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CS 321 Assignment 4

Section 4.1 (Question 1):

The symmetric difference of two sets is a new set that contains every element that is in either set except for the elements that contained within both sets. Therefore the symmetric difference of the sets $\{3, 7, 2, 12, 9\}$ and $\{8, 12, 4, 16, 7, 5\}$, referred to as S1 and S2, can be defined as $S1 \ominus S2 = \{x : x \in S_1$ or $x \in S_2$ but x is not in both $S_1, S_2\}$ therefore $S_1 \ominus S_2 = (S_1 \cap \bar{S}_2) \cup (S_2 \cap \bar{S}_1)$. We also know that regular sets are closed under union with intersection and complement. Therefore if S1 and S2 are regular, then $S_1 \cap \bar{S}_2$ and $S_2 \cap \bar{S}_1$ are also both regular. And $S_1 \ominus S_2 = (S_1 \cap \bar{S}_2) \cup (S_2 \cap \bar{S}_1)$ is also regular. Therefore it can be concluded that a family of regular language is closed under symmetric difference.

Section 4.3 (Question 2):

According to the pumping lemma theorem, a language can be proved whether it is considered regular or not. If a language L is regular, then any string $w = xyz$ of L must satisfy. For every $i \geq 0$, $xyz \in L$. $|y| \geq 0$ and $|xy| \leq m$. Next we must prove that L is not considered to be regular. To do this, let's assume that L is regular. Considering that $w = a^m b^m$, $w \in L$ because in w the number of a's equals the number of b's. Now we must split w such that $x = a^{m-j}$, $y = a^j$, and $z = b^m$ where $j \geq 1$. w satisfies both conditions $|y| \geq 0$ and $|xy| \leq m$. Next we must check that for each $i \geq 0$ that $xyz \in L$. Assume that $i = 2$, then

$$\begin{aligned} xy^2z &= a^{m-j}(a^j)^2 b^m \\ &= a^{m-j}(a^{2j}) b^m \\ &= a^{m+j} b^m \end{aligned}$$

Since $a^{m+j} b^m \notin L$, our assumption is wrong, meaning that L is not regular. Therefore we have proven that $L = \{w : n_a(w) = n_b(w)\}$ is not regular.

Section 4.3 (Question 3):

Given the language $L = \{a^{2^n} \mid n \geq 0\}$, we can assume that L is a regular language. According to the pumping lemma theorem, the L is regular and so the pumping lemma must hold for any string w in L. Let p be the smallest integer such that $2^p > m$. If $w = xyz = a^{2^p}$ and $y = a^k$ for some $1 \leq k \leq m$. This means that the pumping strings will be $w_i = a^{2^p + (i-1)k}$ for some $i = 0, 1, 2, \ldots$ Therefore w_0 is the string that does not contain y. Since $2^p < 2^p + k \leq 2^p + m < 2^p + 2^p = 2^{p+1}$, then $w_2 = a^{2^p + k} \notin L$. This is because $|w_2|$ is between two consecutive powers of 2. Since this is a contradiction, we can confirm that L is not regular.