Abstract Algebra Notes

Definition. A **map** $f : A \to B$ is a subset $f \subset A \times B$ such that for all $a \in A$, there exists a $b \in B$ such that b is unique with $(a, b) \in f$.

Definition. We write $f(a) = b$ if $(a, b) \in f$. A is the **domain** of f and B is the **codomain**.

Definition. A **binary operation** on A is a map $\star : A \times A \to A$ such that $\star(a_1, a_2) = a_1 \star a_2$ for $a_1, a_2 \in A$.

Definition. A binary operation $\star$ is **associative** on A if for all $a, b, c \in A$, $a \star (b \star c) = (a \star b) \star c$.

Definition. An element $e \in A$ is an **identity** element of $\star$ if for each $a \in A$, $e \star a = a \star e = a$.

Definition. An element $a \in A$ has an **inverse** under $\star$ if there exists a $b \in A$ such that $a \star b = b \star a = e$.

Definition. A set A with an associative binary operation $\star$ is a **group** if A has an identity element under $\star$ and every $a \in A$ has an inverse.

Definition

A group is a pair $(G, \star)$ where G is a set and $\star$ is a binary operation on G such that

1. For all $a, b, c \in A$, $a \star (b \star c) = (a \star b) \star c$.
2. There exists an $e \in G$ such that $a \star e = e \star a = a$ for all $a \in G$.
3. For all $a \in G$, there exists a $b \in G$ such that $a \star b = b \star a = e$.