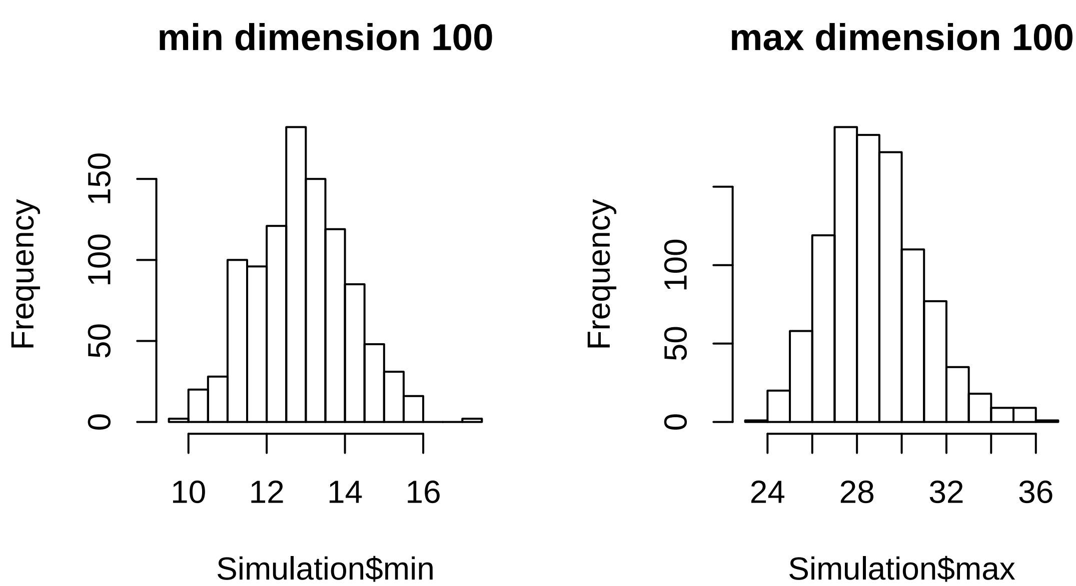
Problem Four

I did this problem with two different dimensions, namely, n = 100, and n = 200, and I feel there are interesting things happening. It seems the min and max(as indicated in the problem) follows a normal distribution with mean as a function of p, and it is illustrated by the hand written sheets attached.

```
library(compiler)
X <- matrix(rnorm(1e+05), nrow = 1000)
rowsqure <- function(x, y) {
    rs <- matrix(NA, nrow <- nrow(y))
    for (i in 1:nrow(y)) {
        rs[i] <- sum((x - y[i, ])^2)
    }
    return(rs)
}
Simulate <- function(n, X) {
    min <- matrix(NA, nrow = n)
    max <- matrix(NA, nrow = n)
    for (k in 1:n) {
        i <- sample(c(1:1000), 1)
        min[k] <- min(rowsqure(X[i, ], X[-i, ]))/10
        max[k] <- max(rowsqure(X[i, ], X[-i, ]))/10
    }
    return(data.frame(min = min, max = max))
}
Simulation <- Simulate(1000, X)
par(mfcol = c(1, 2))
hist(Simulation$min, main = "min dimension 100")
hist(Simulation$max, main = "max dimension 100")
```




```
# Now I increase the dimension of X to 200
X_new <- matrix(rnorm(2e+05), nrow = 1000)
```

```r
Simulation_1 <- Simulate(1000, X_new)
hist(Simulation_1$min, main = "min dimension 200")
hist(Simulation_1$max, main = "max dimension 200")
```