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Theoretische Informatik 1
Übung Blatt 1

Aufgabe 1.1

$\forall A, B, C : \underbrace{A \cap (B \cup C)}_{\Sigma} \Leftrightarrow \underbrace{(A \cap B) \cup (A \cap C)}_{\Sigma'}$

$\Sigma = \{x | (x \in B \vee x \in C) \wedge x \in A\}$
$\Sigma' = \{x | (x \in A \wedge x \in B) \vee (x \in A \wedge x \in C)\}$

1) $B \cup C = \{x | x \in B \vee x \in C\}$
2) $A \cap (B \cup C) = \{x | x \in A \wedge (x \in B \vee x \in C)\}$
3) $A \cap B = \{x | x \in A \wedge x \in B\}$
$A \cap C = \{x | x \in A \wedge x \in C\}$
$(A \cap B) \cup (A \cap C) = \{x | (x \in A \wedge x \in B) \vee (x \in A \wedge x \in C)\}$

q.e.d.

Aufgabe 1.2

add : Nat x Nat → Nat

- add(n,0) = n
- add(n,succ(m)) = succ(add(n,m))

$\forall n, m \in Nat : add(n, m) = add(m, n)$

Nebenbeweis:

IA) add(0,0) = 0
add(0,1) = succ(0) = 1

IS:
IV) add(0,x) = x ⇒ add(0,succ(x)) = succ(x)
IB) add(0,succ(x)) = succ(add(0,x)) = succ(x)

q.e.d.

Hauptbeweis:

IA) add(m,0) = m
add(m,0) = add(0,m)

IS:

IV) add(m,0) = add(0,m) $\Rightarrow$ add (succ(m),0) = add(0,succ(m))

IB) add(succ(m),0) = succ(add(m,0)) = succ(add(0,m))=add(0,succ(m))

add(m,0) = m
add(0,m) = m

IA) add(0,m) = add(m,0)
add(m,n) = add(n,m) $\Rightarrow$ add(m,succ(n)) = add(succ(n),m))
add(m, succ(n)) = succ(add(m,n)) = succ(add(n,m)) = add(succ(n),m))

q.e.d.

Aufgabe 1.3

a)

#:F $\rightarrow \mathbb{N}$
#(A) = 0, falls $A \in T$
$\#(\neg A) = \#(A) + 1$
$\#(A \Diamond B) = \#(A) + \#(B) + 1, wobei \Diamond \in \{\wedge, \vee, \Rightarrow, \Leftrightarrow\}$

b)

$\forall A, B \in T.P(A) \wedge \Big(P(A) \rightarrow P(\neg A)\Big) \wedge \Big(P(A) \wedge P(B) \rightarrow P(A \vee B) \wedge P(A \wedge B) \wedge P(A \Rightarrow B) \wedge P(A \Leftrightarrow B)\Big)$

c)

IA) r(A) = 0
#(A) = 0 ,denn A $\in$ T
r(A) $\leq$ # A(w)

IS:

IV) $r(B) \leq \#(B) \rightarrow r(\neg B) \leq \#(\neg B)$

IB) $r(\neg B) = r(B) + 1 \leq \#(B) + 1 = \#(\neg B)$

IV) $\Big(r(A) \leq \#(A) \wedge r(B) \leq \#(B)\Big) \rightarrow r(A \Diamond B) \leq \#(A \Diamond B), wobei$
$r(A \Diamond B) = max\Big(r(a), r(B)\Big) + 1 \leq r(a) + r(b) + 1 \leq \#(A) + \#(B) + 1 = \#(A \Diamond B),$
wobei $\Diamond \in \{\wedge, \vee, \Rightarrow, \Leftrightarrow\}$.

q.e.d.