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CS 321 Assignment 9

Section 10.1 (Question 1):

As stated within the textbook, 'a turing machine is an automaton whose temporary storage is a tape'. This said tape is then divided into cells, and each of these said cells can carry one symbol. Also connected with each of these tapes is a 'read-write head' which is capable of traveling either to the right or to the left on the tape, they are also capable of writing as well as reading one symbol on each and every move. The turing machine automaton neither has any special output mechanism nor an input file, instead this process is done on the machine's tape. As found in Theorem 10.1 of the textbook, a standard turing machine can simulate on each move either a change in the tape symbol, a read, a write, but never both.

A turing machine is formally defined as $M = (Q, \Sigma, \Gamma, \delta, q_0, \square, F)$.
$M \rightarrow$ the turing machine
$Q \rightarrow$ the set of internal states
$\Sigma \rightarrow$ the input alphabet
$\Gamma \rightarrow$ a finite set of symbols called the tape alphabet
$\delta \rightarrow$ the transition function
$q_0 \rightarrow$ the initial state
$\square \rightarrow$ the special symbol called the blank
$F \rightarrow$ the set of final states

The transition function δ is defined as: $Q \times \Gamma \rightarrow Q \times \Gamma \times \{L,R\}$.

As stated within Theorem 10.1 of the textbook, "to show the converse, let $M = (Q, \Sigma, \Gamma, \delta, q_0, \square, F)$ be a turing machine with a stay-option to be simulated by a standard turing machine $\widehat{M} = (\widehat{Q}, \Sigma, \Gamma, \widehat{\delta}, \widehat{q0}, \square, \widehat{F})$… If the move of M does not involve the stay-option, the simulating machine performs one move, essentially identical to the move to be simulated. If S is involved in the move of M, then $\widehat{M}$ will make two moves: the first will rewrite the symbol and move the read-write head right; the second will move the read-write head left, leaving the tape contents unaltered… every computation of M has a corresponding computation of $\widehat{M}$, so that $\widehat{M}$ can simulate M."

Section 10.3 (Question 2):

A two-stack non-deterministic pushdown automaton containing two independent stacks is dependent on the top elements contained on both of the stacks. These qualities are equivalent to that of a turing machine because everything on the tape before the head is classified as the first stack, resulting in the symbol following immediately after the head on the top of the stack. Another quality making a two-stack non-deterministic pushdown automaton equivalent to a turing machine is the fact that the symbol in the cell where the head is pointing to as well as all of the symbols contained within all of the remaining cells, all join together to form the second stack, this stack contains the symbol in which the head also points on top. Essentially, the first stack contains everything on the tape to the left of the current position, meanwhile the second stack contains everything on the tape to the right of the current position.

Section 10.4 (Question 3):

According to the textbook, “a set is said to be finite if it contains a finite number of elements; otherwise it is infinite.” Also stated within the exploration video regarding countability and uncountability, “an infinite set is countable if its elements have a one-to-one mapping to the positive integers, that is, there is an ordering, or enumeration procedure, for counting every element that belongs to the set.”

If S_1 and S_2 are both countable sets, then $S_1 \cup S_2$ must also be countable since $A \cup B$ essentially only contains what elements are present within both sets, then as long as at least one set either A or B is countable then the whole expression is countable regardless. Thus concluding that if S_1 and S_2 are both countable sets, then $S_1 \cup S_2$ must also be countable.

If S_1 and S_2 are both countable sets, then $S_1 x\ S_2$ must also be countable because all elements will eventually be listed. The statement that the cartesian product of countable sets can be easily proven with induction. For example if $S_1 = \{1,2,3\}$ and $S_2 = \{1,2\}$, which are both countable sets, then the cartesian product of the two sets would be equal to $\{(1,1), (1,2), (2,1), (2,2), (3,1), (3,2)\}$, which is also a countable set. Thus concluding that if S_1 and S_2 are both countable sets, then $S_1 x\ S_2$ must also be countable.