ALTERNATIVE CONTROL LAWS FOR AUTOMOTIVE ACTIVE SUSPENSIONS

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ABSTRACT

A two degree of freedom (1/4 car model) is used to evaluate alternative linear control laws. Control laws considered are full state feedback, sprung mass absolute velocity feedback and an LQG regulator using suspension defelction as the measurement. It is shown that all three can yield improvements to the sprung mass ride quality but that overall the LQG regulator using suspension deflection provides the best trade-off between ride quality, suspension packaging and road holding constraints.

I. Introduction

Electronic suspensions for automobiles have been considered for decades but it's been only recently that the automotive industry has begun to seriously consider modulated, semi-active, and active suspensions [1 - 3]. The Lotus company of England [3] has developed fully active hydraulic actuator suspensions that have been used on formula one race cars with impressive results. Clearly the increasing capability and decreasing cost of electronic components and sensors is motivating strong interest in "smart" suspensions. Early theoretical work on active vehicle suspensions (ref. [4-7]) identified the virtues of absolute or "skyhook" damping as opposed to conventional shock absorber damping (see Figure 1). The active damper using absolute velocity can put damping in the system at the natural frequency without causing high frequency "harshness" problems.

Although educational, the one degree of freedom model shown in Figure 1 can be missleading if conclusions about automotive suspensions are made from it. The two degree of freedom model shown in Figure 2 is called the "1/4 car model" and includes some important features of automotive suspensions, i.e., sprung and unsprung mass, suspension deflection ("rattle space") and tire force variations. Performance measures which must be considered are passenger ride quality, suspension travel and road holding ability. Passenger ride quality is clearly very subjective but it is generally accepted that the acceleration spectrum, particularly in the low frequency range (1—10 Hz) is important. Suspension travel or "rattle space" is important from a packaging point of view; the current trend toward larger tires and lower hood designs puts a premium on keeping, $z_s - z_u$, as small as possible.

The lateral and longitudinal forces available for cornering and/or braking depend on the normal load on the tire, clearly large dynamic variations in the normal tire force ($F_t = k_t(z_u - z_r)$) would be unacceptable. We will use the model shown in Figure 2 to evaluate alternative control laws for active suspensions.

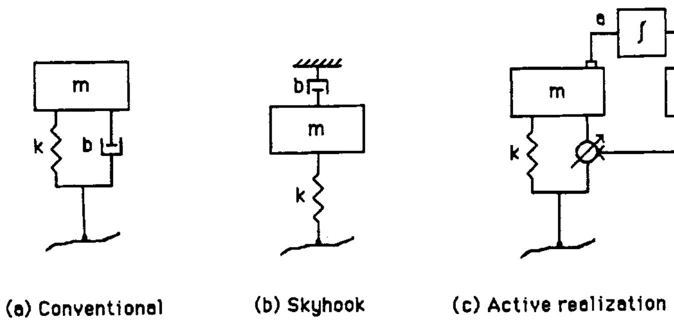

Figure 1 Alternative damping schemes

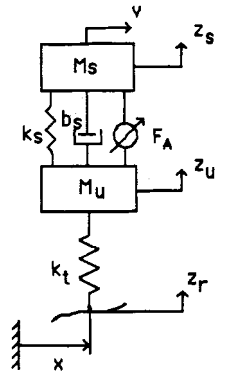

Figure 2 1/4 car model

II. 1/4 Car Model

Because of the arguments made in the previous paragraph the 1/4 car model has been used by many authors to investigate both passive and active automotive suspensions [9 - 12].

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If we assume that the tire does not leave the ground and that z_s and z_u are measured from the static equilibrium position then the linearized equations of motion are:

$$m_s \ddot{z}_s = k_s(z_u-z_s)+b_s(\dot{z}_u-\dot{z}_s)+f_a \tag{1}$$

$$m_u \ddot{z}_u = -k_s(z_u-z_s)-b_s(\dot{z}_u-\dot{z}_s)-f_a+k_t(z_r-z_u) \tag{2}$$

numerical values for a typical passenger car are given in Table 1 [10]

Table 1
VEHICLE PARAMETERS

m_s=240 Kg
m_u=36 Kg
b_s=980 $N{\cdot}sec/m$
k_s=16,000 N/m
k_t=160,000 N/m

A very common road input model that has been used by many authors [4,5,7,10,12] is to assume that the vehicle is traveling at a constant forward speed, V, and to model the road vertical velocity as white noise, i.e.,

$$E[\dot{z}_r(t)] = 0 \tag{3}$$

$$E[\dot{z}_r(t)\dot{z}_r(t+\tau)] = AV\delta(\tau)$$

Equation (3) implies that the road input displacement spectrum is:

$$\phi_{zr}(\omega)=AV/\omega^2 \tag{4}$$

where "A" is a roughness parameter and V is the vehicle speed. For detailed design studies more elaborate road input models can be used ([9, 12]) but the model described by equations (3) or equivalently (4) is adequate to investigate generic properties of active suspensions.

A convenient choice for state variables is,

$x_1 \equiv z_s - z_u$
$x_2 \equiv \dot{z}_s$
$x_3 \equiv z_u - z_r$
$x_4 \equiv \dot{z}_u$

In state variable form we have,

$$\underline{\dot{x}}=\underline{A}\ \underline{x}+\underline{B}\ f_a+\underline{\Gamma}\dot{z}_r \tag{5}$$

where f_a is the scalar active force and A, B, and Γ are the appropriate matrices which are obvious from equation (1) and (2). Using the parameters given in Table 1, the passive system (f_a=0) has the following eigenvalues:

sprung mass mode : $\omega_n = 1.255\ Hz, \xi= .22$
unsprung mass mode : $\omega_n = 11\ Hz, \xi=.20$

III. General Properties

Invariant Point

An interesting property of automotive suspensions (i.e., where the actuator is placed between the sprung and unsprung mass) can be seen by adding equations (1) and (2),

$$m_s \ddot{z}_s+m_u \ddot{z}_u = k_t(z_r-z_u) \tag{6}$$

Equation (6) is independent of k_s, b_s, and f_a. Transforming (6) and setting initial conditions to zero yields:

$$m_s\ddot{z}_s(j\omega) + (k_t-m_u\omega^2)z_u(j\omega)=k_t z_r(j\omega) \tag{7}$$

clearly at the unsprung natural frequency ($\omega=\sqrt{k_t/m_u}$) the sprung mass acceleration ($\ddot{z}_s$) is independent of z_u. If we define,

$$H_A(j\omega)\equiv\frac{\ddot{z}_s(j\omega)}{\dot{z}_r(j\omega)} \tag{8}$$

then we have the results that for $\omega=\sqrt{k_t/m_u}$,

$$|H_A|=\sqrt{k_t m_u}/m_s \tag{9}$$

Equation (9) partly explains the result to be seen later in this paper that active suspensions have very little influence on passenger acceleration at frequencies near the "tire-hop" mode.

Transfer Functions

Three transfer functions are of interest and will be defined as:

$$H_A(s)\equiv\frac{\ddot{z}_s(s)}{\dot{z}_r(s)}$$

$$H_{RS}(s)\equiv\frac{z_s(s)-z_u(s)}{\dot{z}_r(s)}$$

$$H_{TD}(s)\equiv\frac{z_u(s)-z_r(s)}{\dot{z}_r(s)}$$

which are the acceleration, rattle space and tire deflection transfer functions respectively.

First we will consider a full state feedback law for f_a, i.e.,

$$f_a=-g_1x_1-g_2x_2-g_3x_3-g_4x_4 \tag{10}$$

It is straightforward to derive the following transfer functions ([12]):

$$H_A(s)=\frac{s\left\{m_u g_3 s^2+(b_s-g_4)k_t s+(k_s+g_1)k_t\right\}}{d(s)} \tag{11}$$

$$d(s)\equiv m_u m_s s^4+((b_s+g_2)m_u+(b_s-g_4)m_s)s^3+((k_s+g_1)m_u+(k_t+k_s+g_1-g_3)m_s)s^2+(b_s+g_2)k_t s+(k_s+g_1)k_t \tag{12}$$

$$H_{RS}(s)=\frac{(g_3m_u-(k_t-g_3)m_s)s-(g_2+g_4)k_t}{d(s)} \tag{13}$$

$$H_{TD}(s)=-\frac{\alpha(s)}{d(s)} \tag{14}$$

$$\alpha(s)\equiv m_u m_s s^3+((b_s-g_4)m_s+(b_s+g_2)m_u)s^2+(k_s+g_1)(m_u+m_s)s$$

Some interesting asymptotic properties can be derived from equations (11) - (14)

$$\lim_{s\to 0} H_A(s)=s \tag{15}$$

$$\lim_{s\to\infty} H_A(s)=\left(\frac{g_3}{m_s}\right)\frac{1}{s} \tag{16}$$

Equation (15) shows that the low frequency acceleration asymptote is independent of both the passive and active parameters. The high frequency asymptote depends on the active gain, g_3 , and rolls off at 20 db/decade. It is interesting to note that the passive system (f_a=0) has a high frequency asymptote,

$$\lim_{s\to\infty} H_A(s) = \left(\frac{k_t b_s}{m_u m_s}\right)\frac{1}{s^2} \tag{17}$$

that rolls off at 40 db/decade. A comparison of (16) and (17) shows that the presence of g_3 leads to a high frequency "harshness" when compared to the passive system. It will be shown later that setting g_3 to zero does not adversely affect the low frequency performance and eliminates the high frequency "harshness."

The rattle space asymptotes are:

passive:

$$\lim_{s\to 0} H_{RS}(s) = -\frac{m_s s}{k_s} \tag{18}$$

$$\lim_{s\to\infty} H_{RS}(s) = -\left(\frac{k_t}{m_u}\right)\frac{1}{s^3} \tag{19}$$

active:

$$\lim_{s\to 0} H_{RS}(s) = -\frac{g_2+g_4}{k_s+g_1} \tag{20}$$

$$\lim_{s\to\infty} H_{RS}(s) = \left[\frac{g_3 m_u-(k_t-g_3)m_s}{m_u m_s}\right]\frac{1}{s^3} \tag{21}$$

The high frequency asymptotes have the same roll-off rate but the low frequency characteristics are entirely different. Equation (20) shows a general property that full state feedback and absolute velocity feedback control laws have, that of a constant low frequency asymptote whereas the passive systems decreases at low frequencies. Since the road input is modeled as white in $\dot{z}_r$, all frequencies contribute equally to the total r.m.s values, thus this low frequency characteristic will lead to larger rattle space for the state feedback and absolute velocity feedback systems.

The tire deflection asymptotes for both passive and active suspensions are:

$$\lim_{s\to 0} H_{TD}(s) = -\frac{(m_u+m_s)s}{k_t} \tag{22}$$

$$\lim_{s\to\infty} H_{TD}(s) = -1/s \tag{23}$$

It can be seen from equations (22) and (23) that both the high and low frequency asymptotes are independent of the active suspension force.

In this paper we also consider dynamic compensators that use suspension deflection as the measured input. Consider a system as shown in Figure 2 with $b_s=0, k_s\neq 0$ and,

$$f_a(s) \equiv k(s)\cdot(z_u(s)-z_s(s)) \tag{24}$$

where the dynamic compensator is defined by,

$$k(s) \equiv \frac{n_c(s)}{d_c(s)} = \frac{\bar{k}(s^{m_c}+\cdots+\beta_0)}{(s^{n_c}+\cdots+\alpha_0)} \tag{25}$$

for realizable compensators, $n_c \geq m_c$. The asymptotic properties of the acceleration transfer function are

$$\lim_{s\to 0} H_A(s) = s$$

$$\lim_{s\to\infty} H_A(s) = \begin{cases} \dfrac{k_s}{m_s m_u s^3} & \text{for } n_c > m_c \\[2ex] \dfrac{k_s+\bar{k}}{m_s m_u s^3} & \text{for } n_c = m_c \end{cases}$$

Thus the dynamic compensator has the same asymptotic properties as the passive suspension.

As a baseline case for comparison purposes Figures 3,4,5 show the passive acceleration, rattle space and tire deflection transfer functions for the perameters given in table 1.

IV. Full State Feedback

As has been done by many authors [5,8,10,11,12] linear optimal control theory has been used to design full state feedback laws. A quadratic performance index,

$$J = \lim_{T\to\infty}\frac{1}{T}E\left\{\int_0^T \left(\ddot{z}_s^2+\rho_1(z_s-z_u)^2+\rho_2\dot{z}_s^2+\rho_3(z_u-z_r)^2+\rho_3(z_u-z_4)^2+\rho_4\dot{z}_u^2+\rho_5 f_a^2\right)dt\right\} \tag{26}$$

is defined and then the weighting factors $(\rho_1, \ldots, \rho_5)$ are chosen to emphasize either ride quality, suspension travel or tire deflection.

The active force, f_a, which minimizes (26) is the well known linear quadratic regulator problem [13,5,10,11,12] whose solution results in a state feedback law (equation (10)) whose gains $(g_1, \cdots, g_4)$ depend on the weighting factors $(\rho_1, \cdots, \rho_5)$.

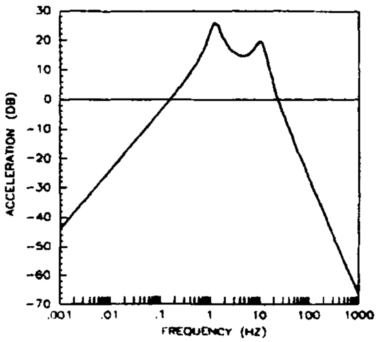

Figure 3. Passive Acceleration Transfer Function

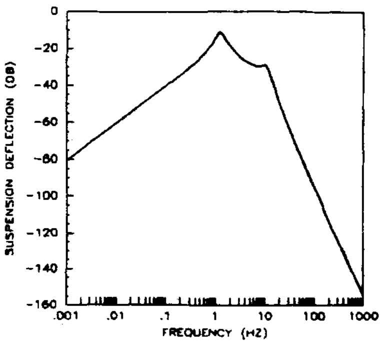

Figure 4. Passive Suspension Deflection Transfer Function

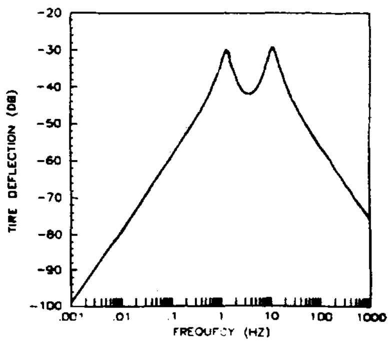

Figure 5. Passive Tire Deflection Transfer Function

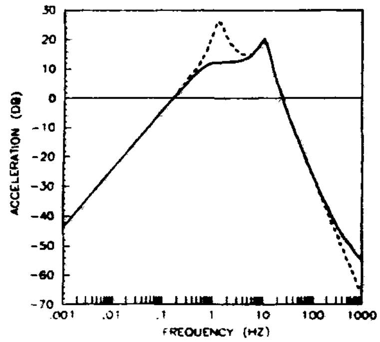

Figure 6. LQR Acceleration Transfer Function

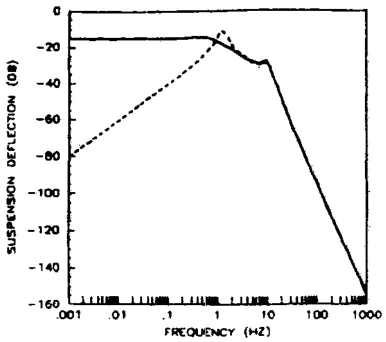

Figure 7. LQR Suspension Delflection Transfer Function

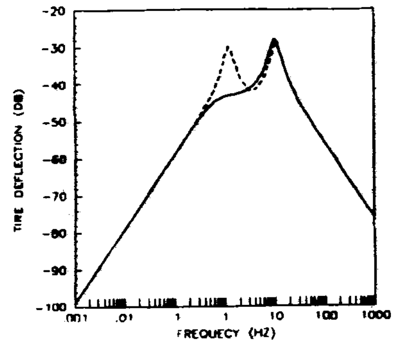

Figure 8. LQR Tire Deflection Transfer Function

Figures 6 - 8 shows the transfer functions for a design that emphasized ride quality. Several interesting characteristics can be pointed out. Figure 6 shows that the active suspension greatly improves the $1Hz$ region while the invariant point ($10.6Hz$) eliminates any effect at the tire-hop mode. Figure 6 also shows that the high frequency performance of the active system is worse than the passive system. Several authors have pointed out that this high frequency harshness is due to the term, $-g_3x_3$, and that g_3 can be set to zero without hurting any of the transfer functions shown in figures 6 - 8. This is fortunate since x_3 (tire deflection) would be hard to measure. Figure 7 illustrates a property of all control laws using absolute sprung mass velocity feedback, a constant low frequency asymptote, this could lead to higher R.M.S. suspension values unless some filtering is added. Figure 8 shows the tire deflection transfer function has been improved at $1Hz$ only.

Figure 9 shows a full state design where ρ_3 (weighting factor on tire deflection) has been substantially increased, clearly the tire force variations at the sprung and unsprung frequency have been reduced; however, this has been done at the expense of greatly increasing the high frequency ride quality harshness (not shown).

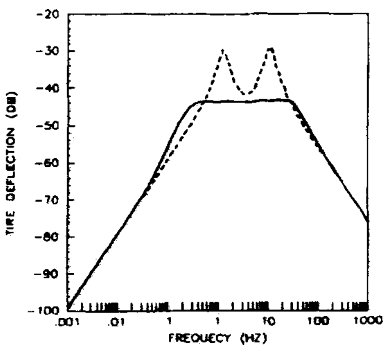

Figure 9. LQR Design for Road Holding

V. Absolute Velocity Feedback

Since most of the performance improvements in ride quality at $1Hz$ are due to the absolute sprung mass velocity feedback term it makes sense to consider a much simpler active control law:

$$f_a = -g_2x_2$$

Increasing g_2 results in a substantial increase in damping of the $1Hz$ mode with very little deterioration of the wheel-hop mode.

In fact the three transfer functions of interest plotted for g_2=3000 $N.sec/m$ would lay right on top of the fully active Figures (6-8), i.e., absolute velocity feedback captures all the nice properties of the full state ride quality design without the high frequency harshness problem. It also has the advantage of being very easy to design since there is only one parameter to choose. The only negative aspects are the need for a possibly expensive accelerometer and the low frequency suspension transfer function problem. In practice the latter problem would be aleviated by bandpass filtering the acceleration signal thus very little low frequency content would be in the filtered integrated acceleration signal.

VI. Dynamic Compensator Design using Suspension Deflection

The previous sections have looked at full state feedback and absolute velocity feedback. This section looks at what can be done if only a measurement of $z_s - z_u$ (suspension deflection) is available. Clearly this is what a passive spring and damper uses and yields results like those in Figures 3-5. Figure 10 shows that by softening k_s from 16,000 N/m to 2000 N/m that we can achieve excellent low frequency acceleration reduction, unfortunately the required suspension travel increases substantially. This is essentially the "Boulevard Ride" property of American luxury cars. Such a soft suspension results in excessive pitch during braking or acceleration and generally produces poor handling. Therefore, this is not a good option.

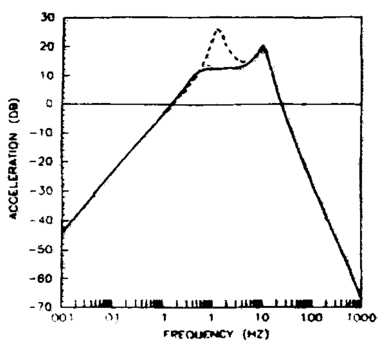

Figure 10. Passive Acceleration Transfer Function Using Soft Spring

Another option is to design a dynamic compensator with suspension travel as the measurement. One popular method for the design of dynamic compensators that behave like full state feedback designs is the LQG method [12,13]. Briefly the problem statement and solution is:

$$\dot{\underline{x}}=\underline{A}\underline{x}+\underline{B}f_a+\underline{\Gamma}\dot{z}_r \tag{5}$$

$$f_a=-g_1\hat{x}_1-g_1\hat{x}_2-g_3\hat{x}_3-g_4\hat{x}_4 \tag{27}$$

$$\dot{\hat{\underline{x}}}=\underline{A}\hat{\underline{x}}+\underline{B}f_a+\underline{H}(y_m-\underline{C}_m\hat{\underline{x}}) \tag{28}$$

where it has been assumed that $\dot{z}_r$ is white noise (equation (3)), y_m is the measured variable, in this case suspension defelction, i.e.,

$$y_m=z_s-z_u+v(t)=x_1+v(t)\equiv\underline{C}_m\underline{x}+v(t) \tag{29}$$

$$\underline{C}_m\equiv[1,0,0,0]$$

and $v(t)$ is the sensor noise which we will assume to be white, i.e.,

$$E[v(t)v(t+\tau)]\equiv\mu\delta(\tau) \tag{30}$$

The optimal control problem is to choose $g_{1,...,}g_4$ and $\underline{H}=(h_1,h_2,h_3,h_4)^T$ to minimize J (equation (26)). The well known LQG solution results in the same control gains as before and $\underline{H}$ (called the Kalman Filter gains) depending only on the system parameters ($\underline{A}\ \underline{B}\ \underline{C}_m$), the road input (A,V) and the sensor noise (μ). For our designs we will use μ as a parameter to shape the three transfer functions of interest along with the weighting factors $\rho_{1,...,}\rho_5$. The overall compensator transfer function between the active force, $f_a(s)$, and the sensor output, $y_m(s)$, is:

$$\frac{f_a(s)}{y_m(s)}=-\underline{G}(sI-\underline{A}+\underline{B}\ \underline{G}+\underline{H}\ \underline{C}_m)^{-1}\underline{H} \tag{31}$$

where
$\underline{G}\equiv(g_1,g_2,g_3,g_4)^T.$

The LQG design procedure is guaranteed to produce closed-loop stable designs if the system is "controllable" and "observable" [13] through f_a and y_m respectively which it is for the system described by equations (5) and (29).

Figures 11,12,13 show the results of an LQG design to emphasize ride quality. Note that the acceleration and tire deflection transfer functions are essentially the same as those for full state feedback (Figures 6,8) while the suspension deflection transfer function has been considerably improved (Figure 12 vs 7), i.e., the LQG compensator provides in a systematic way the same filtering that an "ad-hoc" bandpass filtering of the acceleration signal would.

VII. Conclusions

Three alternative control law formulations for active automotive suspensions have been considered. The active suspension problem is difficult since one control must provide a trade-off between ride quality, suspension travel, and road holding ability. It was shown that both full state feedback and absolute sprung mass velocity feedback could be designed to provide significant improvements in ride quality (near $1Hz$) without degrading road holding, however, some additional filtering would be required to keep the suspension travel from becoming too large. It was shown that designs which emphasize road holding rely on feeding back tire deflection which is very difficult to measure and which causes high frequency harshness problems in the aceleration transfer function.

The best overall designs were achieved by the LQG compensator using the easiest and most inexpensive variable to measure, suspensions deflection. In a controls sense the system using actuator force as the control and suspension deflection as the measurement is both controllable and observable thus we can design for either ride quality or road holding (or alternatively switch between the two depending on operating conditions). It was shown that the design for ride quality captured all the nice features of the full state design and eliminated or reduced some of the bad features (e.g., acceleration harshness and low frequency suspension asymptotes).

Acknowledgements

This research was supported in part by the Mazda Motor Corporation.

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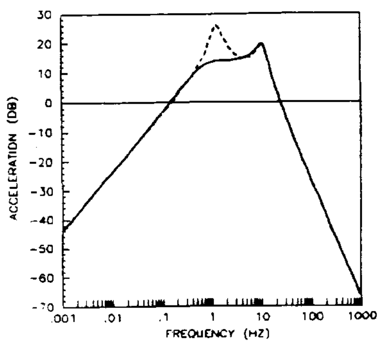

Figure 11. LQG Acceleration Transfer Function

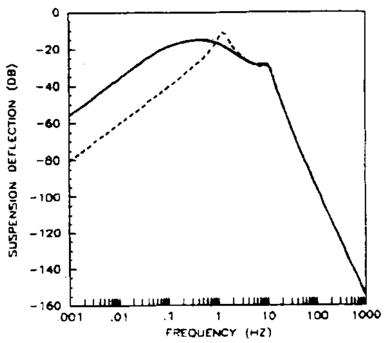

Figure 12. LQG Suspension Deflection Transfer Function

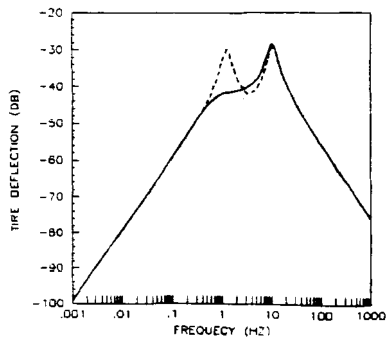

Figure 13. LQG Tire Deflection Transfer Function

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