

Combo Proofs

Prove the following identity.

$$\binom{n}{k} = \binom{n-2}{k} + 2\binom{n-2}{k-1} + \binom{n-2}{k-2}$$

■

Prove the following identity.

$$\binom{n}{k} = \sum_{j=0}^m \binom{m}{j} \binom{n-m}{k-j}$$

■

Prove the following identity for $n \geq 1$.

$$\sum_{k=0}^n k \binom{n}{k} = n2^{n-1}$$

■

Prove the following identity for $n \geq 0$.

$$1 + \binom{n}{1}2 + \binom{n}{2}4 + \dots + \binom{n}{n-1}2^{n-1} + \binom{n}{n}2^n = 3^n$$

■

Prove the following identity for $n \geq 0$.

$$\sum_{k=1}^n f_{2k-1} = f_{2n} - 1$$

■

Prove the following identity for $n \geq 0$.

$$\sum_{k=0}^n \binom{n}{k} F_k = F_{2n}$$

■

Prove the following identity for $n \geq 2$.

$$\sum_{k=0}^n k^2 \binom{n}{k} = n2^{n-1} + n(n-1)2^{n-2}$$

■

Prove the following identity.

$$\sum_{j=r}^n \binom{n}{j} \binom{j}{r} = 2^{n-r} \binom{n}{r}$$

■