

# Functional Programming

## USING HASKELL

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## 1 FUNCTIONS OVER SEQUENCES

- Functions over Natural Numbers
- Functions over Lists
  - Strings

# How does a function work?

In order to ease understanding let us consider the following:

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### DEFINITION

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For the sake of simplicity we shall only be preoccupying ourself with defining functions over these sorts of sequences.

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- ②  $n \in \mathbb{N}_0 \implies n + 1 \in \mathbb{N}_0$

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What this tells us is that Natural numbers can either be zero or the successor of a natural number!

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Where  $\text{nil}$  is the function that creates an empty list and  $\text{cons}$  is the function that adds an element to the beginning of a list.

Haskell uses  $:$  to represent adding an element to the head of a list.

# Are words, sentences and texts sequences?

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### COROLLARY

*Any function applied to generic Lists can be applied to Strings and vice versa.*