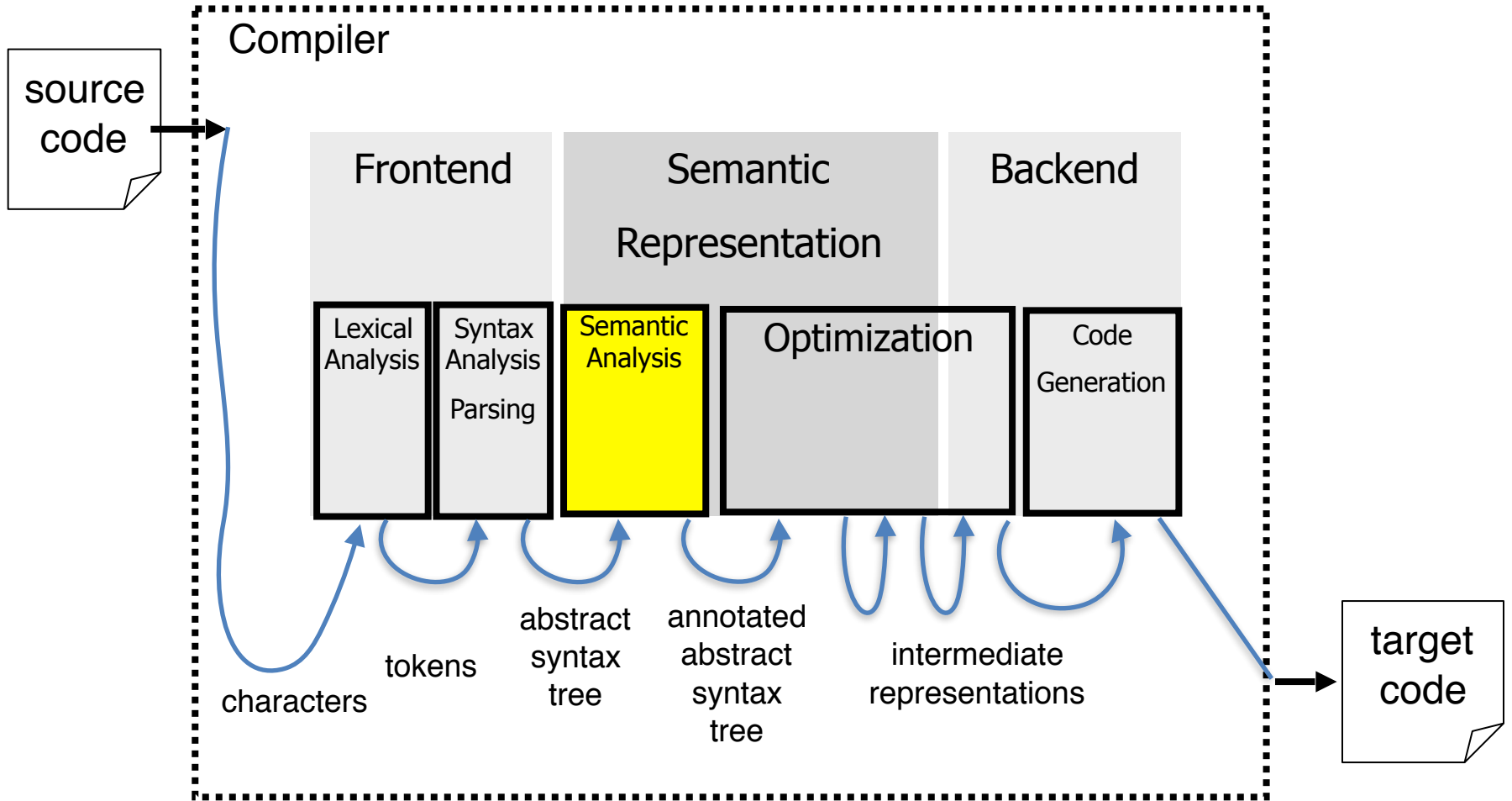


Type Checking



Recap: semantic analysis

- Scope rules
- Symbol tables
- Inheritance graph

Types

- What is a type?
 - simplest answer: a set of values
 - examples: integers, real numbers, booleans, ...
- Why do we care?
 - safety: guarantee that certain errors cannot occur at runtime
 - abstraction: hide implementation details

Type declarations

- Explicit type declarations

```
TYPE Int_Array = ARRAY [Integer 1..42] OF Integer;
```

- Anonymous types

```
Var a : ARRAY [Integer 1..42] OF Real;
```

Anonymous types

```
Var a : ARRAY [Integer 1..42] OF Real;
```



```
TYPE #type01_in_line_73 = ARRAY [Integer 1..42] OF Real;  
Var a : #type01_in_line_73;
```

Forward references

```
TYPE Ptr_List_Entry = POINTER TO List_Entry;  
TYPE List_Entry =  
    RECORD  
        Element : Integer;  
        Next : Ptr_List_Entry;  
    END RECORD;
```

- Type added to symbol table as **forward reference**
- **Update symbol table when the type declaration is met**
- At the end of scope, check that all forward refs have been resolved
- Check for **cycles**

Type equivalence: **name** equivalence

```
Type t1 = ARRAY[Integer] OF Integer;  
Type t2 = ARRAY[Integer] OF Integer;
```

t1 not (name) equivalent to t2

```
Type t3 = ARRAY[Integer] OF Integer;  
Type t4 = t3
```

t3 equivalent to t4

Type equivalence: **structural** equivalence

```
Type t5 = RECORD c: Integer; p: POINTER TO t5; END
RECORD;
Type t6 = RECORD c: Integer; p: POINTER TO t6; END
RECORD;
Type t7 =
  RECORD
    c: Integer;
    p: POINTER TO
      RECORD
        c: Integer;
        p: POINTER to t5;
      END RECORD;
  END RECORD;
```

t5, t6, t7 are all (structurally) equivalent

In practice...

- Almost all modern languages use **name equivalence**
- Why?

Types: strong vs. weak

Output: 73

warning: initialization makes
integer from pointer without a cast

- Coercion
- Strongly typed:
C, C++, Java,...
- Weakly typed:
Perl, PHP, ...
- Not everybody
agrees on this
classification

perl

```
$a=31;  
$b="42x";  
$c=$a+$b;  
print $c;
```

C

```
main() {  
    int a=31;  
    char b[3]="42x";  
    int c=a+b;  
}
```

error: Incompatible type for declaration.
Can't convert java.lang.String to int

Java

```
class A {  
    public static void main() {  
        int a=31;  
        String b ="42x";  
        int c=a+b;  
    }  
}
```

Types: strong vs. weak

Output: 73

warning: initialization makes
integer from pointer without a cast

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C

```
main() {  
    int a=31;  
    char b[3]="42x";  
    int c=a+b;  
}
```

Java

```
public class... {  
    public static void main() {  
        int a=31;  
        String b ="42x";  
        String c=b+a;  
    }  
}
```

OK

Coercions

- Suppose that at some point in the program, we expect a value of type T1 and find a value of type T2
- Is that acceptable?

```
float x = 3.141;  
int y = x;
```

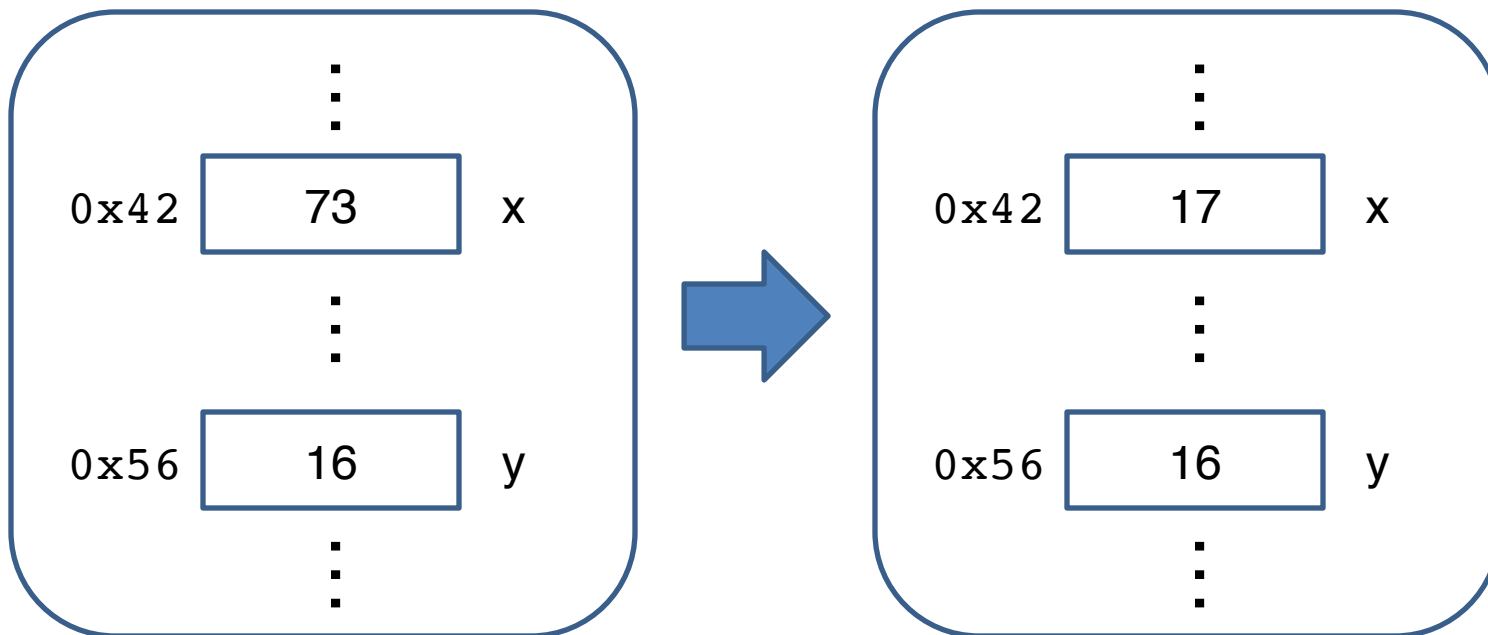
I-values and r-values

`dst := src`

- What is dst? dst is a **memory location** where the value should be stored
- What is src? src is a **value**
- “location” on the left of the assignment called an I-value
- “value” on the right of the assignment is called an r-value

Example: l-values and r-values

$x := y + 1$



Example: l-values and r-values

$x := A + 1$

$x := A[1]$

$x := A[A[1]]$

l-values and r-values

		expected	
found		lvalue	rvalue
	lvalue	ok	deref
	rvalue	error	ok

Type table

- All types in a compilation unit are collected in a type table
- For each type, table entry contains
 - type constructor: basic, record, array, pointer,...
 - size and alignment requirements
 - to be used later in code generation
 - types of components (if applicable)
 - example: types of record fields

Type checking vs inference

- **Type checking**
 - use declared types of variables
 - check types of operands are legal
 - infer types for expressions
- **Type inference**
 - automatically infer types of variables or show that there is no valid typing

Type checking: static vs dynamic

- Static type checking is **conservative**
 - most checking at compile time
 - any program that is determined to be well-typed is free from certain kinds of errors
 - may reject programs that cannot be statically determined as well typed (why?)
- Dynamic type checking
 - most checking at runtime
 - may accept more programs as valid (runtime info)
 - errors not caught at compile time
 - runtime cost

So far...

- Static correctness checking
- **Type Identification**: match applied occurrences of identifier to its defining occurrence
 - symbol table maintains this information
- **Type Checking**
 - which type combinations are legal
 - type equivalence: nominal vs structural
 - type coercion
 - each node in the AST of an expression represents either an l-value (location) or an r-value (value)

Type checking implementation

- How does this magic happen?
- We probably need to go over the AST?
- How does this relate to the clean formalism of the parser?

- Different approaches
 - attribute grammars
 - type systems

Type system (textbook definition)

- “A type system is a tractable **syntactic** method for **proving the absence of certain program behaviors** by classifying phrases according to the kinds of values they compute”

-- Types and Programming Languages
/ Benjamin C. Pierce

Type system

- A type system of a programming language is a way to define how “good” programs behave
- Good programs are well-typed programs
- Bad programs are not well-typed

Type rules

- which types can be combined with certain operator
- assignment of expression to variable
- formal and actual parameters of a method call

string string
"drive" + "drink"
string

int string
42 + "the answer"
ERROR

Type rules

- Specify for each operator
 - types of operands
 - type of result
- Basic types
 - building blocks for the type rules
 - example: int, boolean, (sometimes) string
- Type expressions
 - array types
 - function types
 - record types and classes

Type rules

If $E1$ has type int and $E2$ has type int ,
then $E1 + E2$ has type int

$$\frac{E1 : \text{int} \qquad E2 : \text{int}}{E1 + E2 : \text{int}}$$

Notations for rules

- An inference rule consists of **premises** and **conclusion**

$$\frac{\mathbf{A} \quad \mathbf{B}}{\mathbf{C}}$$

- An inference rule without any premises is an **axiom**

$$\frac{}{\mathbf{A}} \qquad \frac{}{\mathbf{B}}$$

- A proof is a sequence of lines, each of which is either an axiom or follows from earlier lines by an inference rule

$$\frac{\frac{}{\mathbf{A}} \quad \frac{}{\mathbf{B}}}{\mathbf{C}}$$

More type rules

$$\frac{}{\text{true} : \text{boolean}}$$
$$\frac{}{\text{false} : \text{boolean}}$$
$$\frac{}{\text{int-literal} : \text{int}}$$
$$\frac{}{\text{string-literal} : \text{string}}$$
$$\frac{E1 : \text{int} \quad E2 : \text{int}}{E1 \text{ op } E2 : \text{int}}$$
$$\text{op} \in \{ +, -, /, *, \% \}$$
$$\frac{E1 : \text{int} \quad E2 : \text{int}}{E1 \text{ op } E2 : \text{boolean}}$$
$$\text{op} \in \{ <=, <, >, >= \}$$
$$\frac{E1 : T \quad E2 : T}{E1 \text{ op } E2 : \text{boolean}}$$
$$\text{op} \in \{ ==, != \}$$

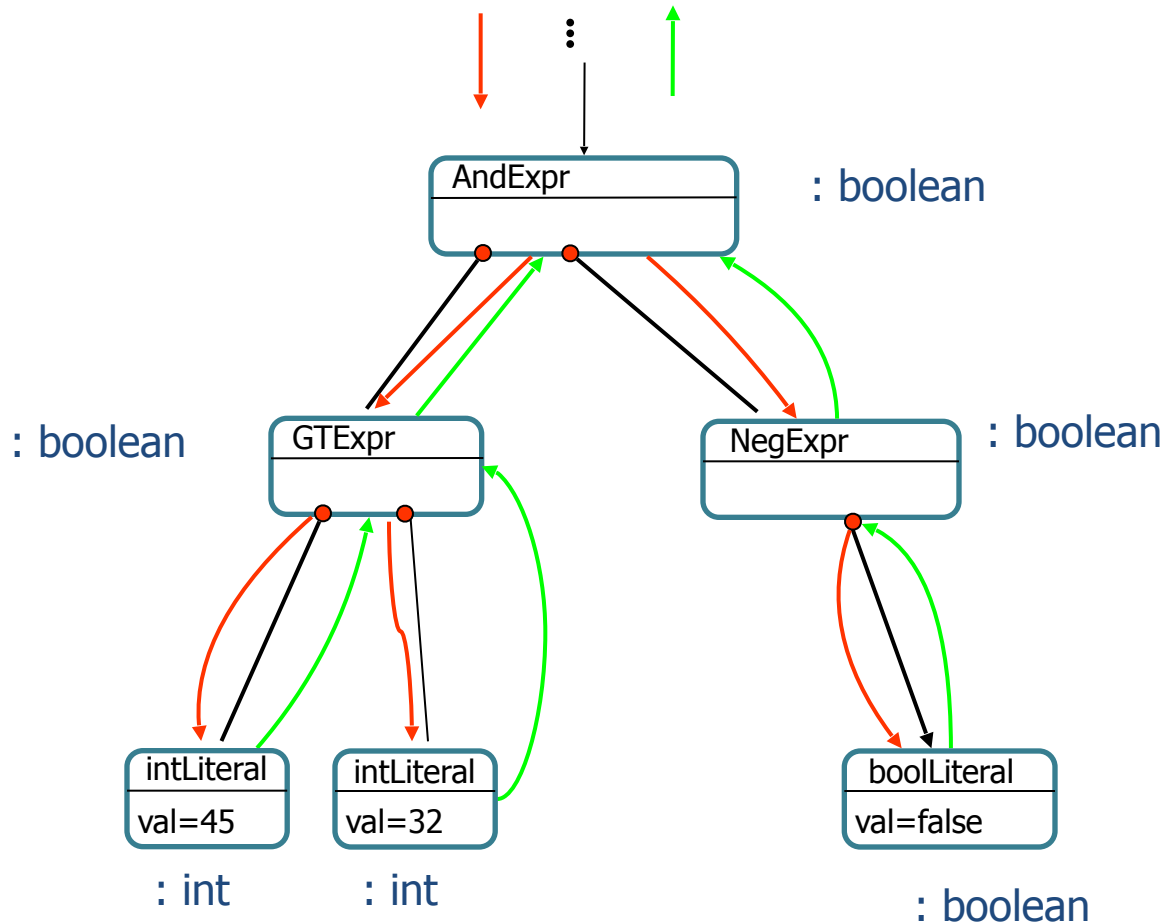
And even more type rules

$$\frac{E1 : \text{boolean} \quad E2 : \text{boolean}}{E1 \text{ op } E2 : \text{boolean}} \quad \text{op} \in \{ \&\&, || \}$$
$$\frac{E1 : \text{int}}{- E1 : \text{int}}$$
$$\frac{E1 : \text{boolean}}{! E1 : \text{boolean}}$$
$$\frac{E1 : T[]}{E1.\text{length} : \text{int}}$$
$$\frac{E1 : T[] \quad E2 : \text{int}}{E1[E2] : T}$$
$$\frac{E1 : \text{int}}{\text{new } T[E1] : T[]}$$

Type checking implementation

- Traverse AST
- Assign types for AST nodes
- Use typing rules to compute node types

Example



`45 > 32 && !false`

$E1 : \text{boolean} \quad E2 : \text{boolean}$

$E1 \text{ op } E2 : \text{boolean}$

$\text{op} \in \{ \&\&, || \}$

$E1 : \text{boolean}$

$!E1 : \text{boolean}$

$E1 : \text{int} \quad E2 : \text{int}$

$E1 \text{ op } E2 : \text{boolean}$

$\text{op} \in \{ \leq, <, >, \geq \}$

`false` : `boolean`

`int-literal` : `int`

Plan

- Cool type rules
- Implementing type checking for Cool

Cool types

- The types are
 - Class names
 - SELF_TYPE
- The user **declares** types for identifiers
- The semantic analysis **infers** types for expressions
 - every expression has a unique type
- Cool **type rules** specify which operations are valid for which types
- The goal of **type checking** is to ensure that operations are used with the correct types
 - enforces intended interpretation of values
- Cool is **statically** typed: type checking during compilation

Notations for rules

Environment₁ ⊢ Statement₁

Environment₂ ⊢ Statement₂

...

Environment_n ⊢ Statement_n

Environment ⊢ Statement

[NAME] conditions

- $A \vdash B$ means “given A, it is provable that B”
- $A \vdash B$ is called a judgement
- A is called context or environment
- B is called statement

Cool type judgements

- Cool type rules have judgements of the form

$$\underbrace{\mathbf{O}, \mathbf{M}, \mathbf{C}}_{\text{type environment}} \vdash e : T$$

- **O** gives types to free identifiers in the current scope
- **M** gives information about the formal parameters and return type of methods
- **C** is the class in which expression *e* appears

Cool type environment

$$\underbrace{O, M, C}_{\text{type environment}} \vdash e : T$$

- **O** mapping Object Id's to types
 - symbol table for the current scope
 - $O(x) = T$
- **M** mapping methods to method signatures
 - $M(K, f) = (A, B, D)$
means there is a method $f(a:A, b:B): D$ defined in class K (or its ancestor)
- **C** the class in which expression e appears
 - used when `SELF_TYPE` is involved

Cool type environment

$$\underbrace{O, M, C}_{\text{type environment}} \vdash e : T$$

- Why separate object/methods?
- In Cool, the method and object identifiers live in different name spaces

Type rules

- Rules are schemas for inferring types of expressions

$$O, M, C \vdash e1 : \text{Int}$$

$$O, M, C \vdash e2 : \text{Int}$$

$$O, M, C \vdash e1 + e2 : \text{Int}$$

$$O, M, C \vdash \text{int_const} : \text{Int}$$

$$O(\text{id}) = T$$

$$O, M, C \vdash \text{id} : T$$

- Infer types by instantiating the schemas

$$O, M, C \vdash 1 : \text{Int}$$

$$O, M, C \vdash 2 : \text{Int}$$

$$O, M, C \vdash 1 + 2 : \text{Int}$$

$$O, M, C \vdash 1 : \text{Int}$$

$$O(y) = \text{Int}$$

$$O, M, C \vdash y : \text{Int}$$

$$O, M, C \vdash y : \text{Int}$$

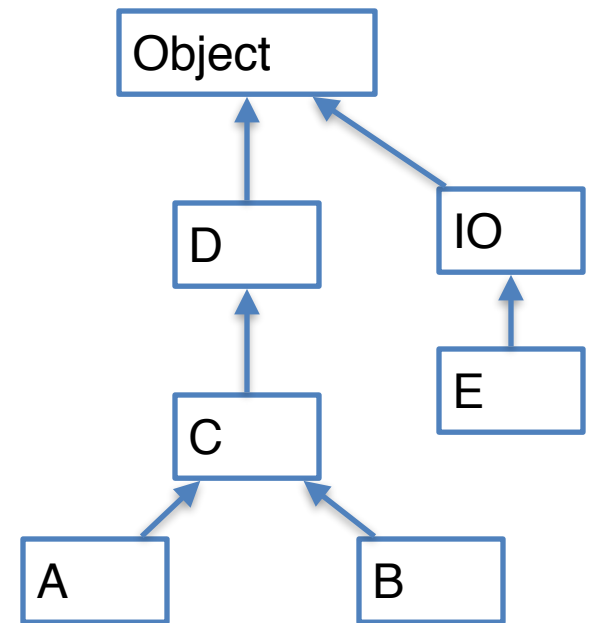
$$O, M, C \vdash (1 + 2) : \text{Int}$$

$$O, M, C \vdash y + (1 + 2) : \text{Int}$$

$$O, M, C \vdash 2 : \text{Int}$$

Subtyping

- Define a relation \leq on classes
 - $X \leq X$
 - $X \leq Y$ if X inherits from Y
 - $X \leq Z$ if $X \leq Y$ and $Y \leq Z$
- Example
 - $A \leq C$
 - $B \leq \text{Object}$
 - $E \not\leq D$ and $D \not\leq E$



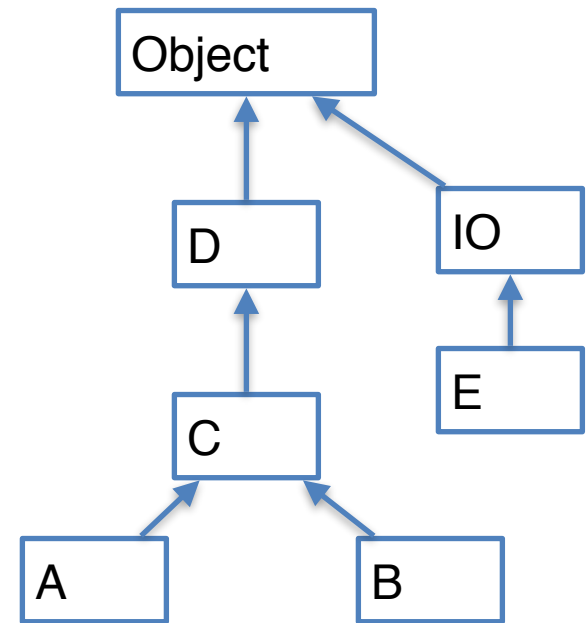
Least upper bounds

- Z is the least upper bound of X and Y
- $\text{lub}(X, Y) = Z$
 - $X \leq Z$ and $Y \leq Z$
 Z is **upper** bound
 - $X \leq Z'$ and $Y \leq Z' \Rightarrow Z \leq Z'$
 Z is the **least** upper bound

Least upper bounds

- In Cool, the least upper bound of two types is their **least common ancestor** in the inheritance tree

- Example
 - $\text{lub}(A, B) = C$
 - $\text{lub}(C, D) = D$
 - $\text{lub}(C, E) = \text{Object}$



Type rules: Assign

$$\frac{\begin{array}{l} O(x) = T_0 \\ O, M, K \vdash e_1 : T_1 \\ T_1 \leq T_0 \end{array}}{O, M, K \vdash x \leftarrow e_1 : T_1} \quad [\text{Assign}]$$

ERROR

OK

OK

```
class A {  
    foo() : A { ... }  
};  
class B inherits A { };  
...  
let x:B in x ← A(new B).foo();  
let x:A in x ← (new B).foo();  
let x:Object in x ← (new B).foo();
```