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Harnessing Multi-Fidelity Design Analysis to Advance Fusion Reactor Design

Introduction: In our world, where wars are fought over oil and Appalachian creeks run orange with acid mine drainage, energy has become the global currency. Because Earth is in a climate crisis, we must prioritize the realization of clean energy sources. Among potential methods of generating clean energy, one of the most ideal is nuclear fusion, which emits no carbon and produces no radioactive waste. However, reactors must be designed to safely handle fusion reactions, including temperatures over 100 million degrees Celsius and extreme plasma density, an extremely difficult design challenge. Prior to lengthy regulatory approvals and expensive construction, fusion reactor designs undergo extensive modeling as system codes. Because of the complexity of the system, there is a significant need for expedited, computationally feasible, and cost-effective design modeling methods.

Multi-fidelity modeling is a method of modeling that has proven successful for modeling complex engineering systems and is commonly used in aerospace engineering [1]. My research applies multi-fidelity modeling to optimize fusion reactor design. I used a Bayesian method of machine learning, Gaussian Process Regression (GPR), to leverage multi-fidelity analysis to optimize fusion reactor design. My project aims to expedite exploration of the design space by employing machine learning to predict new design points while maintaining model accuracy.

Methodology: Multi-fidelity modeling selectively leverages two (or more) models of the same parameter, typically a high-fidelity model, which is more accurate but more computationally expensive, and a low-fidelity model, which is potentially less accurate, but less computationally expensive. A leveraging method is used to determine how to expedite computational exploration of the reactor design space while meeting accuracy specifications.

Last summer, I worked with Dr. Jacob Schwartz at the Princeton Plasma Physics Laboratory to apply multi-fidelity modeling to fusion reactor design, with the goal of testing GPR leveraged multi-fidelity modeling as a successful method of fusion reactor optimization. Fusion rate in a magnetically confined plasma was selected as the parameter to be optimized in order to maximize energy output. This project was completed in a highly independent manner. I developed the theory in conjunction with Dr. Schwartz and I completed the implementation and coding independently.

I selected GPR as the leveraging method because it has strong predictive capabilities. In order to implement multi-fidelity design analysis, I first needed to create mathematical representations of the fusion rate at low- and high-fidelity levels. This research considers magnetically confined plasmas in a tokamak type reactor, meaning the plasma will be confined in a toroidal geometry. The low-fidelity model considers a volume-averaged density (n) and volume-averaged temperature (T) and calculates the fusion rate in the plasma based on averaged values. The high-fidelity model calculates the local fusion rate for each differential volume and integrates over the toroidal plasma volume (V). From these models, I derived equations [1] and [2] respectively. Here, σv is the fusion cross section.

1.) Fusion Rate = $V\,c\ <n^2><T>^{\beta}$

2.) Fusion Rate = $\int_{Plasma\ volume} (\rho)(\sigma\nu)(T)n^2\,dV$

The user inputs several parameters of the fusion device to be modeled: minor radius (a), major radius(R), and central density (ρ_0). These values are to be held constant. The user is additionally prompted to declare a level of acceptable uncertainty in predicted output points. An input point is a three-dimensional point consisting of an additional three reactor parameters: the exponential terms on the density and temperature profiles (α_n,α_T) and the central temperature (T_0). Each input space allows the parameters to vary on realistic ranges of interest. The values of each point are determined in a random, but even, distribution.

In this space, let's say we want to find the fusion rate at several hundred new input points. We could compute the output for each new point using the high-fidelity model, but it would be extremely computationally expensive. Instead, we calculate the output using the "gold standard" high-fidelity model for an order of 10 points; the GPR model is fit to these "training points" of 0 standard deviation. The GPR model is then able to predict new outputs at each new input point; these are called "test points". The output points are represented as the coefficients (c, β) in equation 1. Additionally, the GPR model will return a level of uncertainty, as standard deviation, for each point. Then, the algorithm compares the user-declared uncertainty to the associated uncertainty of each new test point. If the uncertainty of a test point is less than the user-declared uncertainty, then the output point (c, β) is plugged into equation 1, and the fusion rate is calculated using the low-fidelity model, saving computational time without compromising accuracy. If the uncertainty of a test point is greater than the user-declared uncertainty, the high-fidelity model, equation 2, is iterated in order to preserve model accuracy. Because the high-fidelity model was iterated, the algorithm recognizes the test point as a point of zero standard deviation and is therefore able to add the point to the set of training points. This allows the algorithm to get a more comprehensive knowledge of the design space, essentially getting smarter as it goes.

Results: In order to quantify the effectiveness of this method, I implemented a counter of high-fidelity model iterations and low-fidelity model iterations. Each time the low-fidelity model is iterated (referred to as a "hit"), my algorithm is performing more efficiently than the traditional model. The results of my project yielded significant computational time saved.

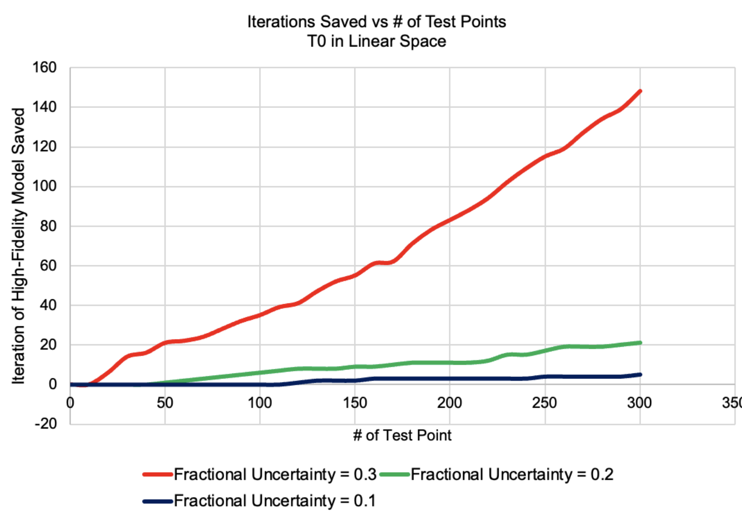
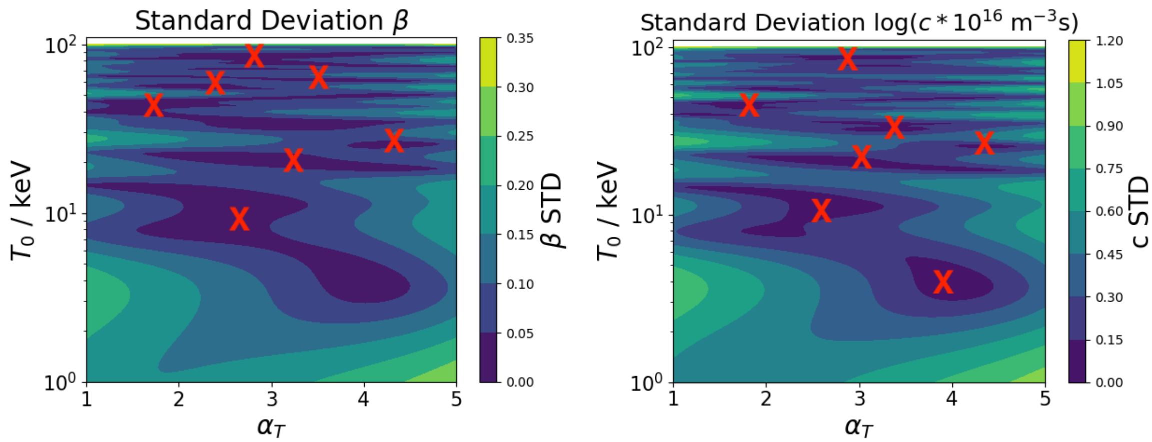


Figure 1. Iterations Saved vs Number of Test Points

As an example, the algorithm was trained on a set of seven input points and tested on a set of 300 test points. I examined the amount of low-fidelity hits, i.e., saved high-fidelity iterations. For 300 points, over 150 of the points were calculated using the low-fidelity model in place of the high-fidelity model, while maintaining user-declared accuracy [Figure 1].

From this example run, I created contour plots of the output space to illustrate the zero standard deviation (marked by red X) observed at the seven training points; these points can be observed in the dark regions of Figures 2 and 3.



Figures 2, 3. Two-dimensional output space for uncertainty parameters B and C respectively. Points with zero standard deviation are the training points, which is characteristic of GPR.

Conclusions and Future Studies: I will continue working on this research for the next 18 months as my honors undergraduate thesis, co-advised by my advisor at the Princeton Plasma Physics Laboratory and a nuclear theory faculty member at Ohio University, Dr. Daniel Phillips. I will explore the use of Bayesian statistics for uncertainty quantification in light-ion fusion reactions, ultimately making the high-fidelity model even more accurate. An amount of uncertainty in fusion reaction design is due to uncertainty in the nuclear reactions occurring in the plasma. I will focus on understanding how these reactions affect the fusion rate and how to optimize the best fuels for magnetically confined plasmas in particular reactor configurations. My research will focus on ions found in plasmas in magnetically confined fusion reactors ($T(d,n)^4He$, $^3He(d,p)^4He$, $D(d,p)T$, and $D(d,n)^3He$) and expand to theoretically fusible materials, such as lithium. To further my experience in plasma physics, this summer I will participate in international research at the Max Planck Institute of Plasma Physics in Germany, where I will assist in building the Electron and Positron Optimized Stellarator (EPOS), a stellarator intended to confine a matter-antimatter plasma . This is an exciting experience in which I hope to apply my experience of theoretically modeling fusion reactors to hands-on practice via this research internship. Long term, I desire to continue studying the design of magnetically confined fusion devices, assisting in the realization of clean nuclear fusion energy

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