# Heisenberg chain dynamics with MPS

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# Outline

# The results

- Dynamics of the monitored chain: strong and weak measurement back-action
- Bayesian smoothing of the signal
- Evolution of the locally driven chain
- Magnetization in quasi-periodic Heisenberg chain

# Measurement scheme

Phys. Rev. A 81, 012120 (2010)

1. evolve state $|\Psi\rangle$ unitarily for a time $\delta t = 1/\kappa$
2. apply the measurement operator $\Omega_\mu$:

$$|\Psi\rangle \to \Omega_\mu |\Psi\rangle / \sqrt{p(\mu)}$$

3. measurement operator is chosen randomly according to the distribution

$$p(\mu) = \langle\Psi| \Omega_\mu^\dagger \Omega_\mu |\Psi\rangle$$

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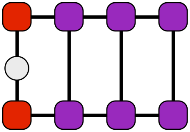
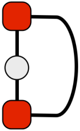
# How we used ITensor



- local operator easily represented in MPO form



- $p(\mu)$ efficiently calculated from the mixed canonical form
- `fitApplyMPO` function with
  `auto args = Args("Cutoff", 1E-9, "Maxm",3000)`
- hamiltonians and measurement operators generated with `AutoMPO` function, but...

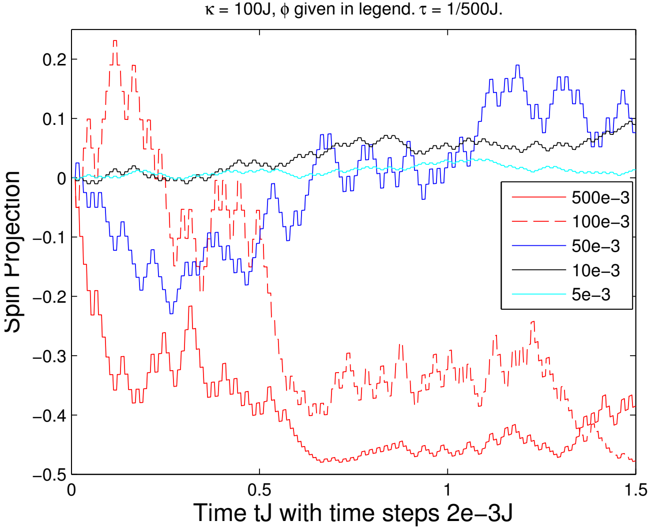
# AutoMPO catch

## Weak Measurement of Phys. Rev. A 81, 012120 (2010)

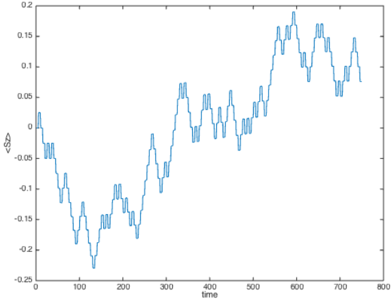
$$\Omega_\mu = \frac{1}{2}(e^{-i\Phi A} + i\mu e^{i\Phi A})$$

```
int indexSite = 1;
auto ampo1 = AutoMPO(sites); // Mu+1 Measurement Operator on indexSite

ampo1 += 0.3608,"Sz",indexSite;            // Assign real value to entry (1,1)
ampo1 += 0.3608/2,"Id",indexSite;          // Assign real value to entry (1,1)
ampo1 += 0.3608*Cplx_i,"Sz",indexSite;     // Assign complex value to entry (1,1)
ampo1 += 0.3608/2*Cplx_i,"Id",indexSite;   // Assign complex value to entry (1,1)

ampo1 += -0.6082,"Sz",indexSite;           // Assign real value to entry (2,2)
ampo1 += 0.6082/2,"Id",indexSite;          // Assign real value to entry (2,2)
ampo1 += -0.6082*Cplx_i,"Sz",indexSite;    // Assign complex value to entry (2,2)
ampo1 += 0.6082/2*Cplx_i,"Id",indexSite;   // Assign complex value to entry (2,2)
```

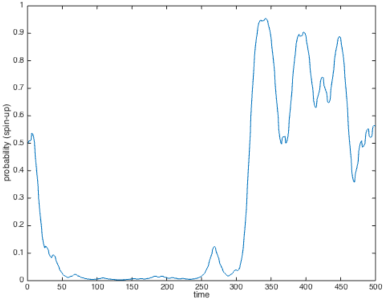
# Back-action results



# Bayesian smoothing

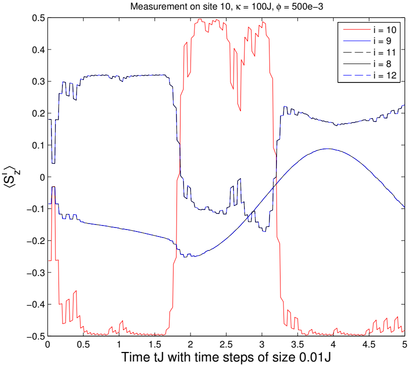



measurement record

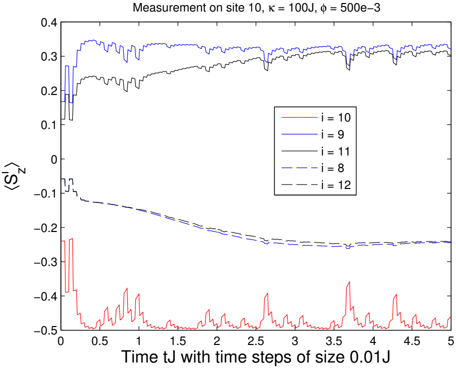



full-bayesian analysis

# Symmetry of the measurement back-action

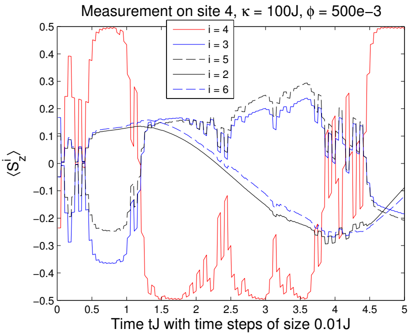



Symmetric case: 19 sites

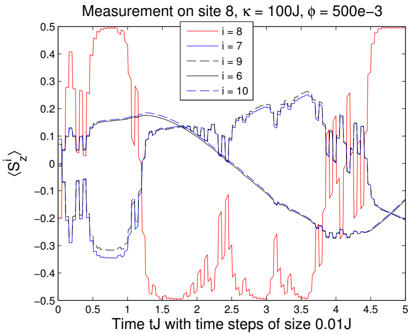



Asymmetric case: 20 sites

# Impact of the Choice of Measured Site




4th site measurement




8th site measurement

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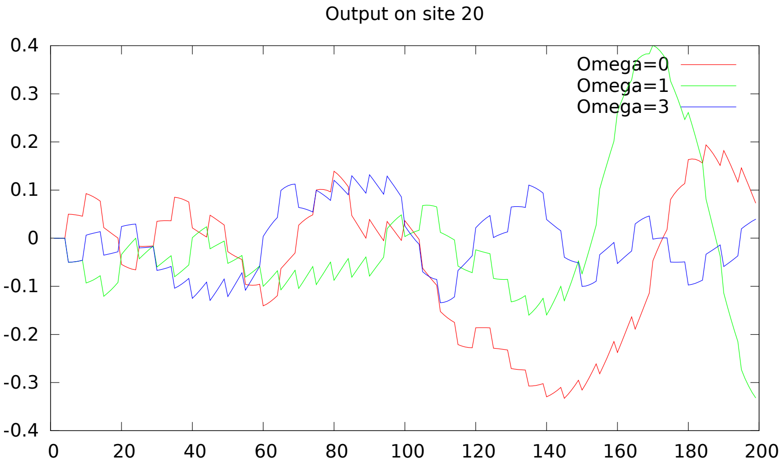
# External driving

- harmonic driving on first site $B(t) = B\cos(\Omega t)$ for $\Omega = 0, 1, 3$
- flip of external field on first site
- global quench

## Code

- at every time step the new hamiltonian is generated with the new external field
- generate the exponential of the hamiltonian with `toExpH` function
- every 5 time step perform a measurement

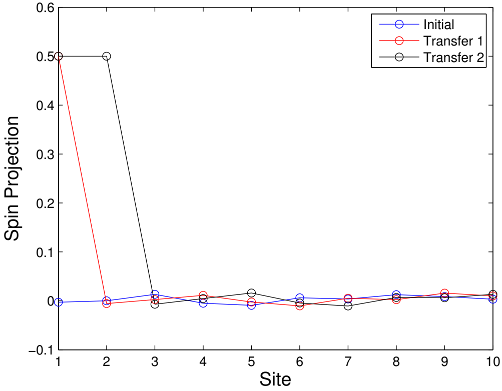
# Output signal



# Quench

## Strong magnetic field on the 1st site

# Spin Transfer



# Alternative approach

## The Operator

$$U_i = S_i^+ \otimes S_{i+1}^- + S_i^- \otimes S_{i+1}^+ + \frac{1}{2}\mathbb{1}_i \otimes \mathbb{1}_{i+1} + S_i^z \otimes S_{i+1}^z$$

$$\downarrow \; \uparrow \; \uparrow \; \downarrow \; \uparrow$$

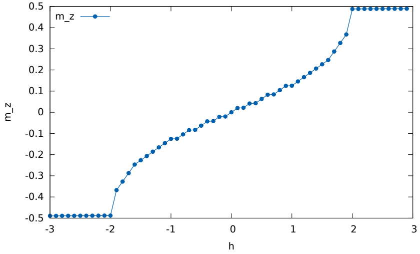
# Quasi-periodic chain

## Perturbation of the Heisenberg model: Scientific Reports 5, 8433 (2015)

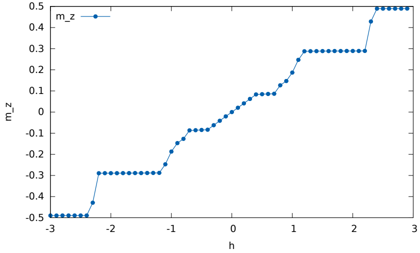
$$H = \sum_i J_i (S_i^x S_{i+1}^x + S_i^y S_{i+1}^y + S_i^z S_{i+1}^z) - h \sum_i S_i^z$$

$$J_i = J(1 - \lambda \cos(2\pi\alpha i + \delta))$$

# Magnetization

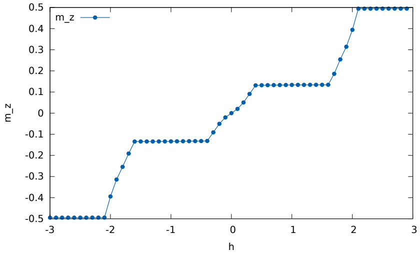



Periodic Heisenberg model

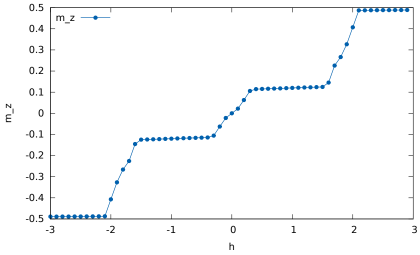



Quasi-periodic: $\alpha = \frac{\sqrt{2}-1}{2}$

# Magnetization

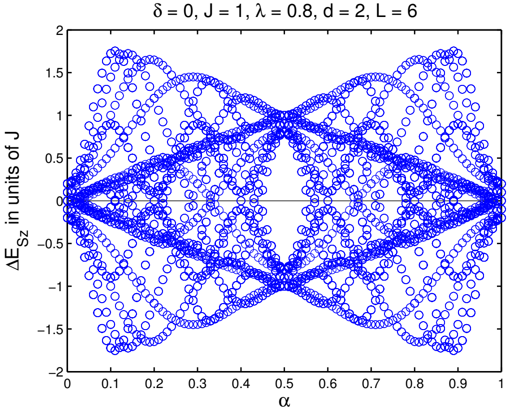



Quasi-periodic: $\alpha = \frac{\sqrt{3}-1}{2}$




Quasi-periodic: $\alpha = \frac{\sqrt{5}-1}{2}$

# Excitation Spectrum




Quasi-periodic Heisenberg model

# Conclusion

- back-action of the strong and the weak measurement in the Heisenberg chain:
  1. asymmetric spin wave propagation
  2. Zeno effect

- driving of the chain

- spin transfer through the motion of the spin chain

- perturbation of the periodicity of the model: topological phase transition