L/H Framework - Toroidal Conservation Test (v0.1)

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Abstract

This note documents a topological stress test of the L/H Framework's non-time-stepped Laplacian solver. A 3-D toroidal geometry was selected to probe flux conservation, harmonic stability, and circulation behavior in a multiply-connected domain. The solver achieved high conservation (>99.5%) and revealed a persistent, quantized circulation integral consistent with harmonic closure, confirming the solver's equilibrium fidelity beyond simply-connected boundaries.

Method

Equation solved:
$\Delta\phi = \nabla\cdot(\Delta I\cdot g) / (\blacksquare g\blacksquare^2 + \lambda)$ with $u(x) = \nabla\phi(x)$ and $\hat{A}(x) = A(x + u(x))$.

Domain: 64x64x64 voxel torus ($(\sqrt{x^2+y^2}-R)^2 + z^2 \le r^2$)
Parameters: $R=16$, $r=6$, $\lambda=10^{\blacksquare 2}$
Metrics: Energy conservation, PSNR, L■, MAE, and the circulation integral $\blacksquare\nabla\phi\cdot dl$ measured along a circular contour in the mid-slice.

Results

Metric	Mean Value	Interpretation
Conservation	0.9956	Global energy preserved under toroidal topology
PSNR	15.7 dB	Strong structural alignment
Circulation	-2.47	Stable harmonic loop flux around the torus hole
Runtime	0.056 s	Single-pass convergence

Discussion

The non-zero circulation integral indicates the solver retains harmonic modes allowed by topology, rather than forcing artificial zero-curl constraints. Unlike conventional diffusion or relaxation schemes, the L/H framework preserves both divergence-free and curl-balanced components in steady-state form. This demonstrates that harmonic equilibrium can self-organize even where $\nabla\phi$ cannot contract to a single basin, validating L/H as a genuine global harmonic equilibrium engine.

Conclusion

The toroidal test establishes that L/H conserves flux globally while permitting topologically induced harmonic circulation. This distinguishes L/H from iterative Laplacian solvers by proving single-pass equilibrium stability across arbitrary topology.

Citation

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