Combo Proofs

1. Prove the following identity.

$$\binom{n}{k} = \binom{n-2}{k} + 2\binom{n-2}{k-1} + \binom{n-2}{k-2}$$

2. Prove the following identity.

$$\binom{n}{k} = \sum_{j=0}^{m} \binom{m}{j}\binom{n-m}{k-j}$$

3. Prove the following identity for $n \geq 1$.

$$\sum_{k=0}^{n} k\binom{n}{k} = n2^{n-1}$$

4. Prove the following identity for $n \geq 0$.

$$1 + \binom{n}{1}2 + \binom{n}{2}4 + \ldots + \binom{n}{n-1}2^{n-1} + \binom{n}{n}2^{n} = 3^{n}$$

5. Prove the following identity for $n \geq 0$.

$$\sum_{k=1}^{n} f_{2k-1} = f_{2n} - 1$$

6. Prove the following identity for $n \geq 0$.

$$\sum_{k=0}^{n} \binom{n}{k} F_k = F_{2n}$$

7. Prove the following identity for $n \geq 2$.

$$\sum_{k=0}^{n} k^2\binom{n}{k} = n2^{n-1} + n(n-1)2^{n-2}$$

8. Prove the following identity.

$$\sum_{k=0}^{n} \binom{n}{k}\binom{k}{m} = \binom{n}{m}2^{n-m}$$