

89 (d) When a body rolls down an inclined plane, it is accompanied by rotational and translational kinetic energies.

$$\text{Rotational kinetic energy, } K_R = \frac{1}{2} I \omega^2$$

where, I is the moment of inertia and ω is the angular velocity.

Translational kinetic energy for pure rolling,

$$= K_T = \frac{1}{2} m v_{CM}^2 = \frac{1}{2} m(r\omega)^2 \quad (\because v_{CM} = r\omega)$$

where, m is mass of the body, v_{CM} is the velocity and ω is the angular velocity.

Given, translational kinetic energy = rotational kinetic energy

$$\therefore \frac{1}{2} m(r^2\omega^2) = \frac{1}{2} I\omega^2 \Rightarrow I = mr^2$$

We know that, mr^2 is the moment of inertia of hollow cylinder about its axis, where m is the mass of hollow cylindrical body and r is the radius of the cylinder.

90 (b) When a body of mass m and radius R rolls down an inclined plane of height h and angle of inclination θ , it loses potential energy. However, it acquired both linear and angular speeds.

$$mgh = \frac{1}{2} m v_{CM}^2 + \frac{1}{2} I\omega^2$$

For pure rolling $\omega = \frac{v_{CM}}{R}$ and using $I = mk^2$,

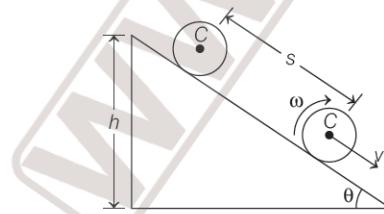
$$mgh = \frac{1}{2} m v_{CM}^2 + \frac{1}{2} mk^2 \frac{v_{CM}^2}{R^2}$$

Velocity at the lowest point, $v_{CM} = \sqrt{\frac{2gh}{1 + (k^2/R^2)}}$

$$\text{For solid sphere, } Mk^2 = \frac{2}{5} MR^2 \Rightarrow \frac{k^2}{R^2} = \frac{2}{5}$$

$$\therefore v = \sqrt{\frac{2 \times 10 \times 7}{1 + (2/5)}} = 10 \text{ ms}^{-1}$$

91 (a) Assuming that no energy is used up against static friction, the loss in potential energy is equal to the total gain in the kinetic energy.



Thus,

$$Mgh = \frac{1}{2} I(v^2/R^2) + \frac{1}{2} Mv^2$$

(where, v = velocity of centre of mass)

$$\text{or } \frac{1}{2} v^2 (M + I/R^2) = Mgh$$

$$\text{or } v^2 = \frac{2Mgh}{M + (I/R^2)} = \frac{2gh}{1 + (I/MR^2)}$$

If s be the distance covered along the plane, then

$$h = s \sin \theta \Rightarrow v^2 = \frac{2g \sin \theta}{1 + (I/MR^2)}$$

$$\text{Now, } v^2 = u^2 + 2as, u = 0 \Rightarrow v^2 = 2as$$

$$\therefore 2as = \frac{2g \sin \theta}{1 + (I/MR^2)} \quad \text{or} \quad a = \frac{g \sin \theta}{1 + (I/MR^2)}$$

92 (c) Given, initial velocity of sphere, $u = 2.8 \text{ m/s}$

Acceleration on the inclined plane,

$$a = \frac{g \sin \theta}{1 + \frac{k^2}{R^2}} \quad \dots(i)$$

where, k and R are the radius of gyration and radius of sphere, respectively.

Moment of inertia of sphere,

$$I = mk^2 = \frac{2}{5} mR^2$$

$$\Rightarrow \frac{k^2}{R^2} = \frac{2}{5} \quad \dots(ii)$$

From Eqs. (i) and (ii), we have

$$a = \frac{g \sin 30^\circ}{1 + \frac{2}{5}} \quad [\because \theta = 30^\circ, \text{ given}]$$

$$= \frac{5g}{14} = \frac{25}{7} \text{ m/s}^2 \quad [\because g = 10 \text{ m/s}^2]$$

If s be the maximum distance travelled by the sphere, then at maximum distance, $v = 0$.

\therefore From third equation of motion,

$$\begin{aligned} v^2 &= u^2 - 2as \\ 0 &= u^2 - 2as \\ \Rightarrow s &= \frac{u^2}{2a} = \frac{(2.8)^2 \times 7}{2 \times 25} \\ &= 1.09 \text{ m} \end{aligned}$$

Hence, the value of maximum distance is very close to option (c), so option (c) is correct.

93 (a) Applying conservation of mechanical energy, we get

Potential energy = Translational kinetic energy + Rotational kinetic energy

$$\Rightarrow mgh = \frac{1}{2} mv^2 + \frac{1}{2} I\omega^2$$

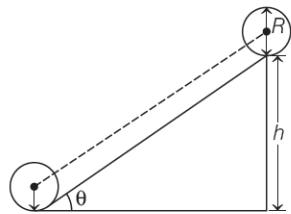
$$\Rightarrow mgh = \frac{1}{2} mv^2 + \frac{1}{2} mk^2 \left(\frac{v}{R} \right)^2 \quad [\because I = mk^2, \omega = \frac{v}{R}]$$

$$mgh = \frac{1}{2} mv^2 \left(1 + \frac{k^2}{R^2} \right)$$

$$\text{or } v^2 = \left(\frac{2gh}{1 + k^2/R^2} \right)$$

where, k and R are the radius of gyration and radius of object, respectively.

Note that velocity v is independent of the mass of the rolling body.



$$\text{For a ring, } k^2 = R^2 \Rightarrow v_{\text{ring}} = \sqrt{\frac{2gh}{1+1}} = \sqrt{gh}$$

$$\text{For a solid cylinder, } k^2 = R^2/2,$$

$$v_{\text{cylinder}} = \sqrt{\frac{2gh}{1+(1/2)}} = \sqrt{\frac{4gh}{3}}$$

$$\text{For a solid sphere, } k^2 = 2R^2/5,$$

$$v_{\text{sphere}} = \sqrt{\frac{2gh}{1+(2/5)}} = \sqrt{\frac{10gh}{7}}$$

From the results obtained, it is clear that among the three bodies, the sphere has the greatest and the ring has the least velocity of the centre of mass at the bottom of the inclined plane.

94 (b) Translational kinetic energy of a rolling body is

$$K_t = \frac{1}{2}mv_{\text{CM}}^2 \quad \dots(\text{i})$$

Total kinetic energy of a rolling body

$$\begin{aligned} &= \text{Rotational kinetic energy } (K_r) + \text{Translational kinetic} \\ &\quad \text{energy } (K_t) \\ &= \frac{1}{2}I\omega^2 + \frac{1}{2}mv_{\text{CM}}^2 \quad \dots(\text{ii}) \end{aligned}$$

For a solid sphere, moment of inertia about its diametric axis, $I = \frac{2}{5}MR^2$

Substituting the value of I in Eq. (ii), we get

$$\begin{aligned} K_t + K_r &= \frac{1}{2}\left(\frac{2}{5}MR^2\right)\omega^2 + \frac{1}{2}mv_{\text{CM}}^2 \\ &= \frac{1}{2}\left(\frac{2}{5}MR^2\right)\left(\frac{v_{\text{CM}}}{R}\right)^2 + \frac{1}{2}mv_{\text{CM}}^2 \\ &\quad [\because v_{\text{CM}} = R\omega] \\ &= \frac{1}{5}mv_{\text{CM}}^2 + \frac{1}{2}mv_{\text{CM}}^2 = \left(\frac{1}{5} + \frac{1}{2}\right)mv_{\text{CM}}^2 \\ &= \frac{7}{10}mv_{\text{CM}}^2 \quad \dots(\text{iii}) \end{aligned}$$

$$\therefore \text{Ratio, } \frac{K_t}{K_t + K_r} = \frac{\frac{1}{2}mv_{\text{CM}}^2}{\frac{7}{10}mv_{\text{CM}}^2} = \frac{1}{2} \times \frac{10}{7} = \frac{5}{7}$$

$$\therefore K_t : (K_t + K_r) = 5 : 7$$

Alternate Method

$$\text{Suppose moment of inertia, } I = xmR^2 \quad \dots(\text{i})$$

$$\text{For solid sphere, moment of inertia, } I = \frac{2}{5}mR^2 \quad \dots(\text{ii})$$

Thus, from Eqs. (i) and (ii), we get

$$x = \frac{2}{5}$$

Since, the ratio of translational energy to the total energy can be written as

$$\frac{K_t}{K_t + K_r} = \frac{\frac{1}{2}mv_{\text{CM}}^2}{\frac{1}{2}mv_{\text{CM}}^2 \left(1 + \frac{k^2}{R^2}\right)} \quad \dots(\text{iii})$$

where, k is called the radius of gyration.

$$\text{As, } k = \sqrt{\frac{I}{m}} \text{ or } k^2 = \frac{I}{m}$$

From Eq. (i), we get

$$k^2 = \frac{xmR^2}{m} = xR^2$$

Substituting the value of k^2 in Eq. (iii), we get

$$\frac{K_t}{K_t + K_r} = \frac{1}{\left(1 + \frac{xR^2}{R^2}\right)} = \frac{1}{1+x}$$

$$\text{Here, } x = \frac{2}{5} \Rightarrow \frac{K_t}{K_t + K_r} = \frac{1}{1+(2/5)} = \frac{5}{7}$$

95 (d) The centre of mass of a body may lie on or outside the body.



Hence, in Fig. (a), centre of mass is on the body and in Fig. (b), centre of mass does not lie on the body.

The centre of mass of an object is the average position of all the parts of the system, weighted according to their masses. Therefore, centre of mass of body lie at the geometric centre of body.

Therefore, Assertion is incorrect but Reason is correct.

96 (c) The motion of centre of mass describes the translational part of the motion.

In translational motion, all points of a moving body move along a straight line, i.e. the relative velocities between any two particles, must be zero.

But it is not necessary that translational motion of body is always in straight line. A parabolic motion of an object without rotation is also translational motion.

Therefore, Assertion is correct but Reason is incorrect.

97 (a) When $\tau_{\text{ext}} = 0 \Rightarrow L = \text{a constant.}$

So, for a system of particles under central force field, the total angular momentum on the system is conserved because torque acting on such a system is zero.

Therefore, Assertion and Reason are correct and Reason is the correct explanation of Assertion.

98 (c) Angular momentum remains constant as particle is moving in a straight line. The angular momentum is constant when particle moves with a uniform velocity. Therefore, Assertion is correct but Reason is incorrect.

99 (d) There is a difference between inertia and moment of inertia of a body. The inertia of a body depends only upon the mass of the body but the moment of inertia of a body about an axis not only depend upon the mass of the body but also upon the distribution of mass about the axis of rotation.

Inertia represents the capacity (ability) of a body to oppose its state of motion or rest.

Therefore, Assertion is incorrect but Reason is correct.

100 (d) Moment of inertia changes with axis chosen.

It is because moment of inertia of a particle depends on its mass and its distance from axis of rotation.

Therefore, Assertion is incorrect but Reason is correct.

101 (c) Friction force between sliding body and inclined plane depends upon the nature of surfaces of both the body and inclined plane, hence if bodies slide down an inclined plane without rolling, then it is not necessary that all bodies reach the bottom simultaneously. Acceleration of all bodies are also not equal due to different values of friction between the surfaces of body and inclined plane. Therefore, Assertion is correct but Reason is incorrect.

102 (a) The work done on a body is given by

$$W = \int F \cdot v dt, \text{ where } F \text{ is force of friction.}$$

For the rolling disc without slipping down an inclined plane, the velocity of the particle on which the friction force is acting, is zero. Hence, work done is zero, i.e. when the disc rolls without slipping, the friction force is required because for rolling condition, velocity of point of contact is zero.

Therefore, Assertion and Reason are correct and Reason is the correct explanation of Assertion.

103 (d) Statement IV is incorrect and it can be corrected as,

The shape of rigid body must not change on application of force.

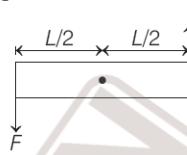
Rest statements are correct.

104 (d) Statement IV is incorrect and it can be corrected as, Moment of inertia changes with axis of rotation.

Rest statements are correct.

105 (a) Using sign convention, we can take anti-clockwise moment (or torque) as positive, while clockwise moment (or torque) as negative.

In case I, net torque about its centre is



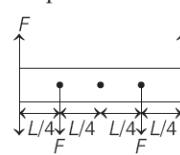
$$\tau = F \times \frac{L}{2} + F \times \frac{L}{2} = FL$$

In case II, net torque about its centre is



$$\tau = F \times \frac{L}{2} + F \times \frac{L}{4} + F \times \frac{L}{2} = \frac{5}{4} FL$$

In case III, net torque about its centre is



$$\tau = -F \times \frac{L}{2} + F \times \frac{L}{4} - F \times \frac{L}{4} + F \times \frac{L}{2} = 0$$

So, case I and II will have a non-zero net torque acting on the rod about its centre.

106 (c) Statements I and II are correct but statement III is incorrect and it can be corrected as,

At equilibrium.

Load arm \times Load = Effort \times Effort arm

$$F_1 d_1 = F_2 d_2 \Rightarrow \frac{F_1}{F_2} = \frac{d_2}{d_1}$$

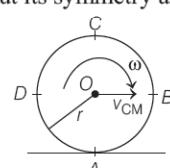
The ratio of $\frac{F_1}{F_2}$ is called mechanical advantage (MA).

107 (d) When a body rolls down on an inclined plane without slipping, then at bottom all objects (cylinder, sphere, ring) possess equal rotational kinetic energy. Because velocity of contact point is zero, hence translational energy is zero.

During rolling motion, kinetic energy is not conserved due to friction. Angular momentum of body during rolling motion without slipping is conserved. Velocity of rolling body at any instant depends upon mass of its own.

So, statement IV is correct, but I, II and III are incorrect.

108 (d) According to figure, the disc rotates with angular velocity ω about its symmetry axis passing through O.



At bottom point A , the linear velocity due to rotation is exactly opposite to the translational velocity v_{CM} , i.e.

$$v_{rot} = r\omega = v_{CM}$$

The point A will be instantaneously at rest, if $v_{CM} = r\omega$, i.e. $v_A = 0$ when $v_{CM} = r\omega$.

Hence, for the disc, the condition for rolling without slipping is $v_{CM} = r\omega$.

At the top point C of the disc, linear velocity due to rotational motion and the translational velocity v_{CM} are in the same direction, parallel to level surface.

Therefore, $v_C = v_{rot} + v_{CM} = v_{CM} + v_{CM} = 2v_{CM} = 2r\omega$

At B and D , linear velocity due to rotation and the translational velocity v_{CM} are perpendicular to each other. So,

$$\begin{aligned} v_B = v_D &= |v_{rot} + v_{CM}| = \sqrt{v_{CM}^2 + v_{CM}^2 + v_{CM}^2 \cos 90^\circ} \\ &= \sqrt{2}v_{CM} = \sqrt{2}r\omega \end{aligned}$$

Therefore, $v_B = v_D < v_C$

So, statements I and III are correct but II and IV are incorrect.

- 109 (d)** In the given situation, projectile could be considered as rigid body before explosion and after explosion, as its fragments are considered as system of particles. Thus, the concept of CM is applicable to both.

Since, the explosion is due to internal forces, so the motion of CM after explosion will follow the same parabolic path as it would have followed if there was no explosion.

Thus, statement given in option (d) is correct, rest are incorrect.

- 110 (a)** For a general rotational motion, where axis of rotation is not symmetric, angular momentum \mathbf{L} and angular velocity ω need not be parallel.

For rotational motion about a fixed axis, angular velocity ω and angular momentum \mathbf{L} are not always parallel.

For a general translational motion, momentum $\mathbf{p} = m\mathbf{v}$, hence \mathbf{p} and \mathbf{v} are always parallel.

For a general translational motion, acceleration \mathbf{a} and velocity \mathbf{v} are not always parallel.

Thus, statement given in option (a) is correct, rest are incorrect.

- 111 (d)** The angular momentum \mathbf{L} of a particle with respect to origin is defined to be $\mathbf{L} = \mathbf{r} \times \mathbf{p}$, where \mathbf{r} is the position vector of the particle and \mathbf{p} is the linear momentum.

The direction of \mathbf{L} is perpendicular to both $d\mathbf{r}$ and \mathbf{p} by right hand rule.

For particle 1, $\mathbf{L}_1 = \mathbf{r}_1 \times m\mathbf{v}$, is out of plane of the paper and perpendicular to \mathbf{r}_1 and $\mathbf{p}(m\mathbf{v})$.

Similarly $\mathbf{L}_2 = \mathbf{r}_2 \times m(-\mathbf{v})$ is into the plane of the paper and perpendicular to \mathbf{r}_2 and $-\mathbf{p}$.

Hence, total angular momentum

$$\mathbf{L} = \mathbf{L}_1 + \mathbf{L}_2 = \mathbf{r}_1 \times m\mathbf{v} + (-\mathbf{r}_2 \times m\mathbf{v})$$

About A , $|\mathbf{L}| = mv d_1 - mvd_2$ as $d_2 > d_1$, total angular momentum will be inward.

$$\text{Hence, } L = m\mathbf{v}(d_2 - d_1) \otimes$$

Thus, statement given in option (d) is correct, rest are incorrect.

- 112 (c)** We know that, torque on a system of particles,

$$\tau = \mathbf{r} \times \mathbf{F} = rF \sin \theta \hat{\mathbf{n}} \quad \dots(i)$$

where, θ is angle between \mathbf{r} and \mathbf{F} and $\hat{\mathbf{n}}$ is a unit vector perpendicular to both \mathbf{r} and \mathbf{F} .

- (a) When forces act radially, from a point on the axis

$$\theta = 0^\circ, \text{ hence } |\tau| = 0 \quad [\text{from Eq. (i)}]$$

- (b) When forces are acting on the axis of rotation,

$$\mathbf{r} = 0, |\tau| = 0 \quad [\text{from Eq. (i)}]$$

- (c) When forces are acting perpendicular to the axis of rotation,

$$\theta = 90^\circ, |\tau| = rF \quad [\text{from Eq. (i)}]$$

- (d) When torque by forces are equal and opposite,

$$\tau_{net} = \tau_1 - \tau_2 = 0$$

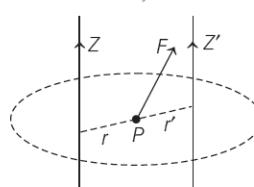
Thus, statement given in option (c) is incorrect, rest are correct.

- 113 (b)**

- (a) Consider the below diagram, where $r > r'$

Torque τ about Z -axis,

$$\tau = \mathbf{r} \times \mathbf{F}, \text{ which is along } \hat{\mathbf{k}}$$



- (b) $\tau' = \mathbf{r}' \times \mathbf{F}$, which is along $-\hat{\mathbf{k}}$

- (c) $|\tau|_Z = Fr_{\perp} = \text{magnitude of torque about } Z\text{-axis}$ where r_{\perp} is perpendicular distance between F and Z -axis.

Similarly, $|\tau|_{Z'} = Fr'_{\perp}$

Clearly, $r_{\perp} > r'_{\perp} \Rightarrow |\tau|_Z > |\tau|_{Z'}$

- (d) We are always calculating resultant torque about a common axis. Hence, total torque $\tau \neq \tau + \tau'$, because τ and τ' are not about common axis.

Thus, statement given in option (b) is correct, rest are incorrect.

- 114 (b)**

- (a) Theorem of perpendicular axes is applicable only for lamina (plane) objects, i.e. $I_Z \neq I_X + I_Y$.

- (b) As, $Z' \parallel Z$ and distance between them

$$= a \frac{\sqrt{2}}{2} = \frac{a}{\sqrt{2}}$$

Now, by theorem of parallel axes

$$I_{Z'} = I_Z + m \left(\frac{a}{\sqrt{2}} \right)^2 = I_Z + \frac{ma^2}{2}$$

(c) Z'' is not parallel to Z , hence theorem of parallel axes cannot be applied, i.e. $I'_Z \neq I_Z + \frac{ma^2}{2}$

(d) As, X and Y -axes are symmetrical.

$$\text{Hence, } I_X = I_Y$$

Thus, statement given in option (b) is correct, rest are incorrect.

115 (b) When a man standing on a rotating platform holds weights in his outstretched arms, draws the weights inwards close to his body, then angular velocity increases. According to law of conservation of angular momentum, $L = I\omega = \text{constant}$, therefore when angular velocity increases, then moment of inertia of the system will decrease, so that L remains constant.

Thus, statement given in option (b) is incorrect, rest are correct.

116 (a) The statement given in option (a) is incorrect and it can be corrected as,

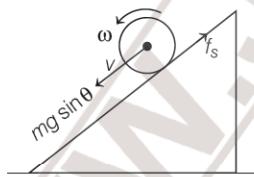
Due to rotation, lower part has acceleration greater than g .

Rest statements are correct.

117 (c) Since, the velocity at point B ($2v$) is more than that at D (zero) or $A(\sqrt{2}v)$. Thus, it can be concluded that, upper portion is moving faster than lower portion. Therefore, section ABC and BC has greater kinetic energy than section ADC , CD or DA .

Thus, statement given in option (c) is incorrect, rest are correct.

118 (a) As we know that, frictional force will be in the opposite direction to the direction of motion. Thus, the given figure can be shown as



where, f_s will be upwards to provide torque.

122 (a) If v is the velocity of CM of the body of radius r , then

Velocity at point A , $v_A = 0$

Velocity at point B , $v_B = v\sqrt{2}$

Velocity at point C , $v_C = v + r\omega = 2v$

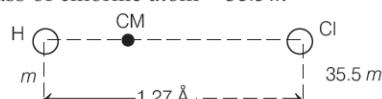
Velocity of point D , $v_D = r\omega = v$

Hence, $A \rightarrow 2$, $B \rightarrow 1$, $C \rightarrow 4$ and $D \rightarrow 3$.

123 (a) Given, separation between the nuclei of H and Cl $= 1.27 \text{ \AA} = 1.27 \times 10^{-10} \text{ m}$

Let the mass of hydrogen atom = m

\therefore Mass of chlorine atom = $35.5m$



Let hydrogen atom be at origin, i.e. position vector of it, $\mathbf{r}_1 = 0$

\therefore Position vector of chlorine atom,

$$\mathbf{r}_2 = 1.27 \times 10^{-10} \text{ m}$$

Position vector of the centre of mass is given by

$$\begin{aligned}\mathbf{r}_{CM} &= \frac{m_1 \mathbf{r}_1 + m_2 \mathbf{r}_2}{m_1 + m_2} \\ &= \frac{m \times 0 + 35.5 m \times 1.27 \times 10^{-10}}{m + 35.5 m} \\ &= \frac{35.5 \times 1.27 \times 10^{-10}}{36.5} \\ &= 1.235 \times 10^{-10} \text{ m} \\ &= 1.24 \text{ \AA}\end{aligned}$$

from hydrogen atom on the line joining H and Cl atoms.

124 (b) The speed of the centre of mass of the (trolley + child) system will remain unchanged, i.e. v . Because the state of any system can be changed only by external force and the forces involved in running on the trolley are internal forces.

125 (d) We know that, $\mathbf{p} = p_x \hat{\mathbf{i}} + p_y \hat{\mathbf{j}} + p_z \hat{\mathbf{k}}$

and

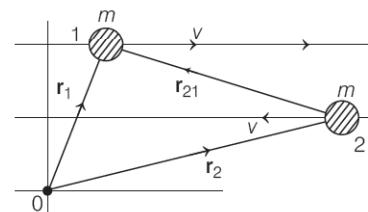
$$\begin{aligned}\mathbf{r} &= x \hat{\mathbf{i}} + y \hat{\mathbf{j}} + z \hat{\mathbf{k}} \\ \mathbf{L} = \mathbf{r} \times \mathbf{p} &= \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ x & y & z \\ p_x & p_y & p_z \end{vmatrix}\end{aligned}$$

If particle remains in xoy -plane, then $z = 0$ and $p_z = 0$.

$$\Rightarrow \mathbf{L} = \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ x & y & 0 \\ p_x & p_y & 0 \end{vmatrix} = \hat{\mathbf{k}} (xp_y - yp_x)$$

So, angular momentum has only z -component.

126 (c) The given situation is as shown below



Angular momentum of system about origin,

$$\begin{aligned}\mathbf{L} &= \mathbf{L}_1 + \mathbf{L}_2 \\ &= \mathbf{r}_1 \times \mathbf{p}_1 + \mathbf{r}_2 \times \mathbf{p}_2 \\ &= \mathbf{r}_1 \times mv \hat{\mathbf{i}} + \mathbf{r}_2 \times mv(-\hat{\mathbf{i}}) \\ &= (\mathbf{r}_2 + \mathbf{r}_{21}) mv \hat{\mathbf{i}} - \mathbf{r}_2 \times mv \hat{\mathbf{i}}\end{aligned}$$

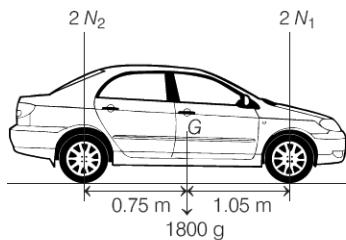
[: using triangle law of vector addition]

$$= \mathbf{r}_{21} mv \hat{\mathbf{i}}$$

which is independent of choice of origin and has same value or constant at any point in space.

127 (b) Moment of force about centre of gravity is zero.

Total mass of the car = 1800 kg



N_1 = reaction by front-one wheel

N_2 = reaction by rear-one wheel

$$g = 9.8 \text{ ms}^{-2}$$

For vertical equilibrium, $2N_1 + 2N_2 = 1800 \text{ g}$

$$\Rightarrow N_1 + N_2 = 900 \text{ g} \quad \dots(\text{i})$$

Taking moments about G,

$$-2N_2(0.75) + 2N_1(1.05) = 0$$

$$\Rightarrow N_2 = \frac{7}{5}N_1 \quad \dots(\text{ii})$$

From Eqs. (i) and (ii), we get

$$N_1 + \frac{7}{5}N_1 = 900 \text{ g}$$

$$\Rightarrow N_1 = \frac{5}{12} \times 900 \text{ g} = 375 \text{ g}$$

$$\Rightarrow N_2 = 525 \text{ g}$$

$$\Rightarrow N_1 = 375 \times 9.8 = 3675 \text{ N}$$

$$\Rightarrow N_2 = 525 \times 9.8 = 5145 \text{ N}$$

128 (a) Using velocity at lowest point, $v = \sqrt{\frac{2gh}{1+k^2/R^2}}$

As, $\frac{k^2}{R^2}$ for sphere is $\frac{2}{5}R$ and for hollow cylinder is R ,

we get v_{sphere} is largest.

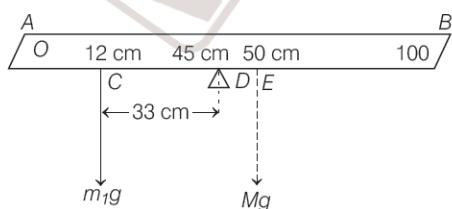
129 (d) From law of conservation of angular momentum,

$$I_1 = I_2 \\ \Rightarrow I_1\omega_1 = I_2\omega_2 \Rightarrow \omega_2 = \frac{I_1\omega_1}{I_2}$$

$$\Rightarrow \omega_2 = \frac{I_1 \times 40}{\frac{2}{5}I_1} = \frac{200}{2} = 100 \text{ rpm}$$

130 (d) Given, $\tau = 180 \text{ N-m}$ and $\omega = 200 \text{ rads}^{-1}$.
Power, $P = \tau\omega = 180 \times 200 = 36 \text{ kW}$

131 (a) Let total mass of the metre stick be M kg..



Distance between mid-point E and new centre of gravity

$$(DE) = 50 - 45 = 5 \text{ cm}$$

Distance between 12 cm mark and the new centre of gravity

$$(CD) = 45 - 12 = 33 \text{ cm}$$

From principle of moments in equilibrium,

$$Mg \times DE = m_1g \times CD = (2 \times 5)g \times CD$$

$$M \times 5 = 10 \times 33 \Rightarrow M = 66 \text{ g}$$

∴ Mass of the metre stick is 66 g.

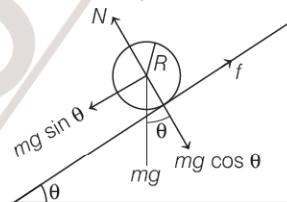
132 (c) Work done = ΔK = Change in rotational kinetic energy + Change in linear kinetic energy

$$= \frac{1}{2}mv_{CM}^2 + \frac{1}{2}I\omega^2 \quad [\because I = mr^2 \text{ and } v_{CM} = r\omega]$$

$$= mv_{CM}^2 = 100 \times (20 \times 10^{-2})^2 = 4 \text{ J}$$

133 (b) Angle of inclination, $\theta = 30^\circ$

Speed of centre of mass, $u = 5 \text{ ms}^{-1}$



Acceleration of the cylinder rolling up the inclined plane is given by the formula, $a = -\left(\frac{g \sin \theta}{1 + k^2/R^2}\right)$

where, k = radius of gyration of cylinder.

Negative sign indicates that acceleration is downwards.

For a cylinder, moment of inertia,

$$I = mk^2 = \frac{1}{2}mR^2 \Rightarrow k^2 = \frac{R^2}{2}$$

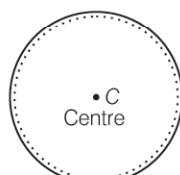
$$\therefore a = -\frac{9.8 \times \sin 30^\circ}{\left(1 + \frac{R^2/2}{R^2}\right)} = \frac{9.8 \times 1/2}{3/2} = -\frac{9.8}{3} \text{ ms}^{-2}$$

Using third equation of motion, $v^2 = u^2 + 2as$

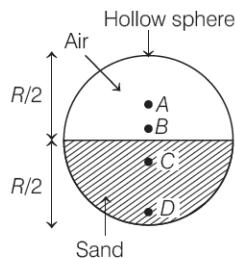
$$0 = (5)^2 + 2\left(-\frac{9.8}{3}\right) \times s$$

$$\therefore s = \frac{25 \times 3}{2 \times 9.8} = 3.83 \text{ m}$$

134 (d) A bangle is in the form of a ring as shown in the below diagram. The centre of mass lies at the centre, which is outside the body (boundary).



- 135** (c) Centre of mass of a system lies towards the part of the system, having bigger mass. In the diagram, lower part is heavier, hence CM of the system lies below the horizontal diameter i.e., at C.



- 136** (b) The initial velocity is $\mathbf{v}_i = v\hat{\mathbf{e}}_y$ and after reflection from the wall, the final velocity is $\mathbf{v}_f = -v\hat{\mathbf{e}}_y$. The trajectory is described as position vector $\mathbf{r} = y\hat{\mathbf{e}}_y + a\hat{\mathbf{e}}_z$. Hence, the change in angular momentum is $\mathbf{r} \times m(\mathbf{v}_f - \mathbf{v}_i) = 2mva\hat{\mathbf{e}}_x$.

- 137** (d) We know that, angular acceleration, $\alpha = \frac{d\omega}{dt}$

where, ω is angular velocity of the disc.

Given, $\omega = \text{constant}$

$$\Rightarrow \alpha = \frac{d\omega}{dt} = \frac{0}{dt} = 0$$

Hence, angular acceleration is zero.

- 138** (b) In the given diagram, when the small piece Q is removed and glued to the centre of the plate, the mass comes closer to the Z-axis, i.e. the axis of rotation and hence moment of inertia decreases.

- 139** (a) Coordinate of centre of mass of non-uniform body is given as,

$$R = \frac{\int xdm}{\int dm}$$

where, $dm = \rho(x)dx$

Here, $\rho(x) = a(1 + bx^2)$

$$\begin{aligned} \Rightarrow R &= \frac{\int_0^L x a(1 + bx^2) \cdot dx}{\int_0^L a(1 + bx^2) \cdot dx} \\ &= \frac{\left[\frac{ax^2}{2} + \frac{abx^4}{4} \right]_0^L}{\left[ax + \frac{abx^3}{3} \right]_0^L} = \frac{\frac{aL^2}{2} + \frac{abL^4}{4}}{aL + \frac{abL^3}{3}} \\ &= \frac{\frac{a}{2} + \frac{ab}{4}}{a + \frac{ab}{3}} \\ &= \frac{3(2+b)}{4(3+b)} \quad [\because \text{given, } L = 1 \text{ m}] \end{aligned}$$

- 140** (a) As no external torque acts on the system, angular momentum should be conserved.

$$\text{Hence, } I\omega = \text{constant} \quad \dots(i)$$

where, I is moment of inertia of the system and ω is angular velocity of the system.

$$\text{From Eq. (i), } I_1\omega_1 = I_2\omega_2$$

where, ω_1 and ω_2 are angular velocities before and after jumping.

$$\Rightarrow I\omega = \frac{I}{2} \times \omega_2$$

(as mass reduced to half, hence moment of inertia also reduced to half)

$$\Rightarrow \omega_2 = 2\omega$$

- 141** (b) As the slope of $\theta-t$ graph is positive and positive slope indicates anti-clockwise rotation.

- 142** (b) Before being brought in contact with the table, the disc was in pure rotational motion, hence $v_{CM} = 0$.

CHAPTER > 08

Gravitation

KEY NOTES

- Each material object has a tendency to be attracted towards the earth. This tendency of an object is governed by few laws, namely Kepler's laws and gravitational law.

Kepler's Laws of Planetary Motion

There are three scientific laws describing the motion of planets around the sun as given below

- Law of Orbits** All planets move in elliptical orbit with sun situated at one of the foci of the ellipse.
- Law of Areas** The line that joins any planet to the sun sweeps equal areas in equal intervals of time. The closest point is called **perihelion** and the farthest point is called **aphelion**.
The law of areas can be understood as a consequence of conservation of angular momentum, which is valid for any central force.
- A **central force** is such that the force on the planet is along the vector joining the sun and the planet.
- The area ΔA swept out by the planet of mass m in time interval Δt is given by

$$\Delta A = \frac{1}{2} (\mathbf{r} \times \mathbf{v} \Delta t)$$

Hence,
$$\frac{\Delta \mathbf{A}}{\Delta t} = \frac{1}{2} \frac{\mathbf{r} \times \mathbf{p}}{m} = \frac{\mathbf{L}}{2m} \quad (\because \mathbf{v} = \frac{\mathbf{p}}{m})$$

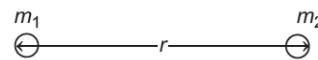
where, \mathbf{v} is the velocity and \mathbf{L} is the angular momentum equal to $(\mathbf{r} \times \mathbf{p})$.

For a central force, which is directed along \mathbf{r} , L is a constant as the planet goes around. Hence, $\Delta \mathbf{A} / \Delta t$ is a constant.

- Law of Periods** The square of the time period of revolution of a planet is proportional to the cube of semi-major axis of the ellipse traced out by the planet, i.e. $T^2 \propto a^3$ where, a = semi-major axis and T = time period.

Universal Law of Gravitation

- It states that every body in the universe attract every other body with a force which is directly proportional to the product of their masses and inversely proportional to the square of the distance between them, i.e.



$$F \propto \frac{m_1 m_2}{r^2}$$

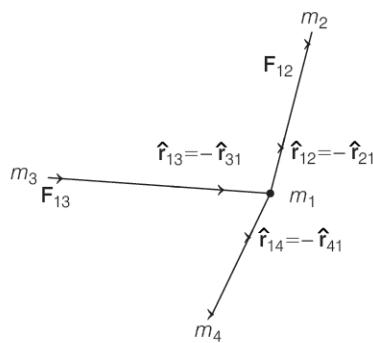
$$F = G \frac{m_1 m_2}{r^2}$$

or
$$\mathbf{F} = -G \frac{m_1 m_2}{|\mathbf{r}|^3} \hat{\mathbf{r}}$$

where, G is the universal gravitational constant and its value is $6.67 \times 10^{-11} \text{ Nm}^2/\text{kg}^2$.

- The gravitational force is attractive in nature, i.e. the force \mathbf{F} is along $-\mathbf{r}$.
- Gravitational force is a central as well as conservative force but the weakest force in nature.
- The universal law of gravitation refers to the point masses whereas we deal with the extended objects which have finite size.

If we have collection of point masses, the force on any one of them is the vector sum of the gravitational forces exerted by the other point masses as shown below



The total force on m_1 is

$$F_1 = \frac{Gm_2m_1}{r_{21}^2} \hat{r}_{21} + \frac{Gm_3m_1}{r_{31}^2} \hat{r}_{31} + \frac{Gm_4m_1}{r_{41}^2} \hat{r}_{41}$$

- The force of attraction between a hollow spherical shell of uniform density and a point mass situated outside is just as if the entire mass of the shell is concentrated at the centre of the shell.
- The force of attraction due to a hollow spherical shell of uniform density, on a point mass situated inside it is zero.

Acceleration Due to Gravity

- The force of gravity acting on a body of unit mass placed near the surface of earth is known as acceleration due to gravity, which is denoted by g and given as $g = \frac{F}{m} = \frac{GM_E}{R_E^2}$.

where, M_E and R_E are the mass and radius of the earth, respectively.

- The value of g at height h from earth's surface is given by

$$g(h) \approx g \left(1 - \frac{2h}{R_E}\right)$$

Therefore, g decreases with height.

- The value of g at depth d from earth's surface is given by

$$g(d) = g \left(1 - \frac{d}{R_E}\right)$$

Therefore, g decreases with depth and becomes zero at earth's centre.

Gravitational Potential

Gravitational potential at a point in the gravitational field is defined as the amount of work done in bringing a body of unit mass from infinity to that point without acceleration.

i.e. $V = -\frac{W}{m}$

Its SI unit is J kg^{-1} and dimensional formula is $[\text{M}^0 \text{L}^2 \text{T}^{-2}]$.

Gravitational Potential Energy

- Gravitational potential energy of a body at a point is the amount of work done in bringing a given body from infinity to a point against the gravitational force.
- If m_1 and m_2 are two masses separated by infinite distance, then gravitational potential energy of the system when they are brought to a separation of r is given by

$$U = -\frac{Gm_1m_2}{r}$$

where, the negative sign indicates that the two bodies are attracting each other.

Escape Velocity

- The minimum speed required for an object to reach infinity (i.e. escape from the earth) is called escape velocity.

It is given by

$$\text{Escape velocity } (v_e)_{\min} = \sqrt{\frac{2GM_E}{R_E}} = \sqrt{2gR_E}$$

Escape velocity at earth is 11.2 kms^{-1} and for the moon is 2.3 kms^{-1} , about five times smaller.

This is the reason that moon has no atmosphere.

Earth Satellites

- Earth satellites are objects which revolve around the earth. Their motion is very similar to the motion of planets around the sun and hence Kepler's laws of planetary motion are equally applicable to them.
- Moon is the only natural satellite of the earth moving in a nearly circular orbit with a time period of approximately 27.3 days which is also roughly equal to the rotational period of the moon about its own axis.
- Orbital velocity of a satellite at a height h from surface of earth is given by $v_o = \sqrt{\frac{GM_E}{R_E + h}}$.

When a satellite revolve near the earth's surface, i.e. $h = 0$, then its orbital velocity,

$$v_o = \sqrt{gR_E} \approx 7.92 \text{ km/h}$$

- Time period of a satellite,

$$T = \frac{2\pi(R_E + h)}{v_o} = \frac{2\pi(R_E + h)^{3/2}}{\sqrt{GM_E}}$$

When a satellite revolve near the earth's surface, then

$$T_o = 2\pi \sqrt{\frac{R_E}{g}} \approx 85 \text{ min}$$

- Kinetic energy of the satellite in a circular orbit with speed v is $KE = \frac{GmM_E}{2(R_E + h)}$.



- **Potential energy** at distance $(R + h)$ from the centre of the earth is $PE = -\frac{GmM_E}{(R_E + h)}$.
 - Total energy of a satellite $= KE + PE = \frac{GmM_E}{2(R_E + h)} - \frac{GmM_E}{(R_E + h)}$
 $= -\frac{GmM_E}{2(R_E + h)}$.
 - The total energy of an circularly orbiting satellite is thus negative, with the potential energy being negative but twice is magnitude of the positive kinetic energy.

Geo-Stationary and Polar Satellites

- Satellites in a circular orbit around the earth in the equilateral plane with $T = 24$ h are called **geo-stationary satellites**.

For $T = 24$ h, the height of satellite above earth's surface is

calculated from, $R_E + h = \left(\frac{T^2 GM_E}{4\pi^2} \right)^{1/3}$

It (height h) comes out to be 35800 km, which is much larger than R_E .

- Low altitude ($h \approx 500$ to 800 km) satellites, which go around the poles of the earth in a north-south direction are called **polar satellites**, whereas the earth rotates around its axis in an east-west direction.

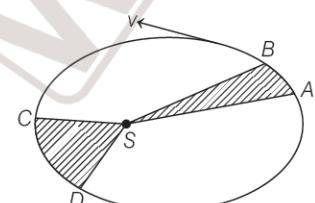
Weightlessness

- When a satellite, which does not produce its own gravity moves around the earth in a circular orbit under the action of gravity. The surface of satellite does not exert any force on the body, hence its apparent weight is zero.
 - In other words, when a body is in free fall, it is said to be **weightless**, since there is no upward force on the body .

Mastering NCERT

MULTIPLE CHOICE QUESTIONS

TOPIC 1 ~ Early Finding and Kepler's Laws



- 4 The law of areas can be understood as a consequence of

 - (a) conservation of angular momentum
 - (b) conservation of energy
 - (c) conservation of linear momentum
 - (d) Both (a) and (b)

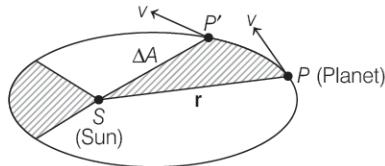
5 A central force is that force which acts along

 - (a) the vector perpendicular to sun
 - (b) the vector joining the sun and the planet
 - (c) the vector tangential to the orbit
 - (d) None of the above

6 The law of areas can be interpreted as

 - (a) $\frac{\Delta A}{\Delta t} = \frac{L}{2m}$
 - (b) $\frac{\Delta A}{\Delta t} = \frac{L}{m}$
 - (c) $\frac{\Delta A}{\Delta t} = 2(r \times p)$
 - (d) $\frac{\Delta A}{\Delta t} = \frac{2L}{m}$

- 7 Let a planet is moving around the sun in an elliptical orbit as shown below.



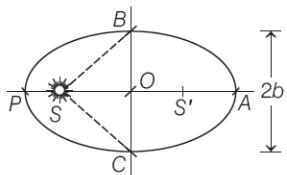
The area swept out by the planet of mass m in time interval Δt is given by

- (a) $\Delta A = \frac{1}{2}(\mathbf{r} \times \mathbf{v}\Delta t)$ (b) $\Delta A = \frac{1}{2}(\mathbf{r} \times \mathbf{v})$
 (c) $\Delta A = \frac{1}{2}(\mathbf{r} \times \mathbf{p})$ (d) All of these

- 8 In planetary motion, the areal velocity of position vector of a planet depends on angular velocity ω and the distance of the planet from sun r . So, the correct relation for areal velocity is

- (a) $\frac{dA}{dt} \propto \omega r$ (b) $\frac{dA}{dt} \propto \omega^2 r$ (c) $\frac{dA}{dt} \propto \omega r^2$ (d) $\frac{dA}{dt} \propto \sqrt{\omega r}$

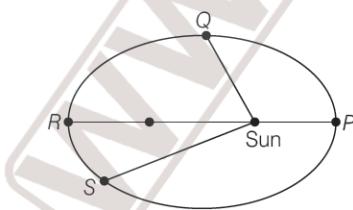
- 9 Let the speed of the planet at the perihelion P in above figure be v_P and the sun planet distance SP be r_P . The corresponding quantity at the aphelion A is (r_A, v_A) . The mass of the planet is m_p .



The magnitude of the angular momentum at P is

- (a) $L_p = \frac{m_p r_p}{v_p}$ (b) $L_p = \frac{m_p v_p}{r_p}$
 (c) $L_p = m_p r_p v_p$ (d) $L_p = m_p v_p$

- 10 A planet is moving around the sun in an elliptical path as shown in the figure. The linear speed of the planet is minimum at



- (a) P (b) S
 (c) Q (d) R

- 11 The maximum and minimum velocities of a planet are $3 \times 10^4 \text{ ms}^{-1}$ and $1 \times 10^3 \text{ ms}^{-1}$, respectively. The minimum distance of the planet from the sun, if maximum distance is $4 \times 10^4 \text{ km}$, is

- (a) $4 \times 10^3 \text{ km}$ (b) $3 \times 10^3 \text{ km}$
 (c) $(4/3) \times 10^3 \text{ km}$ (d) $1 \times 10^3 \text{ km}$

- 12 The kinetic energies of a planet in an elliptical orbit about the sun, at positions A , B and C are K_A , K_B and K_C , respectively. AC is the major axis and SB is perpendicular to AC at the position of the sun S as shown in the figure, then

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- (a) $K_B < K_A < K_C$ (b) $K_A > K_B > K_C$
 (c) $K_A < K_B < K_C$ (d) $K_B > K_A > K_C$

- 13 The time period of revolution T of planet in years (yr) is 84. The semi-major axis a in units of 10^{10} m is 287. The quotient (T^2/a^3) in units of $10^{-34} \text{ yr}^2 \text{ m}^{-3}$ is approximately

- (a) 3 units (b) 4 units (c) 2 units (d) 1 unit

- 14 According to Kepler's law of planetary motion, if T represents time period and r is orbital radius, then for two planets these are related as

- (a) $\left(\frac{T_1}{T_2}\right)^3 = \left(\frac{r_1}{r_2}\right)^2$ (b) $\left(\frac{T_1}{T_2}\right)^{3/2} = \frac{r_1}{r_2}$
 (c) $\left(\frac{T_1}{T_2}\right)^2 = \left(\frac{r_1}{r_2}\right)^3$ (d) $\left(\frac{T_1}{T_2}\right) = \left(\frac{r_1}{r_2}\right)^{2/3}$

- 15 Assuming that earth and mars move in circular orbits around the sun, with the martian orbit being 1.52 times the orbital radius of the earth. The length of the martian year in days is

- (a) $(1.52)^{2/3} \times 365$ (b) $(1.52)^{3/2} \times 365$
 (c) $(1.52)^2 \times 365$ (d) $(1.52)^3 \times 365$

- 16 A planet of radius R has a time period of revolution T . Find the time period of a planet of radius $9R$.

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- (a) $3\sqrt{3} T$ (b) $9T$ (c) $27T$ (d) $9\sqrt{3} T$

- 17 A geostationary satellite is orbiting the earth at a height of $5R$ above the surface of the earth and R being the radius of the earth. The time period of another satellite in hour at a height of $2R$ from the surface of the earth is

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- (a) 5 h (b) 10 h (c) $6\sqrt{2}$ h (d) $6/\sqrt{2}$ h

- 18 Following table shows the data from measurement of planetary motions to confirm Kepler's law of periods.

Planet	a	T	Q
Mercury	5.79	0.24	2.95
Venus	10.8	0.615	<i>A</i>
Earth	15.0	1.00	2.96
Mars	22.8	1.88	2.98

Here, a = semi-major axis in units of 10^{10} m for the elliptical orbits,

T = time-period of revolution of the planet in years (yr) and Q = the quotient (T^2/a^3) in units of 10^{-34} y^2m^{-3})

The missing term A in the given table is approximately equal to

- | | |
|---------------|---------------|
| (a) 3 units | (b) 2 units |
| (c) 2.5 units | (d) 3.5 units |

- 19 Two satellites A and B revolve around the same planet in coplanar circular orbits lying in the same

plane. Their periods of revolutions are 1 h and 8 h, respectively and the radius of the orbit of A is 10^4 km.

The speed of B relative to A (in km/h) when they are close to each other, will be

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- | | | | |
|------------------------|----------|------------------------|-----------------------|
| (a) $3\pi \times 10^4$ | (b) zero | (c) $2\pi \times 10^4$ | (d) $\pi \times 10^4$ |
|------------------------|----------|------------------------|-----------------------|

- 20 If the sun and the earth carried huge amounts of opposite charges, then

- | |
|---|
| (a) all three of Kepler's laws would still be valid |
| (b) only the third law will be valid |
| (c) the second law will change |
| (d) the first law will not be valid |

TOPIC 2 ~ Newton's Law of Gravitation

- 21 Newton's law of gravitation is universal because

- | |
|---|
| (a) it acts on all bodies in the universe |
| (b) it acts on all the masses at all distances and not affected by the medium |
| (c) it is an attractive force |
| (d) it acts only when bodies are in contact |

- 22 Newton arrived at an universal law based on his observation of an apple falling from a tree. Newton's reasoning was that the moon revolving in an orbit of radius R_m was subjected to a centripetal acceleration due to earth's gravity whose magnitude is given by

- | | |
|------------------------------|------------------------------------|
| (a) $a_m = \frac{v^2}{R_m}$ | (b) $a_m = \frac{4\pi^2 R_m}{T^2}$ |
| (c) $a_m = \frac{mv^2}{R_m}$ | (d) Both (a) and (b) |

- 23 On going from one planet to other planet, the value of G (gravitational constant)

- | | |
|------------------|-------------------|
| (a) increases | (b) decreases |
| (c) remains same | (d) None of these |

- 24 There have been suggestions that the value of the gravitational constant G becomes smaller when considered over very large time period (in billions of years) in the future. If that happens, for our earth,
- (a) nothing will change
 - (b) it will become hotter after billions of years
 - (c) it will be going around strictly in closed orbits
 - (d) after sufficiently long time it will leave the solar system

- 25 Kepler's third law states that square of period of revolution T of a planet around the sun, is proportional to third power of average distance r between sun and planet i.e. $T^2 = kr^3$, where k is constant.

If the masses of sun and planet are M and m , respectively, then as per Newton's law of gravitation

force of attraction between them is $F = \frac{GMm}{r^2}$, where

G is gravitational constant.

The relation between G and k is described as

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- | | |
|--------------------|-------------------|
| (a) $k = 1/G$ | (b) $Gk = 4\pi^2$ |
| (c) $GMk = 4\pi^2$ | (d) $k = G$ |

- 26 A planet is revolving around the sun in a circular orbit with a radius r and the time period is T . If the force between the planet and star is proportional to $r^{-3/2}$, then the square of time period is proportional to

- | | |
|---------------|---------------|
| (a) $r^{3/2}$ | (b) r^2 |
| (c) r | (d) $r^{5/2}$ |

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- 27 If the law of gravitation, instead of being inverse

- square law, becomes an inverse cube law, then
- (a) planets will have elliptical orbits
 - (b) Kepler's law of areas does not hold good
 - (c) projectile motion of a stone thrown by hand on the surface of the earth will be approximately parabolic
 - (d) there will be no gravitational force inside a spherical shell of uniform density

- 28 The height at which the weight of a body becomes $(1/16)$ th of its weight on the surface of the earth (radius R), is

- | | | | |
|----------|-----------|----------|----------|
| (a) $5R$ | (b) $15R$ | (c) $3R$ | (d) $4R$ |
|----------|-----------|----------|----------|

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- 29 If mass of a planet is 2% of mass of earth. The ratio of gravitational pull of earth on the planet and that of planet on the earth will be
- (a) $1 : 20$
 - (b) $2 : 5$
 - (c) $1 : 1$
 - (d) $1 : 50$

- 30 Two equal point masses are separated by a distance d_1 . The force of gravitation acting between them is F_1 . If the separation is decreased to d_2 , then the new force of gravitation F_2 is given by

(a) $F_2 = F_1$

(b) $F_2 = F_1 \left(\frac{d_1}{d_2} \right)^2$

(c) $F_2 = F_1 \left(\frac{d_2}{d_1} \right)^2$

(d) $F_2 = F_1 \left(\frac{d_1}{d_2} \right)$

- 31** Two spheres of masses m and M are situated in air and the gravitational force between them is F . The space around the masses is now filled with a liquid of specific gravity 3. The gravitational force will now be

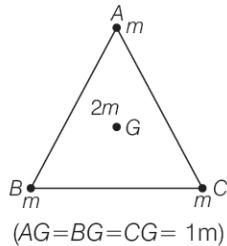
(a) F

(b) $\frac{F}{3}$

(c) $\frac{F}{9}$

(d) $3F$

- 32** Three equal masses of m kg each are fixed at the vertices of an equilateral triangle ABC and a mass $2m$ is placed at centroid G of the triangle as shown below.



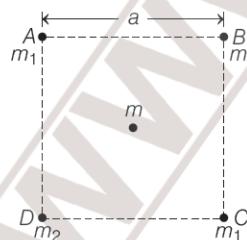
The force acting on a mass $2m$ placed at the centroid G of the triangle is

(a) $\frac{2Gm^2}{\sqrt{2}}$ (b) $2Gm^2$ (c) $6Gm^2$ (d) zero

- 33** Three uniform spheres of mass M and radius R each are kept in such a way that each touches the other two. The magnitude of the gravitational force on any of the spheres due to the other two is

(a) $\frac{\sqrt{3}GM^2}{4R^2}$ (b) $\frac{3GM^2}{2R^2}$ (c) $\frac{\sqrt{3}GM^2}{R^2}$ (d) $\frac{\sqrt{3}GM^2}{2R^2}$

- 34** A point mass m is placed at the centre of the square $ABCD$ of side a units as shown below.



The resultant gravitational force on mass m due to masses m_1 and m_2 placed on the vertices of square is

(a) $\frac{Gm_1m_2}{(a\sqrt{2})^2}$

(b) $\frac{2Gm(m_1 + m_2)}{a^2}$

(c) zero

(d) $\frac{Gm(m_1 + m_2)}{(a\sqrt{2})^2}$

- 35** A mass M is split into two parts, m and $(M - m)$, which are then separated by a certain distance. What ratio of m / M maximizes the gravitational force between the two parts?

(a) 1/3

(b) 1/2

(c) 1/4

(d) 1/5

- 36** Two spherical bodies of masses m and $5m$ and radii R and $2R$ are released in free space with initial separation between their centres equal to $12R$. If they attract each other due to gravitational force only, then the distance covered by the smaller body before collision is

(a) $2.5R$

(b) $4.5R$

(c) $7.5R$

(d) $1.5R$

CBSE AIPMT 2015

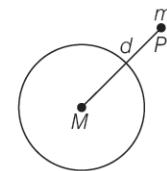
- 37** A point mass m is placed outside a hollow spherical shell of mass M and uniform density at a distance d from centre of the sphere as shown in figure. Gravitational force on point mass m at P is

(a) $\frac{GmM}{d^2}$

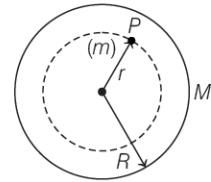
(b) zero

(c) $\frac{2GmM}{d^2}$

(d) Data insufficient



- 38** The force of attraction due to a hollow spherical shell of mass M , radius R and uniform density, on a point mass m situated inside it is



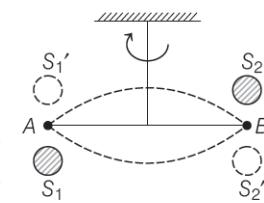
(a) $\frac{GmM}{r^2}$

(b) $\frac{GmM}{R^2}$

(c) zero

(d) Data insufficient

- 39** The figure shown is the schematic representation of Cavendish's experiment to determine the value of the gravitational constant. The bar AB has two small lead spheres attached at its ends. The bar is suspended from a rigid support by a fine wire. Two large lead spheres are brought close to the small ones but on opposite sides as shown.



The big spheres attract the near by small ones by a force which is

(a) equal and opposite

(b) equal but in same direction

(c) unequal and opposite

(d) None of the above

TOPIC 3 ~ Acceleration due to Gravity and its Variation

- 40** If g is the acceleration due to gravity at the surface of the earth. The force acting on the particle of mass m placed at the surface is

- (a) mg
- (b) $\frac{GmM_E}{R_E^2}$
- (c) Data insufficient
- (d) Both (a) and (b)

- 41** The weight of a body at the centre of earth is

- (a) same as on the surface of earth
- (b) same as on the poles
- (c) same as on the equator
- (d) None of the above

- 42** According to Newton's law of gravitation, let \mathbf{F}_1 and \mathbf{F}_2 be the forces between two masses m_1 and m_2 at positions \mathbf{r}_1 and \mathbf{r}_2 , where

$$\mathbf{F}_1 = -\mathbf{F}_2 = -\frac{\mathbf{r}_{12}}{r_{12}^3} GM_0^2 \left(\frac{m_1 m_2}{M_0^2} \right)^n$$

where, M_0 is a constant of dimension of mass, $\mathbf{r}_{12} = \mathbf{r}_1 - \mathbf{r}_2$ and n is a number. In such a case,

- (a) the acceleration due to gravity on the earth will be different for different objects
- (b) None of the three laws of Kepler will be valid
- (c) only the third law will be valid
- (d) None of the above

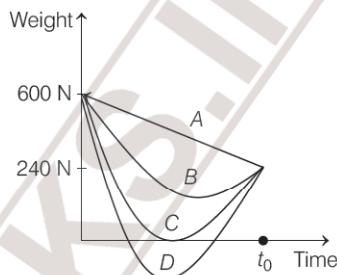
- 43** If the mass of the earth is doubled and its radius halved, then new acceleration due to the gravity g' is
- (a) $g' = 4 g$
 - (b) $g' = 8 g$
 - (c) $g' = g$
 - (d) $g' = 16 g$

- 44** A planet has twice the density of earth but the acceleration due to gravity on its surface is exactly the same as on the surface of earth. Its radius in terms of radius of earth R will be

- (a) $R / 4$
- (b) $R / 2$
- (c) $R / 3$
- (d) $R / 8$

- 45** Suppose, the acceleration due to gravity at the earth's surface is 10 ms^{-2} and at the surface of mars it is 4.0 ms^{-2} . A 60 kg passenger goes from the earth to the mars in a spaceship moving with a constant velocity. Neglect all other objects in the sky.

Which curve best represents the weight (net gravitational force) of the passenger as a function of time?



- (a) A
- (b) B
- (c) C
- (d) D

- 46** If the mass of the sun is ten times smaller and gravitational constant G is ten times larger in magnitude. Then, for earth,

- (a) walking on ground would become more easy
- (b) acceleration due to gravity on the earth will not change
- (c) raindrops will fall much slower
- (d) airplanes will have to travel much faster

- 47** If one assumes that the gravitational force due to the earth decreases in proportion to the inverse square of the distance from the centre of the earth, then which of the following relation between a_m (centripetal acceleration of moon), R_m (distance of the moon from centre of the earth), g (acceleration due to gravity on the surface of the earth) and R_E (radius of earth) is correct?

- (a) $\frac{g}{a_m} = \frac{R_m}{R_E}$
- (b) $\frac{g}{a_m} = \left(\frac{R_m}{R_E} \right)^2$
- (c) $\frac{g}{a_m} = \frac{R_E^{-2}}{R_m^{-2}}$
- (d) Both (b) and (c)

- 48** The time period of the moon is $T = 27.3$ days and radius of orbit is $R_m = 3.84 \times 10^8 \text{ m}$, the value of centripetal acceleration due to the earth's gravity is

- (a) much smaller than the value of acceleration due to gravity g on the surface of the earth
- (b) is equal to the value of acceleration due to gravity g on the surface of the earth
- (c) much larger than the value of acceleration due to gravity g on the surface of the earth
- (d) Either (a) or (b)

- 49** The value of acceleration due to gravity at earth's surface is 9.8 ms^{-2} . The altitude above its surface at which the acceleration due to gravity decreases to 4.9 ms^{-2} , is close to (Take, radius of earth $= 6.4 \times 10^6 \text{ m}$) **JEE Main 2019**
- (a) $9.0 \times 10^6 \text{ m}$ (b) $2.6 \times 10^6 \text{ m}$
 (c) $6.4 \times 10^6 \text{ m}$ (d) $1.6 \times 10^6 \text{ m}$

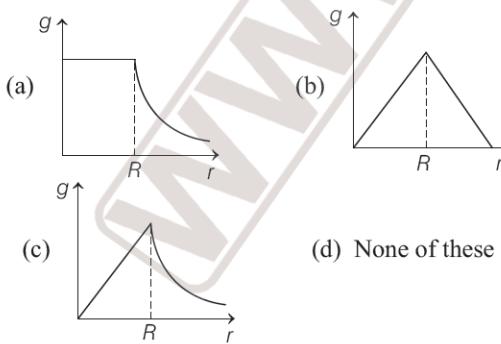
- 50** A body weighs $w \text{ N}$ at the surface of the earth. Its weight at a height equals to half the radius of the earth will be
JEE Main 2020
- (a) $\frac{w}{2}$ (b) $\frac{2w}{3}$ (c) $\frac{4w}{9}$ (d) $\frac{8w}{27}$

- 51** At what height h above the surface of the earth, the value of g becomes $\left(\frac{g}{2}\right)$? (Take, g = acceleration due to gravity at the surface of the earth and R = radius of the earth)
JEE Main 2020
- (a) $3R$ (b) $\sqrt{2}R$
 (c) $(\sqrt{2}-1)R$ (d) $\frac{1}{\sqrt{2}}R$

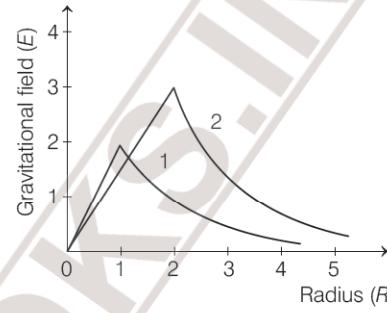
- 52** The radius of earth is R . Height of a point vertically above the earth's surface at which acceleration due to gravity becomes 1% of its value at the surface is
JEE Main 2020
- (a) $8R$ (b) $9R$ (c) $10R$ (d) $20R$

- 53** The weight of a body on the surface of the earth is 45 N . What is the gravitational force on it due to the earth at a height equal to half the radius of the earth?
AIIMS 2018
- (a) 35 N (b) 20 N (c) 18 N (d) 40 N

- 54** Assuming the earth to have a constant density, point out which of the following curves show the variation of acceleration due to gravity from the centre of earth to the points far away from the surface of earth.
NEET 2016, JEE Main 2017



- 55** Consider two solid spheres of radii $R_1 = 1 \text{ m}$, $R_2 = 2 \text{ m}$ and masses M_1 and M_2 , respectively. The gravitational field due to sphere 1 and 2 are shown. The value of $\frac{M_1}{M_2}$ is
JEE Main 2020



- (a) $\frac{1}{3}$ (b) $\frac{1}{2}$
 (c) $\frac{1}{6}$ (d) $\frac{2}{3}$

- 56** Find acceleration due to gravity at a distance of 2000 km from the centre of earth.
AIIMS 2019

- (Given, $R_E = 6400 \text{ km}$, $r = 2000 \text{ km}$ and $M_E = 6 \times 10^{24} \text{ kg}$)
 (a) 1.53 m/s^2 (b) 7.12 m/s^2
 (c) 3.06 m/s^2 (d) 1.8 m/s^2

- 57** A body weighs 200 N on the surface of the earth. How much will it weigh half-way down to the centre of the earth?
NEET 2019

- (a) 200 N (b) 250 N
 (c) 100 N (d) 150 N

- 58** A box weighs 196 N on a spring balance at the north pole. Its weight recorded on the same balance, if it is shifted to the equator is close to (Take, $g = 10 \text{ ms}^{-2}$ at the north pole and the radius of the earth = 6400 km)
JEE Main 2020

- (a) 195.66 N (b) 195.32 N
 (c) 194.66 N (d) 194.32 N

- 59** The acceleration due to gravity at a height 1 km above the earth is the same as at a depth d below the surface of earth. Then,
NEET 2017

- (a) $d = \frac{1}{2} \text{ km}$ (b) $d = 1 \text{ km}$
 (c) $d = \frac{3}{2} \text{ km}$ (d) $d = 2 \text{ km}$

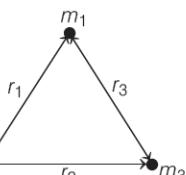


TOPIC 4 ~ Gravitational Potential and Gravitational Potential Energy

- 60** The gravitational potential energy of a system consisting two particles separated by a distance r is
 (a) directly proportional to product of the masses of particles
 (b) inversely proportional to the separation between them
 (c) independent of distance r
 (d) Both (a) and (b)

- 61** Gravitational potential energy of a system of particles as shown in the figure is

- (a) $\frac{Gm_1m_2}{r_1} + \frac{Gm_2m_3}{r_3} + \frac{Gm_1m_3}{r_3}$
 (b) $\left(\frac{-Gm_1m_2}{r_1}\right) + \left(\frac{-Gm_2m_3}{r_2}\right) + \left(\frac{-Gm_1m_3}{r_3}\right)$
 (c) $\frac{-Gm_1m_2}{r_1} - \frac{Gm_2m_3}{r_2} + \frac{Gm_1m_3}{r_3}$
 (d) $\frac{Gm_1m_2}{r_1} + \frac{Gm_2m_3}{r_2} - \frac{Gm_1m_3}{r_3}$



- 62** Two point masses m_1 and m_2 are separated by a distance r . The gravitational potential energy of the system is G_1 . When the separation between the particles is doubled, the gravitational potential energy is G_2 . Then, the ratio of $\frac{G_1}{G_2}$ is
 (a) 1 (b) 2 (c) 3 (d) 4

- 63** The mass of the earth is 6×10^{24} kg and that of the moon is 7.4×10^{22} kg. The potential energy of the

system is -7.79×10^{28} J. The mean distance between the earth and moon is ($G = 6.67 \times 10^{-11} \text{ Nm}^2 \text{kg}^{-2}$)

- (a) 3.8×10^8 m (b) 3.37×10^6 m
 (c) 7.60×10^4 m (d) 1.9×10^2 m

- 64** The work done to raise a mass m from the surface of the earth to a height h , which is equal to the radius of the earth, is

NEET 2019

- (a) $2mgR$ (b) $\frac{1}{2}mgR$ (c) $\frac{3}{2}mgR$ (d) mgR

- 65** The gravitational potential at a distance r from the centre of the earth ($r > R$) is given by (consider, mass of the earth = M_E and radius of the earth = R)

- (a) $\frac{-GM_E}{R}$ (b) $\frac{GM_E}{R}$ (c) $\frac{-GM_E}{r}$ (d) $\frac{+GM_E}{r}$

- 66** At what height from the surface of earth, the gravitation potential and the value of g are $-5.4 \times 10^7 \text{ J kg}^{-2}$ and 6.0 ms^{-2} , respectively?

NEET 2016

- [Take, radius of earth is 6400 km].
 (a) 1600 km (b) 1400 km
 (c) 2000 km (d) 2600 km

- 67** Three particles each of mass m are kept at vertices of an equilateral triangle of side L . The gravitational potential at the centre due to these particles is

- (a) $-\frac{3Gm}{L}$ (b) $-\frac{9Gm}{\sqrt{3}L}$
 (c) $-\frac{3\sqrt{3}Gm}{L}$ (d) Both (b) and (c)



TOPIC 5 ~ Escape Velocity and Satellites

- 68** Gas escapes from the surface of a planet because it acquires an escape velocity. The escape velocity will depend on which of the following factors?
 (a) Mass of the planet
 (b) Mass of the particle escaping
 (c) Temperature of the planet
 (d) None of the above

- 69** Escape velocity of a body on the surface of earth is independent of
 (a) mass
 (b) radius of earth
 (c) direction of projection of body
 (d) Both (a) and (c)

- 70** An object is thrown from the surface of the moon. The escape speed for the object is

- (a) $\sqrt{2g'R_m}$, where g' = acceleration due to gravity on the moon and R_m = radius of the moon

- (b) $\sqrt{2g'R_E}$, where g' = acceleration due to gravity on the moon and R_E = radius of the earth

- (c) $\sqrt{2gR_m}$, where g = acceleration due to gravity on the earth and R_m = radius of the moon

- (d) None of the above

- 71** The escape velocity of a body from the earth is v_e . If the radius of earth contracts to $(1/4)$ th of its value, keeping the mass of the earth constant, escape velocity will be

- (a) doubled (b) halved
 (c) tripled (d) unaltered

JIPMER 2019

- (a) 2.46 km/h (b) 3.46 km/h
 (c) 4.4 km/h (d) None of these

77 The ratio of escape velocity at earth (v_e) to the escape velocity at a planet (v_p) whose radius and mean density are twice as that of earth is **NEET 2016**
 (a) $1:2\sqrt{2}$ (b) $1:4$
 (c) $1:\sqrt{2}$ (d) $1:2$

78 The orbital angular momentum of a satellite revolving at a distance r from the centre is L . If the distance is increased to $16r$, then the new angular momentum will be
 (a) $16L$ (b) $64L$
 (c) $\frac{L}{4}$ (d) $4L$

79 Two satellites A and B have masses m and $2m$ respectively. A is in a circular orbit of radius R and B is in a circular orbit of radius $2R$ around the earth. The ratio of their kinetic energies, T_A / T_B is **JEE Main 2019**
 (a) $\frac{1}{2}$ (b) 2 (c) $\sqrt{\frac{1}{2}}$ (d) 1

80 The kinetic energy of the satellite in a circular orbit with speed v is given as
 (a) $KE = \frac{-GmM_E}{2(R_E + h)}$ (b) $KE = \frac{GmM_E}{(R_E + h)}$
 (c) $KE = \frac{GmM_E}{2(R_E + h)}$ (d) $KE = -\frac{1}{2}mv^2$

81 The radius of the orbit of a satellite is r and its kinetic energy is K . If the radius of the orbit is doubled, then the new kinetic energy K' is
 (a) $2K$ (b) $\frac{K}{2}$
 (c) $4K$ (d) Data insufficient

82 If the gravitational potential energy at infinity is assumed to be zero, the potential energy at distance $(R_E + h)$ from the centre of the earth is
 (a) $PE = \frac{GmM_E}{(R_E + h)}$ (b) $PE = \frac{-GmM_E}{(R_E + h)}$
 (c) $PE = mgh$ (d) $PE = \frac{-GmM_E}{2(R_E + h)}$

83 The potential energy of a satellite is given as

$$PE = \lambda (KE)$$

where, PE = potential energy of the satellite
and KE = kinetic energy of the satellite.

The value of the constant λ is

- 84** A satellite is revolving in a circular orbit at a height h from the earth surface such that $h \ll R$, where R is the radius of the earth. Assuming that the effect of earth's atmosphere can be neglected the minimum increase in the speed required so that the satellite could escape from the gravitational field of earth is

JEE Main 2019

- (a) $\sqrt{\frac{gR}{2}}$ (b) \sqrt{gR} (c) $\sqrt{2gR}$ (d) $\sqrt{gR}(\sqrt{2}-1)$

- 85** The time period of geo-stationary satellite is

- (a) 6 h (b) 12 h (c) 24 h (d) 48 h

- 86** A satellite of mass M is in a circular orbit of radius R about the centre of the earth. A meteorite of the same mass falling towards the earth collides with the satellite completely inelastically. The speeds of the satellite and the meteorite are the same just before the collision. The subsequent motion of the combined body will be

JEE Main 2019

- (a) in the same circular orbit of radius R
 (b) in an elliptical orbit
 (c) such that it escapes to infinity
 (d) in a circular orbit of a different radius

SPECIAL TYPES QUESTIONS

I. Assertion and Reason

■ **Direction** (Q. Nos. 87-98) In the following questions, a statement of Assertion is followed by a corresponding statement of Reason. Of the following statements, choose the correct one?

- (a) Both Assertion and Reason are correct and Reason is the correct explanation of Assertion.
 (b) Both Assertion and Reason are correct but Reason is not the correct explanation of Assertion.
 (c) Assertion is correct but Reason is incorrect.
 (d) Assertion is incorrect but Reason is correct.

- 87 Assertion** The force of attraction between a hollow spherical shell of uniform density and a point mass situated outside is just as if the entire mass of the shell is concentrated at the centre of the shell.

Reason Gravitational force caused by various regions of the shell have components along the line joining the point mass to the centre as well as along a direction perpendicular to this line.

- 88 Assertion** The force of attraction due to a hollow spherical shell of uniform density, on a point mass situated inside it is zero.

Reason Various region of the spherical shell attract the point mass inside it in various directions. These forces cancel each other completely.

- 89 Assertion** The measurement of G by Cavendish's experiment, combined with the knowledge of g and R_E enables one to estimate M_E .

Reason By Newton's second law, the value of g is

$$\text{given by the relation, } g = \frac{GM_E}{R_E^2}.$$

- 90 Assertion** As we go up the surface of the earth, we feel light weighed than on the surface of the earth.
Reason The acceleration due to gravity decreases on going up above the surface of the earth.

- 91 Assertion** The escape speed for the moon is 2.3 kms^{-1} which is five times smaller than that for the earth.

Reason The escape speed depends on acceleration due to gravity on the moon and radius of the moon and both of them are smaller than that of earth.

- 92 Assertion** Moon has no atmosphere.

Reason The escape speed for the moon is much smaller.

- 93 Assertion** The escape velocity for a planet is $v_e = \sqrt{2gR}$. If the radius of the planet is doubled, the escape velocity becomes twice (i.e. $v_e' = 2v_e$)

Reason In the relation for escape velocity, $v_e = \sqrt{2gR}$, the acceleration due to gravity g is inversely proportional to radius of the planet.

$$\text{Thus, } v_e \propto \frac{1}{\sqrt{R}}.$$

- 94 Assertion** The velocity of the satellite decreases as its height above earth's surface increases and is maximum near the surface of the earth.

Reason The velocity of the satellite is inversely proportional to square root of its height above earth's surface.

- 95 Assertion** The total energy of the satellite is always negative irrespective of the nature of its orbit, i.e. elliptical or circular and it cannot be positive or zero.

Reason If the total energy is positive or zero, the satellite would leave its orbit.

- 96 Assertion** The geo-stationary satellite goes around the earth in west-east direction.

Reason Geo-stationary satellites orbits around the earth in the equatorial plane with $T = 24\text{ h}$ same as that of the rotation of the earth around its axis.

- 97 Assertion** In the satellite, everything inside it is in a state of free fall.

Reason Every part and parcel of the satellite has zero acceleration.

- 98 Assertion** An object is weightless when it is in free fall and this phenomenon is called weightlessness.

Reason In free fall, there is no upward force acting on the object.

II. Statement Based Questions

- 99** The scheme of motion put forward by Ptolemy in order to describe the observed motion of the planets is that

- I. the planets are moving in circles with the centre of the circles themselves moving in larger circles.
 - II. the planets are moving in circles with the centre of circles themselves remaining stationary.
 - III. the planets are moving in an elliptical orbit with the centre of the ellipse moving in a circle.

Which of the following statement(s) is/are correct?

- 100** I. All celestial objects, stars, the sun and the planets, revolve around the earth.

II. The only motion that was thought to be possible for celestial objects was motion in a circle.

III. The star, the earth and the planets, all revolve around the sun.

Which of the following statement(s) is/are correct for a geocentric model?

- 102** According to Newton's law of gravitation, the magnitude of the force \mathbf{F} on point masses m_1 and m_2 ,

(say) is given by $|F| = \frac{Gm_1 m_2}{r^2}$.

- I. This equation is not directly applicable for the gravitational force between extended object (like the earth) and a point mass.
 - II. This equation can be directly applied for gravitational force between an extended object and a point mass.
 - III. The equations can be applied to find the gravitational force between an extended object and a point mass provided r is the distance of the point mass from the geometric centre of the extended object.

IV. None of the above

Which of the following statement(s) about this equation is/are correct?

- 103**

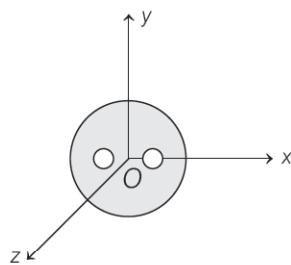
 - I. The centripetal force is directed towards the centre of the earth.
 - II. The centripetal force is provided by the gravitational force.
 - III. The magnitude of the gravitational force acting on the satellite is
$$F_{\text{gravitational}} = \frac{GmM_e}{(R_e + h)^2}, \text{ where } M_e \text{ is the mass of}$$

the earth and it is same as centripetal force.

Which of the given statements is correct for the centripetal force required by the satellite to remain in orbit?

- 106** If the mass of the sun were ten times smaller and the universal gravitational constant were ten times larger in magnitude, which one of the following statement is incorrect? **NEET 2018**
- Time period of a simple pendulum on the earth would decrease.
 - Walking on the ground would become more difficult.
 - Raindrops will fall faster.
 - g on the earth will not change.

- 107** A solid sphere of uniform density and radius 4 units is located with its centre at the origin O of coordinates. Two spheres of equal radii 1 unit with their centres at $A(-2, 0, 0)$ and $B(2, 0, 0)$ respectively are taken out of the solid leaving behind spherical cavities as shown in figure.



- Which one of the following statement with respect to the given situation is incorrect?
- The gravitational force due to this object at the origin is zero.
 - The gravitational force at the point $B(2, 0, 0)$ is zero.
 - The gravitational potential is the same at all points of the circle $y^2 + z^2 = 36$.
 - The gravitational potential is the same at all points on the circle $y^2 + z^2 = 4$.

- 108** Which of the following statement(s) is/are incorrect?
- Acceleration due to gravity decreases with altitude.
 - Acceleration due to gravity increases with increasing depth (assume the earth to be a sphere of uniform density).
 - Acceleration due to gravity increases with latitude.
 - Acceleration due to gravity is independent of the mass of the object.

- 109** The radius and mass of earth are increased by 0.5%. Which one of the following statement is incorrect at the surface of the earth?
- g will increase.
 - g will decrease.

- Escape velocity will remain unchanged.
- Potential energy will remain unchanged.

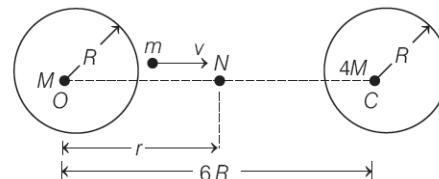
- 110** Which one of the following statement(s) is/are correct?
- A polar satellite goes around the earth's pole in north-south direction.
 - A geo-stationary satellite goes around the earth in east-west direction.
 - A geostationary satellite goes around the earth in north-south direction.
 - A polar satellite goes around the earth in east-west direction.

- 111** Which of the following statement(s) is/are correct about satellites?
- A satellite cannot move in a stable orbit in a plane passing through the earth's centre.
 - Geo-stationary satellites are launched in the equatorial plane.
 - We can use just one geostationary satellite for global communication around the globe.
 - The speed of satellite increases with an increase in the radius of its orbit.

- 112** Which one of the following statement(s) is/are correct?
- The energy required to rocket an orbiting satellite out of earth's gravitational influence is more than the energy required to project a stationary object at the same height (as the satellite) out of earth's influence.
 - If the potential energy is zero at infinity, the total energy of an orbiting satellite is negative of potential energy.
 - The first artificial satellite sputnik I was launched in the year 2001.
 - The time period of rotation of the SYNCOMS (Synchronous communications satellite) is 24 h.

III. Matching Type

- 113** Two uniform solid spheres of equal radii R but mass M and $4M$ have a centre-to-centre separation $6R$ as shown in figure. The two spheres are held fixed. A projectile of mass m is projected from the surface of the sphere of mass M directly towards the centre of the second sphere. N is the point where net gravitational is zero.



With reference to the above situation, match the Column I (quantities) with Column II (mathematical expressions) and select the correct answer from the codes given below.

Column I	Column II
A. Distance r of neutral point N	1. $\sqrt{\frac{3GM}{5R}}$
B. Minimum speed of the projectile, v_{\min} , so that it reaches the surface of the second sphere.	2. $2R$
C. The speed with which the projectile hits the second sphere, if projected with v_{\min} .	3. $\sqrt{\frac{27GM}{5R}}$

A	B	C	A	B	C
(a) 1	2	3	(b) 3	2	1
(c) 2	1	3	(d) 3	1	2

- 114** Two satellites S_1 and S_2 revolve around a planet in coplanar circular orbits in the same sense. Their periods of revolutions are 1 h and 8 h, respectively. The radius of the orbit of S_1 is 10^4 km. With reference to the above situation, match the Column I (quantities) with Column II (approximate values) and select the correct answer from the codes given below.

Column I	Column II
A. Speed of S_2 in kmh^{-1}	1. $\pi/3$
B. Speed of S_1 in kmh^{-1}	2. $2\pi \times 10^4$
C. Velocity of S_2 relative to S_1 when S_2 is closest to S_1 in kmh^{-1}	3. $\pi \times 10^4$
D. Angular speed of S_2 as observed by an astronaut in S_1 when S_2 is closest to S_1 in radh^{-1}	4. $-\pi \times 10^4$

A	B	C	D
(a) 3	2	4	1
(b) 2	3	4	1
(c) 2	3	1	4
(d) 4	2	1	3

- 115** A 400 kg satellite is in a circular orbit of radius $2R_e$ about the earth, where R_e = radius of the earth. With reference to the above situation, match the Column I (quantities) with Column II (calculated values) and select the correct answer from the codes given below.

Column I	Column II
A. Energy required to transfer it to a circular orbit of radius $4R_e$.	1. -6.26×10^9 J
B. Change in KE, ΔK ($\text{KE} \rightarrow \text{Kinetic Energy}$)	2. 3.13×10^9 J
C. Change in potential energy, ΔPE	3. -3.13×10^9 J

A	B	C	A	B	C
(a) 2	3	1	(b) 1	2	3
(c) 3	2	1	(d) 2	1	3

- 116** A satellite of mass m revolving with a velocity v around the earth. With reference to the above situation, match the Column I (types of energy) with Column II (expression) and select the correct answer from the codes given below.

Column I	Column II
A. Kinetic energy of the satellite	1. $-\frac{1}{2}mv^2$
B. Potential energy of the satellite	2. $\frac{1}{2}mv^2$
C. Total energy of the satellite	3. $-mv^2$

A	B	C	A	B	C
(a) 1	2	3	(b) 2	3	1
(c) 3	2	1	(d) 3	1	2

- 117** Match the Column I (quantities) with Column II (approximate values/direction) and select the correct answer from the codes given below.

Column I	Column II
A. Direction of motion of polar satellite	1. 35800 km
B. Direction of motion of geo-stationary satellite	2. West-East
C. Height above the surface of the earth for polar satellite	3. 512 km
D. Height above the surface of the earth for geostationary satellite	4. North-South

A	B	C	D
(a) 2	4	1	3
(b) 4	2	1	3
(c) 4	2	3	1
(d) 2	4	3	1

NCERT & NCERT Exemplar

MULTIPLE CHOICE QUESTIONS

NCERT

- 118** If one of the satellites of jupiter has an orbital period of 1.769 days and the radius of the orbit is 4.22×10^8 m. The mass of Jupiter as compared to that of sun will be

(a) $M_J = \frac{1}{1000} M_S$ (b) $M_J = \frac{1}{10^5} M_S$
(c) $M_J = \frac{1}{10^8} M_S$ (d) $M_J = \frac{1}{10^6} M_S$

- 119** If zero of potential energy is at ∞ , then total energy of satellite is equals to

(a) $-2K$ (b) $-3K$
(c) $-(1/2)K$ (d) $-K$

- 120** A rocket is fired from the earth towards the sun. At what distance from the earth's centre is the gravitational force on the rocket is zero? Mass of the sun = 2×10^{30} kg, mass of the earth = 6.0×10^{24} kg and orbital radius = 1.5×10^{11} m. Neglect the effect of the other planets etc.

(a) 7.30×10^7 m (b) 2.6×10^8 m
(c) 5.30×10^8 m (d) 1.7×10^8 m

- 121** A saturn year is 29.5 times the earth year. How far is the saturn from the sun, if the earth is 1.5×10^8 km away from sun?

(a) 1.4×10^6 km (b) 1.4×10^7 km
(c) 1.4×10^8 km (d) 1.4×10^9 km

- 122** Weight of a body on surface of earth = 63 N. When this object is taken below earth surface at a height equals to half the radius, its weight becomes

(a) 28 N (b) 10 N
(c) 0 (d) 63 N

- 123** A rocket is fired vertically with a speed of 5 kms^{-1} from the earth's surface. How far from the earth does the rocket go before returning to the earth?

Mass of the earth = 6.0×10^{24} kg, mean radius of earth = 6.4×10^6 m and $G = 6.67 \times 10^{-11} \text{ N-m}^2/\text{kg}^2$.
(a) 1600×10^6 km (b) 6.4×10^6 m
(c) 8×10^6 m (d) 9×10^7 m

- 124** The escape velocity of a projectile on earth's surface is 11.2 kms^{-1} . A body is projected out with thrice this speed. What is the speed of the body far away from the earth? Ignore the presence of sun and other planets.

(a) 31.7 kms^{-1} (b) 31.7 kmh^{-1}
(c) $108.7 \text{ kmskms}^{-1}$ (d) 270.6 kms^{-1}

- 125** A satellite orbits the earth at a height of 400 km above the surface. How much energy must be expanded to rocket the satellite out of the earth's gravitational influence? Mass of the satellite = 200 kg, mass of the earth = 6.0×10^{24} kg, radius of the earth = 6.4×10^6 m and $G = 6.67 \times 10^{-11} \text{ N-m}^2/\text{kg}^2$.

(a) 7×10^6 J (b) 8.5×10^9 J
(c) 10×10^9 J (d) 5.9×10^9 J

- 126** A spaceship is stationed on mars. How much energy must be expanded on the spaceship to rocket it out of the solar system? Mass of the spaceship = 1000 kg, mass of sun = 2×10^{30} kg, mass of mars = 6.4×10^{23} kg, radius of mars = 3395 km, radius of orbit of mars = 2.28×10^8 km and $G = 6.67 \times 10^{-11} \text{ N-m}^2/\text{kg}^2$.

(a) 3.1×10^{11} J (b) 6.7×10^{11} J
(c) 3.1×10^{-11} J (d) 6.7×10^{-11} J

- 127** A rocket is fired vertically from the surface of mars with a speed of 2 kms^{-1} . If 20% of its initial energy is lost due to martian atmospheric resistance, how far will the rocket go from the surface of mars before returning to it? Mass of mars = 6.4×10^{23} kg, radius of mars = 3395 km and $G = 6.67 \times 10^{-11} \text{ N-m}^2 \text{ kg}^{-2}$.

(a) 685 km (b) 785 km
(c) 495 km (d) 500 km

NCERT Exemplar

- 128** The earth is an approximate sphere. If the interior contained matter which is not of the same density everywhere, then on the surface of the earth, the acceleration due to gravity

(a) will be directed towards the centre but not the same everywhere
(b) will have the same value everywhere but not directed towards the centre
(c) will be same everywhere in magnitude directed towards the centre
(d) Cannot be zero at any point

- 129** As observed from the earth, the sun appears to move in an approximate circular orbit. For the motion of another planet like mercury as observed from the earth, this would

- (a) be similarly true
 (b) not be true because the force between the earth and mercury is not inverse square law
 (c) not be true because the major gravitational force on mercury is due to the sun
 (d) not be true because mercury is influenced by forces other than gravitational forces

- 130** Different points in the earth are at slightly different distances from the sun and hence experience different forces due to gravitation. For a rigid body, we know that if various forces act at various points in it, the resultant motion is as if a net force acts on the CM (centre of mass) causing translation and a net torque at the CM causing rotation around an axis through the CM. For the earth-sun system (approximating the earth as a uniform density sphere),
 (a) the torque is zero
 (b) the torque causes the earth to spin
 (c) the rigid body result is not applicable since the earth is not even approximately a rigid body
 (d) the torque causes the earth to move around the sun

- 131** Satellites orbiting earth have finite life and sometimes debris of satellites fall to the earth. This is because broken part of satellite
 (a) solar cells and batteries in satellites run out
 (b) laws of gravitation predict a trajectory spiralling inwards
 (c) of viscous forces causing the speed of satellite and hence height to gradually decrease
 (d) of collisions with other satellites

- 132** Both the earth and the moon are subject to the gravitational force of the sun. As observed from the sun, the orbit of the moon
 (a) will be elliptical
 (b) will not be strictly elliptical because the total gravitational force on it is not central
 (c) is not elliptical but will necessarily be a closed curve
 (d) deviates considerably from being elliptical due to influence of planets other than the earth

- 133** In our solar system, the inter-planetary region has chunks of matter (much smaller in size compared to planets) called asteroids. They
 (a) will not move around the sun, since they have very small masses compared to the sun

- (b) will move in an irregular way because of their small masses and will drift away into outer space
 (c) will move around the sun in closed orbits but not obey Kepler's laws
 (d) will move in orbits like planets and obey Kepler's laws

- 134** Choose the incorrect option.

- (a) Inertial mass is a measure of difficulty of accelerating a body by an external force whereas the gravitational mass is relevant in determining the gravitational force on it by an external mass.
 (b) The gravitational mass and inertial mass are equal is an experimental result.
 (c) The acceleration due to gravity on the earth is same for all bodies is due to the equality of gravitational mass and inertial mass.
 (d) Gravitational mass of a particle like proton can depend on the presence of neighbouring heavy objects but the inertial mass cannot.

- 135** Particles of masses $2M$, m and M are respectively at points A , B and C with $AB = 1/2(BC)$. m is much-much smaller than M and at time $t = 0$, they are all at rest as given in figure. At subsequent times before any collision takes place,



- (a) m will remain at rest
 (b) m will move towards M
 (c) m will move towards $2M$
 (d) m will have oscillatory motion

- 136** The centre of mass of an extended body on the surface of the earth and its centre of gravity

- (a) are always at the same point for any size of the body
 (b) are always at the same point only for spherical bodies
 (c) can never be at the same point
 (d) is close to each other for objects, say of sizes less than 100 m

- 137** The eccentricity of earth's orbit is 0.0167. The ratio of its maximum speed in its orbit to its minimum speed.

- | | |
|-----------|-----------|
| (a) 2.507 | (b) 1.033 |
| (c) 8.324 | (d) 1.000 |

Answers

> Mastering NCERT with MCQs

1 (a)	2 (b)	3 (c)	4 (a)	5 (b)	6 (a)	7 (a)	8 (c)	9 (c)	10 (d)
11 (c)	12 (b)	13 (a)	14 (c)	15 (b)	16 (c)	17 (c)	18 (a)	19 (d)	20 (a)
21 (b)	22 (d)	23 (c)	24 (d)	25 (c)	26 (d)	27 (c)	28 (c)	29 (c)	30 (b)
31 (a)	32 (d)	33 (a)	34 (c)	35 (b)	36 (c)	37 (a)	38 (c)	39 (a)	40 (d)
41 (d)	42 (a)	43 (b)	44 (b)	45 (c)	46 (d)	47 (d)	48 (a)	49 (b)	50 (c)
51 (c)	52 (b)	53 (b)	54 (c)	55 (c)	56 (c)	57 (c)	58 (b)	59 (d)	60 (d)
61 (b)	62 (b)	63 (a)	64 (b)	65 (c)	66 (d)	67 (d)	68 (a)	69 (d)	70 (a)
71 (a)	72 (a)	73 (c)	74 (d)	75 (a)	76 (a)	77 (a)	78 (d)	79 (d)	80 (c)
81 (b)	82 (b)	83 (a)	84 (d)	85 (c)	86 (b)				

> Special Types Questions

87 (a)	88 (a)	89 (a)	90 (a)	91 (a)	92 (a)	93 (d)	94 (a)	95 (a)	96 (a)
97 (c)	98 (a)	99 (a)	100 (d)	101 (a)	102 (a)	103 (d)	104 (d)	105 (d)	106 (d)
107 (b)	108 (b)	109 (a)	110 (a)	111 (b)	112 (d)	113 (c)	114 (a)	115 (a)	116 (b)
117 (c)									

> NCERT & NCERT Exemplar MCQs

118 (a)	119 (d)	120 (b)	121 (d)	122 (a)	123 (c)	124 (a)	125 (d)	126 (a)	127 (c)
128 (d)	129 (c)	130 (a)	131 (c)	132 (b)	133 (d)	134 (d)	135 (c)	136 (d)	137 (b)

Hints & Explanations

1 (a) Earth exerts an attractive force on all the material objects as it has a tendency to attract all of them towards it.

So, going uphill is a lot more tiring than going downhill as more efforts are required to work against the attractive force of earth.

3 (c) According to Kepler's second law of motion, the line that joins any planet to the sun sweeps equal area in equal intervals of time.

Given, areas SAB and SCD are equal,

so

$$t_1 = t_2$$

4 (a) The law of areas (Kepler's second law) says that areal velocity is constant, i.e. the line joining the planet sweeps out equal areas in equal interval of time.

The area swept out by the planet of mass m in time interval Δt is given as

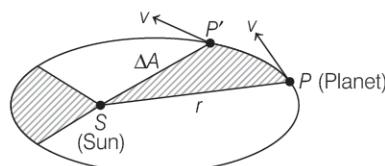
$$\Delta A = \frac{1}{2} r (v dt)$$

$$\text{then areal velocity } = \frac{dA}{dt} = \frac{d}{dt} \left[\frac{1}{2} r (v dt) \right] = \frac{1}{2} rv \\ = \frac{mv}{2m} = \frac{L}{2m} = \text{constant}$$

i.e. angular momentum is conserved. So, we can say that the law of areas can be understood as a consequence of conservation of angular momentum.

7 (a) The area swept out by the given planet in time interval Δt is

$$\Delta A = \frac{1}{2} |\mathbf{r} \times \mathbf{P}\mathbf{P}'|$$



$$\Rightarrow \Delta A = \frac{1}{2} (\mathbf{r} \times \mathbf{v} \Delta t) \quad (\because \text{arc } PP' = v \Delta t)$$

8 (c) From Kepler's second law, the areal velocity can be given as

$$\frac{dA}{dt} = \frac{L}{2m} \\ = \frac{I\omega}{2m} = \frac{mr^2\omega}{2m}$$

$$\Rightarrow \frac{dA}{dt} \propto \omega r^2 \quad \left[\because L = I\omega \text{ and } I = mr^2 \right]$$

9 (c) For a particle, angular momentum is $\mathbf{L} = \mathbf{r} \times \mathbf{p}$

So, magnitude of angular momentum = $|\mathbf{L}|$

$$\Rightarrow |\mathbf{L}| = |\mathbf{r}| |\mathbf{p}| \sin \theta$$

where, θ = angle between vector \mathbf{r} and \mathbf{p} or \mathbf{r} and \mathbf{v} .

$$\Rightarrow |L| = L = r mv \sin \theta \quad (\because p = mv) \\ \text{or} \quad L = mv r \sin \theta$$

When $\theta = 90^\circ$, then

$L = mvr$ = magnitude of angular momentum.

Here, $\theta = 90^\circ$ means r and v are mutually perpendicular.

So, for the planet at position P , $L_P = m_P r_P v_P$.

- 10 (d)** By Kepler's second law of planetary motion, for a moving planet around the sun, angular momentum remains conserved.

$$\begin{aligned} \text{i.e.} \quad m_1 v_1 r_1 &= m_2 v_2 r_2 \\ \text{but} \quad m_1 &= m_2 \\ \therefore \quad v_1 r_1 &= v_2 r_2 \\ \Rightarrow \quad \frac{v_1}{v_2} &= \frac{r_2}{r_1} \end{aligned}$$

For points R and P , $\frac{v_R}{v_P} = \frac{r_P}{r_R}$

Since, $r_R > r_P$
 $\therefore v_R < v_P$

Hence, linear speed of planet is minimum at R .

- 11 (c)** From conservation of angular momentum,

$L = mvr = \text{constant}$ (for planet at all positions) ...(i)

Given, maximum velocity of planet,

$$v_{\max} = 3 \times 10^4 \text{ ms}^{-1}$$

Minimum velocity of planet, $v_{\min} = 1 \times 10^3 \text{ ms}^{-1}$

Maximum distance of the planet from the sun,

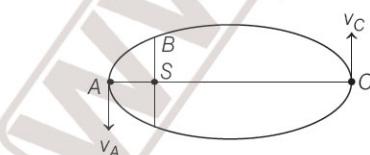
$$r_{\max} = 4 \times 10^4 \text{ km}$$

and minimum distance of the planet from the sun,
 $r_{\min} = ?$

From Eq. (i), $m v_{\max} r_{\min} = m v_{\min} r_{\max}$
(at maximum distance, velocity is minimum and vice-versa)

$$\begin{aligned} \Rightarrow v_{\max} \cdot r_{\min} &= v_{\min} \cdot r_{\max} \\ \text{or} \quad r_{\min} &= \frac{v_{\min} \cdot r_{\max}}{v_{\max}} = \frac{1 \times 10^3 \times 4 \times 10^4}{3 \times 10^4} \\ &= \frac{4}{3} \times 10^3 \text{ km} \end{aligned}$$

- 12 (b)** According to the question,



The figure above shows an ellipse traced by a planet around the sun S . The closest point A is known as **perihelion** and the farthest point C is known as **aphelion**.

Since, as per the result of the Kepler's second law of area, the planet will move slowly (v_{\min}) only when it is farthest from the sun and more rapidly (v_{\max}) when it is nearest to the sun.

If velocities at point A , B and C are v_A , v_B and v_C respectively, thus $v_A = v_{\max}$, $v_C = v_{\min}$

Therefore, we can write

$$v_A > v_B > v_C \quad \dots(\text{i})$$

Kinetic energy (K) of the planet at any point is given as

$$K = \frac{1}{2} mv^2$$

Thus, the kinetic energies, at A , $K_A = \frac{1}{2} mv_A^2$

$$\text{At } B, \quad K_B = \frac{1}{2} mv_B^2$$

$$\text{and at } C, \quad K_C = \frac{1}{2} mv_C^2$$

From Eq. (i), we can write

$$K_A > K_B > K_C$$

- 13 (a)** Given, time period of revolution, $T = 84 \text{ yr}$,

Semi-major axis, $a = 287 \times 10^{10} \text{ m}$

$$\text{Then, } \frac{T^2}{a^3} = \frac{(84)^2}{(287 \times 10^{10})^3} = 2.98 \times 10^{-34} \text{ yr}^2 \text{m}^{-3}$$

$$\approx 3.00 \times 10^{-34} \text{ yr}^2 \text{m}^{-3}$$

Thus, the quotient (T^2/a^3) in units of $10^{-34} \text{ yr}^2 \text{m}^{-3}$ is approximately 3 units.

- 14 (c)** From Kepler's third law of planetary motion,

$$T^2 \propto r^3 \Rightarrow \frac{T^2}{r^3} = \text{constant}$$

then for any two planets, the above equation becomes

$$\Rightarrow \frac{T_1^2}{r_1^3} = \frac{T_2^2}{r_2^3} = \text{constant} \text{ or } \left(\frac{T_1}{T_2}\right)^2 = \left(\frac{r_1}{r_2}\right)^3$$

- 15 (b)** If time period for mass and earth are T_m and T_e , respectively. Then, according to Kepler's third law,

$$\frac{T_m^2}{T_e^2} = \frac{R_{ms}^3}{R_{es}^3}$$

where, R_{ms} is the mars-sun distance and R_{es} is the earth-sun distance.

$$\therefore T_m = \left(\frac{R_{ms}}{R_{es}}\right)^{3/2} \cdot T_e \quad \left[\because \frac{R_{ms}}{R_{es}} = 1.52\right]$$

$$\therefore T_m = (1.52)^{3/2} \times 365 \text{ days}$$

- 16 (c)** By Kepler's third law of planetary motion,

$$\frac{T_1}{T_2} = \left(\frac{a_1}{a_2}\right)^{3/2}$$

$$\text{Given, } T_1 = T, a_1 = R$$

$$T_2 = ? \quad a_2 = 9R$$

$$\therefore \frac{T}{T_2} = \left(\frac{R}{9R}\right)^{3/2}; \frac{T}{T_2} = \left[\left(\frac{1}{3}\right)^2\right]^{3/2}$$

$$\frac{T}{T_2} = \frac{1}{27} \Rightarrow T_2 = 27T$$

\therefore Time period of a planet of radius $9R$ is $27T$.

17 (c) From Kepler's third law, $T^2 \propto r^3$

where, T = time period of satellite and r = radius of elliptical orbit.

Here, $r_1 = R$ (radius of earth) + $5R = 6R$

and $r_2 = R + 2R = 3R$

Hence, $T_1^2 \propto r_1^3$ and $T_2^2 \propto r_2^3$

$$\text{So, } \frac{T_2^2}{T_1^2} = \frac{r_2^3}{r_1^3} = \frac{(3R)^3}{(6R)^3}$$

$$\text{or } \frac{T_2^2}{T_1^2} = \frac{1}{8} \Rightarrow T_2^2 = \frac{1}{8} T_1^2$$

$$T_2 = \frac{24}{2\sqrt{2}} = 6\sqrt{2}\text{h} \quad [\because T_1 = 24\text{ h}]$$

18 (a) Given, the quotient $Q = \frac{T^2}{a^3}$

For A, $a = 10.8$ units = 10.8×10^{10} m, $T = 0.615$ yr

$$\Rightarrow Q = \frac{T^2}{a^3} = \frac{(0.615)^2}{(10.8)^3 \times 10^{30}} \text{ m}^2$$

$$\Rightarrow A = 3.00 \times 10^{-34} \text{ m}^2 \text{ s}^{-3}$$

So, the missing term A is approximately equal to 3 units.

19 (d) If the time period and radius of satellites A and B are T_A , T_B and r_A , r_B , respectively. Then, from Kepler's law,

$$\frac{T_A^2}{T_B^2} = \frac{r_A^3}{r_B^3} \Rightarrow \frac{1^2}{8^2} = \frac{(10^4)^3}{r_B^3}$$

$$[\because T_A = 1\text{ h}, T_2 = 8\text{ h}, r_A = 10^4 \text{ km}, r_B = ?]$$

$$\Rightarrow r_B^3 = 64 \times (10^4)^3$$

$$\therefore r_B = 4 \times 10^4 \text{ km}$$

$$\text{Speed of satellite A, } v_A = \frac{2\pi r_A}{T_A} = \frac{2\pi \times 10^4}{1} = 2\pi \times 10^4 \text{ km/h}$$

$$\text{Speed of satellite B, } v_B = \frac{2\pi r_B}{T_B} = \frac{2\pi \times 4 \times 10^4}{8} = \pi \times 10^4 \text{ km/h}$$

The speed of B relative to A when they are close is

$$\begin{aligned} v_{BA} &= v_A - v_B \\ &= 2\pi \times 10^4 - \pi \times 10^4 \\ &= \pi \times 10^4 \text{ km/h} \end{aligned}$$

20 (a) Due to huge amounts of opposite charges on the sun and the earth, electrostatic force of attraction will be produced. Gravitational force which is already exist between sun and earth is also attractive in nature. So, both forces will be added and obey inverse square law and are central forces. As both the forces are of same nature, hence all the three Kepler's laws will be valid.

22 (d) In a circular motion, the centripetal acceleration,

$$a = \omega^2 R \quad \dots(\text{i})$$

$$\text{where, angular velocity, } \omega = \frac{v}{R} \quad \dots(\text{ii})$$

From Eqs (i) and (ii), we get

$$a = \frac{v^2}{R_m} \quad \dots(\text{iii})$$

where, v = magnitude of velocity and R_m = radius of the moon.

Also, speed of moon, $v = \frac{2\pi R_m}{T}$, where T = time period of revolution.

On putting the above value of v in Eq. (iii), we get

$$a = \left(\frac{2\pi R_m}{T} \right)^2 \times \frac{1}{R_m} = \frac{4\pi^2 R_m}{T^2}$$

24 (d) We know that, gravitational force between the earth and the sun is

$$F_G = \frac{GMm}{r^2}$$

where, M is mass of the sun and m is mass of the earth. When G decreases with time, the gravitational force F_G will become weaker with time. As, F_G is changing with time, so the earth will not be revolving around the sun in a closed orbit strictly. Also, the radius may increase, since the attraction force is becoming weaker. Hence, after long time period, the earth will leave the solar system.

25 (c) As we know that, period of revolution,

$$T = \frac{2\pi r}{v} = 2\pi \frac{r}{\sqrt{GM/r}} = 2\pi \frac{r^{3/2}}{\sqrt{GM}}$$

$$[\because \text{Orbital speed, } v = \sqrt{\frac{GM}{r}}]$$

$$T^2 = \frac{4\pi^2}{GM} r^3 = kr^3$$

$$\therefore k = \frac{4\pi^2}{GM}$$

$$\Rightarrow GMk = 4\pi^2$$

26 (d) According to the question, force = $\frac{GMm}{r^{3/2}} = m\omega^2 r$

$$\Rightarrow \frac{GMm}{r^{3/2}} = \frac{4\pi^2 mr}{T^2} \quad \left[\because T = \frac{2\pi}{\omega} \right]$$

$$\Rightarrow T^2 = \left(\frac{4\pi^2}{GM} \right) \cdot r^{5/2}$$

$$\Rightarrow T^2 \propto r^{5/2}$$

27 (c) If the law of gravitation suddenly changes and becomes an inverse cube law, then the law of areas still holds. This is because the force will still be central force, due to which angular momentum will remain constant.

Also, for a planet of mass m revolving around the sun of mass M , we can write according to the question,

$$F = \frac{GMm}{a^3} = \frac{mv^2}{a}$$

$$[\because \text{Centripetal force, } m\omega^2 a = \frac{mv^2}{a}]$$

where, a is radius of orbiting planet.

$$\text{Orbital speed, } v = \frac{\sqrt{GM}}{a} \Rightarrow v \propto \frac{1}{a}$$

Time period of revolution of a planet,

$$T = \frac{2\pi r}{v}$$

On putting the value of orbital speed, we get

$$T = \frac{2\pi a}{\sqrt{GM}} = \frac{2\pi a^2}{\sqrt{GM}}$$

$$\Rightarrow T^2 \propto a^3$$

Hence, the orbit of the planet around sun will not be elliptical because for elliptical orbit $T^2 \propto a^3$.

$$\text{As force } F = \left(\frac{GM}{a^3}\right)m = g' m, \text{ where } g' = \frac{GM}{a^3}.$$

Since, g' = acceleration due to gravity is constant, hence path followed by a projectile will be approximately parabolic (as $T \propto a^2$).

- 28 (c)** The force of gravity on a body at a height h above the surface of the earth is given by

$$F = \frac{GMm}{(R+h)^2}$$

where, m = mass of the body and M = mass of earth.

As we know that, weight of an object is the force (force of gravity) with which the earth attracts it.

Then, according to the question,

$$\frac{GMm}{(R+h)^2} = \frac{1}{16} \frac{GMm}{R^2}$$

where, $\frac{GM}{R^2}$ = gravitational acceleration.

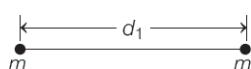
$$\frac{1}{(R+h)^2} = \frac{1}{16R^2}$$

$$\text{or } \frac{R}{R+h} = \frac{1}{4} \quad \text{or} \quad \frac{R+h}{R} = 4$$

$$h = 3R$$

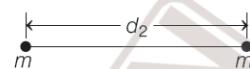
- 29 (c)** The gravitational forces are mutually equal and opposite, hence the ratio of gravitational pull of the earth on the planet and that of planet on the earth will be $1 : 1$.

- 30 (b)** From Newton's law of gravitation, the two equal point masses are separated by a distance d_1 is shown below



$$\text{Gravitational force, } F_1 = \frac{Gm \cdot m}{d_1^2} = \frac{Gm^2}{d_1^2} \quad \dots(i)$$

Similarly,



$$\text{Gravitational force, } F_2 = \frac{Gm^2}{d_2^2} \quad \dots(ii)$$

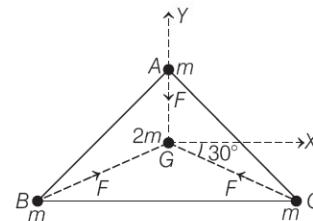
On dividing Eq. (i) by Eq. (ii), we get

$$\begin{aligned} \frac{F_1}{F_2} &= \left(\frac{d_2}{d_1}\right)^2 \\ \therefore F_2 &= F_1 \left(\frac{d_1}{d_2}\right)^2 \end{aligned}$$

- 31 (a)** Gravitational force does not depend on the medium between the masses. So, the liquid filled between the masses of specific gravity 3 will not have any effect on gravitational force.

So, it will remain same, i.e. F .

- 32 (d)** The angle between GC and positive X -axis is 30° and so is the angle between GB and negative X -axis.



From figure, resolving the forces on $2m$ due to masses at B and C along X -axis.

$$\Rightarrow |\mathbf{F}_{x(\text{net})}| = |F \cos 30^\circ \hat{i} - F \cos 30^\circ \hat{i}| = 0$$

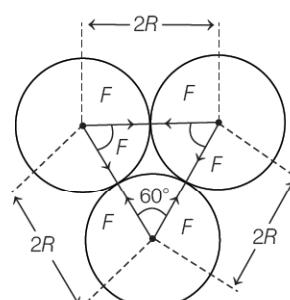
Similarly along Y -axis,

$$\begin{aligned} |\mathbf{F}_{y(\text{net})}| &= |F \hat{j} - (F \sin 30^\circ + F \sin 30^\circ) \hat{j}| \\ &= \left| F \hat{j} - \left(\frac{F}{2} + \frac{F}{2} \right) \hat{j} \right| = |F \hat{j} - F \hat{j}| = 0 \end{aligned}$$

F_{net} = Resultant force on mass at G due to masses at A, B and C

$$\text{and } C = \sqrt{F_{x(\text{net})}^2 + F_{y(\text{net})}^2} = 0$$

- 33 (a)** The figure of three uniform spheres of mass M and radius R is shown below



Force between any two spheres of mass M and radius R is

$$F = G \cdot \frac{M \cdot M}{(2R)^2} = \frac{GM^2}{4R^2} \quad \dots(i)$$

Net force on any sphere due to other two is given by

$$F_{\text{net}} = \sqrt{F^2 + F^2 + 2F^2 \cos 60^\circ} = \sqrt{3} F$$

Putting the value of F from Eq. (i), we get

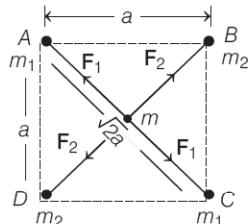
$$F_{\text{net}} = \frac{\sqrt{3}}{4} \frac{GM^2}{R^2}$$

- 34 (c)** A point mass m is located at the centre of the square $ABCD$. Let gravitational force m and m_1 is

$$|\mathbf{F}_1| = \left[\frac{Gmm_1}{(a/\sqrt{2})^2} \right]$$

and between m and m_2 is $|\mathbf{F}_2| = \left[\frac{Gmm_2}{(a/\sqrt{2})^2} \right]$ as shown

below



\therefore Along BD there are two forces of same magnitude F_2 acting between m and m_2 (on B or D)

Net gravitational force along BD ,

$$\mathbf{F}_{\text{net}(BD)} = \mathbf{F}_2 - \mathbf{F}_2 = 0$$

Similarly, net gravitational force along AC ,

$$\mathbf{F}_{\text{net}(AC)} = \mathbf{F}_1 - \mathbf{F}_1 = 0$$

\therefore Net force on m is $|\mathbf{F}_{\text{net}}| = \text{zero}$

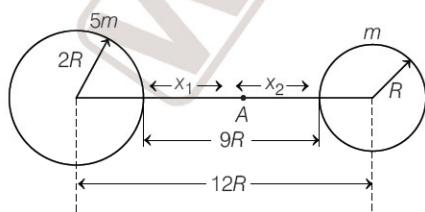
- 35 (b)** Gravitational force between masses m and $(M-m)$ is

$$F = \frac{Gm(M-m)}{r^2}$$

For maximum force, $\frac{dF}{dm} = 0$

$$\Rightarrow \frac{d}{dm} \left(\frac{GmM}{r^2} - \frac{Gm^2}{r^2} \right) = 0 \Rightarrow M - 2m = 0 \Rightarrow \frac{m}{M} = \frac{1}{2}$$

- 36 (c)** The figure of two spherical bodies of masses m and $5m$ and radii R and $2R$ is shown below



Let both spheres collide at point A and after time t , x_1 be the distance travelled by bigger sphere while x_2 be the distance travelled by smaller sphere.

$$\therefore x_1 + x_2 = [12R - (R + 2R)] \quad \dots(i)$$

Gravitational force between both spheres is same and given by

$$F = G \cdot \frac{5m \cdot m}{(12R)^2} \Rightarrow F = \frac{5Gm^2}{(12R)^2} \quad \dots(ii)$$

$$\text{Acceleration of bigger sphere, } a_1 = \frac{F}{5m} = \frac{Gm}{(12R)^2}$$

[using Eq. (ii)]

\therefore Distance travelled by bigger sphere,

$$x_1 = \frac{1}{2} a_1 t^2 = \frac{1}{2} \cdot \frac{Gm}{(12R)^2} \cdot t^2 \quad \dots(iii)$$

Similarly, acceleration of smaller sphere,

$$a_2 = \frac{F}{m} = \frac{5Gm}{(12R)^2}$$

\therefore Distance travelled by smaller sphere,

$$x_2 = \frac{1}{2} a_2 t^2 = \frac{1}{2} \cdot \frac{5Gm}{(12R)^2} \cdot t^2 \quad \dots(iv)$$

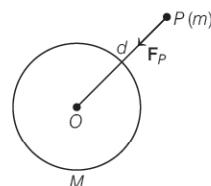
Putting these values of x_1 and x_2 in Eq. (i), we have

$$\begin{aligned} \frac{1}{2} \cdot \frac{Gm}{(12R)^2} \cdot t^2 + \frac{1}{2} \cdot \frac{5Gm}{(12R)^2} \cdot t^2 &= 9R \\ \Rightarrow t^2 &= \frac{3R(12R)^2}{Gm} \end{aligned}$$

From Eq. (iv), we have

$$\begin{aligned} x_2 &= \frac{1}{2} \cdot \frac{5Gm}{(12R)^2} \times \frac{3R(12R)^2}{Gm} \\ &= \frac{15R}{2} = 7.5R \end{aligned}$$

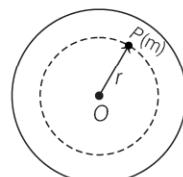
- 37 (a)** For a point outside the spherical shell as shown below



According to Newton's gravitational law, gravitational force on point mass m at P is

$$|\mathbf{F}_P| = \frac{GmM}{d^2}$$

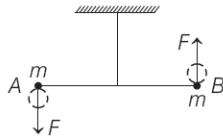
- 38 (c)** Net resultant force at point P inside a hollow spherical shell will be zero because the attractive forces due to various parts of the shell cancel out each other.



i.e.

$$|\mathbf{F}_P(\text{net})| = \text{zero}$$

- 39 (a)** The force of attraction on small spheres due to big sphere are equal and opposite in direction as shown below.



There is no net force on the bar but only a torque which is nearly equal to F times the length of the bar, where F is the force of attraction between a big sphere and the neighbouring small sphere.

- 40 (d)** The force acting on the particle of mass m at surface of the earth

$$F = mg \quad \dots(i)$$

where, g = acceleration due to gravity at the earth's surface.

$$\text{Also, } g = \frac{GM_E}{R_E^2} \quad \dots(ii)$$

Then, from Eqs. (i) and (ii), we get

$$\Rightarrow F = mg = \frac{GmM_E}{R_E^2}$$

Hence, options (a) and (b) are correct.

- 41 (d)** Gravitational acceleration (g) at the centre of earth is zero, hence weight of body ($w = mg$) at the centre of earth becomes zero.

$$\text{42 (a)} \text{ Given, } \mathbf{F}_1 = -\mathbf{F}_2 = \frac{-\mathbf{r}_{12}}{r_{12}^3} GM_0^2 \left(\frac{m_1 m_2}{M_0^2} \right)^n$$

where, $\mathbf{r}_{12} = \mathbf{r}_1 - \mathbf{r}_2$

Acceleration due to gravity,

$$g = \frac{|F|}{\text{mass}} = \frac{GM_0^2(m_1 m_2)^n}{r_{12}^2 (M_0)^{2n}} \times \frac{1}{(\text{mass})}$$

Since, g depends upon position vector \mathbf{r}_{12} , hence it will be different for different objects.

As g is not constant, hence constant of proportionality will not be constant in Kepler's third law.

Hence, Kepler's third law will not be valid.

As the force is of central nature, hence, first two Kepler's laws will be valid.

- 43 (b)** As we know that, acceleration due to gravity is

$$g = \frac{GM}{R^2}$$

Given, $M' = 2M$ [∴ mass gets doubled]

$\Rightarrow R' = (R/2)$ [∴ radius gets halved]

Then, acceleration becomes

$$\Rightarrow g' = \frac{GM'}{R'^2} = \frac{G(2M)}{(R/2)^2} = \frac{8GM}{R^2}$$

$$\therefore g' = 8g$$

Thus, the new acceleration due to gravity g' is 8 times that of g .

- 44 (b)** It is given that, the density of planet is twice the density of earth and both of them have same acceleration due to gravity.

If g and g' are the acceleration due to gravity of earth and planet, then $g = g'$

$$\therefore \frac{G}{R^2} \left[\frac{4}{3} \pi R^3 \rho \right] = \frac{G}{R'^2} \times \frac{4}{3} \pi R'^3 (2\rho)$$

$$\Rightarrow R' = \frac{R}{2}$$

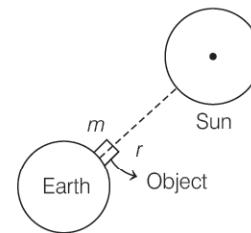
- 45 (c)** Initially, the weight of the passenger at the earth's surface $w = mg = 60 \times 10 = 600 \text{ N}$.

Finally, the weight of the passenger at the surface of the mass $= 60 \times 4 = 240 \text{ N}$ and during the flight in between somewhere its weight will be zero because at that point, gravitational pull of earth and mars will be equal.

Only the curve (c) represents the weight = 0.

So, (c) is correct option.

- 46 (d)** Consider the given diagram,



Force on the object due to the earth,

$$F = \frac{G'M_E m}{R^2} = \frac{10GM_E m}{R^2} \quad (\because G' = 10G)$$

$$= 10 \left(\frac{GM_E m}{R^2} \right) = (10g)m = 10mg \quad \dots(i)$$

$$\left(\because g = \frac{GM_E}{R^2} \right)$$

Now, force on the object due to the sun, $F' = \frac{GM'_s m}{r^2}$

$$= \frac{G(M_s)m}{10r^2} \quad \left(\because M'_s = \frac{M_s}{10} \right)$$

As $r \gg R$ (radius of the earth)

$\Rightarrow F'$ will be very small, so the effect of the sun will be neglected.

Now, as $g' = 10g$

Hence, weight of person $= mg' = 10mg$ [from Eq. (i)] i.e. gravity pull on the person will increase.

Due to it, walking on ground would become more difficult.

Escape velocity v_e is proportional to g ,

i.e. $v_e \propto g$

As, $g' > g \Rightarrow v_e' > v_e$

Hence, rain drops will fall much faster.

To overcome the increased gravitational force of the earth, the airplanes will have to travel much faster.

47 (d) Since, centripetal acceleration of moon,

$$a_m \propto \frac{1}{R_m^2} \quad \dots(i)$$

Acceleration due to gravity,

$$g \propto \frac{1}{R_E^2} \quad \dots(ii)$$

On dividing Eq. (ii) by Eq. (i), we get

$$\frac{g}{a_m} = \frac{R_m^2}{R_E^2} = \left(\frac{R_m}{R_E}\right)^2 = \frac{R_E^{-2}}{R_m^{-2}}$$

Hence, options (b) and (c) are correct.

48 (a) Given, time period of the moon, $T = 27.3$ days

Radius of orbit, $R_m = 3.84 \times 10^8$ m

Centripetal acceleration is given as

$$a = \frac{4\pi^2 R_m}{T^2} \quad \left[\because a = \omega^2 R = \left(\frac{2\pi^2}{T}\right) R_m \right]$$

$$= \frac{4 \times \left(\frac{22}{7}\right)^2 \times (3.84 \times 10^8)}{(27.3 \times 24 \times 60 \times 60)^2}$$

$$\approx 0.0027 \text{ ms}^{-2}$$

From the above value we can say that, centripetal acceleration of the moon due to earth's gravity is much smaller than the value of acceleration due to gravity g on the surface of the earth.

49 (b) Given that at some height h , acceleration due to gravity,

$$g_h = 4.9 \text{ m/s}^2 \approx \frac{g}{2} \quad \dots(i)$$

\therefore The ratio of acceleration due to gravity at earth's surface and at some altitude h is

$$\left(1 + \frac{h}{R_e}\right) = \sqrt{\frac{g}{g_h}} = \sqrt{2} \quad [\text{from Eq. (i)}]$$

$$\therefore \frac{h}{R_e} = \sqrt{2} - 1$$

$$\text{or } h = 0.414 \times R_e$$

$$h = 0.414 \times 6400 \text{ km}$$

(\because given, radius of earth, $R_e = 6400 \text{ km} = 6.4 \times 10^6 \text{ m}$)

$$\text{or } h = 2649.6 \text{ km} = 2.6 \times 10^6 \text{ m}$$

Thus, at $2.6 \times 10^6 \text{ m}$ above the earth's surface, the acceleration due to gravity decreases to 4.9 m/s^2 .

50 (c) Weight of the body at the surface of the earth, $w = mg$.

Weight of the body at any point w.r.t. earth's surface
= Mass of the body \times Acceleration due to gravity at the

point

$$\Rightarrow w' = mg' \quad \dots(i)$$

The gravitational acceleration experienced by that point for height h above the surface of the earth is

$$g' = g \left(\frac{R_E}{R_E + h}\right)^2$$

$$\therefore \text{According to the question, } h = \left(\frac{R_E}{2}\right)$$

$$\therefore g' = g \left[\frac{\frac{R_E}{3R_E}}{\frac{2}{2}}\right]^2 = \frac{4}{9}g \quad \dots(ii)$$

On substituting Eq. (ii) in Eq. (i), we get

$$w' = \frac{4mg}{9} = \frac{4}{9}w$$

51 (c) The acceleration due to gravity for height h above the surface of the earth is

$$g' = g \left(\frac{R}{R + h}\right)^2$$

$$\Rightarrow \frac{g}{2} = g \left(\frac{R}{R + h}\right)^2 \Rightarrow \frac{1}{2} = \left(\frac{R}{R + h}\right)^2$$

$$\text{or } (R + h)^2 = 2R^2$$

$$\text{or } R + h = \sqrt{2}R$$

$$\Rightarrow h = (\sqrt{2} - 1)R$$

$$**52 (b)** Given, $g' = \left(\frac{1}{100}\right)g$ or $g'/g = \frac{1}{100}$$$

For height h above the surface of the earth,

$$g' = g \left(\frac{R}{R + h}\right)^2 \Rightarrow \frac{g'}{g} = \left(\frac{R}{R + h}\right)^2$$

$$\Rightarrow \left(\frac{1}{100}\right) = \left(\frac{R}{R + h}\right)^2 \Rightarrow \frac{R}{R + h} = \frac{1}{10}$$

$$\therefore h = 10R - R = 9R$$

$$**53 (b)** Given, height, $h = \frac{R_E}{2}$$$

Acceleration due to gravity at altitude h is given by

$$g' = \frac{g}{\left(1 + \frac{h}{R_E}\right)^2} = \frac{g}{\left(1 + \frac{R_E/2}{R_E}\right)^2}$$

$$= \frac{g}{\left(1 + \frac{1}{2}\right)^2} = \frac{g}{\left(\frac{3}{2}\right)^2} = \frac{4g}{9} \quad \dots(i)$$

Weight of the body at the earth's surface,

$$w = mg = 45 \text{ N} \quad \dots(ii)$$

Then, weight of the body at altitude h is

$$w' = mg' = \frac{4}{9}mg \quad \dots(iii)$$

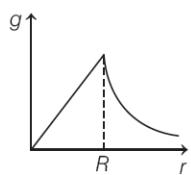
From Eqs. (ii) and (iii), we get

$$w' = \frac{4}{9} \times 45 = 20 \text{ N}$$

- 54 (c)** Assuming earth of radius R , then at a point inside such that $r < R$, acceleration due to gravity, $g \propto r$ and at a point outside such that $r > R$,
- Acceleration due to gravity,

$$g \propto \frac{1}{r^2}$$

So, the variation of g with r can be represented by

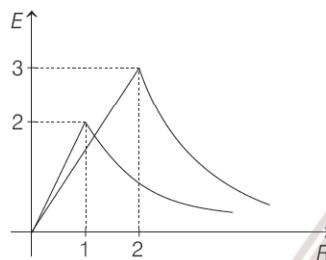


Hence, option (c) is correct.

- 55 (c)** Gravitational field of a solid sphere is maximum at its surface ($r = R$) and its value at surface,

$$E = \frac{GM}{R^2}$$

From graph given in the question,



We can observe that

$$E_1 = \frac{GM_1}{(1)^2} = 2 \text{ and } E_2 = \frac{GM_2}{(2)^2} = 3$$

or

$$GM_1 = 2 \quad \dots(i)$$

and

$$GM_2 = 12 \quad \dots(ii)$$

On dividing Eq. (i) by Eq. (ii), we get

$$\frac{M_1}{M_2} = \frac{2}{12} = \frac{1}{6}$$

- 56 (c)** Given, radius of earth, $R_E = 6400$ km

Distance from centre of earth, $r = 2000$ km

Distance from the surface of earth,

$$\begin{aligned} d &= R_E - r = 6400 - 2000 \\ &= 4400 \text{ km} = 4.4 \times 10^6 \text{ m} \end{aligned}$$

If g' be the acceleration due to gravity below the surface of earth at a depth d , then

$$\begin{aligned} g' &= g \left(1 - \frac{d}{R_E}\right) \\ &= 9.8 \left(1 - \frac{4.4 \times 10^6}{6.4 \times 10^6}\right) \quad [\because g = 9.8 \text{ m/s}^2] \\ &= 9.8 \times 0.3125 = 3.06 \text{ m/s}^2 \end{aligned}$$

- 57 (c)** Given, weight of the body, $w = mg = 200$ N, where m is the mass of the body and g ($\approx 10 \text{ m/s}^2$) is acceleration due to gravity at the surface of the earth. Since, mass m remains constant irrespective of the position of the body on the earth. However, g is not constant and its value at a depth d below the earth's surface is given as

$$g' = g \left(1 - \frac{d}{R}\right) \quad \dots(i)$$

where, R is the radius of the earth.

Multiplying m on both sides of Eq. (i), we get

$$mg' = mg \left(1 - \frac{d}{R}\right)$$

Thus, the weight of the body at half way down

(i.e. $d = \frac{R}{2}$) to the centre of the earth is

$$\begin{aligned} mg' &= 200 \times \left(1 - \frac{R/2}{R}\right) \\ &= 200 \left(1 - \frac{1}{2}\right) = 200 \times \frac{1}{2} \\ &= 100 \text{ N} \end{aligned}$$

\therefore The body will weigh 100 N half-way down to the centre of the earth.

- 58 (b)** Acceleration due to gravity at poles,

$$g_p = 10 \text{ ms}^{-2}$$

Weight of box at poles,

$$w_p = 196 \text{ N}$$

So, mass of box,

$$m = \frac{w_p}{g_p} = 19.6 \text{ kg}$$

Now, due to rotation of earth acceleration due to gravity at equator,

$$g_e = g_p - R\omega^2 \quad \dots(i)$$

Here, $g_p = 10 \text{ ms}^{-2}$, $R = 6400 \times 10^3 \text{ m}$,

$$\omega = \frac{2\pi}{T} \text{ and } T = 24 \times 3600 \text{ s}$$

Now, substituting the given values in Eq. (i), we get

$$\begin{aligned} g_e &= g_p - 0.034 \\ &= 10 - 0.034 \\ &= 9.966 \text{ ms}^{-2} \end{aligned}$$

So, weight of box at equator,

$$\begin{aligned} w_e &= g_e \times m = 9.966 \times 19.6 \\ &= 195.32 \text{ N} \end{aligned}$$

- 59 (d)** Acceleration due to gravity at height h above earth's surface,

$$g_h = g \left(\frac{R}{R+h}\right)^2$$

$$= g \left(\frac{1}{1 + \frac{h}{R}} \right)^2 = g \left(1 + \frac{h}{R} \right)^{-2}$$

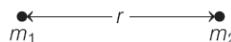
$$= g \left(1 - \frac{2h}{R} \right)$$

Acceleration at depth d below earth's surface,

$$g_d = g \left(1 - \frac{d}{R} \right)$$

Given, when $h = 1$ km, so $g_d = g_h$
or $g \left(1 - \frac{d}{R} \right) = g \left(1 - \frac{2h}{R} \right)$
 $\Rightarrow d = 2h$ or $d = 2$ km

- 60 (d) Two point masses m_1 and m_2 are separated by a distance r is shown as



Gravitational potential energy (U) of the above system is given as

$$U = -\frac{Gm_1m_2}{r}$$

i.e. gravitational potential energy $\propto m_1m_2$ and gravitational potential energy $\propto \frac{1}{r}$ or gravitational potential energy is directly proportional to the product of the masses of particles and inversely proportional to the separation between them.

- 61 (b) For a system of particles, all possible pairs are taken and total gravitational potential energy is the algebraic sum of the potential energies due to each pair, applying the principle of superposition.

Total gravitational potential energy

$$\begin{aligned} &= -\frac{Gm_1m_2}{r_1} - \frac{Gm_2m_3}{r_2} - \frac{Gm_1m_3}{r_3} \\ &= \left(-\frac{Gm_1m_2}{r_1} \right) + \left(-\frac{Gm_2m_3}{r_2} \right) + \left(-\frac{Gm_1m_3}{r_3} \right) \end{aligned}$$

- 62 (b) Two point masses m_1 and m_2 are separated by a distance r is shown as



As we know $G_1 = \frac{-Gm_1m_2}{r}$... (i)

When separation between the particle is doubled, i.e. $2r$, then

$$G_2 = -\frac{Gm_1m_2}{2r} \quad \dots \text{(ii)}$$

On dividing Eq. (i) by Eq. (ii), we get

$$\frac{G_1}{G_2} = 2$$

- 63 (a) Given, potential energy of system,
 $U = -7.79 \times 10^{28}$ J

Mass of the earth, $M_E = 6 \times 10^{24}$ kg

Mass of the moon, $M_m = 7.4 \times 10^{22}$ kg

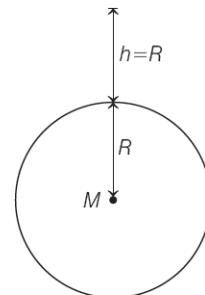
The potential energy of the earth-moon system is

$$U = -\frac{GM_E \cdot M_m}{r}$$

where, r is the mean distance between earth and moon

$$\begin{aligned} \therefore r &= -\frac{GM_E M_m}{U} \\ &= -\frac{6.67 \times 10^{-11} \times 6 \times 10^{24} \times 7.4 \times 10^{22}}{-7.79 \times 10^{28}} \\ &= 3.8 \times 10^8 \text{ m} \end{aligned}$$

- 64 (b) Amount of work done in moving the given body from one point to another against the gravitational force is equal to the change in potential energy of the body. Consider a point mass m at a height $h (= R)$ above the surface of the earth as shown in the figure



As we know, the potential energy of body of mass m on the surface of earth is $V_1 = -\frac{GMm}{R}$... (i)
where, G = gravitational constant,

M = mass of earth and R = radius of earth.

When the mass is raised to a height h from the surface of the earth, then the potential energy of the body becomes

$$V_2 = -\frac{GMm}{(R+h)}$$

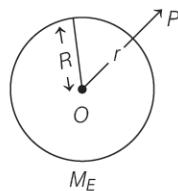
Here, $h = R$ (given)

$$\Rightarrow V_2 = -\frac{GMm}{2R} \quad \dots \text{(ii)}$$

Thus, the change in potential energy, $\Delta V = V_2 - V_1$
Substituting the values from Eqs. (i) and (ii), we get

$$\begin{aligned} \Delta V &= -\frac{GMm}{2R} + \frac{GMm}{R} \\ &= \frac{GMm}{2R} = \frac{gR^2 m}{2R} \quad \left[\because g = \frac{GM}{R^2} \right] \\ &= \frac{mgR}{2} \end{aligned}$$

- 65 (c)** Consider a point P at a distance $r (> R)$ from the centre of the earth (mass M_E and radius R) is as shown below



The gravitational potential at point P is

$$V_g(r) = -\frac{GM_E}{r}$$

- 66 (d)** Gravitational potential at some height h from the surface of the earth is given by

$$V_g = -\frac{GM}{R+h} \quad \dots(i)$$

and acceleration due to gravity at some height h from the earth surface can be given as

$$g' = \frac{GM}{(R+h)^2} \quad \dots(ii)$$

On dividing Eq. (i) by Eq. (ii), we get

$$\Rightarrow \frac{|V_g|}{g'} = R + h \quad \dots(iii)$$

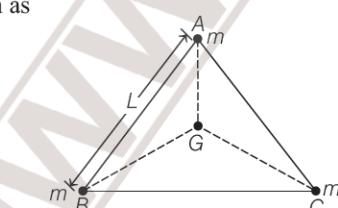
Given, $V_g = -5.4 \times 10^7 \text{ J kg}^{-2}$, $g' = 6.0 \text{ ms}^{-2}$

and radius of earth, $R = 6400 \text{ km}$.

Substituting these values in Eq. (iii), we get

$$\begin{aligned} \frac{5.4 \times 10^7}{6.0} &= R + h \\ \Rightarrow 9 \times 10^6 &= R + h \\ h &= 9 \times 10^6 - R \\ h &= 9 \times 10^6 - 6.4 \times 10^6 \\ \Rightarrow h &= 2.6 \times 10^6 \text{ m} \\ \Rightarrow h &= 2600 \text{ km} \end{aligned}$$

- 67 (d)** Consider three particles A , B and C of equal mass m kept at vertices of an equilateral triangle of side L is shown as



If gravitational potential at point A , B and C are V_A , V_B and V_C respectively, then total gravitational potential at centre, i.e. centroid of triangle G ,

$$\begin{aligned} V_T &= V_A + V_B + V_C \\ \Rightarrow V_T &= -\frac{Gm}{AG} - \frac{Gm}{BG} - \frac{Gm}{CG} \quad \left[\because V = -\frac{GM}{r} \right] \end{aligned}$$

$$\text{Also, } AG = BG = CG = \frac{2}{3} \times \left(\frac{\sqrt{3}L}{2} \right) = \frac{\sqrt{3}L}{3}$$

$$\begin{aligned} \Rightarrow V_T &= -Gm \left(\frac{1}{AG} + \frac{1}{BG} + \frac{1}{CG} \right) \\ &= -Gm \left(\frac{3 \times 3}{\sqrt{3}L} \right) = -\frac{9Gm}{\sqrt{3}L} \text{ or } -\frac{3\sqrt{3}Gm}{L} \end{aligned}$$

Hence, options (b) and (c) are correct.

- 68 (a)** As we know that, escape velocity, $v_e = \sqrt{\frac{2GM}{R}}$... (i)

where, M is mass of planet.

So, on the basis of Eq. (i), it can be said that escape velocity will depend upon the mass of the planet (M).

- 69 (d)** Escape velocity on the surface of earth is given by

$$v = \sqrt{2gR_e}$$

i.e. $v \propto \sqrt{R_e}$

Hence, escape velocity does not depend on the mass and direction of projection of body, it depends on the radius of earth.

Hence, options (a) and (c) are correct.

- 71 (a)** Given, escape velocity on the surface of earth,

$$v_e = \sqrt{\frac{2GM_E}{R_E}}$$

where, M_E = mass of the earth and R_E = radius of the earth.

Now according to the question, radius of earth,

$$R' = R_E / 4$$

$$\Rightarrow v'_e = \sqrt{\frac{2GM_E}{R'}} = \sqrt{4 \left(\frac{2GM_E}{R_E} \right)} = 2 \sqrt{\frac{2GM_E}{R_E}}$$

$$\text{or } v'_e = 2v_e$$

Hence, the escape velocity will be doubled.

- 72 (a)** A black hole is a super dense planetary material formed due to the continued compression.

If R be the approximate radius of super compressed earth such that it behave as a black hole.

In this case,

$v_e = c$, where c is velocity of light ($3 \times 10^8 \text{ m/s}$).

$$v_e = \sqrt{\frac{2GM}{R}} = c$$

$$\begin{aligned} \Rightarrow R &= \frac{2GM}{c^2} = \frac{2 \times 6.67 \times 10^{-11} \times 5.98 \times 10^{24}}{9 \times 10^{16}} \\ &= 8.8 \times 10^{-3} \approx 10^{-2} \text{ m} \end{aligned}$$

- 73 (c)** The kinetic energy of an asteroid of mass m and speed v_i is

$$K_i = \frac{1}{2}mv_i^2 \quad \dots(i)$$