ps-4

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September 2023

github link: https://github.com/gretagoldberg/phys-ua210.git

1 Question 1

The code for this question uses the function outlined in Newman to calculate the integral $\int_0^2 x^4 - 2x + 1\,dx$ first using 10 slices and then 20 slices. As we can see, calculating the error with the formula in Newman produces a "smaller" error than simply calculating the difference between the expected value and calculated value. This is because in the first scenario, we are calculating the error based on two approximations. Therefore this error seems smaller than the true error.

2 Question 2

1. $E = \frac{1}{2}m(\frac{dx}{dt})^2 + V(x)$
2. $E = V(a) \Rightarrow V(a) = \frac{1}{2}m(\frac{dx}{dt})^2 + V(x)$
3. $2\frac{V(a)}{m} - V(x) = (\frac{dx}{dt})^2$
4. $\sqrt{2\frac{V(a)}{m} - V(x)} = (\frac{dx}{dt})$
5. $\int_0^{\frac{1}{4}T} \sqrt{2\frac{V(a)}{m} - V(x)}\,dt = \int dx$
6. $T = \sqrt{8m}\int_0^a \frac{dx}{\sqrt{V(a)-V(x)}}$

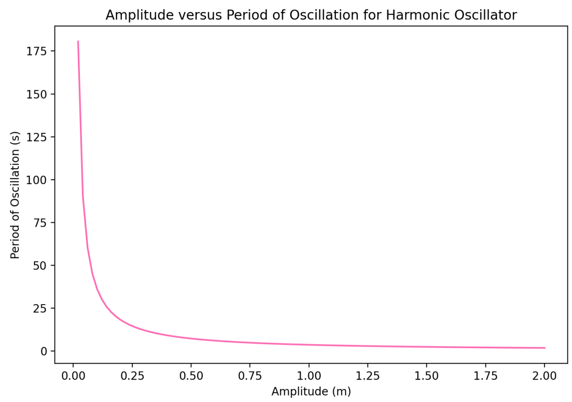

Figure 1: Amplitude versus Period of Harmonic Oscillator

The first phenomena that we see is the the oscillator getting faster as the amplitude increases. This is simply due to conservation of energy. The second phenomena is the apparent divergence of the period as the amplitude approaches zero. This is because of the approximation of the restoring force for small oscillations.

3 Question 3

For this problem I defined a function that calculates the Hermite polynomials for a given n and given x. The function calls itself in order to represent the recursion relation. I then use this function in the function to calculate psi.

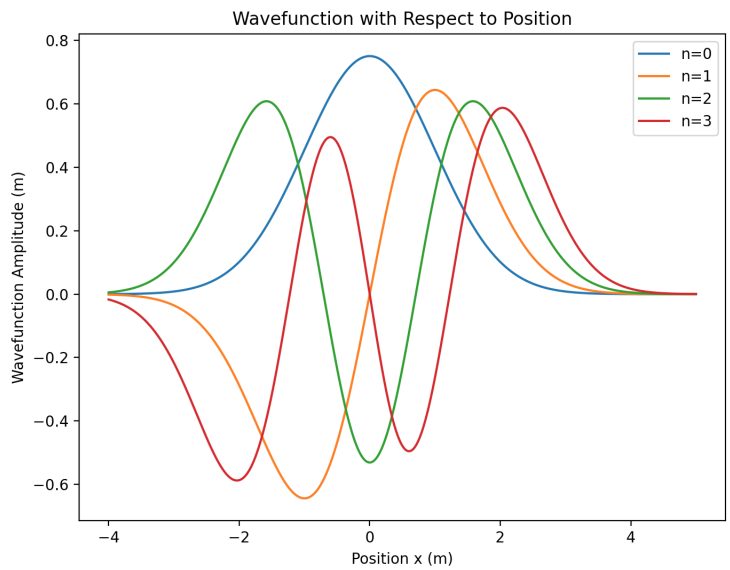

Figure 2: Wavefunction and Position for N up to 3

I then increased N to 30 and increased the bounds to produce the following graph.

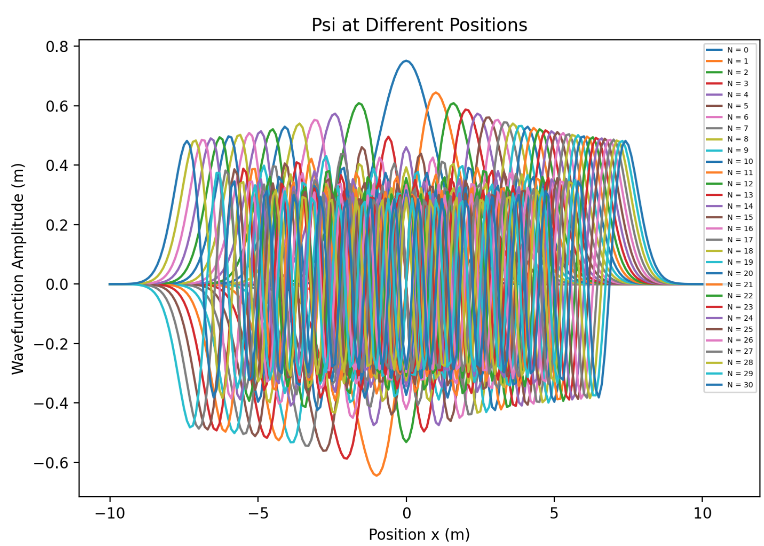

Figure 3: Wavefunction and Position for N up to 30

For this section I used the functions defined in Newman's gaussxw.py. I then defined a function to calculate the integrand for the integral. I then evaluated the integral using the gquad function which calls the gaussxwab function to calculate the particular weights/does the mapping for you. This produced a value very close to 2.3.

For the Gaussian-Hermite section, I used the scipy function special.roots-hermite to calculate the weights and points. Since the bounds for this run from negative infinity to positive infinity, the bounds did not need to be mapped as in the last integral. I then used the same function from the last section (the integrand one) to calculate the value of $< x^2 >$. In theory, this should perfectly integrate to the expected value. However, my output was slightly higher than expected at approximately 2.4. I'm not entirely sure why this is, but the value is highly variable depending on N.