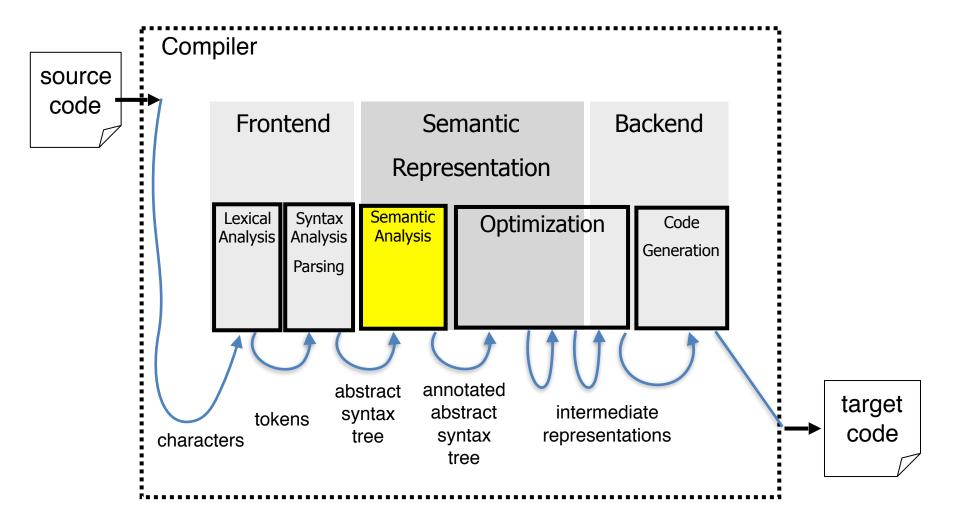
# Type Checking



## Recap: semantic analysis

- Scope rules
- Symbol tables
- Inheritance graph

## **Types**

- What is a type?
  - simplest answer: a set of values
  - examples: integers, real numbers, booleans, ...

- Why do we care?
  - safety: guarantee that certain errors cannot occur at runtime
  - abstraction: hide implementation details

## Type declarations

Explicit type declarations

TYPE Int\_Array = ARRAY [Integer 1..42] OF Integer;

Anonymous types

Var a: ARRAY [Integer 1..42] OF Real;

## Type declarations

Var a : ARRAY [Integer 1..42] OF Real;



TYPE #type01\_in\_line\_73 = ARRAY [Integer 1..42] OF Real; Var a : #type01\_in\_line\_73;

### Forward references

```
TYPE Ptr_List_Entry = POINTER TO List_Entry;
TYPE List_Entry =
RECORD
Element : Integer;
Next : Ptr_List_Entry;
END RECORD;
```

- Forward references must be resolved
- A forward ref is added to the symbol table (as forward ref), and later updated when the type declaration is met
- At the end of scope, check that all forward refs have been resolved
- Must add check for circularity

## Type equivalence: name equivalence

```
Type t1 = ARRAY[Integer] OF Integer;
Type t2 = ARRAY[Integer] OF Integer;
```

t1 not (name) equivalent to t2

```
Type t3 = ARRAY[Integer] OF Integer;
Type t4 = t3
```

t3 equivalent to t4

### Type equivalence: structural equivalence

```
Type t5 = RECORD c: Integer; p: POINTER TO t5; END
RECORD;
Type t6 = RECORD c: Integer; p: POINTER TO t6; END
RECORD;
Type t7 =
 RECORD
   c: Integer;
   p: POINTER TO
     RECORD
       c: Integer;
       p: POINTER to t5;
     END RECORD;
END RECORD;
       t5, t6, t7 are all (structurally) equivalent
```

## In practice...

- Almost all modern languages use name equivalence
- Why?

# Types: strong vs. weak

Output: 73

warning: initialization makes integer from pointer without a cast

Coercion

- Strongly typed:
   C, C++, Java,...
- Weakly typed:
   Perl, PHP, ...
- Not everybody agrees on this classification

```
perl
$a=31;
$b="42x";
$c=$a+$b;
print $c;
```

```
main() {
  int a=31;
  char b[3]="42x";
  int c=a+b;
}
```

error: Incompatible type for declaration. Can't convert java.lang.String to int

```
class A {
  public static void main() {
   int a=31;
   String b ="42x";
  int c=a+b;
  }
}
```

# Types: strong vs. weak

Output: 73

warning: initialization makes integer from pointer without a cast

Coercion

Strongly typed:
 C, C++, Java,...

- Weakly typed:
   Perl, PHP, ...
- Not everybody agrees on this classification

```
perl
$a=31;
$b="42x";
$c=$a+$b;
print $c;
```

```
main() {
  int a=31;
  char b[3]="42x";
  int c=a+b;
}
```

```
public class... {
  public static void main() {
   int a=31;
   String b = "42x";
   String c=b+a;
  }
}
OK
```

### Coercions

- Suppose that at some point in the program, we expect a value of type T1 and find a value of type T2
- Is that acceptable?

```
float x = 3.141;
int y = x;
```

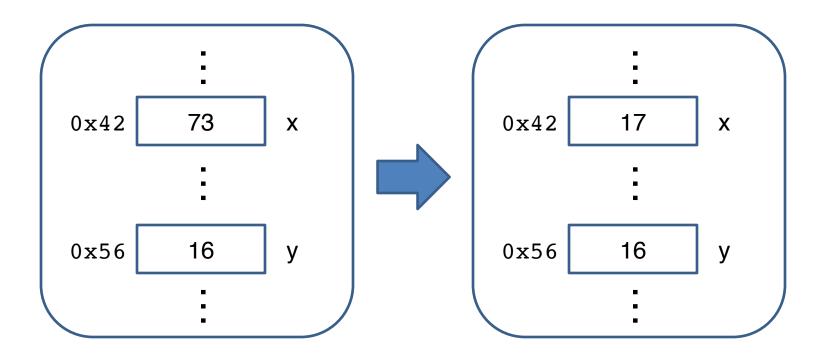
## I-values and r-values

dst := src

- What is dst? dst is a memory location where the value should be stored
- What is src? src is a value
- "location" on the left of the assignment called an I-value
- "value" on the right of the assignment is called an r-value

## Example: I-values and r-values

$$x := y + 1$$



## Example: I-values and r-values

$$x := A + 1$$

$$x := A[1]$$

$$x := A[A[1]]$$

## I-values and r-values

#### expected

found

	Ivalue	rvalue
Ivalue	ok	deref
rvalue	error	ok

## So far...

- Static correctness checking
- Identification: match applied occurrences of identifier to its defining occurrence
  - symbol table maintains this information
- Checking: which type combinations are legal
  - type equivalence: nominal vs structural
  - type coercion
  - each node in the AST of an expression represents either an I-value (location) or an r-value (value)

# Type table

- All types in a compilation unit are collected in a type table
- For each type, its table entry contains
  - type constructor: basic, record, array, pointer,...
  - size and alignment requirements
    - to be used later in code generation
  - types of components (if applicable)
    - example: types of record fields

## How does this magic happen?

We probably need to go over the AST?

 How does this relate to the clean formalism of the parser?

- Different approaches
  - attribute grammars
  - type systems

## **TYPE RULES**

#### Type system (textbook definition)

 "A type system is a tractable syntactic method for proving the absence of certain program behaviors by classifying phrases according to the kinds of values they compute"

-- Types and Programming Languages/ Benjamin C. Pierce

# Type system

- A type system of a programming language is a way to define how "good" programs behave
  - Good programs = well-typed programs
  - Bad programs = not well typed

#### Type checking

- Static typing: most checking at compile time
- Dynamic typing: most checking at runtime

#### Type inference

 Automatically infer types for a program or show that there is no valid typing

# Type checking: static vs. dynamic

- Static type checking is conservative
  - Any program that is determined to be well-typed is free from certain kinds of errors
  - May reject programs that cannot be statically determined as well typed
  - Why?
- Dynamic type checking
  - May accept more programs as valid (runtime info)
  - Errors not caught at compile time
  - Runtime cost

# Type rules

- which types can be combined with certain operator
- assignment of expression to variable
- formal and actual parameters of a method call

```
string string
"drive" + "drink"
string

int string
42 + "the answer"
ERROR
```

# Type rules

- Specify for each operator
  - types of operands
  - type of result
- Basic types
  - building blocks for the type rules
  - example: int, boolean, (sometimes) string
- Type expressions
  - array types
  - function types
  - record types and classes

# Type rules

If E1 has type int and E2 has type int, then E1 + E2 has type int

E1: int E2: int

E1 + E2 : int

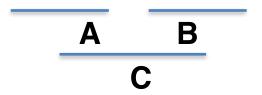
## Notations for rules

An inference rule consists of premises and conclusion

· An inference rule without any premises is an axiom



 A proof is a sequence of lines, each of which is either an axiom or follows from earlier lines by an inference rule



# More type rules

true : boolean false : boolean

int-literal: int string-literal: string

E1: int E2: int

E1 op E2: int

op ∈ { +, -, /, \*, %}

E1 op E2: boolean

E1 op E2 : boolean

op  $\in \{ <=,<,>,>= \}$ 

 $op \in \{ ==,!= \}$ 

# And even more type rules

E1[E2]: T

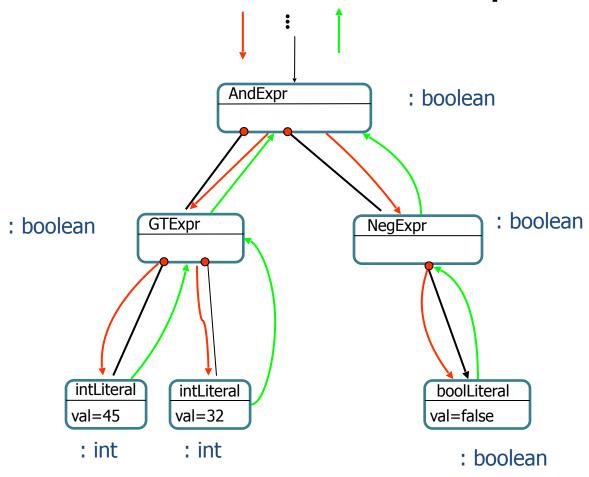
E1.length: int

new T[E1] : T[]

# Type Checking

- Traverse AST
- Assign types for AST nodes
- Use typing rules to compute node types

## Example



E1: boolean

E2: boolean

E1 op E2 : boolean

op  $\in \{ \&\&, || \}$ 

E1: boolean

!E1: boolean

E1: int

E2: int

E1 op E2: boolean

 $op \in \{ <=,<,>,>= \}$ 

false: boolean

int-literal: int

45 > 32 && !false

# Cool Types

- The types are
  - Class names
  - SELF\_TYPE
- The user declares types for identifiers
- The semantic analysis infers types for expressions
  - every expression has a unique type
- Cool type rules specify which operations are valid for which types
- The goal of type checking is to ensure that operations are used with the correct types
  - enforces intended interpretation of values
- Cool is statically typed: type checking during compilation

### Plan

- Cool type rules
- Implementing type checking for Cool

### Notations for rules

- A ⊢ B means "given A, it is provable that B"
- A ⊢ B is called a judgement
- A is called context or environment
- B is called statement

# Cool type judgements

Cool type rules have judgements of the form

$$\underbrace{O,M,C}_{\text{type environment}} \vdash e: T$$

- O gives types to free identifiers in the current scope
- M gives information about the formal parameters and return type of methods
- C is the class in which expression e appears

# Cool type environment

$$\underbrace{O,M,C}_{\text{type environment}} \vdash e: T$$

- O mapping Object Id's to types
  - symbol table for the current scope
  - O(x) = T
- M mapping methods to method signatures
  - M(C, f) = (A, B, D)
    means there is a method f(a:A, b:B): D defined in
    class C (or its ancestor)
- C the class in which expression e appears
  - used when SELF\_TYPE is involved

# Cool type environment

$$O,M,C \vdash e:T$$
type environment

- Why separate object/methods?
- In Cool, the method and object identifiers live in different name spaces

# Type rules

Rules are schemas for inferring types of expressions

O, M, K ⊢ e1 : Int

O, M, K ⊢ e2 : Int

O, M, K  $\vdash$  e1 + e2 : Int

O(id) = T

 $O,M,C \vdash int\_const : Int$ 

 $O,M,C \vdash id : T$ 

Infer types by instantiating the schemas

O, M, K ⊢ 1 : Int

O, M, K ⊢ 2 : Int

O, M,  $K \vdash 1 + 2 : Int$ 

O(y) = Int

 $O,M,C \vdash 1 : Int$   $O,M,C \vdash y : Int$ 

O, M,  $K \vdash y : Int$ 

O, M, K  $\vdash$  (1 + 2) : Int

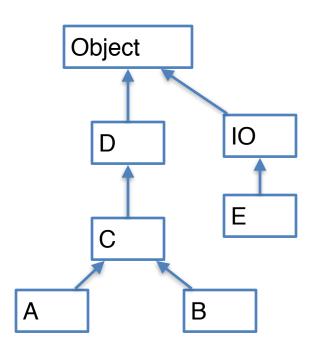
O, M,  $K \vdash y + (1 + 2)$ : Int

O,M,C ⊢ 2 : Int

# Subtyping

- Define a relation ≤ on classes
  - X ≤ X
  - X ≤ Y if X inherits from Y
  - $X \le Z$  if  $X \le Y$  and  $Y \le Z$

- Example
  - A ≤ C
  - B ≤ Object
  - E 
     ⊆ D and D 
     ⊆ E



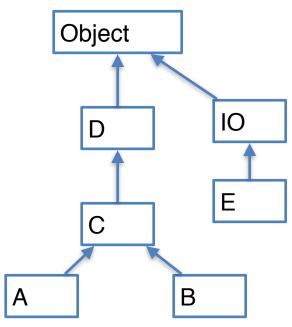
#### Least upper bounds

- Z is the least upper bound of X and Y
- lub(X, Y)=Z
  - $X \le Z$  and  $Y \le Z$ Z is **upper** bound
  - $X \le Z'$  and  $Y \le Z' \Rightarrow Z \le Z'$ 
    - Z is the **least** upper bound

### Least upper bounds

 In Cool, the least upper bound of two types is their least common ancestor in the inheritance tree

- Example
  - lub(A,B) = C
  - lub(C,D) = D
  - lub(C,E) = Object



## Type rules: Assign

```
O (x) = T0

O,M,K \vdash e1 : T1

\frac{T1 \leq T0}{O,M,K \vdash x \leftarrow e1 : T1} [Assign]
```

```
class A {
	foo(): A \{ ... \}
};
class B inherits A \{ \};
...
A
let x:B in x \leftarrow (new B).foo();
let x:A in x \leftarrow (new B).foo();
let x:Object in x \leftarrow (new B).foo();
```

ERROR OK OK

## Example: dynamic vs static types

A variable of static type A
 can hold the value of static type B if B ≤ A

## Types: dynamic vs static

- The dynamic type of an object is the class that is used in the new expression
  - a runtime notion
  - even languages that are statically typed have dynamic types

- The static type of an expression captures all the dynamic types that the expression could have
  - a compile-time notion

#### Soundness

- A type system is sound if for all expressions e dynamic\_type(e) ≤ static\_type(e)
- If the inferred type of e is T
  then in all executions of the program,
  e evaluates to a value of type ≤ T
- We only want sound rules
- But some sound rules are better than others

#### Let rule with initialization

```
\begin{array}{lll} O,M,K \vdash e_0\colon T \\ O(\textbf{T}/x) \vdash e_1 \colon T_1 & \text{[Let Weak Rule]} \\ O,M,K \vdash \text{let } x\colon T \leftarrow e_0 \text{ in } e_1 \colon T_1 & \text{class A } \{ \\ O,M,K \vdash e_0\colon T & \text{foo()}\colon C \; \{\; \dots\; \} \\ O(\textbf{T}_0/x) \vdash e_1 \colon T_1 & \text{} \}; \\ T \leq T_0 & \text{class B inherits A } \{\; \}; \\ O,M,K \vdash \text{let } x\colon T_0 \leftarrow e_0 \text{ in } e_1 \colon T_1 & \text{let } x\colon A \leftarrow \text{new B in } x.\text{foo()}; \end{array}
```

 Both rules are sound but the second one type checks more programs (using subtyping)

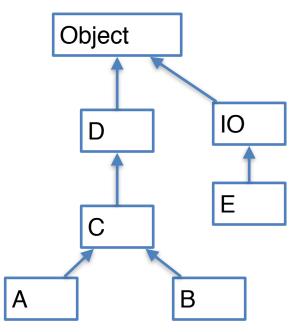
#### Conditional

```
O,M,K \vdash e_0 : Bool
O,M,K \vdash e_1 : T_1
O,M,K \vdash e_2 : T_2
O,M,K \vdash if e_0 then e_1 else e_2 fi : lub(T_1, T_2)
```

```
foo(a:A, b:B, c:C, e:E) : D {
   if (a < b) then e else c fi
}

ERROR
```

lub(E,C) = ObjectIs  $Object \le D$ ?

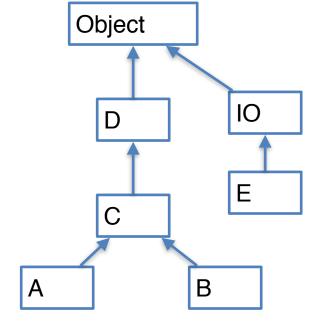


#### Case

```
O,M,K \vdash e : T
O[X/x],M,K \vdash e_1 : E
O[Y/y],M,K \vdash e_2 : F
O[Z/z],M,K \vdash e_3 : G
O,M,K \vdash case e of x: X =>e_1; y:Y =>e_2; z:Z=>e_3 esac : lub(E,F,G)
```

```
foo(d:D) : D {
  case d of
  x : IO => let a:A ← (new A) in x;
  y : E => (new B);
  z : C => z;
  esac
};

ERROR
```

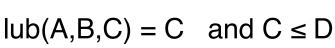


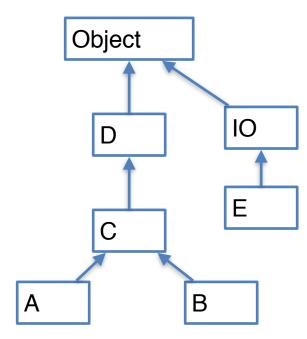
#### Case

```
O,M,K \vdash e : T
O[X/x],M,K \vdash e_1 : E
O[Y/y],M,K \vdash e_2 : F
O[Z/z],M,K \vdash e_3 : G
O[X/z],M,K \vdash e_3 : G
```

 $O,M,K \vdash case e of x: X =>e_1; y:Y =>e_2; z:Z=>e_3 esac: lub(E,F,G)$ 

```
foo(d:D) : D {
   case d of
   x : IO => let a:A ← (new A) in a;
   y : E => (new B);
   z : C => z;
   esac
};
   OK
```





## Cool type rules

- ✓ Arithmetic and boolean expressions
- √ Object identifiers
- √ Conditionals
- ✓ Let
- √ Case
- SELF\_TYPE and self
- Allocation: new
- Dispatch: dynamic and static
- Error handling

#### Motivation for SELF\_TYPE

- What can be the dynamic type of object returned by foo()?
  - any subtype of A

```
class A {
  foo() : SELF_TYPE { self } ;
};
class B inherits A { ... };
class Main {
     B x ← (new B).foo();
};
     OK
```

#### SELF\_TYPE

- Research idea
- Helps type checker to accept more correct programs
  - C,M,K ⊢ (new A).foo() : A
  - C,M,K ⊢ (new B).foo() : B
- SELF\_TYPE is NOT a dynamic type
  - Meaning of SELF\_TYPE depends on where it appears textually
  - SELF TYPE may refer to the class C in which it appears, or any subtype of C

### Where can SELF\_TYPE appear?

- Parser checks that SELF\_TYPE appears only where a type is expected
  - How ?
- But SELF\_TYPE is not allowed everywhere a type can appear

#### Where can SELF\_TYPE appear?

- class T1 inherits T2 { ... }
  - T1, T2 cannot be SELF\_TYPE
- x : SELF\_TYPE
  - attribute
  - let
  - not in case
- new SELF\_TYPE
  - creates an object of the same type as self
- e@T.foo(e1)
  - T cannot be SELF\_TYPE
- foo(x:T1):T2 {...}
  - only T2 can be SELF\_TYPE

### Example: new

## Subtyping for SELF\_TYPE

- SELF\_TYPEc ≤ SELF\_TYPEc
- SELF\_TYPEc ≤ C
- It is always safe to replace SELF\_TYPEc with C
- SELF\_TYPEc ≤T if C≤T
- T ≤ SELF\_TYPEc is always false
  - because SELF\_TYPEc can denote any subtype of C

## lub(T,T') for SELF\_TYPE

 lub(SELF\_TYPEc, SELF\_TYPEc) = SELF\_TYPEc

- lub(T, SELF\_TYPEc) = lub(T,C)
  - the best we can do

# Type rules for self and new

O, M, K ⊢ self : SELF\_TYPEk

O, M, K ⊢ new SELF\_TYPE : SELF\_TYPEk

#### Other rules

- A use of SELF\_TYPE refers to any subtype of the current class
- Except in dispatch
  - because the method return type of SELF\_TYPE might have nothing to do with the current class

### Dispatch

```
O, M, K \vdash c : C
```

O, M, 
$$K \vdash b : B$$

$$M(C, foo) = (A_1, B_1, D_1)$$

$$A \le A_1$$
,  $B \le B_1$ ,  $D_1 \ne SELF\_TYPE...$ 

```
O, M, K \vdash c.foo(a, b) : D_1
```

which class is used to find the declaration of foo()?

```
class C1 {
  foo(a:A1, b:B1) : D1 { new D1 ;
  };
};
class C inherits C1 {...};

E ...
  (new C).foo( (new A) , (new B) );
```

 $O, M, K \vdash c : C$ 

 $O, M, K \vdash a : A$ 

O, M, K ⊢ b : B

 $M(C, foo) = (A1, B1, SELF_TYPE)$ 

 $A \le A1$  ,  $B \le B1$ 

O, M, K  $\vdash$  c.foo(a, b): C

### Example: self

```
class A {
  foo(): A { self };
};
class B inherits A { ... }
...
(new A).foo(); returns A object
(new B).foo(); returns B object
```

### Static Dispatch

```
O, M, K \vdash c : C

O, M, K \vdash a : A

O, M, K \vdash b : B

M(C<sub>1</sub>, f) = (A<sub>1</sub>, B<sub>1</sub>, D<sub>1</sub>)

A \leq A<sub>1</sub>, B \leq B<sub>1</sub>, C \leq C<sub>1</sub>, D<sub>1</sub> \neq SELF_1 (new C)@C1.foo( (new A) , (new B) );
```

if we dispatch a method returning SELF\_TYPE in class C<sub>1</sub>, do we get back C<sub>1</sub>?

No. SELF\_TYPE is the type of self, which may be a subtype of the class in which the method appears

```
B));

O, M, K \vdash c : C

O, M, K \vdash a : A

O, M, K \vdash b : B

M(C<sub>1</sub>, f) = (A<sub>1</sub>, B<sub>1</sub>, SELF_TYPE)

A \leq A<sub>1</sub>, B \leq B<sub>1</sub>, C \leq C<sub>1</sub>

O, M, K \vdash c@C<sub>1</sub>.f@(a, b): C<sub>63</sub>
```

#### SELF\_TYPE Example

```
class A {
  delegate: B;
  callMe() | SELF_TYPE
       { delegate.callMe(); }; ERROR
};
class B {
  callMe(): SELF_TYPE { self };
};
class Main {
       A a \leftarrow (new A).callMe();
};
```

## **Error Handling**

- Error detection is easy
- Error recovery: what type is assigned to an expression with no legitimate type?
  - influences type of enclosing expressions
  - cascading errors

```
let y : Int \leftarrow x + 2 in y + 3
```

- Better solution: special type No\_Type
  - · inheritance graph can be cyclic

# Implementation of Cool Types

- How are types represented?
  - Symbol
  - compare types by comparing Symbol

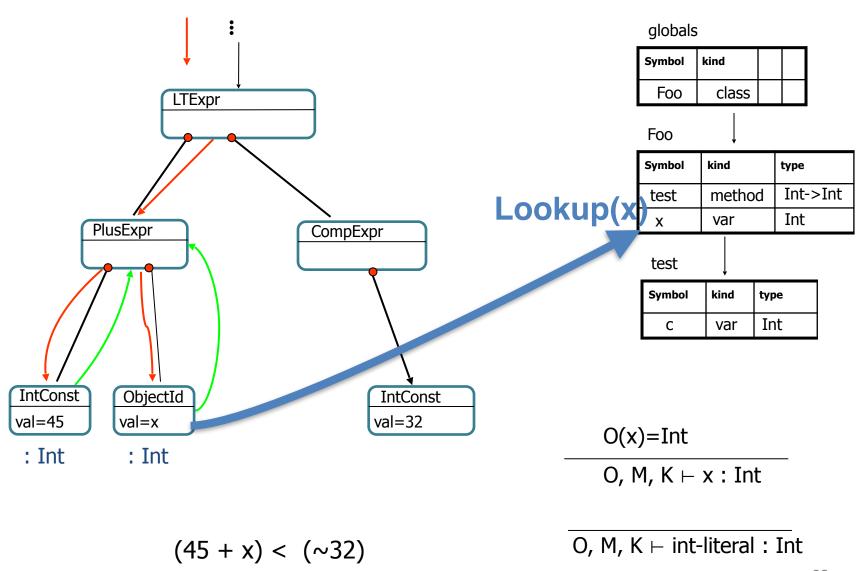
- When are types are created?
  - during lexer/parsing
  - predefined types

# Type Checking Implementation

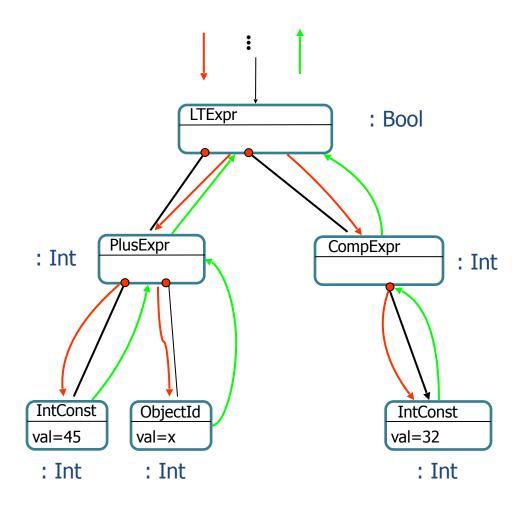
Single traversal over AST

- Types passed up the tree
- Type environment passed down the tree

#### Example



#### Example



O, M, K 
$$\vdash$$
 e1 : Int  
O, M, K  $\vdash$  e2 : Int  
O, M, K  $\vdash$  e1

O, M, K 
$$\vdash$$
 e1 : Int
O, M, K  $\vdash$  e2 : Int
O, M, K  $\vdash$  e1+e2 : Int

$$\frac{O(x)=Int}{O, M, K \vdash x : Int}$$

$$(45 + x) < (\sim 32)$$

O, M, 
$$K \vdash int-literal : Int$$

## Soundness of type rules

- Type is a set of values
- Soundness of static type rules
  - for every expression e, for every value v of e at runtime v ∈values\_of(static\_type(e))
  - static\_type(e) may actually describe more values
  - can reject correct programs
- More complicated with subtyping (inheritance)

#### Static type checking: pros and cons

- Catches many programming errors
- Proves properties of your code
- Avoids the overhead of runtime type checks
- Restrictive: may reject correct programs
- Rapid prototyping is difficult
- Complicates the programming language and the compiler
- In practice, most code is written in statically typed languages with escape mechanisms
  - Unsafe casts in C, Java
  - union in C

## **Types**

- Type checking
  - Static: C, Java, Cool, ML
  - Dynamic: machine code, scripting languages such as python, ruby
  - JavaScript is untyped
- Strong vs weak types (coercion)
  - Python is strongly typed
  - Perl is weakly typed

# Summary: type checking

- Type equivalence: nominal vs structural
- Expressions: locations (I-values) and values (r-values)
- Type coercions
- Types: strong vs weak types
- Types: dynamic vs static
- Type checking: dynamic vs static
- Type checking vs inference
- Subtyping relation and least upper bounds
- Soundness of type rules (conservative, provable)

#### Quick Quiz

- Which type rules use subtyping relation ≤ ?
- Which type rules use lub?
- Which type rules have a special case for SELF\_TYPE?
- Where can SELF\_TYPE appear in Cool program?
- How to extend subtying for SELF\_TYPE?