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AOA Experiment 4

Aim:

To implement & analyze Minimum cost spanning tree - Kruskal's Algorithm:

```

#include <stdio.h>
#define MAX 30

typedef struct edge
{
int src, dest, weight;
} edge;
typedef struct E_list {
edge data[MAX];
int n;
} E_list;
E_list elist;
int Graph[MAX][MAX], n;
E_list spanlist;
void kruskalAlgo();
int find(int belongs[], int v_no);
void Union(int belongs[], int c1, int c2);
void sort();
void print();
void kruskalAlgo() {
int belongs[MAX], i, j, cno1, cno2;
elist.n = 0;
for (i = 1; i < n; i++)
{
for (j = 0; j < i; j++) {
if (Graph[i][j] != 0) {
elist.data[elist.n].src = i;
elist.data[elist.n].dest = j;
elist.data[elist.n].weight = Graph[i][j];
elist.n++;
}
}
}
}

```

```

    sort();
    for (i = 0; i < n; i++)
        belongs[i] = i;
    spanlist.n = 0;
    for (i = 0; i < elist.n; i++) {
        cno1 = find(belongs, elist.data[i].src);
        cno2 = find(belongs, elist.data[i].dest);
        if (cno1 != cno2) {
            spanlist.data[spanlist.n] = elist.data[i];
            spanlist.n = spanlist.n + 1;
            Union(belongs, cno1, cno2);
        }
    }
}

int find(int belongs[], int v_no) {
    return (belongs[v_no]);
}

void Union(int belongs[], int c1, int c2) {
    int i;
    for (i = 0; i < n; i++)
        if (belongs[i] == c2)
            belongs[i] = c1;
}

void sort() {
    int i, j;
    edge temp;
    for (i = 1; i < elist.n; i++)
        for (j = 0; j < elist.n - 1; j++)
            if (elist.data[j].weight > elist.data[j + 1].weight) {
                temp = elist.data[j];
                elist.data[j] = elist.data[j + 1];
                elist.data[j + 1] = temp;
            }
}

```

```

}
void display() {
int i, cost = 0;
printf("The edges in the minimum spanning tree: ");
for (i = 0; i < spanlist.n; i++) {
printf("\n%d - %d : %d", spanlist.data[i].src, spanlist.data[i].dest,
spanlist
cost = cost + spanlist.data[i].weight;
}
printf("\nSpanning tree cost: %d", cost);
}
int main() {
int i, j, total_cost;
printf("Enter the total number of vertices: ");
scanf("%d",&n);
printf("Enter the weight for vertices: \n");
printf("\n");
for(i=0;i<n;i++)
{
for(j=0;j<n;j++)
{
printf("Enter the weight for vertex with src {%d} and dest {%d} : ",
scanf("%d",&Graph[i][j]));
}
printf("\n");
}
kruskalAlgo();
display();
}

```

Output:

```
Enter the weight for vertices:

Enter the weight for vertex with src {0} and dest {0} :      0
Enter the weight for vertex with src {0} and dest {1} :      1
Enter the weight for vertex with src {0} and dest {2} :      3
Enter the weight for vertex with src {0} and dest {3} :      0
Enter the weight for vertex with src {0} and dest {4} :      0

Enter the weight for vertex with src {1} and dest {0} :      1
Enter the weight for vertex with src {1} and dest {1} :      0
Enter the weight for vertex with src {1} and dest {2} :      3
Enter the weight for vertex with src {1} and dest {3} :      6
Enter the weight for vertex with src {1} and dest {4} :      0

Enter the weight for vertex with src {2} and dest {0} :      3
Enter the weight for vertex with src {2} and dest {1} :      3
Enter the weight for vertex with src {2} and dest {2} :      0
Enter the weight for vertex with src {2} and dest {3} :      4
Enter the weight for vertex with src {2} and dest {4} :      2

Enter the weight for vertex with src {3} and dest {0} :      0
Enter the weight for vertex with src {3} and dest {1} :      6
Enter the weight for vertex with src {3} and dest {2} :      4
Enter the weight for vertex with src {3} and dest {3} :      0
Enter the weight for vertex with src {3} and dest {4} :      5

Enter the weight for vertex with src {4} and dest {0} :      0
Enter the weight for vertex with src {4} and dest {1} :      0
Enter the weight for vertex with src {4} and dest {2} :      2
Enter the weight for vertex with src {4} and dest {3} :      5
Enter the weight for vertex with src {4} and dest {4} :      0

The edges in the minimum spanning tree:
1 - 0 : 1
4 - 2 : 2
2 - 0 : 3
3 - 2 : 4
Spanning tree cost: 10
-----
```

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Exp 04 :

Theory :

- Kruskal's algorithm develops the Minimum spanning tree of a given graph by constructing a forest of minimum weighted edges that can be completed into a spanning tree if it is acyclic

Algorithm :

MST Kruskal (G, W)

$A \neq \emptyset$

for each vertex $v \in G.V$

Make-set(v)

// sort the edges $G.E$ into

// non decreasing order by weight

for each edge $(u, v) \in G.E$,

// taken in non-decreasing order

if find-set(v) \neq find-set(u)

$A = A \cup \{(u, v)\}$

UNION(u, v)

return A .

Analysis :

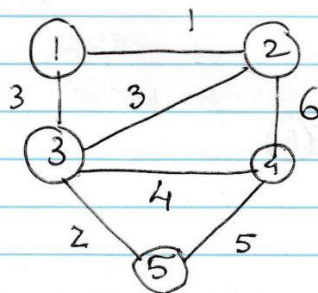
- $G = (V, E)$ be a given graph where V is a set of vertices and E is a set of edges. Kruskal's algorithm uses a priority queue (a min-heap) data structure where priorities are assigned to weights of all the edges of a graph.

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Using this the edges are arranged in ascending order of their weights in time $O(|E| \log |E|)$

- In each step, Kruskal's algorithm connects two partial subtrees in a forest by including the minimum weighted edge without forming a cycle. To do this Kruskal's algorithm performs $|E|$ number of delete operations on a priority queue.
- Since a single delete operation a priority queue takes $O(\log |E|)$, the total time of deleting $|E|$ edges will take $O(|E| \log |E|)$ time.
- The total time complexity of Kruskal's algorithm including construction of a priority queue of edges and deletion of edges from it is given as $O(|E| \log |E|) + O(|E| \log |E|) = O(|E| \log |E|)$

Example :

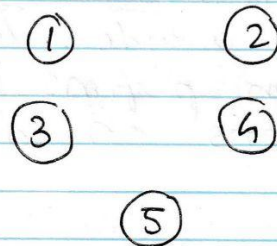


1) We arrange all the edges of a given

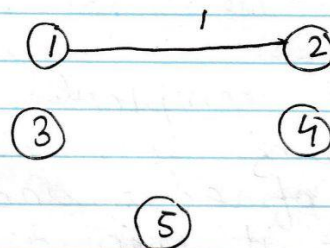
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graph is according order of their weights
as below $\langle 1, 2 \rangle$, $\langle 3, 5 \rangle$, $\langle 1, 3 \rangle$, $\langle 2, 3 \rangle$,
 $\langle 3, 4 \rangle$, $\langle 4, 5 \rangle$, $\langle 2, 4 \rangle$

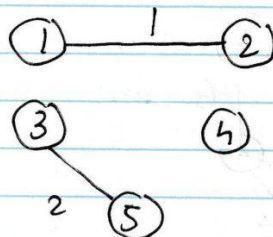
- 2) Construct a forest of partial subtrees
each of them, containing a single node



- 3) Add the first minimum weighted edge
 $\langle 1, 2 \rangle$ to connect nodes 1 & 2



- 4) Include the next minimum weighted edge
 $\langle 3, 5 \rangle$ to connect nodes 3 & 5 in a
forest

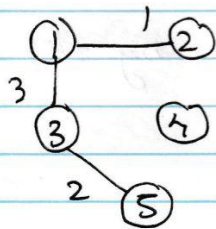


- 5) The next minimum weighted edges are

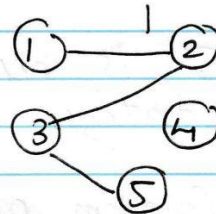
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$\langle 1,3 \rangle$ & $\langle 3,3 \rangle$. By adding either to connect two partial subtrees

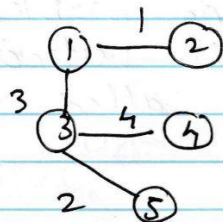
- i) a subtree containing nodes 1 & 2
 - ii) a subtree with nodes 3, 5 we get following two varieties of a forest.
- We cannot as they form a cycle in a forest



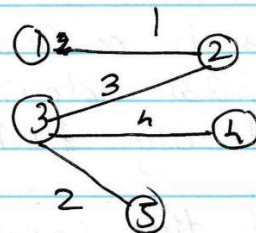
or



- 6) Add the next minimum edge to a forest to connect a subtree with nodes 1, 2, 3, 5, to node 4.



or



- 7) We cannot add the remaining edges $(4,5)$ & $(2,4)$ as they form a cycle in a forest. So the tree obtained in the above step (6) is the final MST of given graph with min cost = 10 units.

= Conclusion: We successfully implemented Kruskal's algorithm & analyzed it.