

Система уравнений:

$$\begin{aligned}
& -\frac{\partial p_1}{\partial t} - a \left(\frac{\partial^2 p_1}{\partial x^2} + \frac{\partial^2 p_1}{\partial y^2} + \frac{\partial^2 p_1}{\partial z^2} \right) + 4\kappa_a \theta^3 (bp_1 - p_2) = \theta - \theta_d, \\
& -\alpha \left(\frac{\partial^2 p_2}{\partial x^2} + \frac{\partial^2 p_2}{\partial y^2} + \frac{\partial^2 p_2}{\partial z^2} \right) + \kappa_a (p_2 - bp_1) = 0, \\
& \Omega = \{(x, y) : 0 < x < L, 0 < y < L, 0 < z < L\}, \\
& -a \frac{\partial p_1}{\partial x} + \beta p_1 \Big|_{x=0} = 0, \quad a \frac{\partial \theta}{\partial x} + \beta p_1 \Big|_{x=L} = 0, \\
& -a \frac{\partial p_1}{\partial y} + \beta p_1 \Big|_{y=0} = 0, \quad a \frac{\partial p_1}{\partial y} + \beta p_1 \Big|_{y=L} = 0, \\
& -a \frac{\partial p_1}{\partial z} + \beta p_1 \Big|_{z=0} = 0, \quad a \frac{\partial p_1}{\partial z} + \beta p_1 \Big|_{z=L} = 0, \\
& -\alpha \frac{\partial p_2}{\partial x} + up_2 \Big|_{x=0} = 0, \quad \alpha \frac{\partial p_2}{\partial x} + up_2 \Big|_{x=L} = 0, \\
& -\alpha \frac{\partial p_2}{\partial y} + up_2 \Big|_{y=0} = 0, \quad \alpha \frac{\partial p_2}{\partial y} + up_2 \Big|_{y=L} = 0, \\
& -\alpha \frac{\partial p_2}{\partial z} + up_2 \Big|_{z=0} = 0, \quad \alpha \frac{\partial p_2}{\partial z} + up_2 \Big|_{z=L} = 0, \\
& p_1|_{t=T} = 0.
\end{aligned}$$

Введем сетку

$$\bar{\omega} = \{(x_i, y_j, z_k, t_m) : x_i = ih, y_j = jh, z_k = kh, t_m = m\tau, i = \overline{0, N}, j = \overline{0, N}, k = \overline{0, N}, m = \overline{0, M}\},$$

$$h = L/N, \quad \tau = T/M.$$

$$0 < i < N, 0 < j < N, 0 < k < N:$$

$$\begin{aligned}
& \frac{p_{1;i,j,k}^m - p_{1;i,j,k}^{m+1}}{\tau} + \frac{1}{2} \left[-\frac{a}{h^2} (p_{1;i-1,j,k}^{m+1} - 2p_{1;i,j,k}^{m+1} + p_{1;i+1,j,k}^m) - \frac{a}{h^2} (p_{1;i,j-1,k}^m - 2p_{1;i,j,k}^m + p_{1;i,j+1,k}^m) - \right. \\
& \quad \left. -\frac{a}{h^2} (p_{1;i,j,k-1}^m - 2p_{1;i,j,k}^m + p_{1;i,j,k+1}^m) + 4\kappa_a (\theta_{i,j,k}^m)^3 (bp_{1;i,j,k}^m - p_{2;i,j,k}^m) \right] + \\
& \quad + \frac{1}{2} \left[-\frac{a}{h^2} (p_{1;i-1,j,k}^{m+1} - 2p_{1;i,j,k}^{m+1} + p_{1;i+1,j,k}^{m+1}) - \frac{a}{h^2} (p_{1;i,j-1,k}^{m+1} - 2p_{1;i,j,k}^{m+1} + p_{1;i,j+1,k}^{m+1}) - \right. \\
& \quad \left. -\frac{a}{h^2} (p_{1;i,j,k-1}^{m+1} - 2p_{1;i,j,k}^{m+1} + p_{1;i,j,k+1}^{m+1}) + 4\kappa_a (\theta_{i,j,k}^{m+1})^3 (bp_{1;i,j,k}^{m+1} - p_{2;i,j,k}^{m+1}) \right] = \frac{1}{2} (\theta_{i,j,k}^m - \theta_d) + \frac{1}{2} (\theta_{i,j,k}^{m+1} - \theta_d), \\
& -\frac{\alpha}{h^2} (p_{2;i-1,j,k}^m - 2p_{2;i,j,k}^m + p_{2;i+1,j,k}^m) - \frac{\alpha}{h^2} (p_{2;i,j-1,k}^m - 2p_{2;i,j,k}^m + p_{2;i,j+1,k}^m) - \\
& \quad -\frac{\alpha}{h^2} (p_{2;i,j,k-1}^m - 2p_{2;i,j,k}^m + p_{2;i,j,k+1}^m) + \kappa_a (p_{2;i,j,k}^m - bp_{1;i,j,k}^m) = 0.
\end{aligned}$$

$i = 0, 0 < j < N, 0 < k < N$:

$$\begin{aligned}
& \frac{1}{2} \left[-\frac{a}{h}(p_{1;1,j,k}^m - p_{1;0,j,k}^m) + \beta p_{1;0,j,k}^m - \frac{a}{2h}(p_{1;0,j-1,k}^m - 2p_{1;0,j,k}^m + p_{1;0,j+1,k}^m) - \right. \\
& \quad \left. -\frac{a}{2h}(p_{1;0,j,k-1}^m - 2p_{1;0,j,k}^m + p_{1;0,j,k+1}^m) + \frac{4\kappa_a h}{2}(\theta_{0,j,k}^m)^3(bp_{1;0,j,k}^m - p_{2;0,j,k}^m) \right] + \\
& \quad + \frac{1}{2} \left[-\frac{a}{h}(p_{1;1,j,k}^{m+1} - p_{1;0,j,k}^{m+1}) + \beta p_{1;0,j,k}^{m+1} - \frac{a}{2h}(p_{1;0,j-1,k}^{m+1} - 2p_{1;0,j,k}^{m+1} + p_{1;0,j+1,k}^{m+1}) - \right. \\
& \quad \left. -\frac{a}{2h}(p_{1;0,j,k-1}^{m+1} - 2p_{1;0,j,k}^{m+1} + p_{1;0,j,k+1}^{m+1}) + \frac{4\kappa_a h}{2}(\theta_{0,j,k}^{m+1})^3(bp_{1;0,j,k}^{m+1} - p_{2;0,j,k}^{m+1}) \right] + \frac{h}{2\tau}(p_{1;0,j,k}^m - p_{1;0,j,k}^{m+1}) = \\
& \quad = \frac{1}{2} \left[\frac{h}{2}(\theta_{0,j,k}^m - \theta_d) \right] + \frac{1}{2} \left[\frac{h}{2}(\theta_{0,j,k}^{m+1} - \theta_d) \right], \\
& \quad -\frac{\alpha}{h}(p_{2;1,j,k}^m - p_{2;0,j,k}^m) + u_{0,j,k}p_{2;0,j,k}^m - \frac{\alpha}{2h}(p_{2;0,j-1,k}^m - 2p_{2;0,j,k}^m + p_{2;0,j+1,k}^m) - \\
& \quad -\frac{\alpha}{2h}(p_{2;0,j,k-1}^m - 2p_{2;0,j,k}^m + p_{2;0,j,k+1}^m) + \frac{\kappa_a h}{2}(p_{2;0,j,k}^m - bp_{1;0,j,k}^m) = 0.
\end{aligned}$$

$i = 0, j = 0, 0 < k < N$:

$$\begin{aligned}
& \frac{1}{2} \left[-\frac{a}{h}(p_{1;1,0,k}^m - p_{1;0,0,k}^m) + \beta p_{1;0,0,k}^m - \frac{a}{h}(p_{1;0,1,k}^m - p_{1;0,0,k}^m) + \beta p_{1;0,0,k}^m - \right. \\
& \quad \left. -\frac{a}{2h}(p_{1;0,0,k-1}^m - 2p_{1;0,0,k}^m + p_{1;0,0,k+1}^m) + \frac{4\kappa_a h}{2}(\theta_{0,0,k}^m)^3(bp_{1;0,0,k}^m - p_{2;0,0,k}^m) \right] + \\
& \quad + \frac{1}{2} \left[-\frac{a}{h}(p_{1;1,0,k}^{m+1} - p_{1;0,0,k}^{m+1}) + \beta p_{1;0,0,k}^{m+1} - \frac{a}{h}(p_{1;0,1,k}^{m+1} - p_{1;0,0,k}^{m+1}) + \beta p_{1;0,0,k}^{m+1} - \right. \\
& \quad \left. -\frac{a}{2h}(p_{1;0,0,k-1}^{m+1} - 2p_{1;0,0,k}^{m+1} + p_{1;0,0,k+1}^{m+1}) + \frac{4\kappa_a h}{2}(\theta_{0,0,k}^{m+1})^3(bp_{1;0,0,k}^{m+1} - p_{2;0,0,k}^{m+1}) \right] + \frac{h}{2\tau}(p_{1;0,0,k}^m - p_{1;0,0,k}^{m+1}) = \\
& \quad = \frac{1}{2} \left[\frac{h}{2}(\theta_{0,0,k}^m - \theta_d) \right] + \frac{1}{2} \left[\frac{h}{2}(\theta_{0,0,k}^{m+1} - \theta_d) \right], \\
& \quad -\frac{\alpha}{h}(p_{2;1,0,k}^m - p_{2;0,0,k}^m) + u_{0,0,k}p_{2;0,0,k}^m - \frac{\alpha}{h}(p_{2;0,1,k}^m - p_{2;0,0,k}^m) + u_{0,0,k}p_{2;0,0,k}^m - \\
& \quad -\frac{\alpha}{2h}(p_{2;0,0,k-1}^m - 2p_{2;0,0,k}^m + p_{2;0,0,k+1}^m) + \frac{\kappa_a h}{2}(p_{2;0,0,k}^m - bp_{1;0,0,k}^m) = 0.
\end{aligned}$$

$i = 0, j = 0, k = 0$:

$$\begin{aligned}
& \frac{1}{2} \left[-\frac{a}{h}(p_{1;1,0,0}^m - p_{1;0,0,0}^m) + \beta p_{1;0,0,0}^m - \frac{a}{h}(p_{1;0,1,0}^m - p_{1;0,0,0}^m) + \beta p_{1;0,0,0}^m - \right. \\
& \left. -\frac{a}{h}(p_{1;0,0,1}^m - p_{1;0,0,0}^m) + \beta p_{1;0,0,0}^m + \frac{4\kappa_a h}{2}(\theta_{0,0,0}^m)^3(bp_{1;0,0,0}^m - p_{2;0,0,0}^m) \right] + \\
& + \frac{1}{2} \left[-\frac{a}{h}(p_{1;1,0,0}^{m+1} - p_{1;0,0,0}^{m+1}) + \beta p_{1;0,0,0}^{m+1} - \frac{a}{h}(p_{1;0,1,0}^{m+1} - p_{1;0,0,0}^{m+1}) + \beta p_{1;0,0,0}^{m+1} - \right. \\
& \left. -\frac{a}{h}(p_{1;0,0,1}^{m+1} - p_{1;0,0,0}^{m+1}) + \beta p_{1;0,0,0}^{m+1} + \frac{4\kappa_a h}{2}(\theta_{0,0,0}^{m+1})^3(bp_{1;0,0,0}^{m+1} - p_{2;0,0,0}^{m+1}) \right] + \frac{h}{2\tau}(p_{1;0,0,0}^m - p_{1;0,0,0}^{m+1}) = \\
& = \frac{1}{2} \left[\frac{h}{2}(\theta_{0,0,0}^m - \theta_d) \right] + \frac{1}{2} \left[\frac{h}{2}(\theta_{0,0,0}^{m+1} - \theta_d) \right], \\
& -\frac{\alpha}{h}(p_{2;1,0,0}^m - p_{2;0,0,0}^m) + u_{0,0,0}p_{2;0,0,0}^m - \frac{\alpha}{h}(p_{2;0,1,0}^m - p_{2;0,0,0}^m) + u_{0,0,0}p_{2;0,0,0}^m - \\
& -\frac{\alpha}{h}(p_{2;0,0,1}^m - p_{2;0,0,0}^m) + u_{0,0,0}p_{2;0,0,0}^m + \frac{\kappa_a h}{2}(p_{2;0,0,0}^m - bp_{1;0,0,0}^m) = 0.
\end{aligned}$$

Для остальных случаев схема записывается симметрично.