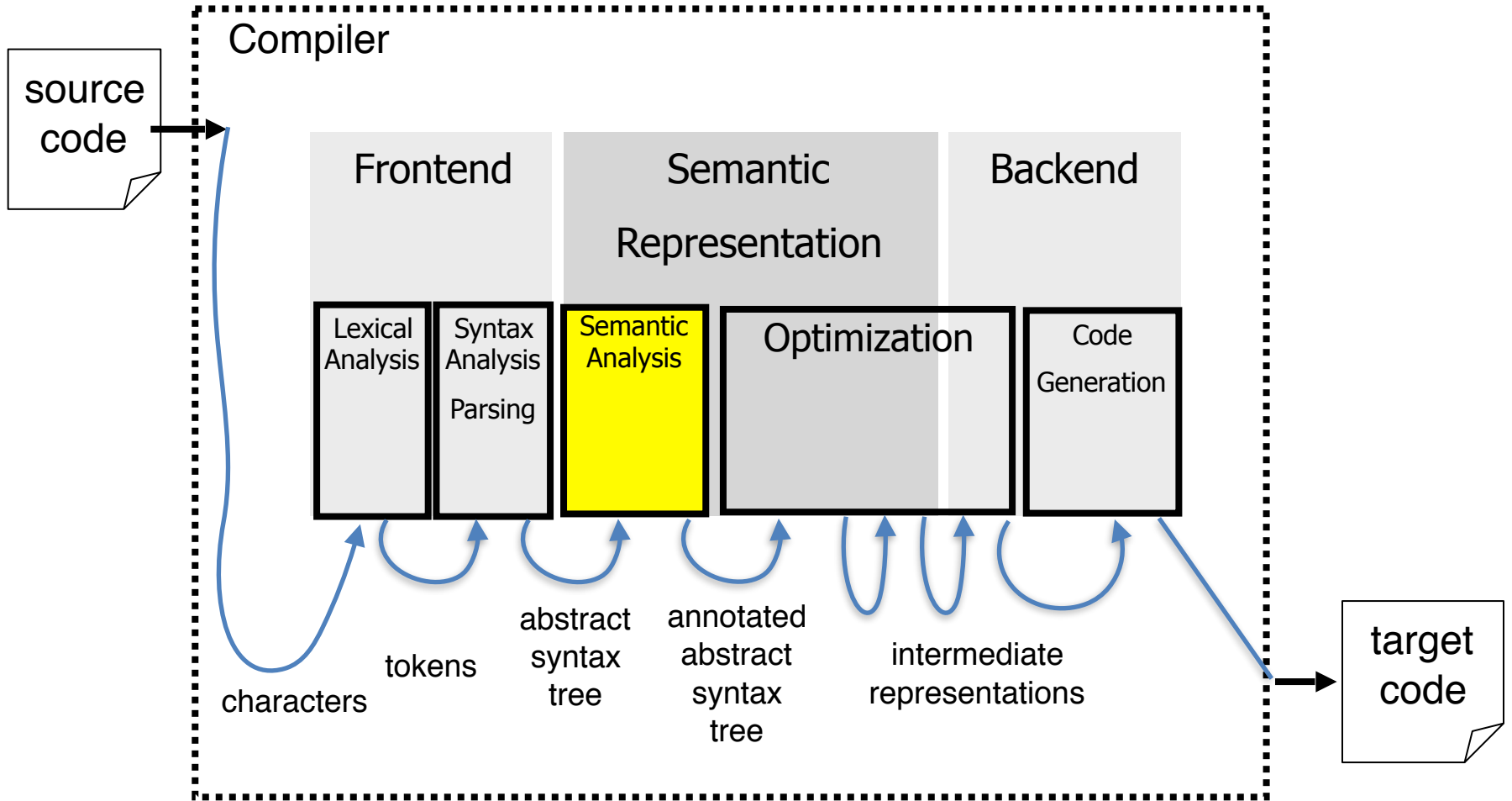


Cool Type Checking



Recap: semantic analysis

- Scope rules
- Symbol tables
- Inheritance graph
- Type is a set of values
- Type equivalence: nominal vs structural
- Expressions: locations (l-values) and values (r-values)
- Type coercions
- Types: strong vs weak types
- Type checking: dynamic vs static
- Type checking vs type inference
- Type rules
- Subtyping relation and least upper bounds

Cool type environment

$$\underbrace{O, M, C}_{\text{type environment}} \vdash e : T$$

- **O** mapping Object Id's to types
 - symbol table for the current scope
 - $O(x) = T$
- **M** mapping methods to method signatures
 - $M(K, f) = (A, B, D)$
means there is a method $f(a:A, b:B): D$ defined in class K (or its ancestor)
- **C** the class in which expression e appears
 - used when `SELF_TYPE` is involved

Soundness of type rules

- For every expression **e**, for every value **v** of **e** at runtime **v** \in **values_of(static_type(e))**
 - values_of(T) is the set of values represented by type T
 - static_type(e) is T when $O, M, C \vdash e : T$
 - static_type(e) may describe more values than e can have in any run
- Static typing can reject correct programs
- More complicated with subtyping (inheritance)

Static type checking: pros and cons

- Catches many programming errors
- Proves properties of your code
- Avoids the overhead of runtime type checks
- Restrictive: may reject correct programs
- Rapid prototyping is difficult
- Complicates the programming language and the compiler
- In practice, most code is written in statically typed languages with escape mechanisms
 - Unsafe casts in C, Java
 - union in C

Types in practice

- Type checking
 - Static: C, Java, Cool, ML
 - Dynamic: machine code, scripting languages (python, ruby)
 - JavaScript is untyped
- Strong vs weak types (coercion)
 - Python is strongly typed
 - Perl is weakly typed

Plan

- Cool type rules
- Implementing type checking for Cool

Type rules

- Rules are **schemas** for inferring types of expressions

$$O, M, C \vdash e1 : \text{Int}$$

$$O, M, C \vdash e2 : \text{Int}$$

$$O, M, C \vdash e1 + e2 : \text{Int}$$

$$O, M, C \vdash \text{int_const} : \text{Int}$$

$$O(\text{id}) = T$$

$$O, M, C \vdash \text{id} : T$$

- Infer types by instantiating the schemas

$$O, M, C \vdash 1 : \text{Int}$$

$$O, M, C \vdash 2 : \text{Int}$$

$$O, M, C \vdash 1 + 2 : \text{Int}$$

$$O, M, C \vdash 1 : \text{Int}$$

$$O(y) = \text{Int}$$

$$O, M, C \vdash y : \text{Int}$$

$$O, M, C \vdash y : \text{Int}$$

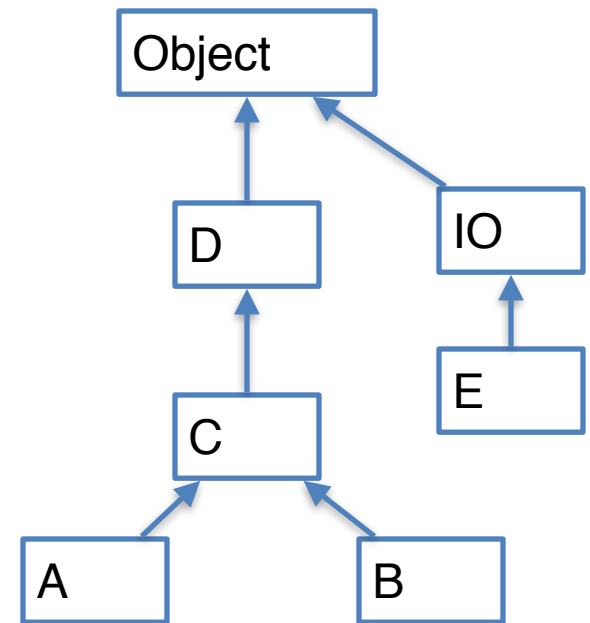
$$O, M, C \vdash (1 + 2) : \text{Int}$$

$$O, M, C \vdash y + (1 + 2) : \text{Int}$$

$$O, M, C \vdash 2 : \text{Int}$$

Subtyping

- Define a relation \leq on classes
 - $X \leq X$
 - $X \leq Y$ if X inherits from Y
 - $X \leq Z$ if $X \leq Y$ and $Y \leq Z$
- Example
 - $A \leq C$
 - $B \leq \text{Object}$
 - $E \not\leq D$ and $D \not\leq E$



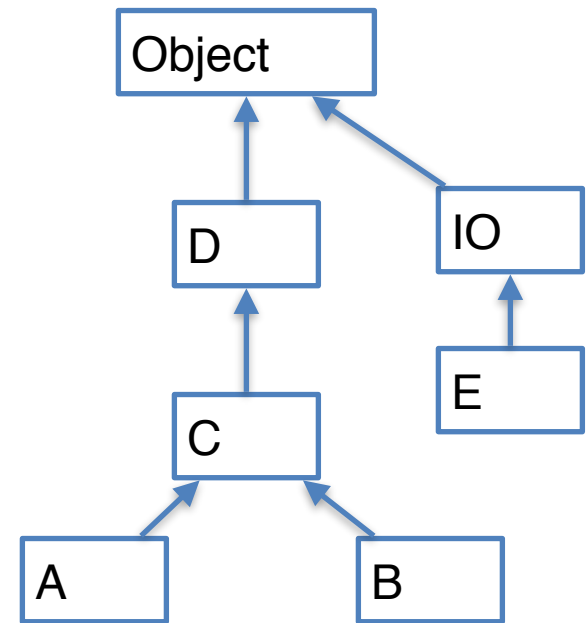
Least upper bounds

- Z is the least upper bound of X and Y
- $\text{lub}(X, Y) = Z$
 - $X \leq Z$ and $Y \leq Z$
 Z is **upper** bound
 - $X \leq Z'$ and $Y \leq Z' \Rightarrow Z \leq Z'$
 Z is the **least** upper bound

Least upper bounds

- In Cool, the least upper bound of two types is their **least common ancestor** in the inheritance tree

- Example
 - $\text{lub}(A, B) = C$
 - $\text{lub}(C, D) = D$
 - $\text{lub}(C, E) = \text{Object}$



Type rules: Assign

$$\frac{\begin{array}{l} O(x) = T_0 \\ O, M, K \vdash e_1 : T_1 \\ T_1 \leq T_0 \end{array}}{O, M, K \vdash x \leftarrow e_1 : T_1} \quad [\text{Assign}]$$

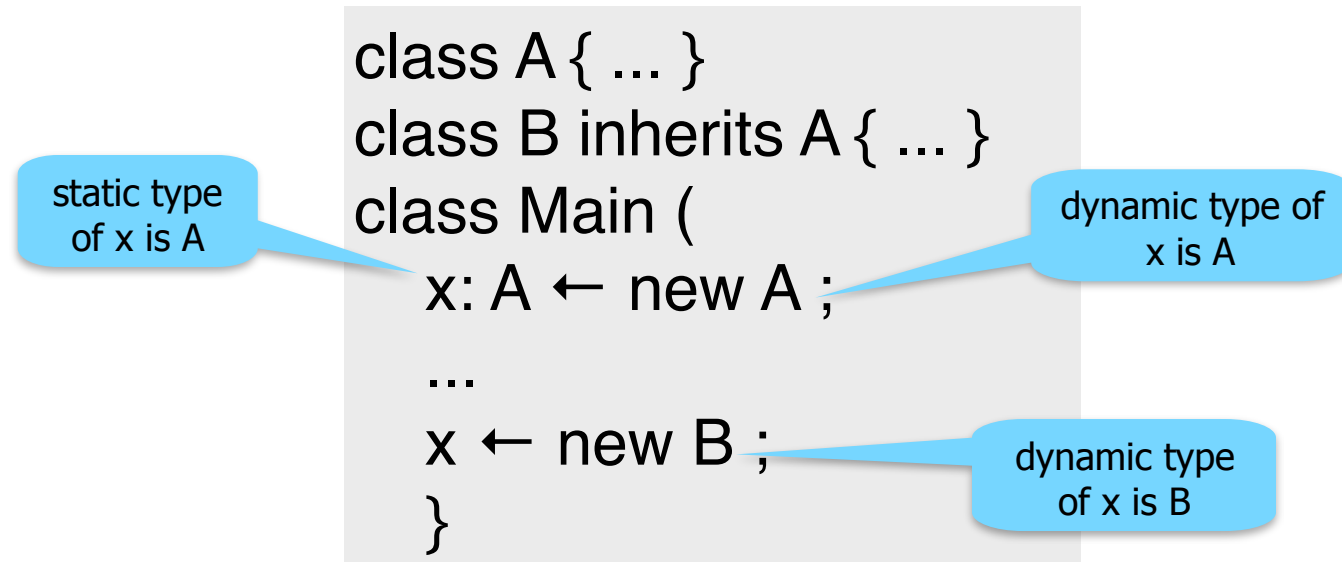
ERROR

OK

OK

```
class A {  
    foo() : A { ... }  
};  
class B inherits A { };  
...  
let x:B in x ← A(new B).foo();  
let x:A in x ← (new B).foo();  
let x:Object in x ← (new B).foo();
```

Example: dynamic vs static types



- A variable of static type A can hold the value of static type B if $B \leq A$

Types: dynamic vs static

- The **dynamic** type of an object is the class that is used in the new expression
 - a runtime notion
 - **even languages that are statically typed have dynamic types**
- The **static** type of an expression captures all the dynamic types that the expression could have
 - a compile-time notion

Soundness with subtyping

- A type system is **sound** if
for all expressions e
 $\text{dynamic_type}(e) \leq \text{static_type}(e)$
- If the inferred type of e is T
then in **all executions** of the program,
 e evaluates to a value of type $\leq T$
- We only want sound rules
- But some sound rules are better than others

Let rule with initialization

$O, M, K \vdash e_0 : T$

$O(\mathbf{T}/x) \vdash e_1 : T_1$ [Let Weak Rule]

$O, M, K \vdash \text{let } x : T \leftarrow e_0 \text{ in } e_1 : T_1$

```
class A {  
    foo():C { ... }  
};  
class B inherits A { };  
...  
let x:A ← new B in x.foo();
```

Let rule with initialization

$$\frac{\begin{array}{l} O, M, K \vdash e_0 : T \\ O(\mathbf{T}/x) \vdash e_1 : T_1 \end{array}}{O, M, K \vdash \text{let } x: T \leftarrow e_0 \text{ in } e_1 : T_1} \text{ [Let Weak Rule]}$$
$$\frac{\begin{array}{l} O, M, K \vdash e_0 : T \\ O(\mathbf{T}_0/x) \vdash e_1 : T_1 \\ T \leq T_0 \end{array}}{O, M, K \vdash \text{let } x: T_0 \leftarrow e_0 \text{ in } e_1 : T_1} \text{ [Let]}$$

```
class A {  
    foo():C { ... }  
};  
class B inherits A { };  
...  
let x:A ← new B in x.foo();
```

- Both rules are sound but the second one type checks more programs (using subtyping)

Conditional

$O, M, K \vdash e_0 : \text{Bool}$

$O, M, K \vdash e_1 : T_1$

$O, M, K \vdash e_2 : T_2$

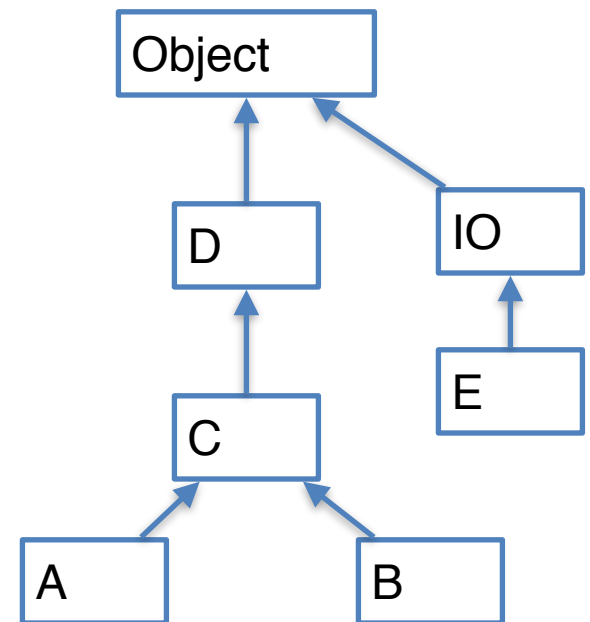
$O, M, K \vdash \text{if } e_0 \text{ then } e_1 \text{ else } e_2 \text{ fi} : \text{lub}(T_1, T_2)$

```
foo(a:A, b:B, c:C, e:E) : D {  
  if (a < b) then e else c fi  
}
```

ERROR

$\text{lub}(E, C) = \text{Object}$

Is $\text{Object} \leq D$?



Case

$O, M, K \vdash e : T$

$O[X/x], M, K \vdash e_1 : E$

$O[Y/y], M, K \vdash e_2 : F$

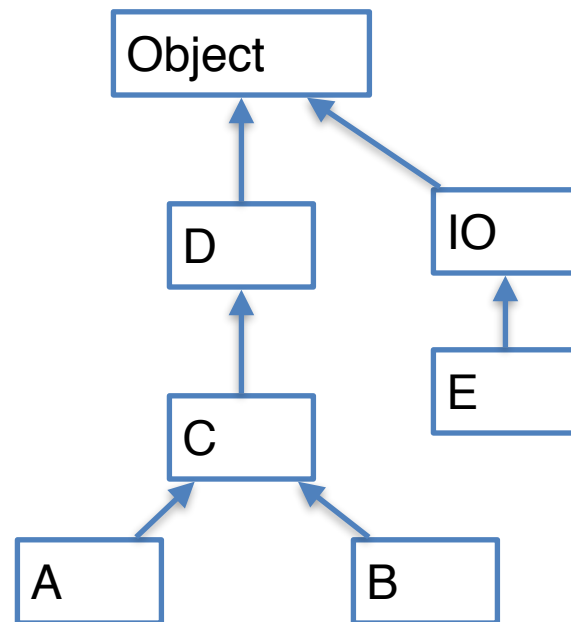
$O[Z/z], M, K \vdash e_3 : G$

$O, M, K \vdash \text{case } e \text{ of } x: X \Rightarrow e_1; y: Y \Rightarrow e_2; z: Z \Rightarrow e_3 \text{ esac} : \text{lub}(E, F, G)$

```
foo(d:D) : D {
  case d of
    x : IO => let a:A ← (new A) in x;
    y : E => (new B);
    z : C => z;
  esac
};
```

ERROR

$\text{lub}(\text{IO}, B, C) = \text{Object}$ and $\text{Object} \leq D$?



Case

$O, M, K \vdash e : T$

$O[X/x], M, K \vdash e_1 : E$

$O[Y/y], M, K \vdash e_2 : F$

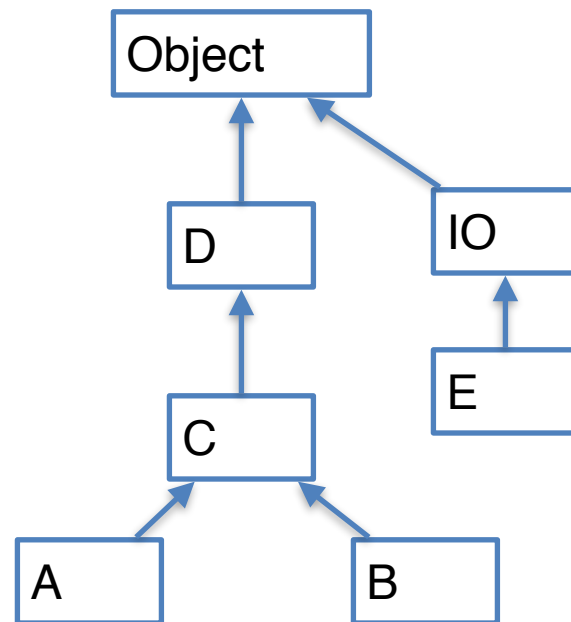
$O[Z/z], M, K \vdash e_3 : G$

$O, M, K \vdash \text{case } e \text{ of } x: X \Rightarrow e_1; y: Y \Rightarrow e_2; z: Z \Rightarrow e_3 \text{ esac} : \text{lub}(E, F, G)$

```
foo(d:D) : D {
  case d of
    x : IO => let a:A ← (new A) in a;
    y : E => (new B);
    z : C => z;
  esac
};
```

OK

$\text{lub}(A, B, C) = C$ and $C \leq D$



Cool type rules

- ✓ Arithmetic and boolean expressions
- ✓ Object identifiers
- ✓ Conditionals
- ✓ Let
- ✓ Case
 - SELF_TYPE and self
 - Allocation: new
 - Dispatch: dynamic and static
 - Error handling

Motivation for SELF_TYPE

- What can be the dynamic type of object returned by foo() ?
 - any subtype of A

```
class A {  
  foo() : A { self } ;  
};  
class B inherits A { ... };  
class Main {  
  B x ← (new B).foo();  
};
```

ERROR

```
class A {  
  foo() : SELF_TYPE { self } ;  
};  
class B inherits A { ... };  
class Main {  
  B x ← (new B).foo();  
};
```

OK

SELF_TYPE

- Research idea
- Helps type checker to accept more correct programs
 - $O, M, K \vdash (\text{new } A).\text{foo}() : A$
 - $O, M, K \vdash (\text{new } B).\text{foo}() : B$
- SELF_TYPE is **NOT a dynamic type**
 - Meaning of SELF_TYPE depends on where it appears textually
 - SELF TYPE may refer to the class K in which it appears, or any subtype of K

Where can SELF_TYPE appear?

- Parser checks that SELF_TYPE appears only where a type is expected (How ?)
- But SELF_TYPE is not allowed everywhere a type can appear

Where can SELF_TYPE appear ?

- `class T1 inherits T2 { ... }`
 - `T1`, `T2` cannot be `SELF_TYPE`
- `x : SELF_TYPE`
 - attribute
 - `let`
 - not in `case`
- `new SELF_TYPE`
 - creates an object of the same type as `self`
- `e@T.foo(e1)`
 - `T` cannot be `SELF_TYPE`
- `foo(x:T1):T2 {...}`
 - only `T2` can be `SELF_TYPE`

Example: new

```
class A {  
    foo() : A { new SELF_TYPE };  
};  
class B inherits A { ... }  
...  
(new A).foo();      creates A object  
(new B).foo();      creates B object
```

Subtyping for SELF_TYPE

- $\text{SELF_TYPE}_c \leq \text{SELF_TYPE}_c$
- $\text{SELF_TYPE}_c \leq C$
- It is always safe to replace SELF_TYPE_c with C
- $\text{SELF_TYPE}_c \leq T$ if $C \leq T$
- $T \leq \text{SELF_TYPE}_c$ is always false
 - because SELF_TYPE_c can denote **any** subtype of C

$\text{lub}(T, T')$ for SELF_TYPE

- $\text{lub}(\text{SELF_TYPE}_c, \text{SELF_TYPE}_c) = \text{SELF_TYPE}_c$
- $\text{lub}(T, \text{SELF_TYPE}_c) = \text{lub}(T, C)$
 - the best we can do

Type rules for **self** and **new**

$O, M, K \vdash \text{self} : \text{SELF_TYPE}_k$

$O, M, K \vdash \text{new SELF_TYPE} : \text{SELF_TYPE}_k$

Other rules

- A use of SELF_TYPE refers to any subtype of the current class
- Except in dispatch
 - because the method return type of SELF_TYPE might have nothing to do with the current class

Dispatch

$O, M, K \vdash c : C$

$O, M, K \vdash a : A$

$O, M, K \vdash b : B$

$M(C, \text{foo}) = (A_1, B_1, D_1)$

$A \leq A_1, B \leq B_1, D_1 \neq \text{SELF_TYPE}$

$O, M, K \vdash c.\text{foo}(a, b) : D_1$

which class is used to find
the declaration of foo() ?

```
class C1 {  
    foo(a:A1, b:B1) : D1 { new D1 ;  
};  
};  
class C inherits C1 {...};  
...  
(new C).foo( (new A) , (new  
B) );
```

$O, M, K \vdash c : C$

$O, M, K \vdash a : A$

$O, M, K \vdash b : B$

$M(C, \text{foo}) = (A_1, B_1, \text{SELF_TYPE})$

$A \leq A_1, B \leq B_1$

$O, M, K \vdash c.\text{foo}(a, b) : C$

Example: self

```
class A {  
    foo() : A { self };  
};  
class B inherits A { ... }  
...  
(new A).foo();      returns A object  
(new B).foo();      returns B object
```

Static Dispatch

$O, M, K \vdash c : C$

$O, M, K \vdash a : A$

$O, M, K \vdash b : B$

$M(C_1, f) = (A_1, B_1, D_1)$

$A \leq A_1, B \leq B_1, C \leq C_1, \mathbf{D_1 \neq SELF_TYPE}$

$O, M, K \vdash c@C_1.f(a, b): D_1$

if we dispatch a method
returning SELF_TYPE in
class C_1 , do we get back C_1 ?

No. SELF_TYPE is the type of self,
which may be a subtype of the class
in which the method appears

```
class C1 {  
  foo(a:A1, b:B1) : D1 { new D1 ; };  
};  
class C inherits C1 {...};  
...  
(new C)@C1.foo( (new A) , (new B) );
```

$O, M, K \vdash c : C$

$O, M, K \vdash a : A$

$O, M, K \vdash b : B$

$M(C_1, f) = (A_1, B_1, \mathbf{SELF_TYPE})$

$A \leq A_1, B \leq B_1, C \leq C_1$

$O, M, K \vdash c@C_1.f@(a, b): C$

SELF_TYPE Example

```
class A {  
  delegate : B;  
  callMe() SELF_TYPE  
    { delegate.callMe(); } ; ERROR  
};  
class B {  
  callMe() : SELF_TYPE { self };  
};  
class Main {  
  A a ← (new A).callMe();  
};
```

Error Handling

- Error detection is easy
- Error recovery: what type is assigned to an expression with no legitimate type ?
 - influences type of enclosing expressions
 - cascading errors

```
let y : Int ← x + 2 in y + 3
```

- Better solution: special type No_Type
 - inheritance graph can be cyclic

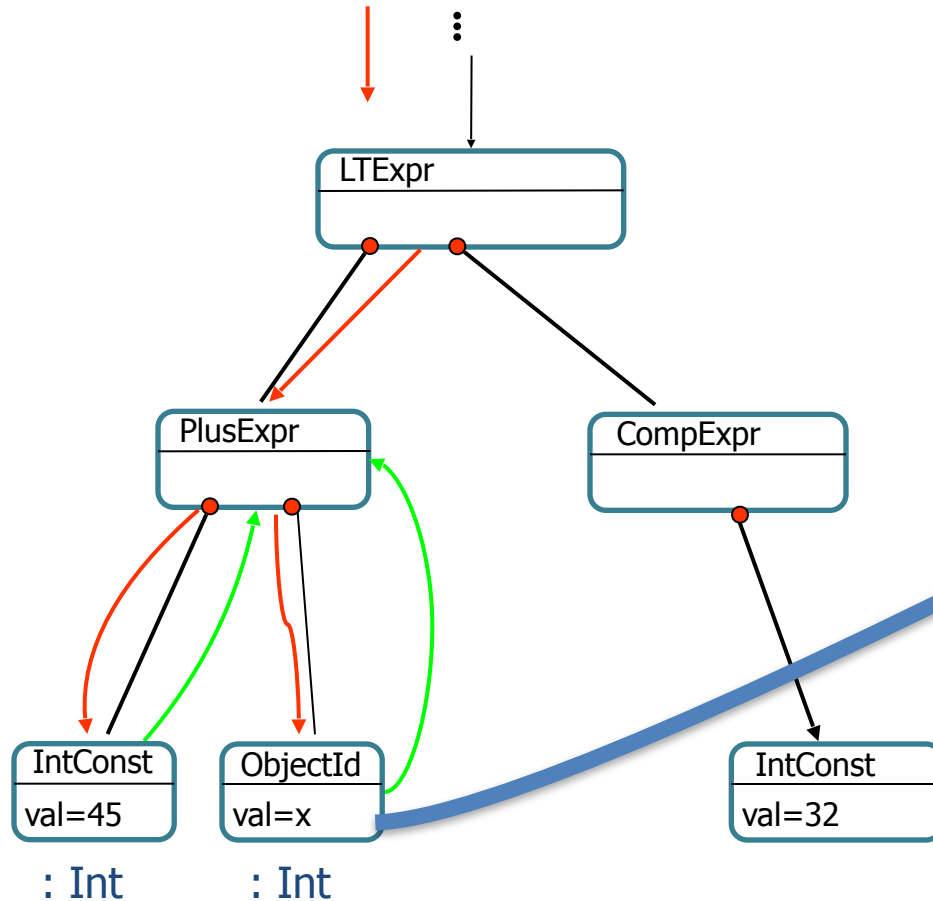
Implementation of Cool Types

- How are types represented ?
 - **Symbol**
 - compare types by comparing **Symbol**
- When are types are created?
 - during lexer/parsing
 - predefined types

Type checking implementation

- Single traversal over AST
- Types passed **up** the tree
- Type environment passed **down** the tree

Example



Lookup(x)

globals

Symbol	kind		
Foo	class		

Foo

Symbol	kind	type
test	method	Int->Int
x	var	Int

test

Symbol	kind	type
c	var	Int

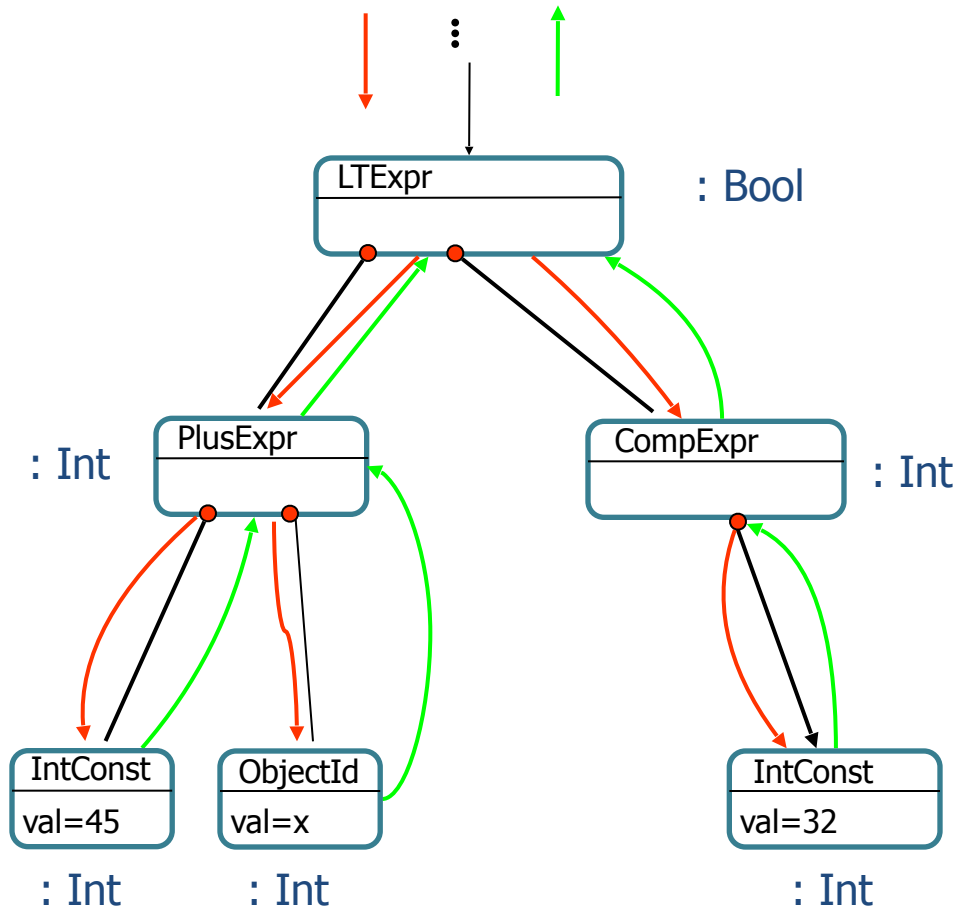
$O(x) = \text{Int}$

$O, M, K \vdash x : \text{Int}$

$O, M, K \vdash \text{int-literal} : \text{Int}$

$(45 + x) < (\sim 32)$

Example



$(45 + x) < (\sim 32)$

$$\frac{O, M, K \vdash e1 : \text{Int} \quad O, M, K \vdash e2 : \text{Int}}{O, M, K \vdash e1 < e2 : \text{Bool}}$$

$$\frac{O, M, K \vdash e : \text{Int}}{O, M, K \vdash \sim e : \text{Int}}$$

$$\frac{O, M, K \vdash e1 : \text{Int} \quad O, M, K \vdash e2 : \text{Int}}{O, M, K \vdash e1 + e2 : \text{Int}}$$

$$\frac{O(x) = \text{Int}}{O, M, K \vdash x : \text{Int}}$$

$$O, M, K \vdash \text{int-literal} : \text{Int}$$

Quick Quiz

- Which type rules use subtyping relation \leq ?
- Which type rules use lub?
- Which type rules have a special case for SELF_TYPE?
- Where can SELF_TYPE appear in Cool program?
- How to extend subtyping for SELF_TYPE?