Краевая задача для 2-х слоев:

$$
\begin{gathered}
-D_1\varphi_1''(\tau) + n_1^2(1-\omega_1)(\varphi_1(\tau) - \theta_1^4(\tau)) = 0, \quad \tau \in (\tau_0, \tau_1), \\
-D_2\varphi_2''(\tau) + n_2^2(1-\omega_2)(\varphi_2(\tau) - \theta_2^4(\tau)) = 0, \quad \tau \in (\tau_1, \tau_2), \\
-D_1\varphi_1'(\tau_0) = n_1^2\gamma_1(\theta_{b1}^4 - \varphi_1(\tau_0)), \quad D_2\varphi_2'(\tau_2) = n_2^2\gamma_2(\theta_{b2}^4 - \varphi_2(\tau_2)), \\
D_1\varphi_1'(\tau_1) = D_2\varphi_2'(\tau_1) = G(\varphi_2(\tau_1) - \varphi_1(\tau_1)), \\
G = \frac{n_1^2\int_0^1 \mu(1-R_{12}(\mu))d\mu}{3\int_0^1 \mu^2(R_{12}(\mu)+R_{21}(\mu))d\mu}, \quad D_j = n_j^2\alpha_j, \quad \alpha_j = \frac{1}{3 - A_j\omega_j}, \\
-N_{c1}\theta_1''(\tau) + (1-\omega_1)(\theta_1^4(\tau) - \varphi_1(\tau)) = 0, \quad \tau \in (\tau_0, \tau_1), \\
-N_{c2}\theta_2''(\tau) + (1-\omega_2)(\theta_2^4(\tau) - \varphi_2(\tau)) = 0, \quad \tau \in (\tau_1, \tau_2), \\
\theta_1(\tau_0) = \theta_{b1}, \quad \theta_2(\tau_2) = \theta_{b2}, \\
\theta_1(\tau_1) = \theta_2(\tau_1), \quad N_{c1}n_1^2\theta_1'(\tau_1) = N_{c2}n_2^2\theta_2'(\tau_1). \\
N_{cj} = \frac{k_j\kappa_j}{4\sigma T_{max}^3 n_j^2}, \quad \gamma_j = \frac{\varepsilon_j}{2(2-\varepsilon_j)}
\end{gathered}
$$

Обозначим $L_1 = \tau_1 - \tau_0$, $L_2 = \tau_2 - \tau_1$. Сделаем замену

$$
\tau \in (\tau_0, \tau_1):\ \tau = \tau_0 + L_1 x, \quad x \in (0,1) \quad \Rightarrow \quad \frac{d}{d\tau} = \frac{1}{L_1}\frac{d}{dx},
$$

$$
\tau \in (\tau_1, \tau_2):\ \tau = \tau_1 + L_2 x, \quad x \in (1,2) \quad \Rightarrow \quad \frac{d}{d\tau} = \frac{1}{L_2}\frac{d}{dx}.
$$

Введем обозначения

$$
B_j = \frac{D_j}{L_j}, \quad C_j = \frac{N_{cj}n_j^2}{L_j}, \quad K_j = L_j n_j^2(1-\omega_j), \quad \widetilde{\gamma}_j = n_j^2\gamma_j.
$$

Получим задачу:

$$
\begin{gathered}
-B_1\varphi_1'' + K_1(\varphi_1 - \theta_1^4) = 0, \quad x \in (0,1), \\
-B_2\varphi_2'' + K_2(\varphi_2 - \theta_2^4) = 0, \quad x \in (1,2), \\
-B_1\varphi_1'(0) = \widetilde{\gamma}_1(\theta_{b1}^4 - \varphi_1(0)), \quad B_2\varphi_2'(2) = \widetilde{\gamma}_2(\theta_{b2}^4 - \varphi_2(2)), \\
B_1\varphi_1'(1) = B_2\varphi_2'(1) = G(\varphi_2(1) - \varphi_1(1)), \\
-C_1\theta_1'' + K_1(\theta_1^4 - \varphi_1) = 0, \quad x \in (0,1), \\
-C_2\theta_2'' + K_2(\theta_2^4 - \varphi_2) = 0, \quad x \in (1,2), \\
\theta_1(0) = \theta_{b1}, \quad \theta_2(2) = \theta_{b2}, \\
\theta_1(1) = \theta_2(1), \quad C_1\theta_1'(1) = C_2\theta_2'(1).
\end{gathered}
$$

Введем на отрезке $[0,2]$ равномерную сетку с $N+1$ узлами $x_0, x_1, \ldots, x_m, \ldots, x_N$ с шагом $h = 1/N$, причем $N = 2m$.

Исходные данные для расчетов:

- параметры слоев: K_j, B_j, C_j;
- параметры границ между слоем и внешней средой: $\widetilde{\gamma}_i$, θ_{bi};
- параметр границы раздела: G.

Аппроксимация уравнений:

$i = 1, \ldots, m-1$:

$$-B_1 \frac{\varphi_{1,i-1} - 2\varphi_{1,i} + \varphi_{1,i+1}}{h^2} + K_1(\varphi_{1,i} - 4(\widetilde{\theta}_{1,i})^3\theta_{1,i} + 3(\widetilde{\theta}_{1,i})^4) = 0,$$
$$-C_1 \frac{\theta_{1,i-1} - 2\theta_{1,i} + \theta_{1,i+1}}{h^2} + K_1(4(\widetilde{\theta}_{1,i})^3\theta_{1,i} - 3(\widetilde{\theta}_{1,i})^4 - \varphi_{1,i}) = 0.$$

$i = m+1, \ldots, N-1$:

$$-B_2 \frac{\varphi_{2,i-1} - 2\varphi_{2,i} + \varphi_{2,i+1}}{h^2} + K_2(\varphi_{2,i} - 4(\widetilde{\theta}_{2,i})^3\theta_{2,i} + 3(\widetilde{\theta}_{2,i})^4) = 0,$$
$$-C_2 \frac{\theta_{2,i-1} - 2\theta_{2,i} + \theta_{2,i+1}}{h^2} + K_2(4(\widetilde{\theta}_{2,i})^3\theta_{2,i} - 3(\widetilde{\theta}_{2,i})^4 - \varphi_{2,i}) = 0.$$

$i = 0$:

$$-B_1 \frac{\varphi_{1,1} - \varphi_{1,0}}{h} - \widetilde{\gamma}_1(\theta_{b1}^4 - \varphi_{1,0}) + \frac{h}{2}K_1(\varphi_{1,0} - 4(\widetilde{\theta}_{1,0})^3\theta_{1,0} + 3(\widetilde{\theta}_{1,0})^4) = 0,$$
$$\theta_{1,0} = \theta_{b1}.$$

$i = N$:

$$B_2 \frac{\varphi_{2,N} - \varphi_{2,N-1}}{h} - \widetilde{\gamma}_2(\theta_{b2}^4 - \varphi_{2,N}) + \frac{h}{2}K_2(\varphi_{2,N} - 4(\widetilde{\theta}_{2,N})^3\theta_{2,N} + 3(\widetilde{\theta}_{2,N})^4) = 0,$$
$$\theta_{2,N} = \theta_{b2}.$$

$i = m$ $(G < \infty)$:

$$\theta_{1,m} = \theta_{2,m},$$
$$C_1 \frac{\theta_{1,m} - \theta_{1,m-1}}{h} - C_2 \frac{\theta_{2,m+1} - \theta_{2,m}}{h} +$$
$$+ \frac{h}{2}K_1(4(\widetilde{\theta}_{1,m})^3\theta_{1,m} - 3(\widetilde{\theta}_{1,m})^4 - \varphi_{1,m}) + \frac{h}{2}K_2(4(\widetilde{\theta}_{2,m})^3\theta_{2,m} - 3(\widetilde{\theta}_{2,m})^4 - \varphi_{2,m}) = 0,$$
$$G(\varphi_{1,m} - \varphi_{2,m}) + B_1 \frac{\varphi_{1,m} - \varphi_{1,m-1}}{h} + \frac{h}{2}K_1(\varphi_{1,m} - 4(\widetilde{\theta}_{1,m})^3\theta_{1,m} + 3(\widetilde{\theta}_{1,m})^4) = 0,$$
$$G(\varphi_{2,m} - \varphi_{1,m}) - B_2 \frac{\varphi_{2,m+1} - \varphi_{2,m}}{h} + \frac{h}{2}K_2(\varphi_{2,m} - 4(\widetilde{\theta}_{2,m})^3\theta_{2,m} + 3(\widetilde{\theta}_{2,m})^4) = 0.$$

$i = m$ $(G = \infty)$:

$$\theta_{1,m} = \theta_{2,m},$$

$$C_1 \frac{\theta_{1,m} - \theta_{1,m-1}}{h} - C_2 \frac{\theta_{2,m+1} - \theta_{2,m}}{h} +$$

$$+ \frac{h}{2} K_1 (4(\widetilde{\theta}_{1,m})^3 \theta_{1,m} - 3(\widetilde{\theta}_{1,m})^4 - \varphi_{1,m}) + \frac{h}{2} K_2 (4(\widetilde{\theta}_{2,m})^3 \theta_{2,m} - 3(\widetilde{\theta}_{2,m})^4 - \varphi_{2,m}) = 0,$$

$$\varphi_{1,m} = \varphi_{2,m},$$

$$B_1 \frac{\varphi_{1,m} - \varphi_{1,m-1}}{h} - B_2 \frac{\varphi_{2,m+1} - \varphi_{2,m}}{h} +$$

$$+ \frac{h}{2} K_1 (\varphi_{1,m} - 4(\widetilde{\theta}_{1,m})^3 \theta_{1,m} + 3(\widetilde{\theta}_{1,m})^4) + \frac{h}{2} K_2 (\varphi_{2,m} - 4(\widetilde{\theta}_{2,m})^3 \theta_{2,m} + 3(\widetilde{\theta}_{2,m})^4) = 0.$$