Redefining Risk in Data-Poor Fisheries

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Supplemental Information

Contents

1 Revisiting existing thresholds

This supplement begins with an exposition of the existing thresholds. In no previous works have the derivation of these thresholds been given, and it is worth calculating these thresholds to verify their use as part of previous implementations of the PSA. The equations to follow generalize the thresholds to be used for any number of subdivisions of the PSA plot into N parts of equal area. Because the area of the plot is fixed at 4 units, setting the threshold equations equal to $\frac{4n}{N}$ gives the desired threshold values $T_n \in T_1, T_2, \ldots, T_{N-1}$. Threshold calculations are given by the piecewise function

$$\frac{4n}{N} = \begin{cases} 1 - \sqrt{T_n^2 - 1} + \frac{T_n^2}{4}(\pi - 4csc^{-1}(T_n)) & T_n \leq \sqrt{10}, \frac{n}{N} \leq 0.659 \\ T_n^2(sin^{-1}(\frac{\sqrt{T_n^2-9}+9}{T_n\sqrt{10}}) - \frac{\pi}{4} - sin^{-1}(\frac{1}{\sqrt{10}})) + 3\sqrt{T_n^2 - 2(\sqrt{T_n^2 - 9} + 4)} - 2 & T_n > \sqrt{10}, \frac{n}{N} > 0.659 \end{cases} \tag{1}$$

PSA plots are traditionally divided by two thresholds into 3 regions of equal area. Vulnerability scores falling below the first threshold signal a stock of low vulnerability, scores between the first and second threshold indicate of stock of medium vulnerability, and scores above the second threshold indicate a highly vulnerable stock. These two thresholds are traditionally given as 2.64 and 3.18, yet no formal calculation exists in the literature. Likely these values were informally approximated through simulation. These approximations are close to the actual values that can be calculated by solving the piecewise function above. The first threshold (T_1) is found by solving:

$$\frac{4}{3} = 1 - \sqrt{T_1^2 - 1} + \frac{T_1^2}{4}(\pi - 4csc^{-1}(T_1)) \tag{2}$$

$$T_1 = 2.645$$

The second threshold (T_2) is found by solving:

$$\frac{8}{3} = T_2^2(sin^{-1}(\frac{\sqrt{T_2^2 - 9} + 9}{T_2\sqrt{10}}) - \frac{\pi}{4} - sin^{-1}(\frac{1}{\sqrt{10}})) + 3\sqrt{T_2^2 - 2(\sqrt{T_2^2 - 9} + 4)} - 2 \tag{3}$$

$$T_2 = 3.173$$

Upon calculation, we find that only small error exists in original approximations of threshold values. The original thresholds were approximations given that vulnerability (V) is calculated as the distance from the origin of points plotted on a PSA plot with axes from 1 to 3 for productivity (P) and susceptibility (S). The origin here is $(0, 0)$, not the origin of the plot $(1, 1)$. The equation for the distance of each point (S_k, P_k) from the origin $(0, 0)$ is

$$V_k = \sqrt{S_k^2 + P_k^2} \tag{4}$$

Calculating vulnerability this way produces thresholds given by circles centered at $(0, 0)$ rather than thresholds given by circles centered at the plot origin $(1, 1)$. This effect is noticed by some, who correctly change the calculation for vulnerability to

$$V_k = \sqrt{(S_k - 1)^2 + (P_k - 1)^2} \tag{5}$$

which shifts the values vulnerability can take to $[0, 2\sqrt{2}]$ rather than $[\sqrt{2}, 3\sqrt{2}]$. Here, P_k is the productivity value for each stock subtracted from 4 to scale axes from 1 to 3. Of course, now the previously calculated thresholds no longer apply if equation 5 is used to calculate vulnerability. There is no simple transformation to make equations 4 and 5 comparable, so provided below are the equations for this formulation of the PSA.

Thresholds for vulnerability can be calculated for equation 5 by solving the piecewise function

$$\frac{4n}{N} = \begin{cases} \frac{\pi T_n^2}{4} & T_n \leq 2, \frac{n}{N} \leq \frac{\pi}{4} \\ \frac{\pi T_n^2}{4} + 2T_n\sqrt{1 - \frac{4}{T_n^2}} - T_n^2 cos^{-1}(\frac{2}{T_n}) & T_n > 2, \frac{n}{N} > \frac{\pi}{4} \end{cases} \tag{6}$$

Dividing the newly scaled PSA plot with axes from 0 to 2 into thirds yields two new thresholds. The first threshold (T_1) is found by solving:

$$\frac{4}{3} = \frac{\pi T_1^2}{4} \tag{7}$$
$$T_1 = 1.303$$

The second threshold (T_2) is found by solving:

$$\frac{8}{3} = \frac{\pi T_2^2}{4} \tag{8}$$

$$T_2 = 1.843$$

To plot these thresholds on a PSA plot with axes from 1 to 3, an arc of radius $T_n + 1$ can be drawn. The thresholds calculated above would take the values $T_1 = 2.303, T_2 = 2.843$. These two thresholds are shown in Figure S1.

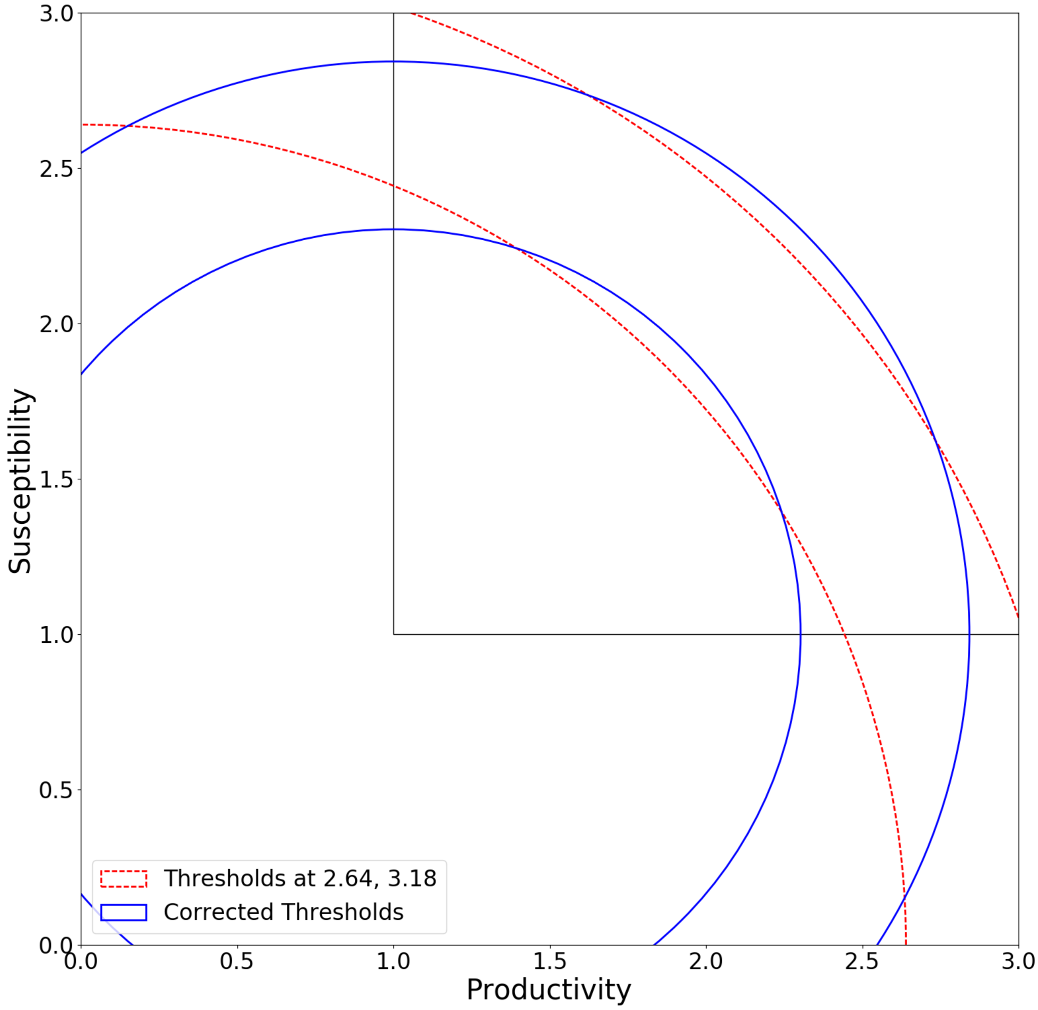

Figure S1: PSA plot in the upper right corner of a square plot with axes from 0 to 3. The existing thresholds are concentric circles centered at the origin (0,0), though some studies calculated vulnerability as the distance from (1,1). The thresholds at V=2.64, 3.18 no longer apply, and the corrected thresholds are given to show the difference in shape. None of these thresholds are valid in the new method.

2 Examples of threshold derivations for the rPSA

Below describes how to use this framework in two example cases to illustrate its use. The first example case applies this method in a simple form where productivity and susceptibility are calculated as arithmetic means for each respective set of attributes. Each attribute is weighted equally, and productivity and susceptibility are composed of equal number of attributes. The second case includes use of weighted attributes, susceptibility and productivity differ in numbers of attributes, and those attributes are assumed multiplicative for susceptibility and additive for productivity.

2.1 Example 1: a simple case

In this first example, productivity is composed of 10 unweighted attributes and susceptibility is composed of 10 unweighted attributes. Both sets are additive, and their means are arithmetic. Because there should be equal probability of assignment of values 1, 2, or 3 to attributes in the null case for step 1:

$$\hat{\mu}_p = \hat{\mu}_s = 2 \qquad\qquad SE_p = SE_s = \sqrt{\frac{2}{3*10}} = \frac{1}{\sqrt{15}}$$

$$\mathbf{\Sigma} = \begin{pmatrix} (\frac{1}{\sqrt{15}})^2 & 0 \\ 0 & (\frac{1}{\sqrt{15}})^2 \end{pmatrix} = \begin{pmatrix} \frac{1}{15} & 0 \\ 0 & \frac{1}{15} \end{pmatrix}$$

The risk axis can be defined by the equation:

$$S - 2 = \frac{\frac{1}{\sqrt{15}}}{\frac{1}{\sqrt{15}}}(P-2) \qquad \Rightarrow \qquad S = P$$

When $SE_p = SE_s$, the risk axis will pass through the origin $(0,0)$. Because the risk axis in this case passes through $(0,0)$, it can be represented simply by any multiple of $[1 \;\; 1]^T$. Projection of the 2-dimensional Gaussian onto the risk axis gives:

$$proj_{\boldsymbol{r}}(\boldsymbol{X}) = \frac{1}{2}\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}\boldsymbol{X}$$

The distance from $\boldsymbol{\mu}$ after projection onto the risk axis is

$$D(\boldsymbol{x_k}) = \frac{[1 \;\; 1]^T \cdot [P_k - \hat{\mu}_p \;\; S_k - \hat{\mu}_s]^T}{\sqrt{1^2+1^2}} = \frac{P_k + S_k - 4}{\sqrt{2}}$$

The resulting standard error is calculated as

$$\hat{SE_r} = \frac{\sqrt{2}(\frac{1}{\sqrt{15}})^2}{\sqrt{(\frac{1}{\sqrt{15}})^2 + (\frac{1}{\sqrt{15}})^2}} = \frac{1}{\sqrt{15}}$$

One can observe that given equal standard error on each coordinate axis, the projection onto the risk axis results in equivalent standard error. To divide the newly linear data into equally probable thirds, we simply multiply this standard error by ± 0.431.

$$T_1 = \frac{-0.431}{\sqrt{15}} \qquad T_2 = \frac{0.431}{\sqrt{15}}$$

We define risk categories as

Low	Medium	High
$D(\boldsymbol{x_k}) \leq \frac{-0.431}{\sqrt{15}}$	$\frac{-0.431}{\sqrt{15}} < D(\boldsymbol{x_k}) < \frac{0.431}{\sqrt{15}}$	$\frac{0.431}{\sqrt{15}} \leq D(\boldsymbol{x_k})$
$\frac{P_k+S_k}{\sqrt{15}} \leq \frac{-0.431\sqrt{2}}{15} + \frac{4}{\sqrt{15}}$	$\frac{-0.431\sqrt{2}}{15} + \frac{4}{\sqrt{15}} < \frac{P_k+S_k}{\sqrt{15}} < \frac{0.431\sqrt{2}}{15} + \frac{4}{\sqrt{15}}$	$\frac{0.431\sqrt{2}}{15} + \frac{4}{\sqrt{15}} \leq \frac{P_k+S_k}{\sqrt{15}}$

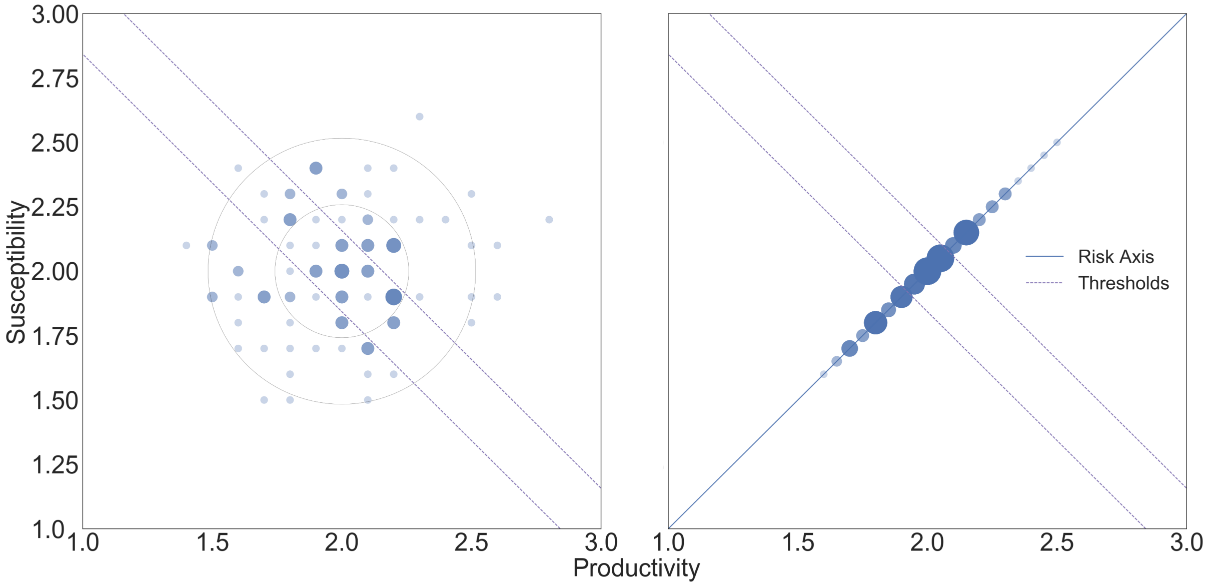

Figure S2: 100 simulated species plotted on a PSA plot (left). P and S are arithmetic means of 10 attribute values each. Circles represent 1 and 2 z-scores from the bivariate mean. A risk axis is defined and the species are orthogonally projected onto this line (right). Those further to right along the line are higher risk. Thresholds are drawn to divide the distribution of species in thirds by probability.

2.2 Example 2: a complex case

In this second example, productivity is composed of 10 weighted attributes and susceptibility is composed of 12 weighted attributes. Productivity is additive, while susceptibility is multiplicative. Here I show how to handle this more complex case.

Let's set the productivity weights as [1, 2, 2, 2, 4, 2, 3, 2, 3, 2] and the susceptibility weights as [2, 2, 2, 3, 4, 4, 1, 2, 1, 2, 2, 3]. As discussed above, the mean for productivity axis is arithmetic, while the mean for the susceptibility axis is geometric.

$$\hat{\mu}_p = 2 \qquad\qquad \hat{\mu}_s = \frac{log(6)}{3}$$

The standard error of the weighted mean is given as

$$\hat{SE}_p = (\frac{2}{3}\sum_{i=1}^{10} w_i^2)^{1/2} = (\frac{2(1^2 + 6*2^2 + 2*3^2 + 4^2)}{3*23^2})^{1/2} = 0.273$$
$$\hat{SE}_s = (0.2058\sum_{j=1}^{12} w_j^2)^{1/2} = 0.2058(\frac{2*1^2 + 6*2^2 + 2*3^2 + 2*4^2}{28^2})^{1/2} = 0.141$$

$$P_k = \sum_{i=1}^{N} w_i p_{i,k} \qquad\qquad S_k = \sum_{j=1}^{N'} w_j log(s_{j,k})$$

$$\mathbf{\Sigma} = \begin{pmatrix} 0.273^2 & 0 \\ 0 & 0.141^2 \end{pmatrix} = \begin{pmatrix} 0.074 & 0 \\ 0 & 0.020 \end{pmatrix}$$

The risk axis can be defined by the equation:

$$S - \frac{log6}{3} = \frac{0.273}{0.141}(P-2)$$

When this line does not pass through $(0,0)$, projection on the 2-dimensional Gaussian onto the line requires an extra transformation after multiplication with the projection matrix to recenter the mean. Projection of the 2-dimensional Gaussian onto the risk axis gives:

$$proj_{\boldsymbol{r}}(\boldsymbol{X}) = \begin{pmatrix} 0.211 & 0.408 \\ 0.408 & 0.788 \end{pmatrix}(\boldsymbol{X} - \boldsymbol{\mu}) + \boldsymbol{\mu}$$

The distance from $\boldsymbol{\mu}$ after projection onto the risk axis is

$$D(\boldsymbol{x_k}) = \frac{[0.141 \quad 0.273]^T \cdot [P_k - \hat{\mu}_p \quad S_k - \hat{\mu}_s]^T}{\sqrt{0.141^2 + 0.273^2}} = \frac{0.141P_k + 0.273S_k - 0.445}{0.307}$$

The resulting standard error is calculated as

$$\hat{SE}_{\boldsymbol{r}} = \frac{\sqrt{2}(0.273)(0.141)}{\sqrt{(0.273)^2 + (0.141)^2}} = 0.177$$

To divide the newly linear data into equally probable thirds, we simply multiply this standard error by ± 0.431.

$$T_1 = -0.431 * 0.177 = -0.076 \qquad T_2 = 0.431 * 0.177 = 0.076$$

We define risk categories as

Low	Medium	High
$D(\boldsymbol{x_k}) \leq -0.076$	$-0.076 < D(\boldsymbol{x_k}) < 0.076$	$0.076 \leq D(\boldsymbol{x_k})$
$0.141P_k + 0.273S_k \leq 0.422$	$0.422 < 0.141P_k + 0.273S_k < 0.469$	$0.469 \leq 0.141P_k + 0.273S_k$

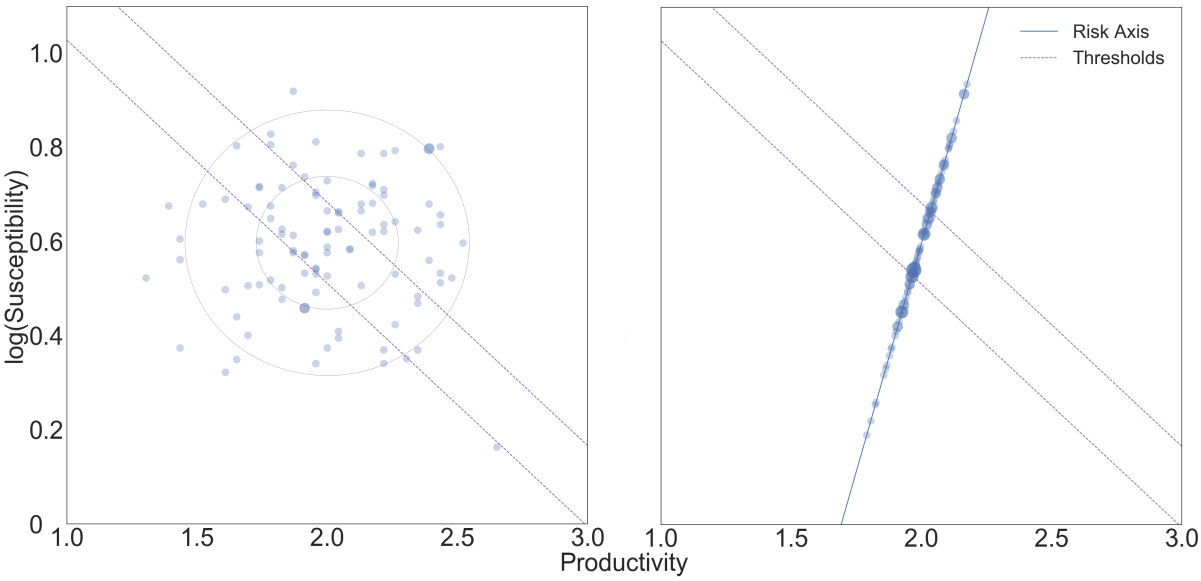

Figure S3: 100 simulated species plotted on a semilog PSA plot (left). P is the weighted arithmetic mean of 10 attribute values, and S is the weighted geometric mean of 12 attribute values. Ellipses represent 1 and 2 z-scores from the bivariate mean. A risk axis is defined and the species are orthogonally projected onto this line (right). Those further to right along the line are higher risk. Thresholds are drawn to divide the distribution of species in thirds by probability. Note that it is the log transformation of the susceptibility axis that produces what appears to be a non-orthogonal projection and skewed thresholds in the right panel.

3 Case study details

Data from 5 empirical case studies used in the analysis and results were collected. In the Patrick et al. NOAA assessment, 10 Productivity and 12 Susceptibility attributes are used to derive Productivity and Susceptibility scores. This data, including labeled attributes is available at https://github.com/grewelle/rPSA. We have also listed the attributes below:

Productivity	**Susceptibility**
Growth Rate (r)	Management Strategy
Maximum Age	Areal Overlap
Maximum Size	Geographic Concentration
vob Bertalanffy Growth Coefficient (k)	Vertical Overlap
Estimated Natural Mortality	Fishing rate relative to M
Measured Fecundity	Biomass of Spawners
Breeding Strategy	Seasonal Migrations
Recruitment Pattern	Schooling-Aggregation
Age at Maturity	Morphology Affecting Capture
Mean Trophic Level	Post Release Mortality
	Value of the Fishery
	Fishery Impact to EFH

We have performed a comparison analysis in Fig. S4 across the four possible treatments of each axis.

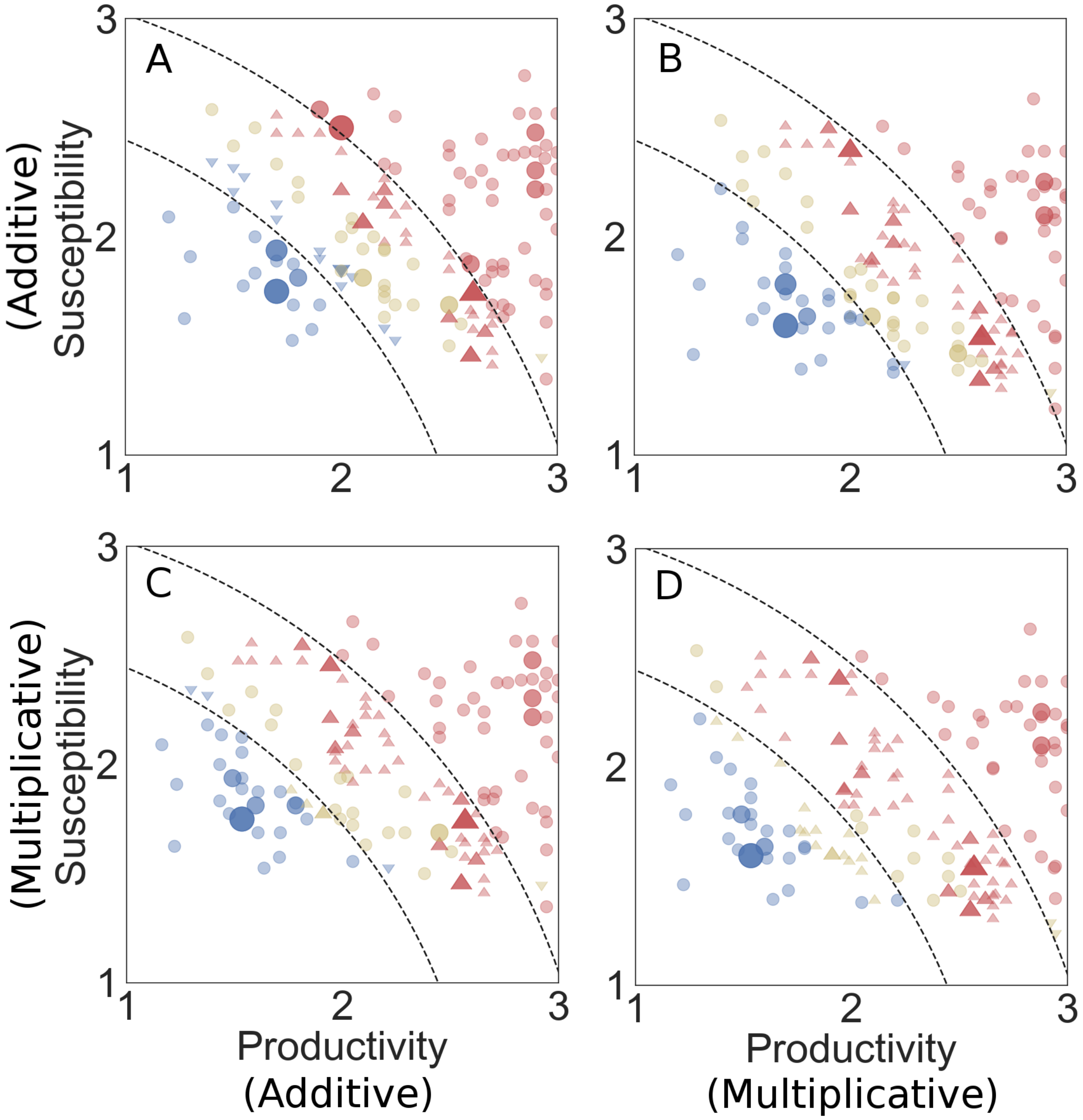

Figure S4: Patrick et al. comparison of the sPSA and rPSA across the four permutations of model choices (additive and multiplicative) for the axes. Down triangles represent lower risk categorization with the rPSA, circles represent the same categorization, and up triangles represent higher risk categorization compare to the sPSA. Colors give the categorization per the rPSA (blue = low, yellow = medium, red = high). Dotted lines show the sPSA thresholds. (A) Both susceptibility and productivity are additive – 16 species categorized lower, 42 higher, 108 same with rPSA; (B) susceptibility is additive, productivity is multiplicative – 2 lower, 61 higher, 103 same; (C) susceptibility is multiplicative, productivity is additive – 4 lower, 63 higher, 99 same; (D) both are multiplicative – 2 lower, 85 higher, 79 same.

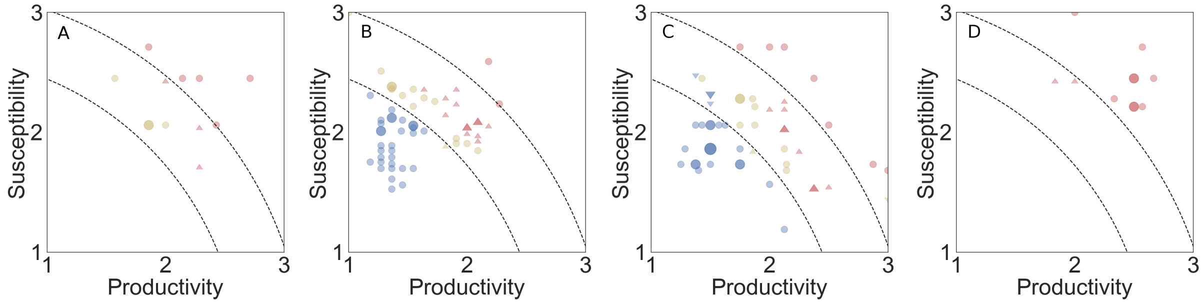

Figure S5: 4 additional case studies evaluated in the main text. (A) Cotter et al., Sri Lanka; (B) Dee et al., aquaria; (C) Lin et al., Taiwan; (D) Ponton-Cevallos et al., Galapagos. Each study varies in species composition and scoring procedure, producing different layouts of points on the plot, which influences the changes observed when applying the rPSA. Point shapes, sizes, and colors are represented according to the system specified in the caption of figure S4.

Figure 6 in the main text reports the changes observed when analyzing all 5 empirical case studies with the sPSA and rPSA. These studies varied in geographical location, species studied and attributes used. Below are the species included in each study.

Cotter et al.
Swordfish
Barracuda
Flying Fishes
Kingfish
Sailfish
Sea Barramundi
Travelly/ Jackfish
Bream
Flying fish
Skipjack Tuna
Yellow fin tuna
Dolphin fish
Dee et al.
Angelfish
Azure demoiselle
Banggai cardinalfish, Indonesia
Bicolor dottyback
Blacktail humbug

Blue devil damselfish
Blueband goby
Brown tang
Butterflyfish
Caerulean damselfish
Canary wrasse
Citron goby
Clarkii clownfish
Cleaner wrasse
Dogface puffer
Dragon wrasse
Elegant firefish
Engineer goby
Fairy wrasses
Fire clownfish
Firefish goby
Flame damselfish
Goldtail demoiselle
Green chromis
Harlequin sweetlips
Koran angelfish
Lagoon triggerfish
Lawnmower blenny
Longhorn cowfish
Lyretail anthias
Mandarinfish
Maroon clownfish
Moon wrasse
Needlespine coral goby
Neon damselfish
Panther grouper
Pennant coralfish

Porcupine puffer
Powder brown tang
Red lionfish
Redtooth triggerfish
Royal dottyback
Saddleback clownfish
Sailfin tang
Scissortail goby
Sergeant major
Six line wrasse
Snowflake moray eel
Spotted prawn goby
Steinitz' prawn goby
Strawberry dottyback
Threespot dascyllus
Tomato clownfish
Tomini tang
Valentinni's sharpnose puffer
Velvet damselfish
Yellow angelfish
Yellow clown goby
Yellow prawn goby
Zebra goby
Zebra lionfish
Lin et al.
Silky shark
Blue sharl
Pacific bluefin tuna
Shortfin mako
Narrow barred mackerel
Japanese Spanish mackerel
Bigeye tuna

Black marlin
Indo-Pacific sailfish
Red seabream
Yellowfin tuna
Yellowback sea bream
Striped marlin
Albacore
Mangrove red snapper
Red Bigeye
Blackhead seabream
Large yellow croaker
Blue marlin
Blackmouth croaker
Japanese jack mackerel
Japanese scad
Escolar
Swordfish
Silver pomfret
Japanese butterfish
Striped bonito
Greater amberjack
Wahoo
Common dolphinfish
Mi-iuy croaker
Dogtooth tuna
Moonfish
Blue mackerel
Chub mackerel
Yellow croaker
Threadfin porgy
Shrimp scad
Crimson snapper

Cobia
Largehead hairtail
Skipjack tuna
Oilfish
Redtail scad
Torpedo scad
Silver-stripe round herring
Bullet tuna
Round herring
Frigate tuna
Butterfish
Flathead grey mullet
Spotted catfish
Patrick et al.
Shortfin mako
Blue shark
Common thresher
Porbeagle
Oceanic whitetip
Bigeye thresher
Longfin mako
Sixgill shark
Sharpnose sevengill shark
Sandbar shark
Blacktip shark
Spinner shark
Silky shark
Bull shark
Tiger shark
Nurse shark
Lemon shark
Scalloped hammerhead

Great hammerhead
Smooth hammerhead
Dusky shark
Caribbean reef shark
Night shark
Bignose shark
Galapagos shark
Sandtiger shark
Bigeye sandtiger shark
White shark
Basking shark
Whale shark
Atlantic sharpnose shark
Bonnethead shark
Blacknose shark
Finetooth shark
Angel shark
Smalltail shark
Caribbean sharpnose shark
Alaska skate
Aleutian skate
Commander skate
Whiteblotched skate
Whitebrow skate
Roughtail skate
Bering skate
Mud skate
Roughshoulder skate
Big skate
Longnose skate
Butterfly skate
Deepsea skate

California sheephead
Cabezon
Kelp greenling
Rock greenling
California scorpionfish
Monkyface prickelback
Black rockfish
Black-and-yellow rockfish
Blue rockfish
Brown rockfish
Calico rockfish
China rockfish
Copper rockfish
Gopher rockfish
Grass rockfish
Kelp rockfish
Olive rockfish
Quillback rockfish
Treefish rockfish
Pacific sardine
Northern Anchovy
Pacific mackerel
Jack mackerel
Market squid
Pacific herring
Pacific bonito
Pacific saury
Albacore
Bigeye Tuna
Black Marlin
Bullet Tuna
Pacific Pomfret

Blue Shark
Bigeye thresher shark
Blue Marlin
Dolphin Fish
Brilliant Pomfret
Kawakawa
Spotted Moonfish
Longfin Mako Shark
Salmon Shark
Striped Marlin
Oilfish
Northern Bluefin Tuna
Roudi Escolar
Pelagic Thresher Shark
Sailfish
Skipjack Tuna
Shortfinned Mako Shark
Short Bill Spearfish
Broadbill Swordfish
Flatheat Pomfret
Dagger Pomfret
Sickle Pomfret
Wahoo
Yellowfin Tuna
Oceanic Whitetip Shark
Silky Shark
Common Thresher Shark
Escolar
Albacore
Bigeye Tuna
Black Marlin
Bullet Tuna

Pacific Pomfret
Blue Shark
Bigeye thresher shark
Blue Marlin
Dolphin Fish
Brilliant Pomfret
Kawakawa
Spotted Moonfish
Longfin Mako Shark
Salmon Shark
Striped Marlin
Oilfish
Northern Bluefin Tuna
Roudi Escolar
Pelagic Thresher Shark
Sailfish
Skipjack Tuna
Shortfinned Mako Shark
Short Bill Spearfish
Broadbill Swordfish
Flatheat Pomfret
Dagger Pomfret
Sickle Pomfret
Wahoo
Yellowfin Tuna
Oceanic Whitetip Shark
Silky Shark
Common Thresher Shark
Escolar
GM Cod
GB Cod
GM Haddock

GB Haddock
Redfish
Pollock
CC-GM Yellowtail Flounder
GB Yellowtail Flounder
SNE Yellowtail Flounder
American Plaice
Witch Flounder
GM Winter Flounder
GB Winter Flounder
SNE-MidA winter Flouder
GM-GB Windowpane
SNE-MA Windowpane
Ocean Pout
White Hake
Halibut
Sand Tilefish
Bar Jack
Rock Sea Bass
Margate
Ponton-Cevallos et al.
Sailfin grouper
Pacific mutton hamlet
Pacific graysby
Leather bass
Olive grouper
Starry grouper
Misty grouper
Pacific creole-fish
Graery threadfin seabass
Grape-eye seabass
White-spotted sandbass

4 Dependency among attributes and between axes

Among the set of attributes chosen for a given analysis, some or all of those attributes may be correlated. If weakly correlated, the method presented in the main text can apply without correction. However, given a subset of attributes that are significantly correlated, the degree of clustering of points will not be as dramatic, and the number of attributes used does not inform completely the degree to which points will cluster. There are two paths to follow: remove hard-to-measure attributes that are highly correlated with other attributes or estimate the number of effectively independent attributes. If attributes are not removed from analysis due to correlation, a principal component analysis can be used to describe the effective number of independent attributes. All attribute values should be normalized by dividing by 3 and subtracting 0.5.

$$\bar{P}_{i,k} = \frac{P_{i,k}}{3} - \frac{1}{2} \qquad\qquad \bar{S}_{i,k} = \frac{S_{i,k}}{3} - \frac{1}{2} \tag{9}$$

Two covariance matrices can be produced: one for productivity attributes ($\boldsymbol{\Sigma_P}$) and another for susceptibility attributes ($\boldsymbol{\Sigma_S}$). $\boldsymbol{\Sigma_P}$ will be an NxN matrix, and $\boldsymbol{\Sigma_S}$ will be an MxM matrix. Let $\boldsymbol{\lambda_P}$ be the set of N eigenvalues of $\boldsymbol{\Sigma_P}$ and $\boldsymbol{\lambda_S}$ be the set of N' eigenvalues of $\boldsymbol{\Sigma_S}$. Then let

$$\bar{\boldsymbol{\lambda}}_P = \frac{N\boldsymbol{\lambda_P}}{\sum_{i=1}^{N} \boldsymbol{\lambda_P}_i} \qquad\qquad \bar{\boldsymbol{\lambda}}_S = \frac{M\boldsymbol{\lambda_S}}{\sum_{j=1}^{M} \boldsymbol{\lambda_S}_i} \tag{10}$$

The number of effectively independent attributes for productivity and susceptibility are given respectively as

$$N_P = \alpha_P + N - \sum_{i=1}^{\alpha_P} \boldsymbol{\lambda_P}_i \qquad\qquad N_S = \alpha_S + M - \sum_{i=1}^{\alpha_S} \boldsymbol{\lambda_S}_i \tag{11}$$

where α_P and α_S represent the number of values greater than 1 in the vectors $\boldsymbol{\lambda_P}$ and $\boldsymbol{\lambda_S}$, respectively. The analysis can be performed as described preceding this subsection by replacing N and M with N_P and N_S, respectively, in all equations. We recommend in future work that standardized covariance matrices be created to describe the relationships among attributes to be used across studies. In their absence, covariance matrices can be created for each study, particularly for studies including more than 20 species.

Equation 14 in the main text gives the transformation of the standard error of the bivariate gaussian distribution $\boldsymbol{X}$ to the risk axis. This equation assumes independent contributions of the component standard errors $\hat{SE}_p$ and $\hat{SE}_s$. However, this equation can be generalized to cases in which productivity and susceptibility are not independent of each other. Consider the covariance matrix ($\mathbf{\Sigma}$) expressed in equation 10 (main text). Equation 14 can be expressed in terms of this covariance matrix:

$$\hat{SE}_{\boldsymbol{r}} = \sqrt{\frac{2det(\mathbf{\Sigma})}{\hat{SE}_p^2 + \hat{SE}_s^2}} \tag{12}$$

Accounting for expected covariance between productivity and susceptibility allows for calculation of V_p via equations 15 and 21 (main text). V is a reliable metric regardless of the covariance relationship between P and S, and therefore needs no recalculation when P and S are dependent because covariance is symmetrical.

5 Example tables

Presented below are example tables to rapidly assign a risk category to each species analyzed. For each species, find the cell in the table corresponding to number of productivity attributes used and the number of susceptibility attributes used. There are four numbers in that cell. Multiply the top (first) number (n_1) in the cell by the productivity score (P) and the second number (n_2) by the susceptibility score (S). Add these products together to yield a standardized risk score (vulnerability). If the vulnerability value is lower than the third value in the cell, the species is low risk. If the vulnerability value is between the third and fourth value in the cell, the species is medium risk. High risk species will have vulnerability scores above this fourth value. The vulnerability value is standardized and can be used to rank species according to risk, even if differing numbers of attributes are used for each species.

$$V = n_1 S + n_2 P \tag{13}$$

The first table presented assumes attributes are equally weighted for productivity and susceptibility and that productivity and susceptibility scores are arithmetic means of these attribute values. The second table presented assumes attributes are equally weighted for productivity and susceptibility and that the productivity score is the arithmetic mean of productivity attribute values. The susceptibility score, however, is the geometric mean of the susceptibility attribute values. Similar tables can be created to represent any variant of assumptions using equation 19 in the main text. Otherwise, computation can be handled using the software associated with this paper.

Number of Productivity Attributes

Number of Susceptibility Attributes

	3	4	5	6	7	8	9	10	11	12	13	14	15
3	0.471 0.471 1.75 2.021	0.408 0.471 1.642 1.877	0.365 0.471 1.568 1.778	0.333 0.471 1.514 1.705	0.309 0.471 1.471 1.649	0.289 0.471 1.437 1.603	0.272 0.471 1.409 1.565	0.258 0.471 1.385 1.533	0.246 0.471 1.364 1.506	0.236 0.471 1.346 1.482	0.226 0.471 1.331 1.461	0.218 0.471 1.317 1.442	0.211 0.471 1.304 1.425
4	0.471 0.408 1.642 1.877	0.408 0.408 1.531 1.735	0.365 0.408 1.456 1.638	0.333 0.408 1.4 1.566	0.309 0.408 1.357 1.511	0.289 0.408 1.322 1.466	0.272 0.408 1.293 1.429	0.258 0.408 1.269 1.397	0.246 0.408 1.248 1.37	0.236 0.408 1.229 1.347	0.226 0.408 1.213 1.326	0.218 0.408 1.199 1.307	0.211 0.408 1.186 1.291
5	0.471 0.365 1.568 1.778	0.408 0.365 1.456 1.638	0.365 0.365 1.379 1.542	0.333 0.365 1.323 1.471	0.309 0.365 1.279 1.416	0.289 0.365 1.243 1.372	0.272 0.365 1.214 1.335	0.258 0.365 1.189 1.304	0.246 0.365 1.168 1.277	0.236 0.365 1.149 1.254	0.226 0.365 1.133 1.234	0.218 0.365 1.118 1.215	0.211 0.365 1.105 1.199
6	0.471 0.333 1.514 1.705	0.408 0.333 1.4 1.566	0.365 0.333 1.323 1.471	0.333 0.333 1.266 1.401	0.309 0.333 1.221 1.347	0.289 0.333 1.185 1.303	0.272 0.333 1.156 1.266	0.258 0.333 1.131 1.236	0.246 0.333 1.109 1.209	0.236 0.333 1.09 1.186	0.226 0.333 1.074 1.166	0.218 0.333 1.059 1.147	0.211 0.333 1.045 1.131
7	0.471 0.309 1.471 1.649	0.408 0.309 1.357 1.511	0.365 0.309 1.279 1.416	0.333 0.309 1.221 1.347	0.309 0.309 1.176 1.292	0.289 0.309 1.14 1.249	0.272 0.309 1.11 1.213	0.258 0.309 1.085 1.182	0.246 0.309 1.063 1.156	0.236 0.309 1.044 1.133	0.226 0.309 1.028 1.113	0.218 0.309 1.013 1.095	0.211 0.309 0.999 1.079
8	0.471 0.289 1.437 1.603	0.408 0.289 1.322 1.466	0.365 0.289 1.243 1.372	0.333 0.289 1.185 1.303	0.309 0.289 1.14 1.249	0.289 0.289 1.104 1.205	0.272 0.289 1.074 1.17	0.258 0.289 1.048 1.139	0.246 0.289 1.026 1.113	0.236 0.289 1.007 1.09	0.226 0.289 0.99 1.07	0.218 0.289 0.975 1.052	0.211 0.289 0.962 1.036
9	0.471 0.272 1.409 1.565	0.408 0.272 1.293 1.429	0.365 0.272 1.214 1.335	0.333 0.272 1.156 1.266	0.309 0.272 1.11 1.213	0.289 0.272 1.074 1.17	0.272 0.272 1.044 1.134	0.258 0.272 1.018 1.104	0.246 0.272 0.996 1.078	0.236 0.272 0.977 1.055	0.226 0.272 0.96 1.035	0.218 0.272 0.945 1.017	0.211 0.272 0.931 1.001
10	0.471 0.258 1.385 1.533	0.408 0.258 1.269 1.397	0.365 0.258 1.189 1.304	0.333 0.258 1.131 1.236	0.309 0.258 1.085 1.182	0.289 0.258 1.048 1.139	0.272 0.258 1.018 1.104	0.258 0.258 0.992 1.073	0.246 0.258 0.97 1.048	0.236 0.258 0.951 1.025	0.226 0.258 0.934 1.005	0.218 0.258 0.918 0.987	0.211 0.258 0.905 0.971
11	0.471 0.246 1.364 1.506	0.408 0.246 1.248 1.37	0.365 0.246 1.168 1.277	0.333 0.246 1.109 1.209	0.309 0.246 1.063 1.156	0.289 0.246 1.026 1.113	0.272 0.246 0.996 1.078	0.258 0.246 0.97 1.048	0.246 0.246 0.948 1.022	0.236 0.246 0.928 0.999	0.226 0.246 0.911 0.979	0.218 0.246 0.896 0.962	0.211 0.246 0.882 0.946
12	0.471 0.236 1.346 1.482	0.408 0.236 1.229 1.347	0.365 0.236 1.149 1.254	0.333 0.236 1.09 1.186	0.309 0.236 1.044 1.133	0.289 0.236 1.007 1.09	0.272 0.236 0.977 1.055	0.258 0.236 0.951 1.025	0.246 0.236 0.928 0.999	0.236 0.236 0.909 0.977	0.226 0.236 0.892 0.957	0.218 0.236 0.876 0.939	0.211 0.236 0.863 0.923
13	0.471 0.226 1.331 1.461	0.408 0.226 1.213 1.326	0.365 0.226 1.133 1.234	0.333 0.226 1.074 1.166	0.309 0.226 1.028 1.113	0.289 0.226 0.99 1.07	0.272 0.226 0.96 1.035	0.258 0.226 0.934 1.005	0.246 0.226 0.911 0.979	0.236 0.226 0.892 0.957	0.226 0.226 0.875 0.937	0.218 0.226 0.859 0.919	0.211 0.226 0.845 0.904
14	0.471 0.218 1.317 1.442	0.408 0.218 1.199 1.307	0.365 0.218 1.118 1.215	0.333 0.218 1.059 1.147	0.309 0.218 1.013 1.095	0.289 0.218 0.975 1.052	0.272 0.218 0.945 1.017	0.258 0.218 0.918 0.987	0.246 0.218 0.896 0.962	0.236 0.218 0.876 0.939	0.226 0.218 0.859 0.919	0.218 0.218 0.844 0.902	0.211 0.218 0.83 0.886
15	0.471 0.211 1.304 1.425	0.408 0.211 1.186 1.291	0.365 0.211 1.105 1.199	0.333 0.211 1.045 1.131	0.309 0.211 0.999 1.079	0.289 0.211 0.962 1.036	0.272 0.211 0.931 1.001	0.258 0.211 0.905 0.971	0.246 0.211 0.882 0.946	0.236 0.211 0.863 0.923	0.226 0.211 0.845 0.904	0.218 0.211 0.83 0.886	0.211 0.211 0.816 0.87

Number of Productivity Attributes

Number of Susceptibility Attributes	3	4	5	6	7	8	9	10	11	12	13	14	15
3	0.471 0.262 0.73 0.881	0.408 0.262 0.702 0.833	0.365 0.262 0.684 0.8	0.333 0.262 0.67 0.776	0.309 0.262 0.659 0.757	0.289 0.262 0.65 0.742	0.272 0.262 0.643 0.73	0.258 0.262 0.637 0.719	0.246 0.262 0.632 0.71	0.236 0.262 0.627 0.702	0.226 0.262 0.623 0.695	0.218 0.262 0.619 0.689	0.211 0.262 0.616 0.683
4	0.471 0.227 0.67 0.8	0.408 0.227 0.641 0.754	0.365 0.227 0.621 0.722	0.333 0.227 0.607 0.699	0.309 0.227 0.595 0.681	0.289 0.227 0.586 0.666	0.272 0.227 0.579 0.654	0.258 0.227 0.572 0.644	0.246 0.227 0.567 0.635	0.236 0.227 0.562 0.627	0.226 0.227 0.558 0.62	0.218 0.227 0.554 0.614	0.211 0.227 0.55 0.609
5	0.471 0.203 0.629 0.746	0.408 0.203 0.599 0.7	0.365 0.203 0.579 0.669	0.333 0.203 0.564 0.646	0.309 0.203 0.552 0.628	0.289 0.203 0.542 0.614	0.272 0.203 0.535 0.602	0.258 0.203 0.528 0.592	0.246 0.203 0.522 0.583	0.236 0.203 0.517 0.576	0.226 0.203 0.513 0.569	0.218 0.203 0.509 0.563	0.211 0.203 0.506 0.558
6	0.471 0.185 0.599 0.705	0.408 0.185 0.568 0.66	0.365 0.185 0.547 0.63	0.333 0.185 0.532 0.607	0.309 0.185 0.52 0.59	0.289 0.185 0.51 0.575	0.272 0.185 0.502 0.564	0.258 0.185 0.495 0.554	0.246 0.185 0.49 0.545	0.236 0.185 0.485 0.538	0.226 0.185 0.48 0.531	0.218 0.185 0.476 0.525	0.211 0.185 0.472 0.52
7	0.471 0.171 0.575 0.674	0.408 0.171 0.544 0.629	0.365 0.171 0.523 0.599	0.333 0.171 0.507 0.577	0.309 0.171 0.495 0.559	0.289 0.171 0.485 0.545	0.272 0.171 0.477 0.534	0.258 0.171 0.47 0.524	0.246 0.171 0.464 0.516	0.236 0.171 0.459 0.508	0.226 0.171 0.454 0.502	0.218 0.171 0.45 0.496	0.211 0.171 0.447 0.491
8	0.471 0.16 0.556 0.648	0.408 0.16 0.525 0.604	0.365 0.16 0.503 0.575	0.333 0.16 0.487 0.552	0.309 0.16 0.475 0.535	0.289 0.16 0.465 0.521	0.272 0.16 0.457 0.51	0.258 0.16 0.45 0.5	0.246 0.16 0.444 0.492	0.236 0.16 0.438 0.485	0.226 0.16 0.434 0.478	0.218 0.16 0.43 0.472	0.211 0.16 0.426 0.467
9	0.471 0.151 0.541 0.627	0.408 0.151 0.509 0.584	0.365 0.151 0.487 0.554	0.333 0.151 0.471 0.532	0.309 0.151 0.458 0.515	0.289 0.151 0.448 0.501	0.272 0.151 0.44 0.49	0.258 0.151 0.433 0.48	0.246 0.151 0.427 0.472	0.236 0.151 0.421 0.465	0.226 0.151 0.417 0.459	0.218 0.151 0.413 0.453	0.211 0.151 0.409 0.448
10	0.471 0.143 0.527 0.61	0.408 0.143 0.495 0.566	0.365 0.143 0.473 0.537	0.333 0.143 0.457 0.515	0.309 0.143 0.444 0.498	0.289 0.143 0.434 0.485	0.272 0.143 0.426 0.473	0.258 0.143 0.419 0.464	0.246 0.143 0.412 0.455	0.236 0.143 0.407 0.448	0.226 0.143 0.402 0.442	0.218 0.143 0.398 0.436	0.211 0.143 0.394 0.431
11	0.471 0.137 0.516 0.594	0.408 0.137 0.483 0.551	0.365 0.137 0.461 0.522	0.333 0.137 0.445 0.5	0.309 0.137 0.432 0.484	0.289 0.137 0.422 0.47	0.272 0.137 0.413 0.459	0.258 0.137 0.406 0.449	0.246 0.137 0.4 0.441	0.236 0.137 0.395 0.434	0.226 0.137 0.39 0.428	0.218 0.137 0.386 0.422	0.211 0.137 0.382 0.417
12	0.471 0.131 0.506 0.581	0.408 0.131 0.473 0.538	0.365 0.131 0.451 0.509	0.333 0.131 0.434 0.488	0.309 0.131 0.422 0.471	0.289 0.131 0.411 0.457	0.272 0.131 0.403 0.446	0.258 0.131 0.395 0.437	0.246 0.131 0.389 0.429	0.236 0.131 0.384 0.421	0.226 0.131 0.379 0.415	0.218 0.131 0.375 0.41	0.211 0.131 0.371 0.405
13	0.471 0.126 0.497 0.569	0.408 0.126 0.464 0.527	0.365 0.126 0.442 0.498	0.333 0.126 0.425 0.476	0.309 0.126 0.412 0.46	0.289 0.126 0.402 0.446	0.272 0.126 0.393 0.435	0.258 0.126 0.386 0.426	0.246 0.126 0.38 0.418	0.236 0.126 0.374 0.41	0.226 0.126 0.369 0.404	0.218 0.126 0.365 0.399	0.211 0.126 0.361 0.394
14	0.471 0.121 0.489 0.559	0.408 0.121 0.456 0.516	0.365 0.121 0.434 0.488	0.333 0.121 0.417 0.466	0.309 0.121 0.404 0.45	0.289 0.121 0.394 0.436	0.272 0.121 0.385 0.425	0.258 0.121 0.378 0.416	0.246 0.121 0.371 0.408	0.236 0.121 0.366 0.401	0.226 0.121 0.361 0.394	0.218 0.121 0.357 0.389	0.211 0.121 0.353 0.384
15	0.471 0.117 0.482 0.549	0.408 0.117 0.449 0.507	0.365 0.117 0.426 0.478	0.333 0.117 0.41 0.457	0.309 0.117 0.397 0.441	0.289 0.117 0.386 0.427	0.272 0.117 0.377 0.416	0.258 0.117 0.37 0.407	0.246 0.117 0.364 0.399	0.236 0.117 0.358 0.392	0.226 0.117 0.353 0.386	0.218 0.117 0.349 0.38	0.211 0.117 0.345 0.375