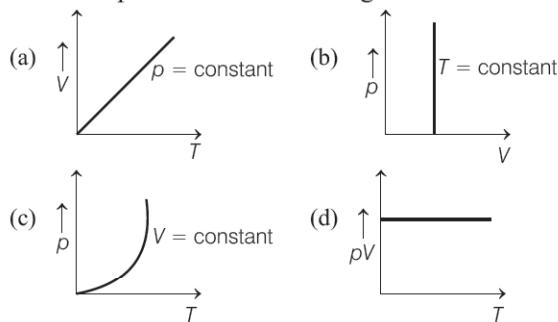


- 20** A perfect gas at 27°C is heated at constant pressure, so as to double its volume. The increase in temperature of the gas will be
(a) 300°C (b) 54°C (c) 327°C (d) 600°C

21 When the temperature of a gas increases by 1°C at constant volume, its pressure increases by 0.4%. What is its initial temperature?
(a) 250°C (b) 125°C (c) 195°C (d) 329°C

22 The tyre of a motorcar contains air at 15°C . If the temperature increases to 35°C , the approximate percentage increase in pressure is (ignore the expansion of tyre)
(a) 7 (b) 9 (c) 11 (d) 13

23 Which of the following diagrams (graphs) is the correct representation of ideal gas behaviour?



- 24** Consider a mixture of non-interacting ideal gases having μ_1 mole of gas 1, μ_2 mole of gas 2 and μ_3 mole of gas 3 in a vessel of volume V at temperature T and pressure p . The total pressure of mixture of gases is given by **JEE Main 2013**

 - $$\frac{\mu_1\mu_2\mu_3 V}{RT}$$
 - $$\frac{RT}{\mu_1\mu_2\mu_3 V}$$
 - $$(\mu_1 + \mu_2 + \mu_3) \frac{RT}{V}$$
 - $$\mu_1\mu_2\mu_3 \frac{RT}{V}$$

TOPIC 2 ~ Kinetic Theory of an Ideal Gas

25 Kinetic theory of gases

- (a) correctly explains specific heat capacities of many gases
- (b) relates properties of gases such as viscosity, conduction, etc. with molecular parameters
- (c) Both (a) and (b)
- (d) None of the above

26 The collisions of the molecules of an ideal gas are

- (a) elastic
- (b) inelastic
- (c) completely inelastic
- (d) partially elastic

27 In collision of a molecule of an ideal gas with the wall of container,

- (a) only total kinetic energy remains conserved
- (b) only total momentum remains conserved
- (c) total kinetic energy and total momentum remains conserved
- (d) Neither total kinetic energy nor total momentum is conserved

28 A gas at 300K is enclosed in a container. Now, the container is placed on a fast moving train. While the train is in motion, the temperature of the gas

- (a) rises above 300 K
- (b) falls below 300K
- (c) remains unchanged
- (d) becomes unsteady

29 When molecules of an isotropic gas travelling in cubical box along +x-direction collides with the wall of box elastically, the pressure exerted on the walls due to gas is (where, n = number of molecules per unit volume, m = mass and \bar{v}^2 = mean of squared speed)

- (a) $3nm\bar{v}^2$
- (b) $\frac{1}{3}nm\bar{v}^2$
- (c) $nm\bar{v}^2$
- (d) None of these

30 The internal energy of an ideal gas is in the form of

- (a) kinetic energy of molecules

(b) potential energy of molecules

- (c) Both kinetic and potential energy of molecules
- (d) gravitational potential energy of molecules

31 Increase in temperature of a gas, filled in a container would lead to

NEET 2019

- (a) increase in its kinetic energy
- (b) decrease in its pressure
- (c) decrease in intermolecular distance
- (d) increase in its mass

32 If 10^{22} gas molecules each of mass

10^{-26} kg collide with a surface (perpendicular to it) elastically per second over an area 1 m^2 with a speed 10^4 m/s , the pressure exerted by the gas molecules will be of the order of

- (a) 10^4 N/m^2
- (b) 10^8 N/m^2
- (c) 10^3 N/m^2
- (d) 10^{16} N/m^2

JEE Main 2019

33 The average translational kinetic energy of the molecule of a gas is

- (a) $\frac{3}{2}k_B T$
- (b) $k_B T$
- (c) $\frac{1}{2}k_B T$
- (d) $\frac{2}{3}k_B T$

34 Oxygen and hydrogen are at the same temperature T . The ratio of the mean kinetic energy of oxygen molecules to that of the hydrogen molecules will be

- (a) 16 : 1
- (b) 1 : 1
- (c) 4 : 1
- (d) 1 : 4

35 At what temperature, the kinetic energy of gas molecule is half of the value at 27°C ?

- (a) 13.5°C
- (b) 150°C
- (c) 75 K
- (d) -123°C

36 The ratio of molecular masses of two gases in a container at a temperature T is 3 : 2. The ratio of the kinetic energies of the molecule of two gases is

- (a) 3 : 2
- (b) 9 : 4
- (c) 1 : 1
- (d) 4 : 9

TOPIC 3 ~ Various Speeds of Gases

37 At constant pressure, which of the following is true for an ideal gas? (where, ρ = density of the gas and \bar{v} = rms speed of the gas)

- (a) $\bar{v} \propto \sqrt{\rho}$
- (b) $\bar{v} \propto \frac{1}{\rho}$
- (c) $\bar{v} \propto \rho$
- (d) $\bar{v} \propto \frac{1}{\sqrt{\rho}}$

38 The root mean square speed of a nitrogen molecule at 300K is (take, $k_B = 1.38 \times 10^{-23} \text{ JK}^{-1}$ and $N_A = 6.02 \times 10^{23}$)

- (a) $\sqrt{534} \text{ ms}^{-1}$
- (b) 534 ms^{-1}
- (c) 16.34 ms^{-1}
- (d) $\sqrt{216} \text{ ms}^{-1}$

39 The rms velocity of a gas molecule is 300 ms^{-1} . The rms velocity of the molecules of gas with twice the molecular weight and half the absolute temperature, is

JEE Main 2019

- (a) 300 ms^{-1}
- (b) 600 ms^{-1}
- (c) 75 ms^{-1}
- (d) 150 ms^{-1}

40 For a given gas at 1 atm pressure, rms speed of the molecules is 200 m/s at 127°C . At 2 atm pressure and at 227°C , the rms speed of the molecules will be

JEE Main 2019

- (a) $100\sqrt{5} \text{ m/s}$
- (b) 80 m/s
- (c) 100 m/s
- (d) $80\sqrt{5} \text{ m/s}$

- 41** If the mass of He is 4 times that of hydrogen, then mean velocity of He at the same temperature is
 (a) 2 times of H-mean value
 (b) $(1/2)$ times of H-mean value
 (c) 2 times of H-mean value
 (d) same as H-mean value
- 42** The molecules of a given mass of a gas have rms velocity of 200 ms^{-1} at 27°C and $1.0 \times 10^5 \text{ Nm}^{-2}$ pressure. When the temperature and pressure of the

gas are respectively, 127°C and $0.05 \times 10^5 \text{ Nm}^{-2}$, the rms velocity of its molecules (in ms^{-1}) is **NEET 2016**
 (a) $\frac{400}{\sqrt{3}}$ (b) $\frac{100\sqrt{2}}{3}$ (c) $\frac{100}{3}$ (d) $100\sqrt{2}$

- 43** An ideal gas ($\gamma = 1.5$) is expanded adiabatically. How many times the gas has to be expanded to reduce the root mean square velocity of the molecules 2.0 times?
 (a) 4 times (b) 16 times (c) 8 times (d) 2 times

TOPIC 4 ~ Specific Heat Capacities : Law of Equipartition of Energy : Mean Free Path

- 44** Law of equipartition of energy is used to
 (a) predict the specific heats of gases
 (b) predict the specific heats of solids
 (c) Both (a) and (b)
 (d) Neither (a) nor (b)
- 45** Total number of degree of freedom for a molecule moving along a straight line is
 (a) one (b) two (c) three (d) four
- 46** The average energy associated with a monoatomic molecule is
 (a) $k_B T$ (b) $\frac{1}{2} k_B T$ (c) $\frac{3}{2} k_B T$ (d) $2k_B T$
- 47** If γ is the ratio of specific heats and R is the universal gas constant, then the molar specific heat at constant volume C_V is given by **CBSE AIPMT 2013**
 (a) γR (b) $\frac{(\gamma - 1)R}{\gamma}$ (c) $\frac{R}{\gamma - 1}$ (d) $\frac{\gamma R}{\gamma - 1}$
- 48** The amount of heat energy required to raise the temperature of 1 g of helium at NTP, from T_1 Kelvin to T_2 Kelvin is **CBSE AIPMT 2013**
 (a) $\frac{3}{8} N_A k_B (T_2 - T_1)$ (b) $\frac{3}{2} N_A k_B (T_2 - T_1)$
 (c) $\frac{3}{4} N_A k_B (T_2 - T_1)$ (d) $\frac{3}{4} N_A k_B \left(\frac{T_2}{T_1} \right)$
- 49** A balloon has 5.0 g mol of helium at 7°C . Calculate the total internal energy of the system.
 (a) $5 \times 10^4 \text{ J}$ (b) $2.9 \times 10^4 \text{ J}$
 (c) $1.74 \times 10^4 \text{ J}$ (d) $8 \times 10^4 \text{ J}$
- 50** Diatomic molecule (rigid rotator) has
 (a) 3 translational degrees of freedom
 (b) 2 rotational degrees of freedom
 (c) Both (a) and (b)
 (d) All of the above

- 51** The value of energy for the O_2 molecule (rigid rotator) is
 (a) $(3/2) k_B T$ (b) $(5/2) k_B T$ (c) $(1/2) k_B T$ (d) $4k_B T$
- 52** 1 kg of a diatomic gas is at a pressure of $8 \times 10^4 \text{ Nm}^{-2}$ and the density of the gas is 4 kgm^{-3} . What is the energy of the gas due to its thermal motion?
 (a) $3 \times 10^4 \text{ J}$ (b) $5 \times 10^4 \text{ J}$ (c) $6 \times 10^4 \text{ J}$ (d) $7 \times 10^4 \text{ J}$
- 53** A gas consisting of rigid diatomic molecules was initially under standard condition, then gas was compressed adiabatically to one-fifth of its initial volume. What will be the mean kinetic energy of a rotating molecule in the final state? **AIIMS 2018**
 (a) 1.44 J (b) 4.55 J
 (c) $787.98 \times 10^{-23} \text{ J}$ (d) $757.3 \times 10^{-23} \text{ J}$
- 54** A gas mixture consists of 2 mol of O_2 and 4 mol of Ar at temperature T . Neglecting all vibrational modes, the total internal energy of the system is **NEET 2017**
 (a) $4RT$ (b) $15RT$ (c) $9RT$ (d) $11RT$
- 55** The value of $\gamma = \frac{C_p}{C_V}$, for hydrogen, helium and another ideal diatomic gas X (whose molecules are not rigid but have an additional vibrational mode), are respectively equal to **NEET (Odisha) 2019**
 (a) $\frac{7}{5}, \frac{5}{3}, \frac{9}{7}$ (b) $\frac{5}{3}, \frac{7}{5}, \frac{9}{7}$ (c) $\frac{5}{3}, \frac{7}{5}, \frac{7}{5}$ (d) $\frac{7}{5}, \frac{5}{3}, \frac{7}{5}$
- 56** Molecule of CO at moderate temperature have energy
 (a) $\frac{7}{2} k_B T$ (b) $\frac{5}{2} k_B T$ (c) $\frac{3}{2} k_B T$ (d) $\frac{1}{2} k_B T$
- 57** The specific heats, C_p and C_V of a gas of diatomic molecules, A are given (in units of $\text{J mol}^{-1} \text{ K}^{-1}$) by 29 and 22, respectively. Another gas of diatomic molecules B, has the corresponding values 30 and 21. If they are treated as ideal gases, then **JEE Main 2019**

- (a) A has a vibrational mode but B has none
 (b) Both A and B have a vibrational mode each
 (c) A has one vibrational mode and B has two
 (d) A is rigid but B has a vibrational mode
- 58** The value of total energy for one mole of polyatomic molecules is
 (a) $\left(\frac{3}{2}k_B T + \frac{f}{2}k_B T\right)N_A$ (b) $\left(3k_B T + \frac{f}{2}k_B T\right)N_A$
 (c) $(3k_B T + fk_B T)N_A$ (d) $\left(\frac{3}{2}k_B T + fk_B T\right)N_A$
- 59** A gas mixture consists of 3 moles of oxygen and 5 moles of argon at temperature T . Considering only translational and rotational modes, the total internal energy of the system is **JEE main 2019**
 (a) $12RT$ (b) $15RT$ (c) $20RT$ (d) $4RT$
- 60** Experimental values of specific heats are usually more than predicted values because
 (a) in predicted values we usually do not consider rotational mode
 (b) in predicted values we usually do not consider vibrational mode
 (c) in predicted values we usually do not consider translational mode
 (d) None of the above
- 61** As temperature tends to zero, i.e. $T \rightarrow 0K$
 (a) specific heat of all substances approaches zero
 (b) specific heat of all substances approaches infinity
 (c) specific heat of all substances may be zero or infinity
 (d) None of the above
- 62** We took two separate gases with the same number densities for both. If the ratio of the diameters of their molecules is $4 : 1$, then ratio of their mean free paths is
 (a) $1 : 4$ (b) $4 : 1$ (c) $2 : 1$ (d) $1 : 16$
- 63** The mean free path for air molecule with average speed 18.5 ms^{-1} at STP is ($d = 2 \times 10^{-10} \text{ m}$ and $n = 2.7 \times 10^{25} \text{ m}^{-3}$)
 (a) $3.5 \times 10^{-7} \text{ m}$ (b) $4 \times 10^{-7} \text{ m}$
 (c) $2.05 \times 10^{-7} \text{ m}$ (d) $5 \times 10^{-7} \text{ m}$
- 64** Consider an ideal gas confined in an isolated closed chamber. As the gas undergoes an adiabatic expansion, the average time of collision between molecules increases as V^q , where V is the volume of the gas. The value of q is $\left(\gamma = \frac{C_p}{C_V}\right)$
 (a) $\frac{3\gamma + 5}{6}$ (b) $\frac{3\gamma - 5}{6}$ (c) $\frac{\gamma + 1}{2}$ (d) $\frac{\gamma - 1}{2}$

SPECIAL TYPES QUESTIONS

I. Assertion and Reason

- **Direction** (Q. Nos. 65-70) In the following questions, a statement of Assertion is followed by a corresponding statement of Reason. Of the following statements, choose the correct one.
- (a) Both Assertion and Reason are correct and Reason is the correct explanation of Assertion.
 (b) Both Assertion and Reason are correct but Reason is not the correct explanation of Assertion.
 (c) Assertion is correct but Reason is incorrect.
 (d) Assertion is incorrect but Reason is correct.
- 65 Assertion** In gases, the molecules move faster and long distances without colliding.
Reason In gases, the interatomic forces are negligible.
- 66 Assertion** A gas that satisfy $pV = \mu RT$ at all pressures and temperatures is known as ideal gas.
Reason A real gas at low pressure and high temperature behaves like ideal gas.

- 67 Assertion** The total translational kinetic energy of all the molecules of a given mass of an ideal gas is 1.5 times the product of its pressure and volume.
Reason The molecules of a gas collide with each other and the velocities of the molecules change due to the collision.
- 68 Assertion** If a gas chamber containing a gas is moved translationally, then temperature of gas will increase.
Reason Total kinetic energy of the chamber will increase by the translational motion.
- 69 Assertion** Pressure of an ideal gas is equal to $(1/3) nm \bar{v}^2$.
Reason Pressure of an ideal gas increases with increase of volume.

- 70 Assertion** Degree of freedom of a monoatomic gas is always three, whether we consider vibrational effect or not.

Reason At all temperatures (low or high), vibrational kinetic energy of an ideal gas is zero.

II. Statement Based Questions

- 71** I. Atomic theory relates molecular properties with bulk properties of a gas.

II. According to kinetic theory, the interatomic forces are negligible in gases.

III. According to kinetic theory, the gas consists of rapidly moving atoms.

Which of the following statements are correct?

- 72** I. In solids and liquids, the interatomic forces are important.

II. In solids and liquids, the atoms attract when they are at a few angstroms but repel when they come closer.

Which of the following statements are correct?

- (a) Only I
 - (b) Only II
 - (c) Both I and II
 - (d) Neither I nor II

- 73** Choose the correct statement.

- (a) Avogadro's law with Dalton theory could explain Gay-Lussac's law.
 - (b) Dalton's atomic theory could also be termed as molecular theory as well.
 - (c) Initially Dalton's theory was not accepted by other scientists.
 - (d) All of the above

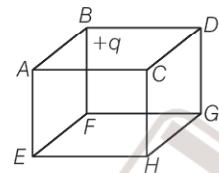
- 74** Which of the following statement is correct about the liquid?

- (a) In liquids, the atoms are not as rigidly fixed as in solid.
 - (b) In liquids, the atoms are more rigidly fixed as in gas.
 - (c) In liquids, the separation between atoms is about 1 Å.
 - (d) All of the above

- 75** Choose the correct statement.

- (a) The pressure of an ideal gas depends on the volume of the gas.
 - (b) The pressure of an ideal gas depends on the temperature of the gas.
 - (c) Both (a) and (b)
 - (d) Neither (a) nor (b)

- 76** $ABCDEFGH$ is a hollow cube as shown in figure made of an insulator with face $ABCD$ having positive charge ($+q$) on it. Inside the cube, we have ionised hydrogen.



If p is the usual kinetic theory expression for pressure, then choose the correct statement.

- (a) p will be valid.
 - (b) p will not be valid, since the ions would experience forces other than due to collisions with the walls.
 - (c) p will not be valid, since collisions with walls would not be elastic.
 - (d) p will not be valid because isotropy is constant.

- 77** Choose the correct statement.

- (a) Maxwell and Boltzmann were among the scientists who developed kinetic theory.
 - (b) Kinetic theory gives molecular interpretation of pressure and temperature of a gas.
 - (c) Kinetic theory is consistent with gas laws and Avogadro's hypothesis.
 - (d) All of the above

- 78** Choose the correct statement.

- (a) While finding out pressure of a gas inside a vessel, the shape of the vessel does not matter.
 - (b) Pressure of gas in equilibrium is same everywhere.
 - (c) While deriving the expression of pressure, we neglect the collisions between the molecules as it does not make much difference.
 - (d) All of the above

- 79** Choose the correct statement.

- (a) Each translational mode contributes $\frac{1}{2} k_B T$ average energy.

(b) Each rotational mode contributes $\frac{1}{2} k_B T$ average energy.

(c) Each vibrational mode contributes $k_B T$ average energy.

(d) All of the above

- 80** Choose the correct statement.

- (a) $C_p - C_V = R$, is true for monoatomic ideal gas.
 (b) $C_p - C_V = R$, is true for polyatomic ideal gas.
 (c) $C_p - C_V = R$, is true for any type of ideal gas.
 (d) None of the above

- 81** From the following statements, concerning ideal gas at any given temperature T , select the incorrect one.

- (a) The coefficient of volume expansion at constant pressure is same for all ideal gases.
 - (b) The average translational kinetic energy per molecule of oxygen gas is $k_B T$ (k_B being Boltzmann constant).
 - (c) In a gaseous mixture, the average translational kinetic energy of the molecules of each component is same.
 - (d) The mean free path of molecules increases with the decrease in pressure.

III. Matching Type

- 82** Match the Column I (relation of p, V and T) with Column II (related laws) and select the correct answer from the codes given below.

Column I	Column II
A. $pV = k_B NT$	1. Dalton's law
B. $p \propto 1/V,$ $T = \text{constant}$	2. Boyle's law
C. $V \propto T,$ $p = \text{constant}$	3. Ideal gas equation
D. $p = p_1 + p_2 + \dots$	4. Charles' law

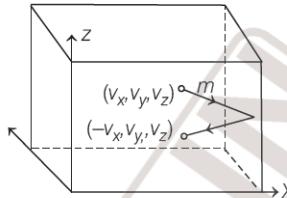
	A	B	C	D
(a)	3	2	1	4
(b)	3	3	1	4
(c)	3	2	4	1
(d)	3	2	1	4

- # **NCERT & NCERT Exemplar**

MULTIPLE CHOICE QUESTIONS

NCERT

- 83** A molecule of gas collide with the wall of a container elastically as shown in figure below

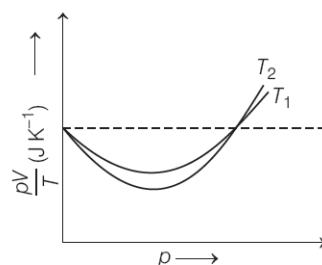


- Match the Column I (physical quantity) with Column II (formula) and select the correct answer from the codes given below.

	Column I					Column II			
A.	Change in momentum on the wall				1.	$nm\bar{v}_x^2$			
B.	Total momentum due to v_x				2.	$2mv_x$			
C.	Pressure due to collision along X -axis				3.	$(nmv_x^2)A\Delta t$			
D.	Total pressure along X -axis				4.	nmv_x^2			
A	B	C	D		A	B	C	D	
a) 2	3	1	4		(b)	2	3	4	1
c) 1	4	2	3		(d)	1	2	4	3

speed of a helium gas atom at -20°C ? (Take, atomic mass of Ar = 39.9 u and H = 4.0 u).

- 88** The figure below shows the plot of $\frac{pV}{T}$ versus p for one mole of oxygen gas at two different temperatures.



Read the following statements concerning the above curves and choose the correct one.

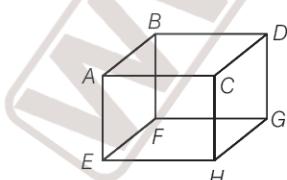
- I. The dotted line corresponds to the ideal behaviour.
 - II. $T_1 > T_2$.

NCERT Exemplar

- 91** A cubic vessel (with face horizontal + vertical) contains an ideal gas at NTP. The vessel is being carried by a rocket which is moving at a speed of 500 ms^{-1} in vertical direction. The pressure of the gas inside the vessel as observed by us on the ground

 - remains the same because 500 ms^{-1} is very much smaller than v_{rms} of the gas
 - remains the same because motion of the vessel as a whole does not affect the relative motion of the gas molecules w.r.t. the walls
 - will increase by a factor equal to $(v_{\text{rms}}^2 + (500)^2) / v_{\text{rms}}^2$ where, v_{rms} was the original mean square velocity of the gas
 - will be different on the top wall and bottom wall of the vessel

92 1 mol of an ideal gas is contained in a cubical volume V , $ABCDEFGH$ at 300K (see figure). One face of the cube $EFGH$ is made up of a material which totally absorbs any gas molecule incident on it. At any given time,



- (a) the pressure on $EFGH$ would be zero
 - (b) the pressure on all the faces will be equal
 - (c) the pressure of $EFGH$ would be double of the pressure on $ABCD$
 - (d) the pressure on $EFGH$ would be half of that on $ABCD$

- 93** Boyle's law is applicable for an
(a) adiabatic process (b) isothermal process
(c) isobaric process (d) isochoric process

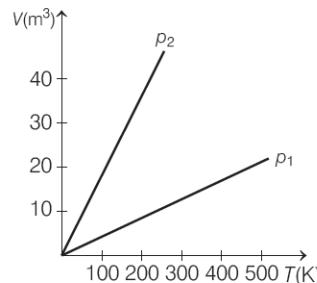
94 A cylinder containing an ideal gas is in vertical position and has a piston of mass M that is able to move up or down without friction (see figure). If the temperature is increased, then **JEE Main 2014**

JEE Main 2014



- (a) both p and V of the gas will change
 - (b) only p will increase according to Charles' law
 - (c) V will change but not p
 - (d) p will change but not V

- 95** Volume *versus* temperature graphs for a given mass of an ideal gas are shown in figure, at two different values of constant pressure. What can be inferred about relation between p_1 and p_2 ?



- (a) $p_1 > p_2$ (b) $p_1 = p_2$
 (c) $p_1 < p_2$ (d) Data is insufficient

- 96** 1 mol of H_2 gas is contained in a box of volume $V = 1.00 \text{ m}^3$ at $T = 300 \text{ K}$. The gas is heated to a temperature of $T = 3000 \text{ K}$ and the gas gets converted to a gas of hydrogen atoms. The final pressure would be (considering all gases to be ideal)

 - (a) same as the pressure initially
 - (b) 2 times the pressure initially
 - (c) 10 times the pressure initially
 - (d) 20 times the pressure initially

- 97** A vessel of volume V contains a mixture of 1 mol of hydrogen and 1 mol oxygen (both considered as ideal). Let $f_1(v)dv$ denote the fraction of molecules with speed between v and $(v + dv)$ with $f_2(v)dv$, similarly for oxygen. Then,

Answers

> Mastering NCERT with MCQs

1 (c)	2 (d)	3 (b)	4 (b)	5 (b)	6 (a)	7 (a)	8 (b)	9 (b)	10 (d)
11 (c)	12 (b)	13 (b)	14 (c)	15 (d)	16 (a)	17 (b)	18 (c)	19 (c)	20 (a)
21 (a)	22 (a)	23 (a)	24 (c)	25 (c)	26 (a)	27 (c)	28 (c)	29 (b)	30 (a)
31 (a)	32 (*)	33 (a)	34 (b)	35 (d)	36 (c)	37 (d)	38 (c)	39 (d)	40 (a)
41 (b)	42 (a)	43 (b)	44 (c)	45 (a)	46 (c)	47 (c)	48 (a)	49 (c)	50 (c)
51 (b)	52 (b)	53 (c)	54 (d)	55 (a)	56 (a)	57 (a)	58 (c)	59 (b)	60 (b)
61 (a)	62 (d)	63 (c)	64 (c)						

> Special Types Questions

65 (a) 66 (b) 67 (b) 68 (d) 69 (c) 70 (c) 71 (c) 72 (c) 73 (d) 74 (a)
 75 (c) 76 (b) 77 (d) 78 (d) 79 (d) 80 (c) 81 (b) 82 (c) 83 (b)

> NCERT & NCERT Exemplar MCQs

84 (b) 85 (a) 86 (b) 87 (c) 88 (d) 89 (a) 90 (b) 91 (b) 92 (d) 93 (b)
 94 (c) 95 (a) 96 (d) 97 (b) 98 (d) 99 (c) 100 (a)

Hints & Explanations

- 4 (b)** Given, $V_1 = 4.0 \text{ L}$, $\mu_1 = 0.75 \text{ mol}$, when 0.25 mol of gas is added, then $\mu_2 = \mu_1 + 0.25 = 0.75 + 0.25 = 1 \text{ mol}$. Since, gas constant is same for all gases, i.e.

$$\frac{pV_1}{T_1\mu_1} = \frac{p_2V_2}{T_2\mu_2} \Rightarrow \frac{V_1}{\mu_1} = \frac{V_2}{\mu_2} \quad (\because p \text{ and } T \text{ are same.})$$

$$\Rightarrow \frac{4}{0.75} = \frac{V_2}{1}$$

$$\Rightarrow V_2 = \frac{4}{0.75} = 5.33 \text{ L}$$

- 6 (a)** ∵ The number of moles of a gas,

$$\mu = \frac{M}{M_0} = \frac{\text{Total mass}}{\text{Molecular mass}}$$

Given, for two gases $\frac{(M_0)_1}{(M_0)_2} = \frac{7}{8}$ and M is same, so ratio of number of moles of two gases

$$\frac{\mu_1}{\mu_2} = \frac{M/(M_0)_1}{M/(M_0)_2} = \frac{(M_0)_2}{(M_0)_1} = \frac{8}{7}$$

- 7 (a)** Number of moles,

$$\mu = \frac{\text{Total mass}}{\text{Molecular weight}} = \frac{M}{M_0} = \frac{5}{32}$$

(∵ For O_2 , $M_0 = 2 \times 16 = 32$)

So, from ideal gas equation, $pV = \mu RT$

$$\Rightarrow pV = \frac{5}{32} RT$$

- 8 (b)** For first vessel having O_2 , number of moles,

$$\mu_1 = \frac{m_1}{M_1} = \frac{32}{32} = 1$$

When volume = V and temperature = T , then

$$p_1V = RT \quad \dots(\text{i})$$

For second vessel having H_2 , number of moles,

$$\mu_2 = \frac{m_2}{M_2} = \frac{4}{2} = 2$$

When volume = V and temperature = $2T$, then

$$p_2V = 2R(2T) = 4RT \quad \dots(\text{ii})$$

From Eqs. (i) and (ii), we get

$$p_2 = 4p_1 = 4p \quad [\because p_1 = p \text{ (given)}]$$

- 9 (b)** Let p_1 and p_2 are the initial and final pressures of the gas filled in vessel A , then

$$p_1 = \frac{\mu_A RT}{V} \text{ and } p_2 = \frac{\mu_A RT}{2V}$$

$$\therefore \Delta p = p_2 - p_1 = -\frac{\mu_A RT}{2V}$$

$$\Rightarrow \Delta p = -\left(\frac{m_A}{M}\right) \frac{RT}{2V} \quad \dots(\text{i})$$

where, M is the atomic weight of the gas.

Similarly, for gas in vessel B

$$1.5\Delta p = -\left(\frac{m_B}{M}\right) \frac{RT}{2V} \quad \dots(\text{ii})$$

On dividing Eq. (ii) by Eq. (i), we get

$$1.5 = \frac{m_B}{m_A} \quad \text{or} \quad \frac{3}{2} = \frac{m_B}{m_A}$$

$$\Rightarrow 3m_A = 2m_B$$

- 10 (d)** The perfect gas equation can be written as

$$pV = \mu RT \quad \dots(\text{i})$$

where,

and

$$R = N_A k_B = \text{universal constant} \dots(\text{ii})$$

Here,

$$\mu = \frac{m}{M_0} = \frac{N}{N_A} \quad \dots(\text{iii})$$

where,

m = mass of gas, M_0 = molar mass,

N = number of molecules

and

N_A = Avogadro's number.

Using, Eqs. (i), (ii) and (iii), we get

$$pV = k_B NT \text{ or } p = k_B nT$$

where, n = number density = $\frac{N}{V}$

$$\text{Also, } p = \frac{\rho RT}{M_0} \quad \left[\because V = \frac{M_0}{\rho} \right]$$

where, ρ = density.

Hence, all given options are correct.

- 11 (c)** Number of moles of the gas,

$$\mu = \frac{pV}{RT} = \frac{\text{Mass (m)}}{\text{Molar mass (M}_0)}$$

$$\Rightarrow \frac{\text{Mass}}{V} = \frac{(\text{Molar mass})p}{RT} \quad \dots(\text{i})$$

$$\text{As, Density} = \frac{\text{Mass}}{\text{Volume}} = \frac{(\text{Molar mass})p}{RT}$$

$$\text{Since, number of moles, } \mu = \frac{m}{M} = \frac{N}{N_A}$$

where, N = number of molecules

and N_A = Avogadro's number.

Molar mass, $M_0 = N_A m$, for $N = 1$

$$\therefore \text{Density} = \frac{(mN_A)p}{RT} \quad [\text{from Eq. (i)}]$$

$$= \frac{pm}{kT} \quad [\because R = kN_A]$$

- 12 (b)** According to given situation, $\rho_A = 1.5 \rho_B$

$$p_A = 2 p_B$$

According to ideal gas equation, we have

$$\text{Pressure, } p = \frac{\rho RT}{M}$$

where, M is molecular weight of ideal gas.

$$\Rightarrow M = \frac{\rho RT}{p}$$

where, R and T are constants.

$$\text{So, } M \propto \frac{\rho}{p} \Rightarrow \frac{M_A}{M_B} = \frac{\rho_A}{\rho_B} \times \frac{p_B}{p_A}$$

$$= 1.5 \times \frac{1}{2} = 0.75 = \frac{3}{4}$$

13 (b) Ideal gas equation can be written as

$$pV = \mu RT \quad \dots(\text{i})$$

$$\text{From Eq. (i), } \frac{\mu}{V} = \frac{p}{RT} = \text{constant}$$

[\because pressure and temperature are same]

So, at constant pressure and temperature, the gases in both balloons will contain same number of molecules per unit volume.

14 (c) From ideal gas equation,

$$\text{Initially, } pV = \mu RT \quad \dots(\text{i})$$

$$\text{Finally, } p\Delta V = \mu_1 R \Delta T$$

$$\text{Given, } \Delta V = \left(1 - \frac{10}{100}\right)V = \frac{90}{100}V = \frac{9}{10}V$$

$$\text{and } \Delta T = \left(1 + \frac{25}{100}\right)T = \frac{125}{100}T = \frac{5}{4}T$$

$$\therefore p \times \frac{9}{10}V = \mu_1 R \times \frac{5}{4}T \quad \dots(\text{ii})$$

On dividing Eq. (ii) by Eq. (i), we get

$$\frac{9}{10} = \frac{\mu_1}{\mu} \times \frac{5}{4} \Rightarrow \frac{\mu_1}{\mu} = \frac{18}{25}$$

\therefore Percentage of gas leaked

$$= \frac{\mu - \mu_1}{\mu} \times 100 = \left(1 - \frac{18}{25}\right) \times 100 = 28\%$$

15 (d) For an ideal gas keeping the temperature same throughout,

$$pV = \text{constant}$$

Hence, for a given mass, the graph between pV and V will be a straight line parallel to V -axis, whatever may be the volume.

Hence, graph (d) represents the behaviour of an ideal gas.

16 (a) According to Boyle's law, $pV = \text{constant}$

$$\Rightarrow p_2 = \frac{p_1 V_1}{V_2} = \frac{p_1 V_1}{V_1/2} = 2p_1$$

Thus, the pressure will be doubled.

17 (b) Given, $p_1 = 100 \text{ mm}$, $V_1 = 200 \text{ mL}$ and $p_2 = 400 \text{ mm}$

$$\text{From Boyle's law, } p_1 V_1 = p_2 V_2$$

$$\Rightarrow V_2 = \frac{p_1 V_1}{p_2} = \frac{100 \times 200}{400}$$

$$\Rightarrow V_2 = 50 \text{ mL}$$

$$\therefore \text{Volume occupied by two moles of gas} \\ = 2 \times 50 = 100 \text{ mL}$$

18 (c) According to Boyle's law, $p_1 V_1 = p_2 V_2$

As the pressure is decreased by 20%, so

$$p_2 = \frac{80}{100} p_1$$

$$p_1 V_1 = \frac{80}{100} p_1 V_2$$

$$\Rightarrow V_1 = \frac{80}{100} V_2 \Rightarrow V_2 = \frac{100}{80} V_1$$

\therefore Percentage increase in volume

$$= \frac{V_2 - V_1}{V_1} \times 100 \\ = \frac{100 - 80}{80} \times 100 = 25\%$$

19 (c) At constant atmospheric pressure, the volume of a given mass of a gas is directly proportional to its absolute temperature (T).

$$\text{i.e. } \frac{V}{T} = \text{constant}$$

Hence, correct graph is given in option (c).

20 (a) According to Charles' law, $\frac{V_1}{V_2} = \frac{T_1}{T_2}$

$$\text{Given, } V_1 = V, V_2 = 2V \quad T_1 = 27^\circ \text{C} + 273 = 300 \text{ K},$$

$$\therefore \frac{1}{2} = \frac{300 \text{ K}}{T_2}$$

$$\Rightarrow T_2 = 600 \text{ K} = 600 - 273 = 327^\circ \text{C}$$

Increase in temperature, $\Delta T = 327 - 27 = 300^\circ \text{C}$

21 (a) If the volume remains constant, then

$$\frac{p_1}{p_2} = \frac{T_1}{T_2} \Rightarrow \frac{p}{p + \frac{0.4}{100}p} = \frac{T}{T+1}$$

$$\Rightarrow \frac{T}{T+1} = \frac{250}{251} \Rightarrow 251T = 250T + 250$$

$$\therefore T = 250^\circ \text{C}$$

22 (a) Temperatures, $T_1 = 15^\circ \text{C} = 15 + 273 = 288 \text{ K}$

$$T_2 = 35^\circ \text{C} = 35 + 273 = 308 \text{ K}$$

As, volume remains constant.

$$\therefore \frac{p_1}{T_1} = \frac{p_2}{T_2} \Rightarrow \frac{p_1}{p_2} = \frac{T_1}{T_2} \Rightarrow \frac{p_1}{p_2} = \frac{288}{308}$$

$$\Rightarrow \frac{p_2}{p_1} = \frac{308}{288}$$

\therefore Percentage increase in pressure

$$= \frac{p_2 - p_1}{p_1} \times 100 = \frac{308 - 288}{288} \times 100 \\ = 6.94\% \approx 7\%$$

23 (a) We know that, ideal gas equation is

$$pV = nRT \quad \dots(\text{i})$$

(a) When pressure, $p = \text{constant}$

From Eq. (i), volume $V \propto$ temperature T

So, the graph is a straight line passing from origin.

(b) When temperature, $T = \text{constant}$

$$\text{From Eq. (i), } pV = \text{constant, i.e. } p \propto \frac{1}{V}$$

So, the graph is straight line having negative slope.

(c) When volume, $V = \text{constant}$

$$\text{From Eq. (i), i.e. } p \propto T$$

So, the graph is a straight line which passes through the origin.

(d) From Eq. (i), $pV \propto T$

$$\text{i.e. } \frac{pV}{T} = \text{constant}$$

So, the graph is a straight line which passes through origin.

Hence, only graph (a) is the correct representation of ideal gas behaviour.

24 (c) Since, mixture of gases are non-interacting ideal gases, hence according to Dalton's law of partial pressures, the total pressure of mixture of gases,

$$p = p_1 + p_2 + p_3 \quad \dots(\text{i})$$

where, p_1 , p_2 and p_3 are partial pressures of gases 1, 2 and 3, respectively.

$$\therefore p_1 = \mu_1 \frac{RT}{V}, p_2 = \mu_2 \frac{RT}{V}, p_3 = \mu_3 \frac{RT}{V}$$

$$\begin{aligned} \text{From Eq. (i), } p &= \mu_1 \frac{RT}{V} + \mu_2 \frac{RT}{V} + \mu_3 \frac{RT}{V} \\ &= (\mu_1 + \mu_2 + \mu_3) \frac{RT}{V} \end{aligned}$$

27 (c) In collision of a molecule of an ideal gas with the wall of container, total kinetic energy and total momentum will remain conserved, as the collisions between molecules among themselves or between molecules and the walls are elastic.

28 (c) According to kinetic theory, molecules of a gas are in a state of continuous random motion. The temperature of a gas depends on the average KE of these random motion of molecules. As, when the container is placed on a fast moving train, the average KE of container increases, while that of gas molecules remain same. So, temperature of the gas remains unchanged.

29 (b) When molecules collide with the wall in x -direction momentum imparted to the wall = $2mv_x$

If A be area of wall, then on an average half of molecules in volume $Av_x \Delta t$ will hit the wall. The total momentum transferred to the wall in time Δt is

$$(2mv_x) \left(\frac{1}{2} nAv_x \Delta t \right) = nmv_x^2 A \Delta t$$

where, n = number of molecules per unit volume.

$$\text{So, pressure} = \frac{\text{force}}{\text{area}} = \frac{\text{momentum}}{\text{area} \times \text{time}}$$

$$\Rightarrow p = \frac{nmv_x^2 A \Delta t}{A \Delta t} = nmv_x^2$$

Total pressure, $p = nm\bar{v}_x^2$

where, \bar{v}_x^2 = average of v_x^2 .

$$\text{As gas is isotropic, } \bar{v}_x^2 = \bar{v}_y^2 = \bar{v}_z^2 = \frac{\bar{v}^2}{3}$$

$$\text{where, } \bar{v}^2 = \text{mean of squared speed} \Rightarrow p = \frac{1}{3} nm \bar{v}^2$$

30 (a) As in an ideal gas, the molecular interaction is negligible, so the potential energy of molecules is not considered. Thus, the internal energy is in the form of only the kinetic energy of the molecules.

$$\text{It is given by } E = N \times \frac{1}{2} m \bar{v}^2$$

31 (a) As the temperature of gas in the container is increased, the kinetic energy also increases. This is because, the average kinetic energy of a gas is given by

$$KE = \frac{3}{2} k_B T \quad \dots(\text{i})$$

where, k_B = Boltzmann constant

and T = absolute temperature of the gas.

From Eq. (i), we get

$$KE \propto T$$

∴ Increase in temperature of a gas filled in a container would lead to increase in its KE. Thus, option (a) is correct.

However, option (b) is incorrect as increase in temperature will lead to increase in pressure as $p \propto T$. Other options (c) and (d) are also incorrect as molecular distance increases, while mass remains the same for increase in temperature.

32 (*) Momentum imparted to the surface in one collision,

$$\Delta p = (p_i - p_f) = mv - (-mv) = 2mv \quad \dots(\text{i})$$

Force on the surface due to n collision per second,

$$\begin{aligned} F &= \frac{n}{t} (\Delta p) = n \Delta p \quad (\because t = 1\text{s}) \\ &= 2 mnv \quad \text{from Eq. (i)]} \end{aligned}$$

So, pressure on the surface,

$$p = \frac{F}{A} = \frac{2 mnv}{A}$$

Here, $m = 10^{-26}$ kg, $n = 10^{22}$ s⁻¹, $v = 10^4$ ms⁻¹, $A = 1\text{ m}^2$

$$\therefore \text{Pressure, } p = \frac{2 \times 10^{-26} \times 10^{22} \times 10^4}{1} = 2 \text{ N/m}^2$$

So, pressure exerted is of order of 10^0 .

33 (a) We know that, pressure of gas, $p = \frac{1}{3} nm \bar{v}^2$

$$\begin{aligned} pV &= \frac{1}{3} nm V \bar{v}^2 = \frac{2}{3} (nV) \frac{1}{2} m \bar{v}^2 \\ &= \frac{2}{3} N \left(\frac{1}{2} m \bar{v}^2 \right) \quad (\because N = nV) \end{aligned}$$

or $pV = \frac{2}{3}E$... (i)
 where, E = total internal energy of the gas
 $= N(1/2)m\bar{v}^2$

We know that, ideal gas equation, $pV = k_BNT$... (ii)
 From Eqs. (i) and (ii), we get

$$\frac{2}{3}E = k_BNT \Rightarrow \frac{E}{N} = \frac{3}{2}k_B T$$

∴ Average translational kinetic energy of the molecule of gas $= \frac{3}{2}k_B T$

34 (b) The mean kinetic energy for gas molecules,

$$E = \frac{3}{2}k_B T \Rightarrow E \propto T$$

$$\therefore \frac{E_O}{E_H} = \frac{T_1}{T_2} \quad \dots (i)$$

According to the question, both gases are at the same temperature T .

So, $\frac{E_O}{E_H} = \frac{T}{T} = \frac{1}{1} \Rightarrow E_O : E_H = 1 : 1$

35 (d) Kinetic energy of a gas molecule, $E = (3/2)k_B T$

where, k_B is Boltzmann's constant.

$$\therefore E \propto T \text{ or } \frac{E_1}{E_2} = \frac{T_1}{T_2}$$

$$\Rightarrow \frac{E}{E/2} = \frac{300}{T_2} \quad [:: T_1 = 273 + 27 = 300 \text{ K}]$$

$$\therefore T_2 = 150 \text{ K} = 150 - 273 = -123^\circ\text{C}$$

36 (c) Average kinetic energy of a molecule

$$KE = \frac{3}{2}k_B T$$

does not depend on the molecular mass and total mass of the gas. It depends only on the temperature of the gas.

As temperature T is same for two gases, so ratio of KE is

$$\frac{(KE)_1}{(KE)_2} = \frac{1}{1}$$

37 (d) Pressure due to an ideal gas is given by

$$pV = \frac{1}{3}mN\bar{v}^2 \Rightarrow p = \frac{mN}{3V}\bar{v}^2 \quad \dots (i)$$

where, m = mass of gas molecule and N = total number of gas molecules.

Putting $\frac{m}{V} = \rho$ (density of gas) in Eq. (i)

$$p = \frac{1}{3}\rho\bar{v}^2N \Rightarrow \bar{v} = \sqrt{\left(\frac{3p}{\rho N}\right)}$$

As N and p are constants, so

$$\therefore \bar{v} \propto \frac{1}{\sqrt{\rho}}$$

38 (c) For one molecule, $v_{rms} = \sqrt{\frac{3k_B T}{m}}$

Given, $k_B = 1.38 \times 10^{-23} \text{ J K}^{-1}$, $T = 300 \text{ K}$

and $m = \frac{M_{N_2}}{N_A} = \frac{28}{6.02 \times 10^{23}}$
 $= 4.65 \times 10^{-23} \text{ kg}$
 $\therefore v_{rms} = \sqrt{\frac{3 \times 1.38 \times 10^{-23} \times 300}{4.65 \times 10^{-23}}} \approx 16.34 \text{ m s}^{-1}$

Hence, root mean square speed of a nitrogen molecule at 300 K is 16.34 ms^{-1} .

39 (d) Given, $v_{rms} = 300 \text{ ms}^{-1}$

Root mean square velocity, $v_{rms} = \sqrt{\frac{3k_B T}{m}} \quad \dots (i)$

When molecular weight is twice and absolute temperature is half, then

$$v'_{rms} = \sqrt{\frac{3k_B (T/2)}{2m}} = \frac{1}{2}\sqrt{\frac{3k_B T}{m}}$$

$$= \frac{v_{rms}}{2}$$

$$= \frac{300}{2} = 150 \text{ ms}^{-1}$$

[from Eq. (i)]

40 (a) Let unknown rms speed be $v_{rms,2}$

at $T_2 = 227^\circ\text{C}$ (or 500 K)

and at $T_1 = 127^\circ\text{C}$ (or 400 K)

$$v_{rms,1} = 200 \text{ m/s}$$

∴ Using the relation $v_{rms} \propto \sqrt{T}$, we can write

$$\frac{v_{rms,2}}{v_{rms,1}} = \sqrt{\frac{T_2}{T_1}} \quad \dots (i)$$

Substituting these given values in Eq. (i), we get

$$\therefore v_{rms,2} = \sqrt{\frac{500}{400}} \times 200 \text{ m/s}$$

$$= \frac{1}{2}\sqrt{5} \times 200 \text{ m/s} = 100\sqrt{5} \text{ m/s}$$

41 (b) Average speed or mean speed of gas molecules,

$$\bar{v} = \sqrt{\frac{3k_B T}{M}}$$

⇒ At same temperature $\bar{v} \propto \frac{1}{\sqrt{M}}$

$$\Rightarrow \frac{\bar{v}_{He}}{\bar{v}_{H_2}} = \sqrt{\frac{M_{He}}{M_{H_2}}}$$

$$\text{Given, } M_{He} = 4M_{H_2}$$

$$\therefore \frac{\bar{v}_{He}}{\bar{v}_{H_2}} = \sqrt{\frac{4}{1}} = 2 \Rightarrow \bar{v}_{He} = \frac{1}{2}\bar{v}_{H_2}$$

Hence, mean velocity of He at the same temperature is $\left(\frac{1}{2}\right)$ times of H-mean value.

42 (a) Given, $v_{rms} = 200 \text{ ms}^{-1}$, $T_1 = 27 + 273 = 300 \text{ K}$,

$$p_1 = 1.0 \times 10^5 \text{ N m}^{-2}$$

$$T_2 = 127 + 273 = 400 \text{ K}$$

$$\text{and } p_2 = 0.05 \times 10^5 \text{ N m}^{-2}$$

As, rms velocity of gas molecule,

$$v_{\text{rms}} \propto \sqrt{T} \quad \left(\because v_{\text{rms}} = \sqrt{\frac{3k_B T}{m}} \right)$$

For two different cases,

$$\begin{aligned} \frac{(v_{\text{rms}})_1}{(v_{\text{rms}})_2} &= \sqrt{\frac{T_1}{T_2}} \\ \Rightarrow \frac{200}{(v_{\text{rms}})_2} &= \sqrt{\frac{300}{400}} = \sqrt{\frac{3}{4}} = \frac{\sqrt{3}}{2} \\ \Rightarrow (v_{\text{rms}})_2 &= \frac{2}{\sqrt{3}} \times 200 = \frac{400}{\sqrt{3}} \text{ ms}^{-1} \end{aligned}$$

43 (b) As, root mean square speed of gas molecule is

$$v_{\text{rms}} = \sqrt{\frac{3k_B T}{m}} \quad \text{or} \quad v_{\text{rms}} \propto \sqrt{T}$$

v_{rms} is to reduce two times, i.e. the temperature of the gas will have to reduce four times or $\frac{T'}{T} = \left(\frac{v'_{\text{rms}}}{v_{\text{rms}}}\right)^2 = \frac{1}{4}$

During adiabatic process, $TV^{\gamma-1} = T'V'^{\gamma-1}$

$$\begin{aligned} \text{or} \quad \frac{V'}{V} &= \left(\frac{T}{T'}\right)^{\frac{1}{\gamma-1}} \\ \Rightarrow \frac{V'}{V} &= (4)^{\frac{1}{1.5-1}} = 4^{\frac{1}{0.5}} = 16 \\ \therefore V' &= 16V \end{aligned}$$

Hence, gas has to be expanded to 16 times.

47 (c) From Mayer's formula,

$$\begin{aligned} C_p - C_V &= R & \dots(i) \\ \text{and} \quad \gamma &= \frac{C_p}{C_V} \Rightarrow \gamma C_V = C_p & \dots(ii) \end{aligned}$$

On substituting Eq. (ii) in Eq. (i), we get

$$\begin{aligned} \gamma C_V - C_V &= R \\ \Rightarrow C_V (\gamma - 1) &= R \\ \therefore C_V &= \frac{R}{\gamma - 1} \end{aligned}$$

48 (a) Energy of a gas, $E = \frac{3}{2} N k_B T$

$$\begin{aligned} \therefore N &= \mu N_A \\ \therefore E &= \frac{3}{2} \cdot \mu \cdot N_A \cdot k_B \cdot T \end{aligned}$$

where, N_A = Avogadro's number and k_B = Boltzmann constant.

For helium molecule, $\mu = \frac{m}{M} = \frac{1}{4}$

\therefore Heat energy required to raise temperature from T_1 to T_2 is

$$\Delta E = \frac{3}{8} N_A k_B \Delta T = \frac{3}{8} N_A k_B (T_2 - T_1)$$

49 (c) Given, number of moles of helium = 5

Number of atoms (He is monoatomic)
= Number of moles \times Avogadro's number

$$\begin{aligned} &= 5 \times 6.023 \times 10^{23} = 30.015 \times 10^{23} \\ &= 3.0 \times 10^{24} \text{ atoms} \end{aligned}$$

Now, average kinetic energy per molecule = $\frac{3}{2} k_B T$

where, k_B = Boltzmann's constant

Temperature, $T = 273 + 7 = 280 \text{ K}$

\therefore Total energy of all the atoms = Total internal energy

$$\begin{aligned} &= \frac{3}{2} k_B T \times \text{Number of atoms} \\ &= \frac{3}{2} \times 1.38 \times 10^{-23} \times 280 \times 3.0 \times 10^{24} \\ &= 1.74 \times 10^4 \text{ J} \end{aligned}$$

52 (b) Given, mass = 1 kg, density = 4 kg m^{-3}

$$\therefore \text{Volume} = \frac{\text{Mass}}{\text{Density}} = \frac{1}{4} \text{ m}^3$$

$$\text{Pressure} = 8 \times 10^4 \text{ N m}^{-2}$$

Thermal energy corresponds to internal energy,
i.e. internal energy of diatomic gas,

$$\begin{aligned} E &= \frac{5}{2} k_B N T = \frac{5}{2} p \times V \quad (\because pV = k_B NT) \\ &= \frac{5}{2} \times 8 \times 10^4 \times \frac{1}{4} \text{ J} \\ &= 5 \times 10^4 \text{ J} \end{aligned}$$

53 (c) For diatomic gas, $\gamma = \frac{7}{5}$

At standard condition, $T_1 = 27^\circ \text{C} = 273 + 27 = 300 \text{ K}$

For adiabatic process, $TV^{\gamma-1} = \text{constant}$

$$\begin{aligned} T_1 V_1^{\gamma-1} &= T_2 V_2^{\gamma-1} \\ (300) V_1^{\frac{2}{5}-1} &= T_2 \left(\frac{V_1}{5}\right)^{\frac{7}{5}-1} \\ \Rightarrow T_2 &= \frac{300 \times V_1^{2/5}}{V_1^{\frac{2}{5}} \times \left(\frac{1}{5}\right)^{2/5}} = \frac{300}{\frac{-2}{5^2}} = 300 \times 5^{2/5} \\ &= 300 \times 1.903 = 570.9 \approx 571 \text{ K} \end{aligned}$$

Mean kinetic energy of a rotating molecules

$$= k_B T = 1.38 \times 10^{-23} \times 571$$

$$= 787.98 \times 10^{-23} \text{ J}$$

54 (d) Total internal energy of system = Internal energy of oxygen (diatomic) molecules + Internal energy of argon (monoatomic) molecules

$$\Rightarrow E = 2 \times \frac{5}{2} RT + 4 \times \frac{3}{2} RT = 11RT$$

55 (a) For hydrogen or diatomic gas, the internal energy is given by

$$U = \frac{5}{2} k_B T \times N_A = \frac{5}{2} RT$$

$$C_V = \frac{dU}{dT} = \frac{5}{2} R$$

$$\Rightarrow C_p = \frac{5}{2}R + R = \frac{7}{2}R \quad [\text{from } C_p = C_V + R]$$

$$\therefore \gamma = \frac{C_p}{C_V} = \frac{7}{5}$$

where, C_p = molar heat capacity at constant pressure and C_V = molar heat capacity at constant volume.

For helium (or monoatomic) gas, the internal energy is given by

$$U = \frac{3}{2}k_B TN_A = \frac{3}{2}RT$$

$$\Rightarrow C_V = \frac{dU}{dT} = \frac{3}{2}R$$

$$C_p = \frac{3}{2}R + R = \frac{5}{2}R$$

$$\therefore \gamma = \frac{C_p}{C_V} = \frac{5}{3}$$

For ideal diatomic gas X (non-rigid, having an additional vibrational mode) the internal energy is given by

$$U = \frac{7}{2}k_B TN_A = \frac{7}{2}RT$$

$$\Rightarrow C_V = \frac{dU}{dT} = \frac{7}{2}R$$

$$C_p = \frac{7}{2}R + R = \frac{9}{2}R$$

$$\therefore \gamma = \frac{C_p}{C_V} = \frac{9}{7}$$

56 (a) Molecule of CO at moderate temperature possess vibrational modes, i.e. it oscillates like one-dimensional oscillator.

$$\therefore \text{Vibrational energy} = \frac{1}{2}m\left(\frac{dy}{dt}\right)^2 + \frac{1}{2}ky^2$$

As, each vibrational mode contributes two square terms, so this mode will contribute $2\left(\frac{1}{2}k_B T\right)$ energy.

CO molecule has 3 translational and 2 rotational modes and each of them contributes $\frac{1}{2}k_B T$ to the energy.

\therefore Total energy,

$$E = \underbrace{3\left(\frac{1}{2}k_B T\right)}_{\text{Translational mode}} + \underbrace{2\left(\frac{1}{2}k_B T\right)}_{\text{Rotational mode}} + \underbrace{2\left(\frac{1}{2}k_B T\right)}_{\text{Vibrational mode}}$$

$$= \frac{7}{2}k_B T$$

57 (a) Now, in given case,

For gas A, $C_p = 29, C_V = 22$

For gas B, $C_p = 30, C_V = 21$

By using

$$\gamma = \frac{C_p}{C_V} = 1 + \frac{2}{f}$$

We have,

For gas A,

1 + \frac{2}{f} = \frac{29}{22} \approx 1.3 \Rightarrow f = 6.67 \approx 7

So, gas A has vibrational mode of degree of freedom.

For gas B,

1 + \frac{2}{f} = \frac{30}{21} \approx 1.4 \Rightarrow f = 5

Hence, gas B does not have any vibrational mode of degree of freedom.

58 (c) A polyatomic molecule has 3 translational, 3 rotational degrees of freedom and a certain number f degree of freedom of vibrational modes. Therefore, total energy is

$$U = \left(\frac{3}{2}k_B T + \frac{3}{2}k_B T + fk_B T \right) N_A$$

$$= (3k_B T + fk_B T) N_A$$

59 (b) Internal energy of a gas with f degree of freedom is

$$U = \frac{n_f RT}{2}, \text{ where } n \text{ is the number of moles.}$$

Internal energy due to O₂ gas which is a diatomic gas is

$$U_1 = \frac{n_1 f_1 RT}{2} = 3 \times \frac{5}{2} RT$$

($\because n_1 = 3$ moles, degree of freedom for a diatomic gas $f_1 = 5$)

Internal energy due to Ar gas which is a monoatomic gas is

$$U_2 = \frac{n_2 f_2 RT}{2} = 5 \times \frac{3}{2} RT$$

($\because n_2 = 5$ moles, degree of freedom for a monoatomic gas $f_2 = 3$)

$$\therefore \text{Total internal energy} = U_1 + U_2$$

$$\Rightarrow U = 15 RT$$

61 (a) The prediction of specific heats, based on law of equipartition of energy, are independent of temperature. But as we go to low temperatures, there is a marked departure from this prediction. Specific heat of all substances approach zero as $T \rightarrow 0$ K.

This is related to the fact that degrees of freedom get frozen and ineffective at low temperatures.

62 (d) As, mean free path,

$$l = \frac{1}{\sqrt{2}n\pi d^2}$$

$$\Rightarrow l \propto \frac{1}{d^2} \quad (\because n_1 = n_2)$$

$$\Rightarrow \frac{l_1}{l_2} = \frac{d_2^2}{d_1^2} = \left(\frac{1}{4}\right)^2 = \frac{1}{16}$$

- 63 (c)** For air at STP, $n = 2.7 \times 10^{25} \text{ m}^{-3}$
and $d = 2 \times 10^{-10} \text{ m}$

$$\text{Mean free path, } l = \frac{1}{\sqrt{2} n \pi d^2}$$

On putting values, we get

$$l = \frac{1}{\sqrt{2} \times 2.7 \times 10^{25} \times 3.14 \times (2 \times 10^{-10})^2} \\ = 2.05 \times 10^{-7} \text{ m}$$

- 64 (c)** We know that, average time of collision between molecules, $\tau = \frac{1}{n \pi \sqrt{2} v_{\text{rms}} d^2}$

where, n = number of molecules per unit volume and v_{rms} = rms velocity of molecules.

$$\Rightarrow \tau \propto \frac{1}{n v_{\text{rms}}}$$

$$\text{As, } n \propto \frac{1}{V} \text{ and } v_{\text{rms}} \propto \sqrt{T}$$

Thus, we can write $n = k_1 V^{-1}$ and $v_{\text{rms}} = k_2 T^{1/2}$ where, k_1 and k_2 are constants.

$$\Rightarrow \tau \propto \frac{V}{\sqrt{T}}$$

For adiabatic process, $T V^{\gamma-1} = \text{constant}$

$$\text{or } T \propto V^{1-\gamma}$$

Thus, we can write $\tau \propto V T^{-1/2} \propto V (V^{1-\gamma})^{-1/2}$

$$\Rightarrow \tau \propto V^{\frac{\gamma+1}{2}}$$

As, given that average time of collision between molecules increases as V^q , so on comparing, we get

$$q = \frac{\gamma+1}{2}$$

- 65 (a)** The interatomic forces are negligible in gases. Therefore in gases, the molecules are free and move long distances without colliding, with faster speed. Therefore, Assertion and Reason are correct and Reason is the correct explanation of Assertion.

- 66 (b)** A real gas at low pressure and high temperature behaves like ideal gas and satisfies $pV = \mu RT$.

Therefore, Assertion and Reason are correct but Reason is not the correct explanation of Assertion.

- 67 (b)** Total translational kinetic energy of all molecules (n moles) of a given mass of an ideal gas

$$= \frac{3}{2} \mu R T = \frac{3}{2} p V = 1.5 p V$$

$$[\because pV = \mu RT]$$

Due to collision of molecules of gas with each other, their velocities continuously change.

Therefore, Assertion and Reason are correct but Reason is not the correct explanation of Assertion.

- 68 (d)** The translational motion of the gas chamber increases the total kinetic energy of the chamber but it does not contribute to the internal energy of the system. So, the temperature of gas in the chamber remains same. Therefore, Assertion is incorrect but Reason is correct.

- 69 (c)** Pressure of an ideal gas = $\frac{1}{3} n m v^2$

$$\text{Also, pressure of an ideal gas, } p = \frac{\mu R T}{V}$$

$$\text{If } \mu \text{ and } T \text{ are constants, } p \propto \frac{1}{V}$$

So, pressure of an ideal gas increases with the decrease of volume.

Therefore, Assertion is correct but Reason is incorrect.

- 70 (c)** In an ideal gas, there is no inter-particle interaction, so vibrational kinetic energy remains constant, irrespective of temperature.

If the gas is monoatomic, no vibrational energy is there, so the degree of freedom is always three.

Therefore, Assertion is correct but Reason is incorrect.

- 71 (c)** Statements II and III are correct but I is incorrect and it can be corrected as,

The kinetic theory relates the molecular properties like viscosity, conduction and diffusion with bulk properties of a gas.

- 74 (a)** In liquids, the atoms are not as rigidly fixed as in solid and can move around. This enables a liquid to flow. In liquids the separation between atoms is about 2\AA . In case of gases, the atoms are not rigidly fixed but are free to move.

Thus, the statement given in option (a) is correct, rest are incorrect.

- 75 (c)** An ideal gas is that which satisfy the equation,

$$pV = \mu RT$$

where, p = pressure, V = volume,

μ = number of moles, R = gas constant

and T = temperature.

Here, μ and R are constant for a gas, so

$$p \propto \frac{T}{V}$$

Hence, pressure of an ideal gas depends on both the temperature and volume of the gas. Thus, both the statements given in options (a) and (b) are correct.

- 76 (b)** Due to the presence of external positive charge on the face $ABCD$, the usual expression for pressure p on the basis of kinetic theory will not be valid.

This is because ions present inside cube would also experience electrostatic forces other than the forces due to collisions with the walls of the container. Also due to presence of positive charge, the isotropy is lost.

Thus, the statement given in option (b) is correct, rest are incorrect.

78 (d) Pressure of a gas,

$$p = \frac{1}{3} nm\bar{v}^2$$

where, n = number of molecules per unit volume,
 m = mass of gas and \bar{v}^2 = mean of squared speed.

Thus we can conclude that, while finding out pressure, the shape of the vessel does not matter, as A and Δt do not appear in the above formula.

By Pascal's law, the pressure of the gas in equilibrium is the same everywhere.

Also, while deriving the expression of pressure, we neglect the collisions between the molecules as it does not make much difference. This is because the collisions are too frequent and time spent is negligible.

Thus, all the statements given in options (a), (b) and (c) are correct.

81 (b) All statements are correct except the statement given in option (b) and it can be corrected as,

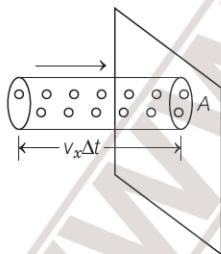
The average translational KE per molecule is same for molecules of all gases like here O_2 and for each molecule it is $\frac{3}{2} k_B T$.

83 (b) A → As only x -component of velocity changes, so change in momentum of molecule

$$= -mv_x - (mv_x) = -2mv_x$$

As collision is elastic, the total momentum of molecule and wall is conserved. So, change in momentum of wall (momentum imparted to wall) = $2mv_x$.

B → In time Δt , the number of molecules with velocity v_x that could reach the wall is $nAv_x\Delta t$ within volume of cylinder of length $v_x\Delta t$ as shown below, where n → number density.



But on an average half of these molecules will hit the wall in time, $\Delta t = \frac{1}{2}nAv_x\Delta t$

The total momentum transferred to the wall in time Δt

$$= (2mv_x) \left(\frac{1}{2}nAv_x\Delta t \right) = (nmv_x^2)A\Delta t$$

$$\text{C} \rightarrow \text{Pressure} = \frac{F}{A} = \frac{\text{Momentum}}{\text{Area} \times \text{Time}}$$

$$= \frac{(nmv_x^2)A\Delta t}{A\Delta t} = nmv_x^2$$

D → As the x -component of velocities of all the molecules are not same. So, we take average of \bar{v}_x^2 .

$$\therefore \text{Total pressure, } p = nm\bar{v}_x^2$$

Hence, A → 2, B → 3, C → 4 and D → 1.

84 (b) Given, diameter of O_2 molecule = 3\AA

$$\therefore \text{Radius of } O_2 = 1.5 \text{\AA} = 1.5 \times 10^{-8} \text{ cm}$$

$$\text{Volume of each molecule, } V = \frac{4}{3}\pi r^3$$

$$\begin{aligned} \text{Number of molecules in one mole of gas} &= N_A \\ &= \text{Avogadro's number} = 6.023 \times 10^{23} \text{ molecules/mol} \end{aligned}$$

$$\therefore \text{Volume of total molecules at STP, } V = \frac{4}{3}\pi r^3 N_A$$

$$\begin{aligned} V &= \frac{4}{3} \times 3.14 (1.5 \times 10^{-8})^3 \times 6.023 \times 10^{23} \\ &= 8.51 \text{ cm}^3 \end{aligned}$$

We know that, the volume of one mole of gas at STP

$$\begin{aligned} &= 22.4 \text{ L} = 22.4 \times 10^3 \text{ cm}^3 \\ &= 22400 \text{ cm}^3 \end{aligned}$$

Fraction of molecular volume to the actual volume

$$\begin{aligned} &= \frac{8.51}{22400} = 3.8 \times 10^{-4} \\ &\approx 4.0 \times 10^{-4} \end{aligned}$$

85 (a) Given, pressure, $p = 1 \text{ atm}$

$$= 1.013 \times 10^5 \text{ N m}^{-2}$$

$$\text{Gas constant, } R = 8.314 \text{ J mol}^{-1} \text{ K}^{-1}$$

$$\text{Number of mole, } \mu = 1$$

$$\text{Temperature, } T = 273.15 \text{ K}$$

$$\text{Using ideal gas equation, } pV = \mu RT$$

$$\Rightarrow V = \frac{\mu RT}{p} = \frac{1 \times 8.314 \times 273.15}{1.013 \times 10^5} = 0.0224 \text{ m}^3 = 22.4 \times 10^{-3} \text{ m}^3 = 22.4 \text{ L}$$

86 (b) Given, volume of the room, $V = 25.0 \text{ m}^3$

$$\text{Temperature, } T = 27^\circ \text{C} = (273 + 27) \text{ K} = 300 \text{ K}$$

$$\text{Pressure, } p = 1 \text{ atm} = 1.013 \times 10^5 \text{ Pa}$$

$$\text{Boltzmann constant, } k_B = 1.38 \times 10^{-23}$$

$$\text{Using the relation, } pV = \mu k_B T$$

$$\Rightarrow \mu = \frac{pV}{k_B T} = \frac{1.013 \times 10^5 \times 25}{1.38 \times 10^{-23} \times 300} = 6.11 \times 10^{26} \text{ molecules}$$

87 (c) Root mean square speed of argon atom,

$$(v_{\text{rms}})_{\text{Ar}} = \sqrt{\frac{3k_B T_{\text{Ar}}}{M_{\text{Ar}}}} \quad \dots(\text{i})$$

Root mean square speed of helium atom,

$$(v_{\text{rms}})_{\text{He}} = \sqrt{\frac{3k_B T_{\text{He}}}{M_{\text{He}}}} \quad \dots(\text{ii})$$

Dividing Eq. (i) by Eq. (ii), we get

$$\frac{(v_{\text{rms}})_{\text{Ar}}}{(v_{\text{rms}})_{\text{He}}} = \sqrt{\frac{3k_B T_{\text{Ar}}}{M_{\text{Ar}}} \times \frac{M_{\text{He}}}{3k_B T_{\text{He}}}}$$

Given, $(v_{\text{rms}})_{\text{Ar}} = (v_{\text{rms}})_{\text{He}}$

and $T_{\text{He}} = 273.15 - 20 = 253.15 \text{ K}$

$$\Rightarrow 1 = \sqrt{\left(\frac{T_{\text{Ar}}}{T_{\text{He}}}\right) \cdot \left(\frac{M_{\text{He}}}{M_{\text{Ar}}}\right)}$$

$$\Rightarrow \frac{M_{\text{He}}}{M_{\text{Ar}}} = \frac{T_{\text{He}}}{T_{\text{Ar}}}$$

$$\therefore T_{\text{Ar}} = T_{\text{He}} \left(\frac{M_{\text{Ar}}}{M_{\text{He}}} \right) = 253.15 \left(\frac{39.9}{4} \right) \\ = 2525.171 \text{ K} = 2.52 \times 10^3 \text{ K}$$

88 (d)

- I. The dotted line in the diagram shows that there is no deviation in the value of $\frac{pV}{T}$ for different temperatures T_1 and T_2 for increasing pressure, so this gas behaves ideally. Hence, dotted line corresponds to ideal gas behaviour.
 - II. At high temperature, the deviation of the gas is less and at low temperature the deviation of gas is more. In the graph, deviation for T_2 is greater than for T_1 . Thus, $T_1 > T_2$.
 - III. Since, the two curves intersect at dotted line, so the value of $\frac{pV}{T}$ at that point on the Y-axis is same for all gases.
- So, all statements are correct.

89 (a) Absolute pressure, $p_1 = (15 + 1) \text{ atm}$

$$(\because \text{absolute pressure} = \text{gauge pressure} + 1 \text{ atm}) \\ = 16 \times 1.013 \times 10^5 \text{ Pa}$$

$$V_1 = 30 \text{ L} = 30 \times 10^{-3} \text{ m}^3$$

$$T_1 = 27 + 273.15 = 300.15 \text{ K}$$

Using ideal gas equation, $pV = \mu RT$

$$\Rightarrow \mu = \frac{pV}{RT}$$

Initially number of moles,

$$\mu_1 = \frac{p_1 V_1}{RT_1} = \frac{16 \times 1.013 \times 10^5 \times 30 \times 10^{-3}}{8.314 \times 300.15}$$

$$= 19.48$$

$$\text{Finally, } p_2 = (11 + 1) = 12 \text{ atm}$$

$$= 12 \times 1.013 \times 10^5 \text{ Pa}$$

$$V_2 = 30 \text{ L} = 30 \times 10^{-3} \text{ m}^3$$

$$T_2 = 17 + 273.15 = 290.15 \text{ K}$$

Number of moles,

$$\mu_2 = \frac{p_2 V_2}{RT_2} = \frac{12 \times 1.013 \times 10^5 \times 30 \times 10^{-3}}{8.314 \times 290.15} \\ = 15.12$$

$$\text{Hence, moles removed} = \mu_1 - \mu_2 \\ = 19.48 - 15.12 = 4.36$$

$$\text{Mass removed} = 4.36 \times 32 \text{ g} \\ = 139.52 \text{ g} = 0.1396 \text{ kg}$$

90 (b) Given, $T = 27^\circ\text{C} = (273.15 + 27)\text{K} = 300.15 \text{ K}$

$$\text{As average thermal energy, } E = \frac{3}{2} k_B T$$

$$\text{where, } k_B = \text{Boltzmann's constant} \\ = 1.38 \times 10^{-23} \text{ JK}^{-1}.$$

$$\therefore E = \frac{3}{2} \times 1.38 \times 10^{-23} \times 300.15 \\ = 6.21 \times 10^{-21} \text{ J}$$

91 (b) As the motion of the vessel as a whole does not effect the relative motion of the gas molecules with respect to the walls of the vessel, hence pressure of the gas inside the vessel, as observed by us, on the ground remains the same.

92 (d) In an ideal gas, when a molecule collides elastically with a wall, the momentum transferred to each molecule will be twice the magnitude of its normal momentum. However, as the face $EFGH$ absorbs the gas molecule incident on it, thus the momentum transferred will be only half.

Therefore, the pressure on $EFGH$ would be half of that on $ABCD$.

93 (b) Boyle's law is applicable when temperature is constant,

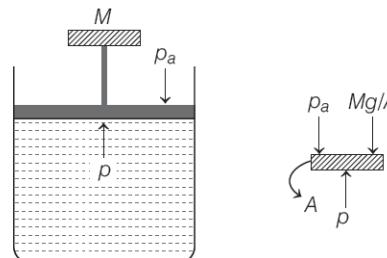
$$\text{i.e. } pV = \mu RT = \text{constant}$$

$$\Rightarrow pV = \text{constant} \text{ (at constant temperature)}$$

$$\Rightarrow p \propto \frac{1}{V} \text{ (where, } p = \text{pressure and } V = \text{volume)}$$

As, this process is called as isothermal process, so Boyle's law is applicable for isothermal process.

94 (c) Consider the diagram, where an ideal gas is contained in a cylinder, having a piston of mass M (friction is absent here).



The pressure inside the gas will be

$$p = p_a + Mg/A$$

where, p_a = atmospheric pressure,

A = area of cross-section of the piston

and Mg = weight of piston.

As the mass remains constant, so the pressure p of the gas remains the same.

As $pV = \mu RT \Rightarrow V \propto T$,
So, when temperature T increases, volume V increases at constant pressure.
Hence, V will change but not p .

95 (a) We know for an ideal gas,

$$pV = \mu RT \\ \Rightarrow V = \left(\frac{\mu R}{p} \right) T$$

where, p = pressure, V = volume,
 μ = number of moles of gas,
 R = gas constant and T = temperature.

Slope of the $V-T$ graph,

$$m = \frac{dV}{dT} = \frac{\mu R}{p} \\ \Rightarrow m \propto \frac{1}{p} \quad (\because nR = \text{constant}) \\ \text{or} \quad p \propto \frac{1}{m} \\ \text{Hence,} \quad \frac{p_1}{p_2} = \frac{m_2}{m_1} > 1$$

where, m_1 is slope of the graph corresponding to p_1 and similarly m_2 is slope corresponding to p_2 .

$$\Rightarrow p_2 < p_1 \text{ or } p_1 > p_2$$

96 (d) Consider the diagram, when the molecules breaks into atoms, the number of moles would become twice.

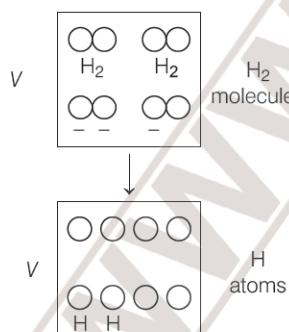
$$\text{So, } \mu_2 = 2\mu_1$$

Now, by ideal gas equation,

$$pV = \mu RT$$

where, p = pressure of gas, μ = number of moles,
 R = gas constant and T = temperature.

As volume V of the container is constant.



Given, $T_1 = 300 \text{ K}$ and $T_2 = 3000 \text{ K}$

$$\text{So,} \quad p \propto \mu T$$

$$\Rightarrow \frac{p_2}{p_1} = \frac{\mu_2 T_2}{\mu_1 T_1}$$

$$= \frac{(2\mu_1)(3000)}{\mu_1(300)} = 20$$

$$\Rightarrow p_2 = 20p_1$$

Hence, final pressure of the gas would be 20 times the pressure initially.

97 (b) As the vessel contains 1 mole of hydrogen and 1 mol of oxygen, therefore as per Maxwell's law of speed distribution, speed of any molecule depend on its mass. The mass of hydrogen and oxygen are different, so their speed will also be different. Hence, $f_1(v)$ and $f_2(v)$ will obey the law separately.

98 (d) We know for an ideal gas, $pV = \mu RT$

where, μ = number of moles, p = pressure,
 V = volume, R = gas constant, and T = temperature.

$$\Rightarrow \mu = \frac{pV}{RT}$$

As number of moles of the gas remains fixed, hence we can write

$$\frac{p_1 V_1}{RT_1} = \frac{p_2 V_2}{RT_2}$$

$$\Rightarrow p_2 = p_1 V_1 \left(\frac{T_2}{V_2 T_1} \right)$$

Given $p_1 = p, V_2 = 1.05V$ and $T_2 = 1.1T$

$$\Rightarrow p_2 = \frac{(p)(V)(1.1T)}{(1.05)V(T)}$$

$$= p \times \left(\frac{1.1}{1.05} \right)$$

$$= p(1.0476)$$

$$\approx 1.05p$$

Hence, final pressure p_2 lies between p and $1.1p$.

99 (c) According to kinetic theory, we assume that the walls only exert perpendicular forces on molecules. They do not exert any parallel force, hence there will not be any type of rotation present and the pressure due to collision relates only to the change in momentum.

The wall produces only change in translational motion. Hence, in the equation,

$$pV = \frac{2}{3} E \quad \left(\text{where, } p = \text{pressure} \right) \quad \left(\text{and } V = \text{volume} \right)$$

E is representing only translational part of energy.

100 (a) When an ideal gas is compressed adiabatically, the total work done on the gas increases its internal energy which in turn increases the kinetic energy of gas molecules. It is because of the collisions of molecules with the moving parts of the wall only.

CHAPTER > 14

Oscillations



Periodic and Oscillatory Motion

- In uniform circular motion and orbital motion of planets in the solar system, the motion is repeated after a certain interval of time, hence it is called **periodic motion**.
- If the body is given a small displacement from the equilibrium position, a force comes into play which tries to bring the body back to the equilibrium position, giving rise to **oscillations or vibrations**.
- A motion in which a body moves to and fro about a mean position or back and forth about a mean position, termed as **oscillatory motion**.
- Every oscillatory motion is periodic but every periodic motion need not to be oscillatory. e.g. Circular motion is a periodic motion, but it is not oscillatory.
- The description of a periodic motion, in general and oscillatory motion in particular, requires some fundamental concepts, like period, frequency, displacement, amplitude and phase.

Period and Frequency

- The smallest interval of time after which the motion is repeated is called its **period**.
- The reciprocal of period T gives the number of repetitions that occur per unit time. This quantity is called the **frequency** of the periodic motion.
- The relation between frequency v and period T is given as
$$v = 1 / T$$

The SI unit of frequency is hertz.

Displacement

- The displacement can be represented by a mathematical function of time.
- In case of periodic motion, this function is periodic in time. One of the simplest periodic function is given by

$$f(t) = A \cos\omega t$$

If the argument of this function ωt is increased by an integral multiple of 2π radians, the value of function remains the same.

The function $f(t)$ is then periodic and its period T is given by

$$T = \frac{2\pi}{\omega}$$

- A linear combination of sine and cosine functions like

$$f(t) = A \sin\omega t + B \cos\omega t \quad \dots(i)$$

is also a periodic function with same period T .

Eq. (i) can be written as

$$f(t) = D \sin(\omega t + \phi)$$

where, $D = \sqrt{A^2 + B^2}$

and $\phi = \tan^{-1}\left(\frac{B}{A}\right)$.

- Therefore, any periodic function can be expressed as a superposition of sine and cosine functions of different time periods with suitable coefficients.

Simple Harmonic Motion

- It is the simplest form of oscillatory motion. This motion arises when the force on the oscillating body is directly proportional to its displacement from the mean position and its direction is always towards the mean position.
- A particle oscillating back and forth about the origin of an X-axis between the limits $+A$ and $-A$ is as shown in figure



This oscillatory motion is said to be **simple harmonic**, if the displacement x of the particle from the origin varies with time as

$$x(t) = A \cos(\omega t + \phi)$$

where, $x(t)$ = displacement x as a function of time t ,

A = amplitude,

$$\omega = \text{angular frequency} \left(= \frac{2\pi}{T}\right)$$

$(\omega t + \phi)$ = phase (time-dependent)

and ϕ = initial phase constant.

- The **amplitude** A of SHM is the magnitude of maximum displacement of the particle.

Simple Harmonic Motion and Uniform Circular Motion

- Simple harmonic motion can be defined as the projection of uniform circular motion on any diameter of a cycle of reference.
- The particle **velocity and acceleration** during SHM as functions of time are given by

$$v(t) = \frac{d}{dt}x(t) = -\omega A \sin(\omega t + \phi)$$

$$a(t) = \frac{d}{dt}v(t) = -\omega^2 A \cos(\omega t + \phi) = -\omega^2 x(t)$$

- Therefore, we can say that both velocity and acceleration of a body executing simple harmonic motion are periodic functions.
- Velocity amplitude is $v_{\max} = \omega A$ and acceleration amplitude is $a_{\max} = \omega^2 A$.

Force Law and Energy in SHM

- The force acting on a particle of mass m in SHM is given as

$$F(t) = ma = -m\omega^2 x(t)$$

i.e.

$$F(t) = -kx(t)$$

where,

$$k = m\omega^2$$

\Rightarrow

$$\omega = \sqrt{\frac{k}{m}}$$

- Like acceleration, force is always directed towards the mean position, hence it is sometimes called the **restoring force** in SHM.

- When a particle oscillating under such a force which is linearly proportional to displacement $x(t)$, then it is called **linear harmonic oscillator**.
- In the real world, the force may contain small additional terms proportional to x^2 , x^3 , etc. these are called **non-linear oscillators**.
- Both kinetic and potential energies of a particle in SHM vary between zero and their maximum values.
- Kinetic energy** K of particle performing SHM is given by

$$\begin{aligned} K &= \frac{1}{2}mv^2 = \frac{1}{2}m\omega^2 A^2 \sin^2(\omega t + \phi) \\ &= \frac{1}{2}kA^2 \sin^2(\omega t + \phi) \end{aligned}$$

- Kinetic energy in SHM is also a periodic function of time, being zero when the displacement is maximum and maximum when the particle is at the mean position. Since, the sign of velocity v is immaterial in K , the period of K is $\frac{T}{2}$.
- The **potential energy** of a particle executing simple harmonic motion is

$$U(x) = \frac{1}{2}kx^2 = \frac{1}{2}kA^2 \cos^2(\omega t + \phi)$$

- The potential energy of a particle executing simple harmonic motion is also periodic, with period $\frac{T}{2}$, being zero at the mean position and maximum at the extreme displacements.
- Total energy of a particle** in SHM is given as

$$E = U + K = \frac{1}{2}kA^2 = \frac{1}{2}m\omega^2 A^2$$

- The total mechanical energy of a harmonic oscillator is thus independent of time as expected for motion under any conservative force.
- Both kinetic and potential energies peak twice during each period of SHM.
- For $x = 0$ (mean position), the energy is kinetic and at the extremes $x = \pm A$, it is all potential energy. This means, in the course of motion between these limits, kinetic energy increases at the expense of potential energy or vice-versa.

Some Systems Executing SHM

- There are no physical examples of absolutely pure simple harmonic motion. In practice, we come across systems that execute simple harmonic motion approximately under certain conditions.

The simplest observable example of SHM is the small oscillations due to a spring and simple pendulum.

Oscillation Due to a Spring

- When a linear simple harmonic oscillator consisting of a block of mass m attached to a spring is pulled or pushed and released, executes simple harmonic motion, whose period of oscillation is given as

$$T = 2\pi \sqrt{\frac{m}{k}}$$

where, k is spring constant.

- Stiff springs have high value of k , while soft springs have low value of k .

Simple Pendulum

- A simple pendulum in practice, consists of a heavy but small sized metallic bob suspended by a light, inextensible and flexible string.
- The motion of a simple pendulum is simple harmonic whose time period is given by

$$T = 2\pi \sqrt{\frac{L}{g}}$$

Damped Simple Harmonic Motion

- When the motion of simple pendulum, swinging in air, dies out eventually because the air drag and the friction at the support oppose the motion of the pendulum and dissipate its energy gradually, then the pendulum is said to be **executing damped oscillations**.
- The displacement equation of the motion of block (mass m) under the influence of damping force is given as

$$x(t) = Ae^{-bt/2m} \cdot \cos(\omega't + \phi)$$

where, A is amplitude and ω' is the angular frequency of the damped oscillator is given by $\omega' = \sqrt{\frac{k}{m} - \frac{b^2}{4m^2}}$.

- In damped oscillations, the energy of the system is dissipated continuously but for small damping, the oscillations remain approximately periodic.
- The mechanical energy E of the damped oscillator is given by $E(t) = \frac{1}{2}kA^2e^{-bt/m}$
- The mechanical energy in a real oscillating system decreases during oscillation because external force, such as drag inhibit the oscillations and transfer mechanical energy to thermal energy.
- The damping force depends on the nature of the surrounding medium. If the block is immersed in a liquid, the magnitude of damping will be much greater and the dissipation of energy much faster.
- The damping force is generally proportional to velocity of the bob and it acts opposite to the direction of velocity. If the damping force is denoted by F_d ,

we have,

$$F_d = -b\mathbf{v}$$

where, the positive constant b depends on characteristics of the medium, size and shape of block.

Note The oscillation whose amplitude does not change with time are called **undamped oscillations**.

Forced Oscillations and Resonance

- When a system (such as a simple pendulum or block attached to a spring) is displaced from its equilibrium position and released, it oscillates with its natural frequency ω and the oscillations are called **free oscillations**.
- All free oscillations eventually die out because of the ever present damping forces. However, an external agency can maintain these oscillations, these are called **forced or driven oscillations**.
- When an external periodic force is applied, then displacement equation of forced oscillation is given as $x(t) = A \cos(\omega_d t + \phi)$ where, t is the time measured from the moment when we apply the periodic force.
- The amplitude A is a function of the forced frequency ω_d and the natural frequency ω and given as

$$A = \frac{F_0}{\{m^2(\omega^2 - \omega_d^2)^2 + \omega_d^2 b^2\}^{1/2}}$$

where, F_0 is the amplitude of external periodic force and m is mass of the particle.

- The value of phase difference ϕ is given as

$$\tan \phi = \frac{-v_0}{\omega_d x_0}$$

where, v_0 and x_0 are the velocity and the displacement of the particle at $t = 0$.

- For small damping, driving frequency far from natural frequency**

In this case, $\omega_d b$ will be much smaller than $m(\omega^2 - \omega_d^2)$, hence amplitude A is given as

$$A = \frac{F_0}{m(\omega^2 - \omega_d^2)}$$

- When driving frequency is close to natural frequency**

If ω_d is very close to ω , $m(\omega^2 - \omega_d^2)$ would be much less than $\omega_d b$, for any reasonable value of b , then amplitude A becomes

$$A = \frac{F_0}{\omega_d b}$$

- The phenomenon of increase in amplitude, when the driving force is close to the natural frequency of the oscillator is called **resonance**.



Mastering NCERT

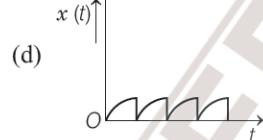
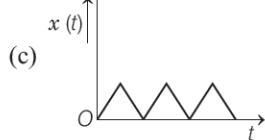
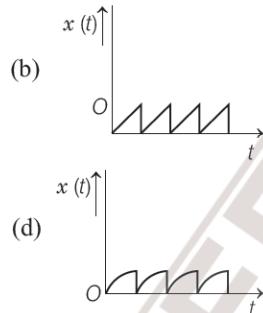
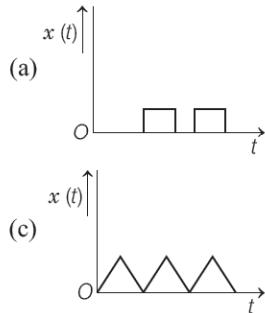
MULTIPLE CHOICE QUESTIONS

TOPIC 1 ~ Periodic and Oscillatory Motion

1 The rotation of earth about its axis is

- (a) periodic motion
- (b) simple harmonic motion
- (c) non-periodic motion
- (d) None of the above

2 Choose $x-t$ graph for an insect climbing up a ramp uniformly and sliding down then comes back to initial point and repeats the process identically.



3 On an average, a human heart is found to beat 75 times in a minute. The beat frequency and period of human heart are respectively

- (a) 1 Hz, 0.8 s
- (b) 1 Hz, 1 s
- (c) 1.25 Hz, 0.8 s
- (d) 2 Hz, 0.5 s

4 The periodic function $f(t) = A \sin \omega t$ repeats itself after

- (a) 2π
- (b) 3π
- (c) π
- (d) $\pi/2$

5 A particle perform oscillatory motion with amplitude 4 cm. If the time period of particle is 1 s, then time taken by the particle to reach 2 cm from the mean position is given by

- (a) $(1/2)$ s
- (b) $(1/4)$ s
- (c) $(1/12)$ s
- (d) $(1/6)$ s

6 Displacement of a particle in periodic motion is expressed as $x(t) = 20 \cos \omega t$. If the time period of particle motion is 4 s, then displacement of the particle in 1 s will be

- (a) 10 m
- (b) 15 m
- (c) 0
- (d) 20 m

7 The function $\log \omega t$

- (a) is a periodic function
- (b) is a non-periodic function
- (c) could represents oscillatory motion
- (d) can represents circular motion

8 Choose the periodic function from the following.

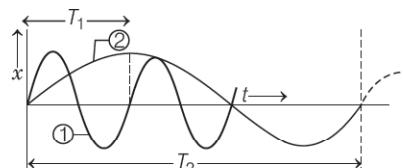
- (a) $A \sin^3(\omega t)^2$
- (b) $\sin \omega t + \cos \omega t$
- (c) $\tan(\omega t)^3$
- (d) $e^{\omega t}$

TOPIC 2 ~ Simple Harmonic Motion

9 Motion of a ball bearing inside a smooth curved bowl, when released from a point slightly above the lower point, is

- (a) simple harmonic motion (SHM)
- (b) non-periodic motion
- (c) parabolic
- (d) periodic but not SHM

10 The relation between the time period of two simple harmonic motions represented by two curves is



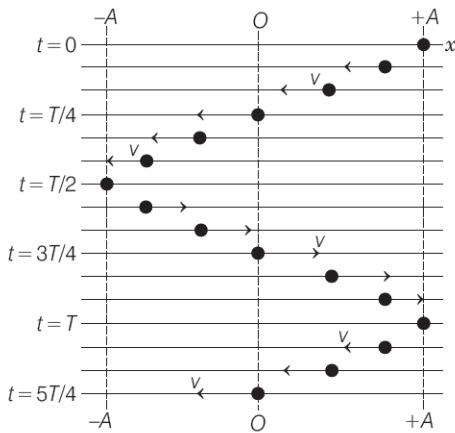
- (a) $T_1 = 4T_2$
- (b) $T_2 = 4T_1$
- (c) $T_2 = 2T_1$
- (d) $T_1 = 2T_2$

- 11** The distance covered by a particle undergoing SHM in one time period is (amplitude = A)

NEET (Odisha) 2019

- (a) zero (b) A (c) $2A$ (d) $4A$

- 12** Figure shows snapshots of a particle moving between $+A$ and $-A$ about origin (at $x = 0$) at different instants. The particle moves in a way that velocity is maximum at $x = 0$ and minimum at $x = \pm A$.



The correct displacement equation for the motion of the particle is

- (a) $A \sin \omega t$ (b) $A \cos \omega t$
 (c) $A \tan \omega t$ (d) $A \cot \omega t$

- 13** When two displacements represented by $y_1 = a \sin \omega t$ and $y_2 = b \cos \omega t$ are superimposed, the motion is

CBSE AIPMT 2015

- (a) not a simple harmonic
 (b) simple harmonic with amplitude $\frac{a}{b}$
 (c) simple harmonic with amplitude $\sqrt{a^2 + b^2}$
 (d) simple harmonic with amplitude $\frac{(a+b)}{2}$

- 14** The displacement of a particle executing simple harmonic motion is given by
 $y = A_0 + A \sin \omega t + B \cos \omega t$.

Then, the amplitude of its oscillation is given by

- NEET (National) 2019**
 (a) $\sqrt{A^2 + B^2}$ (b) $\sqrt{A_0^2 + (A+B)^2}$
 (c) $A + B$ (d) $A_0 + \sqrt{A^2 + B^2}$

- 15** Two simple harmonic motions are represented by
 $y_1 = 5 (\sin 2\pi t + \sqrt{3} \cos 2\pi t)$

and $y_2 = 5 \sin \left(2\pi t + \frac{\pi}{4} \right)$.

The ratio of their amplitudes is

- (a) 1 : 1 (b) 2 : 1 (c) 1 : 3 (d) $\sqrt{3} : 1$

- 16** Which of the following is incorrect when function $x = A \sin^2 \omega t + B \cos^2 \omega t + C \sin \omega t \cos \omega t$ represents SHM?

- (a) if $A = 0, B = 0$, it will be in SHM
 (b) if $A = -B, C = 2B$, amplitude = $|B\sqrt{2}|$
 (c) if $A = B, C = 0$, amplitude = $|B|$
 (d) if $A = B, C = 2B$, it will be in SHM

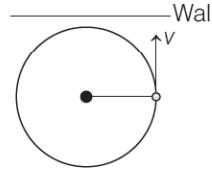
- 17** A body executing simple harmonic motion has a periodic time of 3 s. After how much time from $t = 0$, its displacement will be half of its amplitude?

- (a) $(1/8)$ s (b) $(1/6)$ s (c) $(1/4)$ s (d) $(1/3)$ s

- 18** Two particles execute SHM of the same amplitude and frequency along the same straight line. If they pass one another when going in opposite directions, each time their displacement is half their amplitude, the phase difference between them is

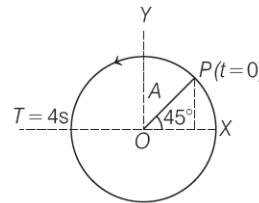
- (a) $\frac{\pi}{3}$ (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{6}$ (d) $\frac{2\pi}{3}$

- 19** A ball is moving in uniform circular motion in a horizontal plane, the shadow of ball on the wall will execute



- (a) projectile motion
 (b) uniform circular motion
 (c) simple harmonic motion
 (d) non-uniform circular motion

- 20** Figure depicts a circular motion. The radius of the circle, period of revolution, initial position and the sense of revolution are indicated on the figure.



The simple harmonic motion of the X -projection of the radius vector of the rotating particle P is

- (a) $A \cos \left(\frac{2\pi}{4} t + \frac{\pi}{4} \right)$ (b) $A \cos \left(\frac{\pi}{4} t + \frac{\pi}{4} \right)$
 (c) $A \cos \left(\frac{\pi}{3} t + \frac{\pi}{2} \right)$ (d) None of these

- 21** The equation of a simple harmonic motion is given by $y = 3 \sin (50t - x)$, where x and y are in metres and t is in seconds, the maximum particle velocity in ms^{-1} is

- (a) 3 (b) 50 (c) 150 (d) 25

- 22** The piston in the cylinder head of a locomotive has a stroke (twice the amplitude) of 2.0 m. If the piston moves with simple harmonic motion with an angular frequency of 100 rad min^{-1} , what is its maximum speed?
 (a) 50 m min^{-1} (b) 100 m min^{-1}
 (c) 200 m min^{-1} (d) 75 m min^{-1}

- 23** Maximum acceleration in SHM is **JIPMER 2019**

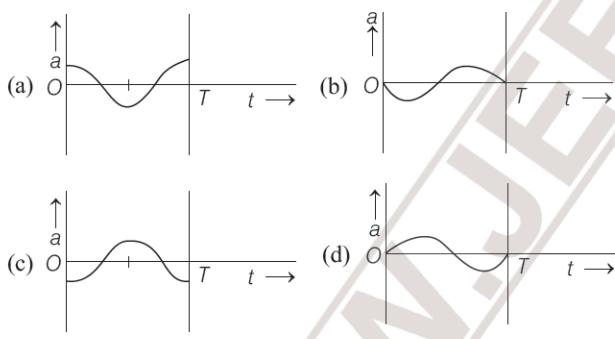
- (a) $\omega^2 A$ (b) $\frac{\omega^2 A}{2}$
 (c) $\omega^2 A^2$ (d) 0

- 24** Two simple harmonic motions of angular frequency 100 rads^{-1} and 1000 rads^{-1} have the same displacement amplitude. The ratio of their maximum accelerations is

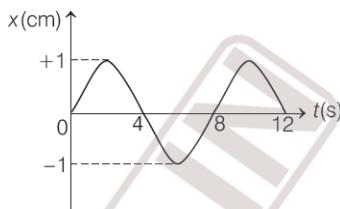
- (a) $1 : 10$ (b) $1 : 10^2$
 (c) $1 : 10^3$ (d) $1 : 10^4$

- 25** The oscillation of a body on a smooth horizontal surface is represented by the equation $x = A \cos \omega t$, where x = displacement at time t and ω = frequency of oscillation.

Which one of the following graphs shows correctly the variation of a with t ? **CBSE AIPMT 2014**



- 26** The $x-t$ graph of a particle undergoing simple harmonic motion is shown below.



The acceleration of the particle at $t = \frac{4}{3} \text{ s}$ is

- (a) $\frac{\sqrt{3}}{32} \pi^2 \text{ cms}^{-2}$ (b) $-\frac{\pi^2}{32} \text{ cms}^{-2}$
 (c) $\frac{\pi^2}{32} \text{ cms}^{-2}$ (d) $-\frac{\sqrt{3}}{32} \pi^2 \text{ cms}^{-2}$

- 27** A pendulum is hung from the roof of a sufficiently high building and is moving freely to and fro like a simple harmonic oscillator. The acceleration of the bob of the pendulum is 20 ms^{-2} at a distance of 5 m from the mean position. The time period of oscillation is **NEET 2018**

- (a) 2 s (b) π s (c) 2π s (d) 1 s

- 28** A particle is executing a simple harmonic motion. Its maximum acceleration is α and maximum velocity is β , then its time period of vibration will be

CBSE AIPMT 2015

- (a) $\frac{\beta^2}{\alpha^2}$ (b) $\frac{\alpha}{\beta}$ (c) $\frac{\beta^2}{\alpha}$ (d) $\frac{2\pi\beta}{\alpha}$

- 29** A particle executes linear simple harmonic motion with an amplitude of 3 cm. When the particle is at 2 cm from the mean position, the magnitude of its velocity is equal to that of its acceleration. Then, its time period in seconds is **NEET 2017**

- (a) $\frac{\sqrt{5}}{\pi}$ (b) $\frac{\sqrt{5}}{2\pi}$ (c) $\frac{4\pi}{\sqrt{5}}$ (d) $\frac{2\pi}{\sqrt{3}}$

TOPIC 3 ~ Force Law and Energy in SHM

- 30** In simple harmonic motion, the force

- (a) is constant in magnitude only
 (b) is constant in direction only
 (c) varies in magnitude as well as in direction
 (d) is constant in both magnitude and direction

- 31** For simple harmonic motion of an object of mass m ,

- (a) $\mathbf{F} = -m\omega^2 x$
 (b) $\mathbf{F} = -m\omega x$

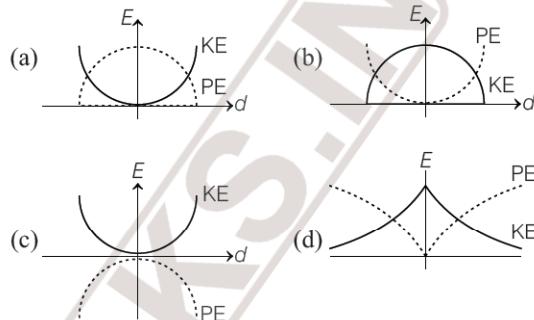
- (c) force always acts in the opposite direction of displacement
 (d) Both (a) and (c)

- 32** In SHM,

- (a) PE is stored due to elasticity of system
 (b) KE is stored due to inertia of system
 (c) Both KE and PE are stored by virtue of elasticity of system.
 (d) Both (a) and (b)

- 33** A mass of 1 kg is executing SHM which is given by $x = 6.0 \cos\left(100t + \frac{\pi}{4}\right)$ cm. What is the maximum kinetic energy? **JIPMER 2018**
 (a) 3 J (b) 6 J (c) 9 J (d) 18 J
- 34** The expression for displacement of an object in SHM is $x = A \cos \omega t$. The potential energy at $t = T/4$ is
 (a) $\frac{1}{2}kA^2$ (b) $\frac{1}{8}kA^2$
 (c) $\frac{1}{4}kA^2$ (d) zero
- 35** In simple harmonic motion, let the time period of variation of potential energy is T_1 and time period of variation of position is T_2 , then relation between T_1 and T_2 is **JEE Main 2017**
 (a) $T_1 = T_2$ (b) $T_1 = 2T_2$
 (c) $2T_1 = T_2$ (d) None of these
- 36** The total energy of a particle executing simple harmonic motion depends on its
 I. amplitude.
 II. period.
 III. displacement.
 Choose the correct option from the following options.
 (a) Both I and II (b) Both II and III
 (c) Both I and III (d) I, II and III

- 37** For a particle executing SHM, a graph is plotted between its kinetic energy and potential energy against its displacement d . Which one of the following represents these correctly? (graphs are schematic and not drawn to scale) **JEE Main 2015**



- 38** A particle free to move along the X -axis has potential energy given as

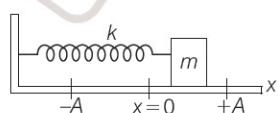
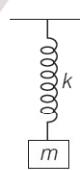
$$U(x) = k[1 - \exp(-x^2)] \quad (\text{for } -\infty < x < +\infty)$$

where, k is a positive constant of appropriate dimensions. Then,

- (a) at points away from origin, the particle is in equilibrium
- (b) for any finite non-zero value of x , there is a force directed away from the origin
- (c) its total mechanical energy is $k/2$ and it is equal to its kinetic energy at origin
- (d) the motion of the particle is simple harmonic

TOPIC 4 ~ Some Systems Executing SHM

- 39** A block is left in the equilibrium position as shown in the figure. If now it is stretched by $\frac{mg}{k}$, the net stretch of the spring is
 (a) $\frac{mg}{k}$ (b) $\frac{mg}{2k}$
 (c) $\frac{2mg}{k}$ (d) $\frac{mg}{4k}$
- 40** A block is in SHM on a frictionless surface as shown in the figure. The position $x=0$ show the unstretched position of the spring. If the spring is, then stretched and the block is released at $x=A$. Then, the speed of block at $x=0$ is
 (a) 0 (b) $A^2 \sqrt{\frac{k}{m}}$ (c) $A^2 \sqrt{\frac{2k}{m}}$ (d) $A \sqrt{\frac{k}{m}}$

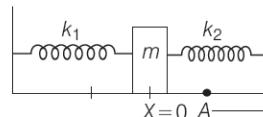


- 41** The mass m as shown in the figure oscillates in simple harmonic motion with amplitude A . The extension in the spring with spring constant k_1 is



- (a) $\frac{k_1 A}{k_2}$ (b) $\frac{k_2 A}{k_1}$ (c) $\frac{k_1 A}{k_1 + k_2}$ (d) $\frac{k_2 A}{k_1 + k_2}$

- 42** In the figure shown, the block is moved side ways by a distance A . The magnitude of net force on the block is

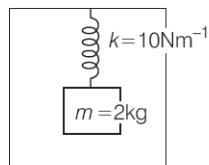


- (a) $(k_1 - k_2)A$ (b) $(k_2 - k_1)A$
 (c) $(k_1 + k_2)A$ (d) $\left(\frac{k_1 + k_2}{2}\right)A$

- 43** A system containing a ball is oscillating on a frictionless horizontal plane. The position of the mass when its potential energy and its kinetic energy both are equal, is (let A is the amplitude of oscillation)
- A
 - $A/\sqrt{2}$
 - $A/2$
 - $A/\sqrt{3}$

- 44** A block attached to a spring is executing SHM. Let the time period of variation of velocity is T_1 and time period of variation of kinetic energy is T_2 . The relation between T_1 and T_2 is
- $T_1 = T_2$
 - $T_1 = 2T_2$
 - $T_1 = T_2/2$
 - None of these

- 45** A spring mass system is hanging from a ceiling of an elevator in equilibrium as shown. The elevator suddenly starts accelerating with acceleration 9 ms^{-2} , then the frequency of oscillation is

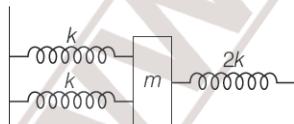


- 10 Hz
 - 4 Hz
 - 2.8 Hz
 - 0 Hz
- 46** Two identical blocks A and B , each of mass m resting on smooth floor, are connected by a light spring of natural length L and the spring constant k , with the spring at its natural length. A third identical block at C (mass m) moving with a speed v along the line joining A and B collides with A . (Consider the collision to be elastic in nature)

The maximum compression in the spring is equal to

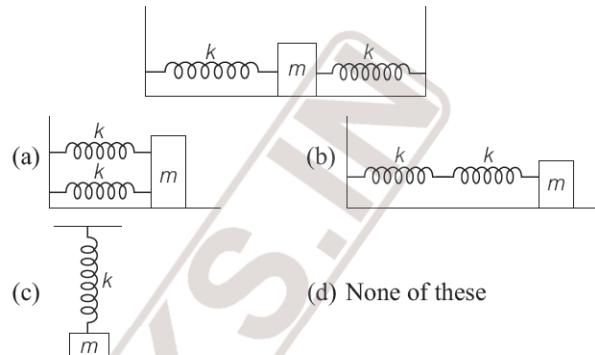
- $v \sqrt{\frac{m}{2k}}$
- $m \sqrt{\frac{v}{2k}}$
- $\sqrt{\frac{mv}{k}}$
- $\frac{mv}{2k}$

- 47** The time period of system shown below is



- $2\pi \sqrt{\frac{m}{3k}}$
- $2\pi \sqrt{\frac{2m}{k}}$
- $2\pi \sqrt{\frac{m}{4k}}$
- $2\pi \sqrt{\frac{m}{k}}$

- 48** The time period of the given spring mass system is equal to that of system

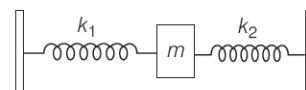


- 49** A body mass m is attached to the lower end of a spring whose upper end is fixed. The spring has negligible mass. When the mass m is slightly pulled down and released, it oscillates with a time period of 3s. When the mass m is increased by 1 kg, the time period of oscillations becomes 5s. The value of m in kg is

- $\frac{3}{4}$
- $\frac{4}{3}$
- $\frac{16}{9}$
- $\frac{9}{16}$

NEET 2016

- 50** Two spring of force constants k_1 and k_2 are connected to a mass m as shown in figure. The frequency of oscillation of the mass is f . If both k_1 and k_2 are made four times their original values, the frequency of oscillation becomes



- $f/2$
- $f/4$
- $4f$
- $2f$

- 51** When a body of mass 1.0 kg is suspended from a certain light spring hanging vertically, its length increases by 5 cm. By suspending 2.0 kg block to the spring and if the block is pulled through 10 cm and released, the maximum velocity in it (in ms^{-1}) is (use, acceleration due to gravity = 10 ms^{-2})

- 0.5
- 1
- 2
- 4

- 52** The ratio of frequencies of two pendulums are $2 : 3$, then their lengths are in ratio

- $\sqrt{2/3}$
- $\sqrt{3/2}$
- $4/9$
- $9/4$

- 53** A mass falls from a height h and its time of fall t is recorded in terms of time period T of a simple pendulum. On the surface of earth, it is found that $t = 2T$. The entire set up is taken on the surface of another planet whose mass is half of earth and radius the same. Same experiment is repeated and corresponding times are noted as t' and T' .

NEET (Odisha) 2019

- (a) $t' = \sqrt{2} T'$ (b) $t' > 2 T'$ (c) $t' < 2 T'$ (d) $t' = 2 T'$

- 54** A simple pendulum is suspended from the ceiling of a lift. When the lift is at rest, its time period is T . With what acceleration should the lift be accelerated upwards in order to reduce its period to $T/2$? (Take, g is acceleration due to gravity).

- (a) $2g$ (b) $3g$
(c) $4g$ (d) g

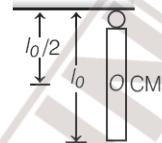
- 55** A simple pendulum has a time period T_1 when it is on the earth's surface and T_2 when it is taken to a height $2R$ above the earth's surface, where R is the radius of the earth. The value of T_1/T_2 is

- (a) $1/9$ (b) $1/3$
(c) $\sqrt{3}$ (d) 9

AIIMS 2018

- 56** A uniform rod of mass m and length l_0 is pivoted at one end and is hanging in the vertical direction. The period of small angular oscillations of the rod is

JIPMER 2018



- (a) $T = 3\pi \sqrt{\frac{2l_0}{3g}}$ (b) $T = 4\pi \sqrt{\frac{l_0}{3g}}$
(c) $T = 4\pi \sqrt{\frac{2l_0}{3g}}$ (d) $T = 2\pi \sqrt{\frac{2l_0}{3g}}$

- 57** A simple pendulum oscillating in air has period T . The bob of the pendulum is completely immersed in a non-viscous liquid. The density of the liquid is $\frac{1}{16}$ th of the material of the bob. If the bob is inside liquid all the time, its period of oscillation in this liquid is

JEE Main 2019

- (a) $2T \sqrt{\frac{1}{10}}$ (b) $2T \sqrt{\frac{1}{14}}$ (c) $4T \sqrt{\frac{1}{14}}$ (d) $4T \sqrt{\frac{1}{15}}$

TOPIC 5 ~ Free, Forced and Damped Oscillations; Resonance

- 58** The natural frequencies of vibration of a building depend on

- (i) its height and other size parameters.
(ii) the nature of building materials.

The correct option is/are

- (a) Only (i) (b) Only (ii)
(c) Both (i) and (ii) (d) Neither (i) nor (ii)

- 59** The value of amplitude of the forced oscillation when damping is small and ω_d is far away from ω , where ω_d = driving frequency, ω = natural frequency and F_0 = amplitude of applied periodic force.

- (a) $\frac{F_0}{m\omega^2}$
(b) $\frac{F_0}{m(\omega^2 - \omega_d^2)}$
(c) $\frac{F_0}{\omega_d b}$
(d) $\frac{F_0}{m\omega^2 d}$

- 60** The value of maximum possible amplitude in the case of forced oscillations when driving frequency is close to natural frequency, is

- (a) $\frac{F_0}{m(\omega^2 - \omega_d^2)}$ (b) $\frac{F_0}{\omega_d b}$
(c) $\frac{F_0}{m\omega}$ (d) $\frac{-F_0}{m\omega_d^2}$

- 61** The amplitude of a damped oscillator decreases to 0.9 times its original magnitude in 5 s. In another 10 s, it will decrease to α times its original magnitude, where α equals to

- JEE Main 2013**
(a) 0.7 (b) 0.81 (c) 0.729 (d) 0.6

- 62** A damped harmonic oscillator has a frequency of 5 oscillations per second. The amplitude drops to half its value for every 10 oscillations. The time it will take to drop to $\frac{1}{1000}$ of the original amplitude is close to

- JEE Main 2019**
(a) 20 s (b) 50 s (c) 100 s (d) 10 s

SPECIAL TYPES QUESTIONS

I. Assertion and Reason

■ **Direction** (Q. Nos. 63-77) In the following questions, a statement of Assertion is followed by a corresponding statement of Reason. Of the following statements, choose the correct one.

- (a) Both Assertion and Reason are correct and Reason is the correct explanation of Assertion.
- (b) Both Assertion and Reason are correct but Reason is not the correct explanation of Assertion.
- (c) Assertion is correct but Reason is incorrect.
- (d) Assertion is incorrect but Reason is correct.

63 Assertion Vibrations and oscillations are two different types of motion.

Reason For vibration, frequency is more and for oscillation, the frequency is less.

64 Assertion $x(t) = A \sin \omega t$ is periodic in nature but cannot represent an oscillatory motion.

Reason $\sin \theta$ is a sinusoidal periodic function.

65 Assertion $x = A \cos \omega t$ and $x = A \sin \omega t$ can represent same motion depending on initial position of particle.

Reason If the argument of $x = A \cos \omega t$, i.e. ωt is increased by 2π radian the value of x remains same.

66 Assertion $x = A \cos \omega t$ represents a periodic function. The value of x varies between $+A$ and $-A$.

Reason Amplitude is a vector quantity.

67 Assertion For an oscillatory motion, the equilibrium position can be represented by saying that it is a point for periodic motion where net external force on the body is zero.

Reason For an oscillatory motion, if the body is displaced from the equilibrium position, a restoring force will arise that will try to bring back the body to equilibrium position.

68 Assertion If the amplitude of SHM of a spring mass system is increased, then time period of SHM will remain constant.

Reason If amplitude is increased, body will have to travel more distance to complete one oscillation.

69 Assertion A block of mass m attached to a stiff spring have large oscillation frequency.

Reason Stiff spring have high value of spring constant k .

70 Assertion If a pendulum is falling freely, then its time period becomes zero.

Reason Freely falling body has the acceleration equal to g .
AIIMS 2018

71 Assertion In damped oscillations, the motion is periodic.

Reason In damped oscillations, the amplitude decreases due to dissipative forces.

72 Assertion In damped oscillations, the total mechanical energy remain constant.

Reason Total mechanical energy of oscillator executing SHM is given by $\frac{1}{2} kA^2$, where A is amplitude at time t .

73 Assertion The motion of a simple pendulum dies out gradually due to air drag and friction at the support.

Reason For small damping also, the oscillations are non-periodic in nature.

74 Assertion Free oscillations cannot die out with time.

Reason Swinging of a child in a swing (with an external push) is the example of forced oscillation.

75 Assertion In forced oscillation, the external force is constant.

Reason In forced oscillation, external force helps in sustaining the oscillations.

76 Assertion In resonance, amplitude is infinity.

Reason At resonance, driving frequency is equal to natural frequency of the system.

77 Assertion The army troops are suggested to break their march on a hanging bridge.

Reason Due to resonance, the bridge may collapse.

II. Statement Based Questions

- 78**
- I. In sitar and guitar, the strings vibrate and produce sound.
 - II. Sound wave propagate due to vibration of air molecules.
 - III. In solids, atoms oscillate to produce the sensation of temperature.
 - IV. In antennas of TV and satellites transmitters, electrons oscillate to convey information.

Which of the following statement(s) is/are correct?

- (a) I, III and IV
- (b) Both II and III
- (c) Both III and IV
- (d) I, II, III and IV