PS 5

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1 Problem 1

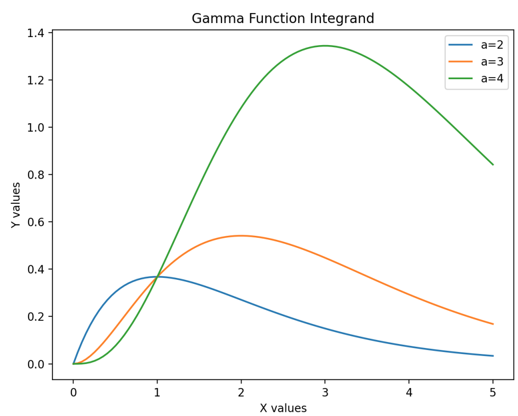

Figure 1: Plot of Gamma Function Integrand

b) to find max/min

$$\frac{d}{dx} = 0$$

$$-e^{-x}x^{a-1} + (a-1)x^{a-2}e^{-x} = 0$$

$$\ln((a-1)x^{a-2}e^{-x} = e^{-x}x^{a-1})$$

$$\ln((a-1)x^{a-2}) + \ln(e^{-x}) = \ln(e^{-x}) + \ln(x^{a-1})$$

$$\ln((a-1)x^{a-2}) - x = -x + \ln(x^{a-1})$$

$$\ln((a-1)x^{a-2}) = \ln(x^{a-1})$$

$$\ln(\frac{(a-1)x^{a-2}}{x^{a-1}}) = 0$$

$$\ln((a-1)x^{-1}) = 0$$

$$\frac{(a-1)}{x} = e^0$$

$$a - 1 = x$$

we know this is a maximum because we're asked to maximize the function so it must be a maximum rather than a minimum (I also checked graphically)

c)

$$\frac{1}{2} = \frac{x}{c+x}$$

$$\frac{1}{2}c + \frac{1}{2}x = x$$
$$\frac{1}{2}c = \frac{1}{2}x$$
$$c = x$$

so c must be $a - 1$

d)

$$x^{a-1} = e^{(a-1)\ln(x)}$$
$$\rightarrow e^{(a-1)\ln(x)}e^{-x}$$

This transformation makes the power function into a log of x which behaves better for a larger range of x. You can see this simply by plotting both functions for a range of a values.

2 Problem 2

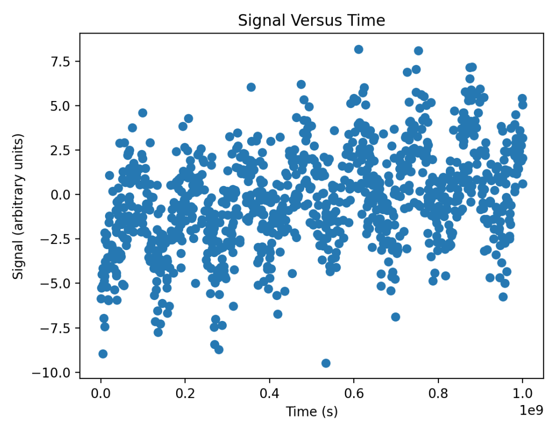

Figure 2: Plot of Signal Data

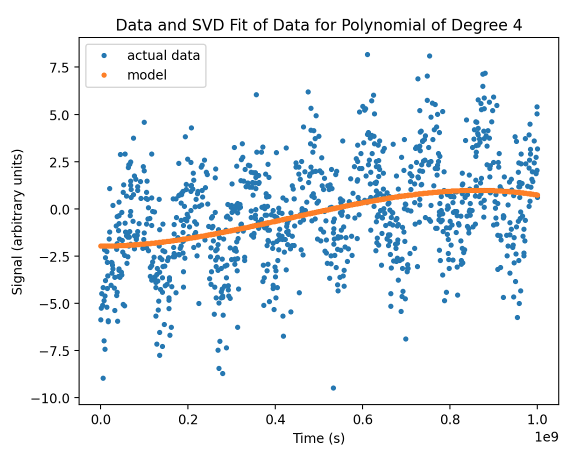

Figure 3: Plot of SVD Fit with Polynomial of Degree 4

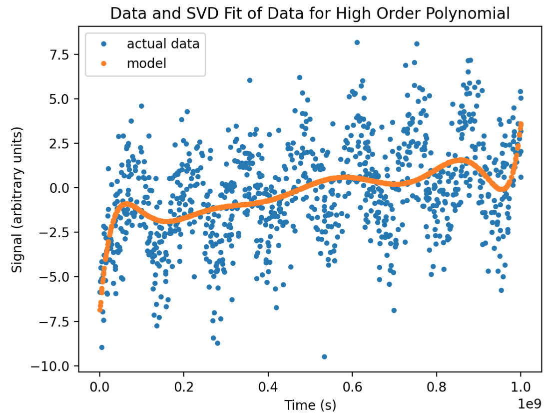

Figure 4: Plot of SVD Fit with Polynomial of Degree 10

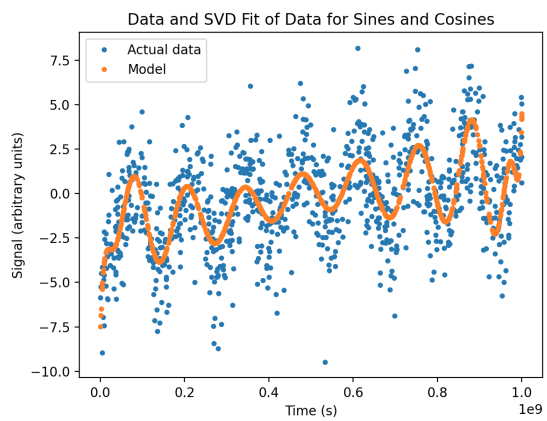

Figure 5: Plot SVD Fit with Sines and Cosines

c) After plotting the residuals, we can see that they are all over, instead of being centered around zero. We know that if the fit were good, the residuals would not be centered around zero.

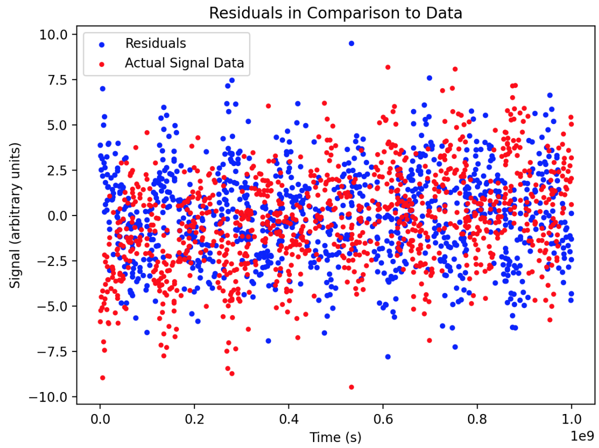

Figure 6: Residuals and Data

d) For our low order polynomial, the condition number is extremely high. For our high order polynomial the condition number is also extremely high. For this reason, we can tell that there definitely is no viable polynomial fit.

e) This fit does a better job of fitting the data which we can see from the residuals. Additionally, the ideal frequency is the frequency of the tenth harmonic.