MAST90083 Assignment 2

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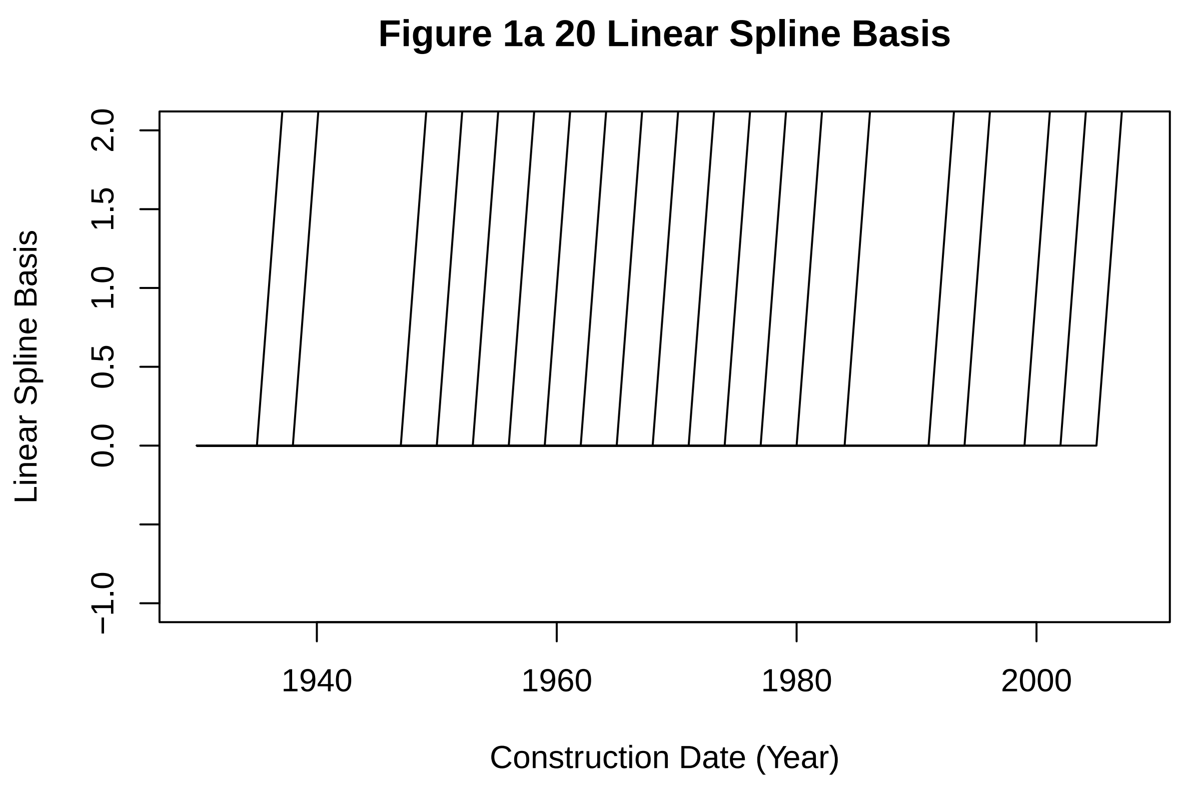
Question 1.1

```r
library(HRW)
data(WarsawApts)
# Reorder the dataset base on x
WarsawApts <- WarsawApts[order(WarsawApts$construction.date), ]

x <- WarsawApts$construction.date
y <- WarsawApts$areaPerMzloty

# Exclude extreme values of 0 and 1
probVec <- seq(0, 1, length = 22)[2:21]
k <- quantile(unique(x), probs = probVec)
```

Question 1.2

```r
construct_basis <- function(x, knot) {
  z_matrix <- matrix(0, length(x), length(knot))
  for (i in 1:length(knot)) {
    for (j in 1:length(x)) {
      z_matrix[j, i] <-  x[j] - knot[i]
    }
  }
  z_matrix[z_matrix < 0] <- 0
  return(z_matrix)
}

z <- construct_basis(sort(x), k)
```

```r
plot(x, z[,1], type = "l", ylim = c(-1, 2), xlab = "Construction Date (Year)",
     ylab = "Linear Spline Basis", main = "Figure 1a 20 Linear Spline Basis")
for (i in 2:20) {
  lines(x, z[, i])
}
```



Question 1.3

```r
intercept <- rep(1, length(x))
Z <- construct_basis(x, k)
# Construct matrix C
C <- cbind(intercept, x, Z)

# Construct matrix D
D <- diag(1, dim(C)[2], dim(C)[2])
# Replace first two columns with 0s since we don't want to penalise them
D[, c(1, 2)] <- 0

# Tuning parameter
lambda <- seq(0, 50, length = 100)
```

Question 1.4

```r
fit_penalised_spline <- function(X, D, lambda, y) {
  RSS_error <- rep(0, length(lambda))
  fitted_df <- rep(0, length(lambda))
  GCV <- rep(0, length(lambda))
  fitted_y <- matrix(0, length(y), length(lambda))
  for (i in 1:length(lambda)) {
    betaHat <- solve(t(X) %*% X + lambda[i] * D) %*% t(X) %*% y
    fitted_y[, i] <- X %*% betaHat
    RSS_error[i] <- sum((y - fitted_y[, i])^2)
    fitted_df[i] <- sum(diag(solve(t(X) %*% X + lambda[i] * D) %*% t(X) %*% X))
    GCV[i] = RSS_error[i] / (1 - fitted_df[i]/length(y))^2
  }

  return(list(RSS_error = RSS_error, fitted_y = fitted_y, fitted_df = fitted_df, GCV = GCV))
}
```

```r
# Fit penalised spline regression for each lambda
penalised_spline_20 <- fit_penalised_spline(C, D, lambda, y)

# RSS error associated with 100 tuning parameters
(RSS_error_20 <- penalised_spline_20$RSS_error)
```

```
##   [1] 119307.9 119359.8 119475.7 119620.3 119776.3 119934.8 120091.3 120243.5
##   [9] 120390.4 120531.5 120666.9 120796.7 120921.1 121040.5 121155.1 121265.3
##  [17] 121371.4 121473.6 121572.2 121667.4 121759.5 121848.7 121935.1 122019.0
##  [25] 122100.4 122179.5 122256.5 122331.5 122404.5 122475.7 122545.1 122613.0
##  [33] 122679.2 122744.0 122807.4 122869.4 122930.1 122989.5 123047.8 123104.9
##  [41] 123161.0 123216.0 123270.0 123323.0 123375.0 123426.2 123476.5 123526.0
##  [49] 123574.6 123622.5 123669.6 123716.0 123761.6 123806.6 123850.9 123894.5
##  [57] 123937.5 123979.9 124021.8 124063.0 124103.7 124143.8 124183.4 124222.5
##  [65] 124261.0 124299.1 124336.7 124373.9 124410.5 124446.8 124482.6 124518.0
##  [73] 124552.9 124587.5 124621.7 124655.4 124688.8 124721.9 124754.6 124786.9
##  [81] 124818.9 124850.5 124881.8 124912.8 124943.5 124973.8 125003.9 125033.6
##  [89] 125063.1 125092.3 125121.1 125149.8 125178.1 125206.2 125234.0 125261.5
##  [97] 125288.9 125315.9 125342.7 125369.3
```

```r
# Degrees of freedom associated with 100 tuning parameters
(df_20 <- penalised_spline_20$fitted_df)
```

```
##   [1] 22.00000 21.28811 20.68134 20.15438 19.68998 19.27589 18.90308 18.56472
##   [9] 18.25550 17.97123 17.70855 17.46471 17.23744 17.02486 16.82535 16.63758
##  [17] 16.46036 16.29270 16.13373 15.98268 15.83888 15.70174 15.57073 15.44539
##  [25] 15.32530 15.21008 15.09939 14.99293 14.89042 14.79161 14.69628 14.60419
##  [33] 14.51518 14.42906 14.34567 14.26485 14.18648 14.11041 14.03655 13.96476
##  [41] 13.89496 13.82704 13.76092 13.69652 13.63375 13.57255 13.51284 13.45456
##  [49] 13.39766 13.34207 13.28774 13.23462 13.18266 13.13183 13.08207 13.03334
##  [57] 12.98561 12.93885 12.89301 12.84806 12.80398 12.76074 12.71830 12.67664
##  [65] 12.63573 12.59555 12.55608 12.51729 12.47917 12.44169 12.40484 12.36858
##  [73] 12.33292 12.29783 12.26329 12.22929 12.19581 12.16284 12.13037 12.09838
##  [81] 12.06685 12.03579 12.00516 11.97498 11.94521 11.91585 11.88690 11.85834
##  [89] 11.83016 11.80235 11.77491 11.74782 11.72108 11.69468 11.66860 11.64286
##  [97] 11.61742 11.59230 11.56748 11.54296
```

```r
# Generalised CV associated with 100 tuning parameters
(GCV_20 <- penalised_spline_20$GCV)
```

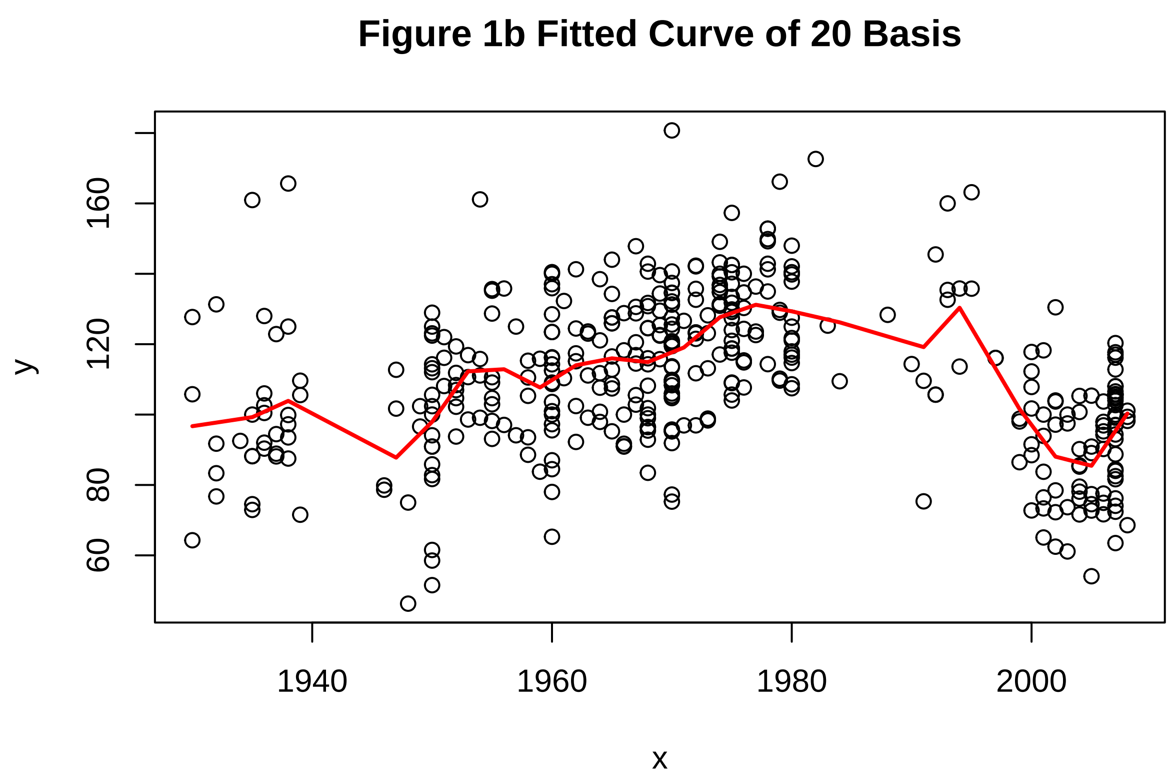
```
##   [1] 133258.2 132827.0 132540.8 132341.8 132198.4 132092.2 132011.9 131950.2
##   [9] 131902.4 131865.1 131836.0 131813.5 131796.2 131783.2 131773.9 131767.5
##  [17] 131763.7 131762.1 131762.3 131764.2 131767.5 131772.1 131777.8 131784.4
##  [25] 131791.9 131800.1 131809.1 131818.6 131828.7 131839.3 131850.3 131861.6
##  [33] 131873.4 131885.4 131897.7 131910.3 131923.1 131936.0 131949.2 131962.5
##  [41] 131975.9 131989.4 132003.1 132016.8 132030.6 132044.5 132058.5 132072.4
##  [49] 132086.4 132100.5 132114.5 132128.6 132142.6 132156.7 132170.7 132184.8
##  [57] 132198.8 132212.8 132226.8 132240.7 132254.7 132268.6 132282.4 132296.2
##  [65] 132310.0 132323.7 132337.4 132351.0 132364.6 132378.1 132391.6 132405.0
##  [73] 132418.4 132431.7 132444.9 132458.1 132471.3 132484.4 132497.4 132510.4
##  [81] 132523.3 132536.1 132548.9 132561.6 132574.3 132586.9 132599.4 132611.9
##  [89] 132624.3 132636.7 132649.0 132661.2 132673.4 132685.5 132697.6 132709.6
##  [97] 132721.5 132733.4 132745.2 132757.0
```

Question 1.5

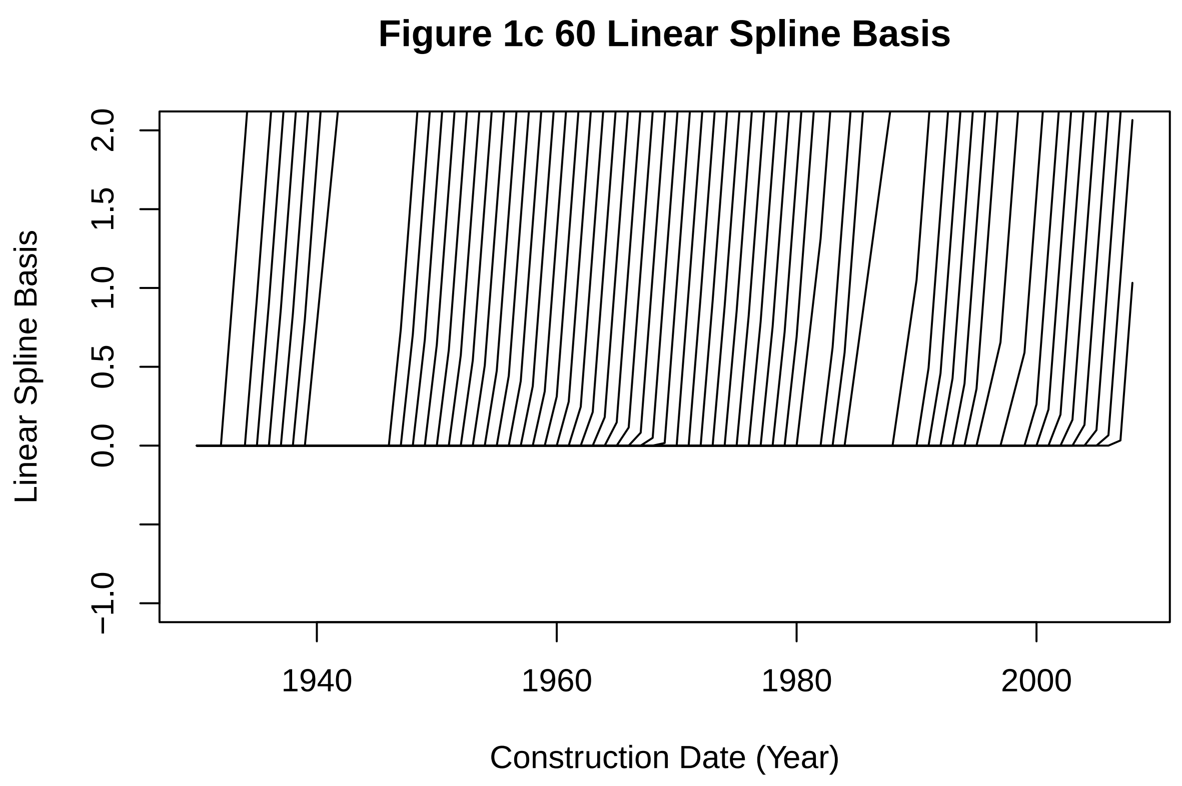
```
# Obtain lambda corresponding to the minimum GCV
min_20 <- which(GCV_20 == min(GCV_20))
(min_lambda_20 <- lambda[min_20])
```

```
## [1] 8.585859
```

```
# Overlay the true plot with a fit of y
plot(x, y, main = "Figure 1b Fitted Curve of 20 Basis")
lines(x, penalised_spline_20$fitted_y[, min_20], col = "red", lwd = 2)
```



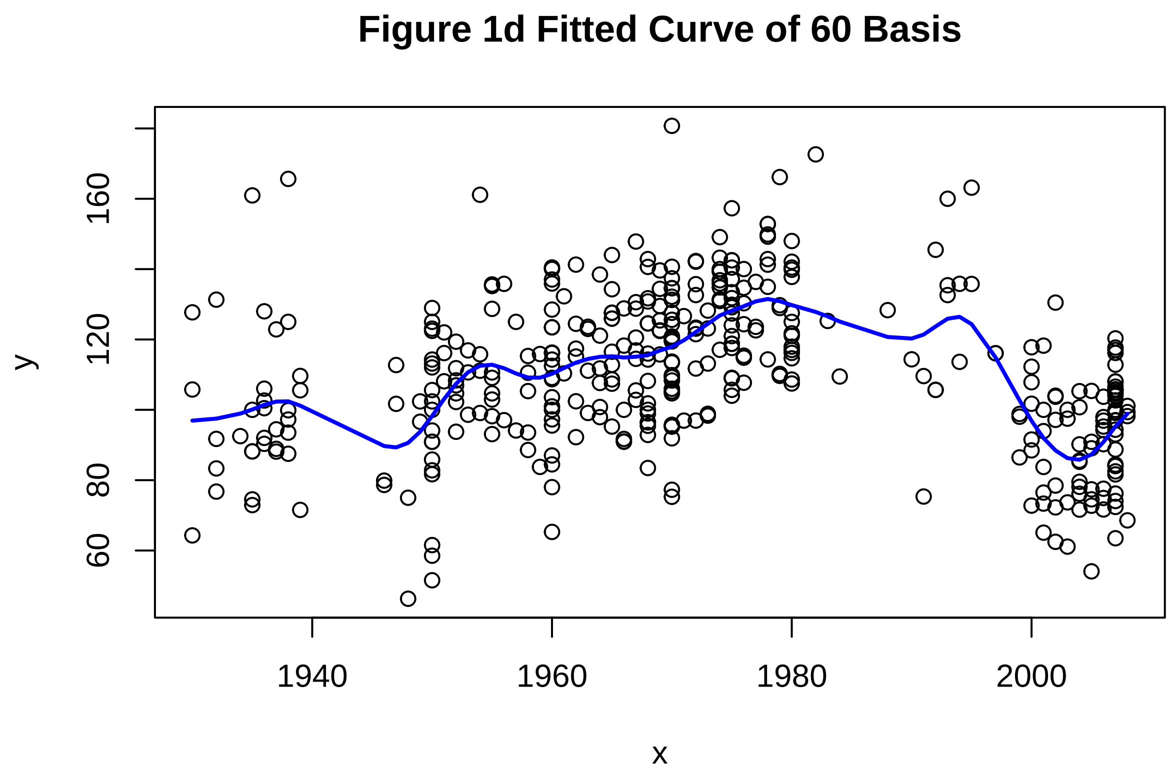
```r
# Increase the number of basis to 60
# Exclude extreme values of 0 and 1
probVec <- seq(0, 1, length = 62)[2:61]
k <- quantile(unique(x), probs = probVec)

z <- construct_basis(x, k)

plot(x, z[,1], typ = "l", ylim = c(-1,2), xlab = "Construction Date (Year)",
     ylab = "Linear Spline Basis", main = "Figure 1c 60 Linear Spline Basis")
for (i in 2:60) {
  lines(x, z[, i])
}
```



```
intercept <- rep(1, length(x))
Z <- construct_basis(x, k)
# Construct C matrix C
C <- cbind(intercept, x, Z)

# Construct matrix D
D <- diag(1, dim(C)[2], dim(C)[2])
# Replace first two columns with 0s since we don't want to penalise them
D[, c(1, 2)] <- 0
# Fit penalised spline regression for each lambda
penalised_spline_60 <- fit_penalised_spline(C, D, lambda, y)
GCV_60 <- penalised_spline_60$GCV

# Obtain lambda corresponding to the minimum GCV
min_60 <- which(GCV_60 == min(GCV_60))
(min_lambda_60 <- lambda[min_60])
```

```
## [1] 31.31313
```

```
# Overlay the true plot with a fit of y
plot(x, y, main = "Figure 1d Fitted Curve of 60 Basis")
lines(x, penalised_spline_60$fitted_y[, min_60], col = "blue", lwd = 2)
```



```
# Compute MSE for basis 20 and 60
paste("MSE for 20 Linear Spline Basis:", RSS_error_20[min_20]/length(x))
```

```
## [1] "MSE for 20 Linear Spline Basis: 297.00135815963"
```

```
paste("MSE for 60 Linear Spline Basis:", penalised_spline_60$RSS_error[min_60]/length(x))
```

```
## [1] "MSE for 60 Linear Spline Basis: 297.466954069915"
```

As we can see from Figure 1d. the fitted curve for 60 linear spline basis appears to be more smooth compared to the one for 20 linear spline basis. However, from the computed mean squared square errors, we cannot see an improvement for the 60 spline basis, and it is even slight higher than the mean square error of the 20 spline basis.

Question 2.1

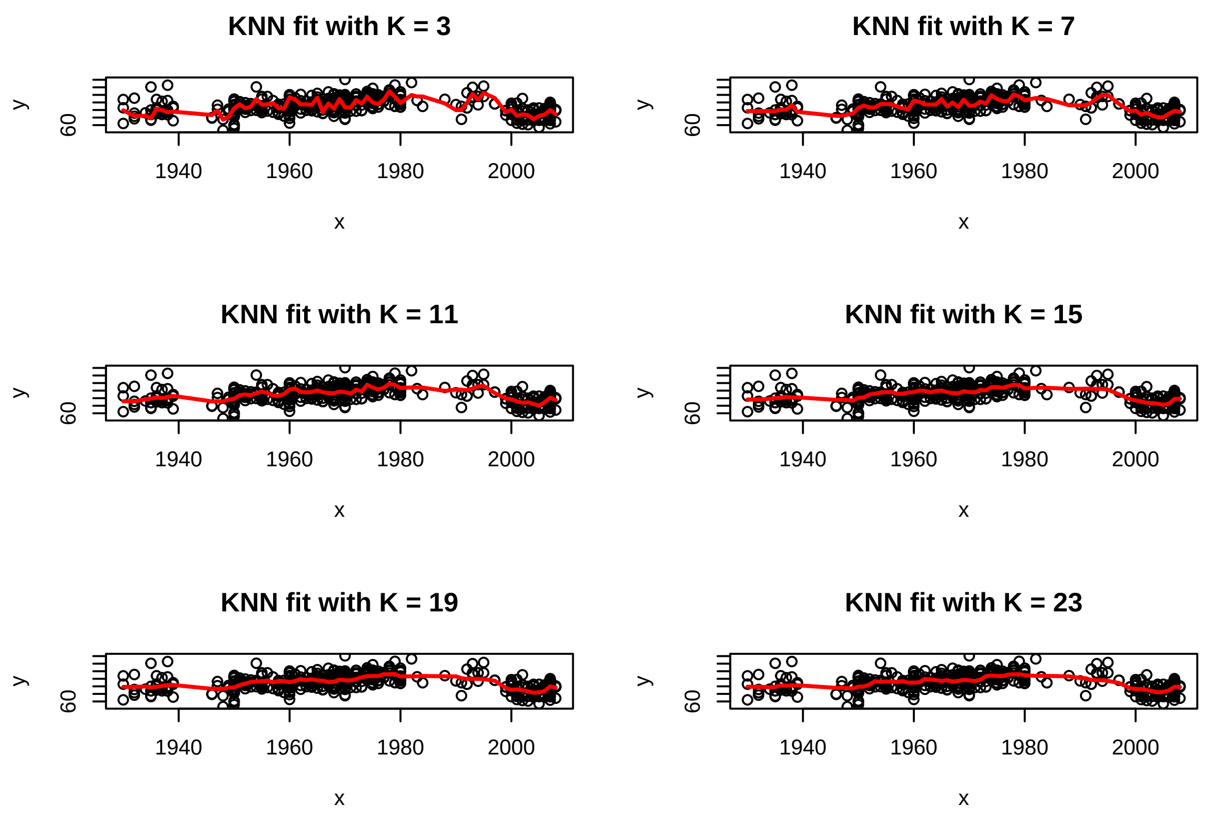
```
# Using 6 values of k selected from 3 to 23
k <- seq(3, 23, length = 6)

KNN <- function(x, cur_x, k, y) {
  abs_dif <- abs(cur_x - x)
```

```r
  # Sort the absolute difference and find the k nearest neighbors
  indices <- sort(abs_dif, index.return = TRUE)$ix[c(1:k)]

  # Use these indices and take their mean as the i-th estimate.
  return(mean(y[indices]))
}
```

Question 2.2

```r
prediction <- matrix(0, length(y), length(k))
MSE <- rep(0, length(k))

for (i in 1:length(k)) {
  y_hat <- rep(0, length(y))
  count <- 1
  for (j in x) {
    y_hat[count] <- KNN(x, j, k[i], y)
    count <- count + 1
  }
  prediction[, i] <- y_hat
  # Compute the mean square error for each k
  MSE[i] <- sum((y - y_hat)^2)/length(y)
}
```

```r
par(mfrow=c(3,2))

for (i in 1:length(k)) {
  plot(x, y, main = paste("KNN fit with K =", k[i]))
  lines(x, prediction[, i], col = "red", lwd = 2)
}
```



```
# Obtain k corresponding to the minimum MSE
rbind(k, MSE)
```

```
##         [,1]     [,2]     [,3]     [,4]     [,5]     [,6]
## k     3.0000   7.0000  11.0000  15.0000  19.0000  23.0000
## MSE 333.3363 293.9181 297.3904 300.3149 302.7898 309.1673
```

As we can see from the table, the minimum mean squared error is 293.9181 when $K = 7$, which is quite similar to the one we obtained using penalised spline regression, but slightly lower. And we can see that the MSE increases as k increases.

Question 2.3

```
kernel_name <- c('epanechnikov', 'gaussian', 'biweight', 'triweight', 'uniform', 'tricube')
# Function that accommodate all six kernels
kernel_collection <- function(x) {
  gaussian <- (2 * pi)^(-1/2) * exp(-(x^2)/2)

  if (abs(x) < 1) {
    epanechnikov <- (3/4) * (1 - x^2)
    biweight <- (15/16) * (1 - x^2)^2
    triweight <- (35/32) * (1 - x^2)^3
    uniform <- 1/2
```

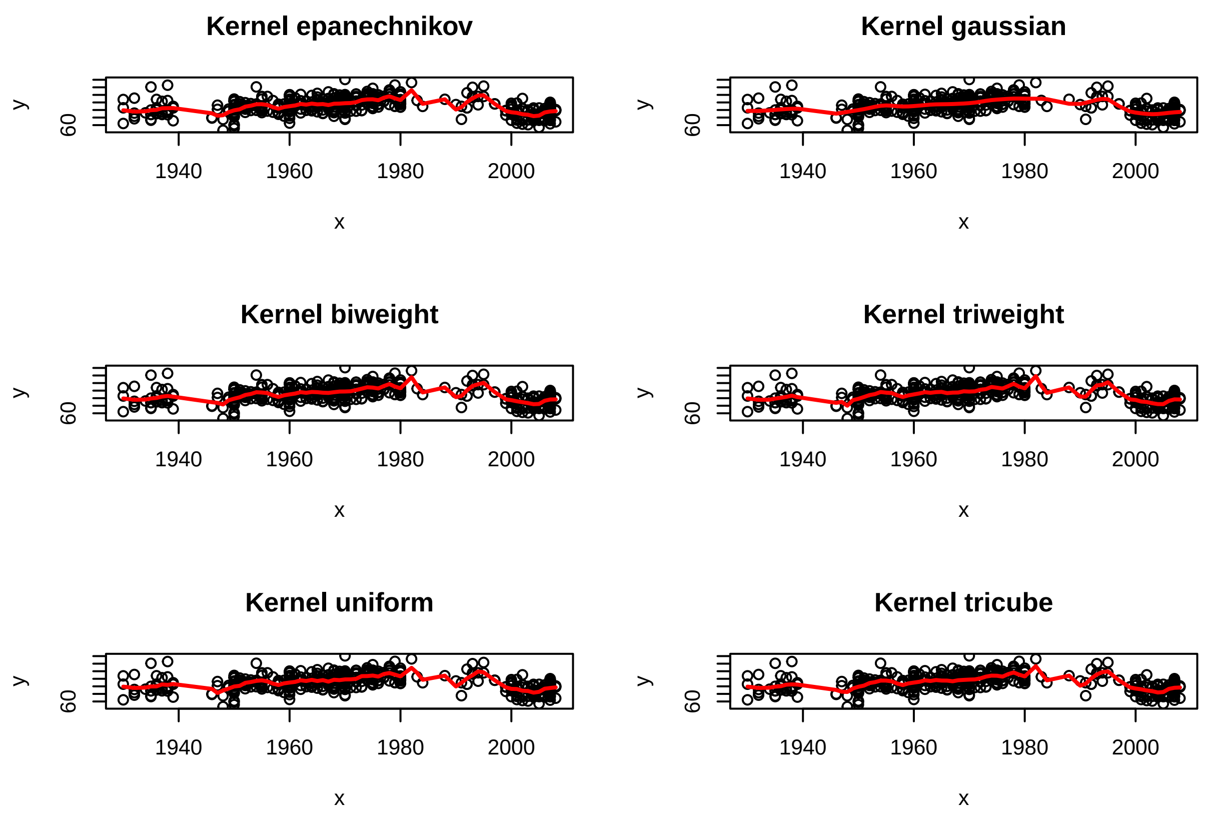
```r
    tricube <- (70/81) * (1 - abs(x)^3)^3
  }
  else {
    epanechnikov <- biweight <- triweight <- uniform <- tricube <- 0
  }
  return(c(epanechnikov, gaussian, biweight, triweight, uniform, tricube))
}
```

Question 2.4

```r
# Bandwidth
h <- 2

kernel_regression <- function(x, y) {
  prediction <- matrix(0, length(y), 6)

  for (i in 1:length(y)) {
    weight <- matrix(0, length(y), 6)
    nw <- 0
    for (j in 1:length(y)) {
      weight[j, ] = kernel_collection((x[j] - x[i])/h)
      nw <- nw + (weight[j, ] * y[j])
    }
    prediction[i,] <-  nw / colSums(weight)
  }
  colnames(prediction) <- kernel_name
  return(prediction)
}
```

Question 2.5

```r
prediction <- kernel_regression(x, y)
par(mfrow=c(3,2))

for (i in 1:6) {
  plot(x, y, main = paste("Kernel", kernel_name[i]))
  lines(x, prediction[, i], col = "red", lwd = 2)
}
```



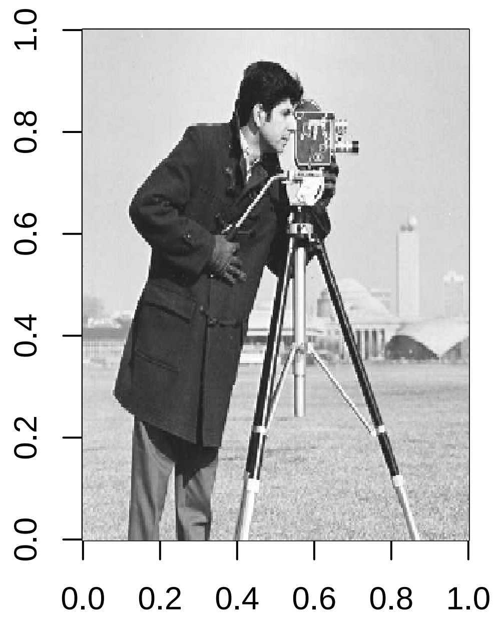
```r
# Estimate the MSE for each kernel
colSums(((y - prediction)^2)/length(y))
```

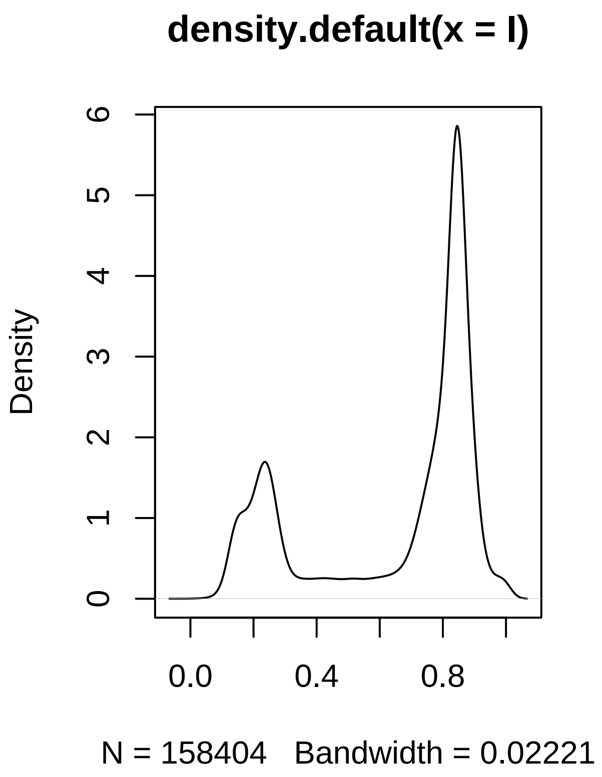
```
## epanechnikov     gaussian     biweight    triweight      uniform      tricube
##     285.5042     300.2880     278.8030     272.9163     292.8334     282.7872
```

As we can see, triweight kernel gives the lowest mean square error at 272.9163.

Question 3.1

```r
# Load libraries and image
library(plot.matrix)
library(png)
library(fields)

I <- readPNG("CM.png")
I <- I[,, 1]
I <- t(apply(I, 2, rev))

par(mfrow=c(1, 2))
image(I, col = gray((0:255) / 255))
plot(density(I))
```





Question 3.2 and 3.3

Here the expectation step will be computing

$$P(Z_i = k|X_i) = \frac{P(Z_i = k)P(X_i|Z_i = k)}{\sum_{k=1}^{K} P(Z_i = k)P(X_i|Z_i = k)}$$

The maximisation step will be updating the parameter estimates base on the following equations

$$\hat{\pi_k} = \frac{\sum_{i=1}^{N} P(Z_i = k|X_i)}{N} \quad \hat{\mu}_k = \frac{\sum_{i=1}^{N} P(Z_i = k|X_i) \cdot x_i}{\sum_{i=1}^{N} P(Z_i = k|X_i)} \quad \hat{\sigma_k^2} = \frac{\sum_{i=1}^{N} P(Z_i = k|X_i)(x_i - \hat{\mu}_k)^2}{\sum_{i=1}^{N} P(Z_i = k|X_i)}$$

```
compute.posterior <- function(X, p.curr, mu.curr, sigma.curr) {
  # compute the posterior of the latent variable given the current estimates and data
  p<- matrix(NA, nrow = length(X), ncol = length(p.curr))
  for (k in 1:length(p.curr)) {
    p[, k] <- p.curr[k] * dnorm(X, mean = mu.curr[k], sd = sigma.curr[k])
  }
  return(list(p = p))
}

EM.iter <- function(X, p.curr, mu.curr, sigma.curr) {

  # Expectation step
  p <- compute.posterior(X, p.curr, mu.curr, sigma.curr)$p
```

```r
  # Compute the posterior of latent variable given the observation
  gamma_i <- p / rowSums(p)

  # Maximisation step: compute new parameter estimates
  p.new <- colSums(gamma_i) / sum(gamma_i)
  mu.new <- colSums(gamma_i * X) / colSums(gamma_i)

  temp <- gamma_i * (replicate(length(mu.new), X) - t(replicate(length(X), mu.new)))^2
  sigma.new <- sqrt(colSums(temp) / colSums(gamma_i))

  return(list(p.new = p.new, mu.new = mu.new, sigma.new = sigma.new))
}

EM <- function(X, p, mu, sigma, epsilon = 1e-6) {
  p.curr <- p
  mu.curr <- mu
  sigma.curr <- sigma
  e.curr <- 0

  delta <- 1
  count <- 1

  # keep updating until convergence
  while (delta > epsilon) {
    # run EM to compute new estimates for the parameters
    EM.out <- EM.iter(X, p.curr, mu.curr, sigma.curr)
    count <- count + 1
    # print the new estimates of the current iteration
    print(round(c(EM.out$mu.new, EM.out$sigma.new, EM.out$p.new), 4))

    # compute the stopping criteria
    e.new <- sum(EM.out$mu.new - mu.curr) + sum(EM.out$sigma.new - sigma.curr)
    delta <- abs(e.new - e.curr)

    # replace current values with new estimates
    p.curr <- EM.out$p.new
    mu.curr <- EM.out$mu.new
    sigma.curr <- EM.out$sigma.new
    e.curr <- e.new
  }
  return(list(p = p.curr, mu = mu.curr, sigma = sigma.curr))
}
```

```r
# Define the initial values for the mixture model
p <- c(0.2, 0.3, 0.5)
mu <- c(0, 0.4, 0.7)
sigma <- c(0.2, 0.2, 0.2)
# Run EM
result <- EM(as.vector(I), p, mu, sigma)
```

```
## [1] 0.2024 0.3936 0.7970 0.0650 0.2464 0.1356 0.0852 0.2227 0.6921
## [1] 0.2102 0.3786 0.8197 0.0495 0.2216 0.0813 0.1395 0.1772 0.6832
## [1] 0.2120 0.3923 0.8289 0.0473 0.1888 0.0629 0.1732 0.1502 0.6766
## [1] 0.2119 0.4177 0.8322 0.0478 0.1785 0.0578 0.1886 0.1416 0.6697
## [1] 0.2115 0.4438 0.8336 0.0486 0.1772 0.0560 0.1985 0.1375 0.6640
## [1] 0.2115 0.4673 0.8344 0.0494 0.1774 0.0550 0.2061 0.1350 0.6589
## [1] 0.2119 0.4877 0.8350 0.0500 0.1775 0.0542 0.2120 0.1337 0.6544
## [1] 0.2122 0.5054 0.8354 0.0506 0.1777 0.0536 0.2165 0.1334 0.6501
## [1] 0.2126 0.5207 0.8358 0.0511 0.1779 0.0530 0.2200 0.1340 0.6460
## [1] 0.2130 0.5341 0.8362 0.0515 0.1784 0.0525 0.2227 0.1353 0.6420
## [1] 0.2133 0.5459 0.8364 0.0519 0.1792 0.0519 0.2249 0.1371 0.6380
## [1] 0.2136 0.5565 0.8367 0.0522 0.1803 0.0514 0.2265 0.1394 0.6341
## [1] 0.2138 0.5663 0.8368 0.0524 0.1816 0.0509 0.2278 0.1421 0.6301
## [1] 0.2140 0.5753 0.8370 0.0526 0.1831 0.0503 0.2288 0.1452 0.6260
## [1] 0.2142 0.5838 0.8371 0.0528 0.1847 0.0497 0.2296 0.1486 0.6218
## [1] 0.2144 0.5919 0.8372 0.0529 0.1863 0.0491 0.2302 0.1523 0.6175
## [1] 0.2145 0.5995 0.8373 0.0531 0.1877 0.0485 0.2307 0.1562 0.6131
## [1] 0.2146 0.6067 0.8374 0.0532 0.1888 0.0478 0.2312 0.1602 0.6086
## [1] 0.2147 0.6135 0.8375 0.0533 0.1896 0.0472 0.2317 0.1641 0.6042
## [1] 0.2148 0.6198 0.8377 0.0534 0.1901 0.0465 0.2321 0.1681 0.5998
## [1] 0.2149 0.6257 0.8378 0.0535 0.1903 0.0459 0.2325 0.1719 0.5956
## [1] 0.2150 0.6311 0.8379 0.0536 0.1901 0.0453 0.2330 0.1756 0.5915
## [1] 0.2151 0.6361 0.8381 0.0537 0.1898 0.0448 0.2334 0.1790 0.5875
## [1] 0.2152 0.6407 0.8382 0.0538 0.1892 0.0442 0.2339 0.1823 0.5838
## [1] 0.2152 0.6450 0.8384 0.0539 0.1885 0.0437 0.2344 0.1854 0.5803
## [1] 0.2153 0.6488 0.8385 0.0540 0.1877 0.0433 0.2348 0.1883 0.5769
## [1] 0.2154 0.6524 0.8387 0.0541 0.1869 0.0429 0.2353 0.1909 0.5738
## [1] 0.2155 0.6557 0.8388 0.0542 0.1860 0.0425 0.2357 0.1935 0.5708
## [1] 0.2156 0.6587 0.8390 0.0543 0.1851 0.0421 0.2362 0.1958 0.5680
## [1] 0.2157 0.6615 0.8391 0.0544 0.1842 0.0418 0.2366 0.1980 0.5654
## [1] 0.2158 0.6641 0.8393 0.0545 0.1833 0.0414 0.2370 0.2001 0.5629
## [1] 0.2159 0.6664 0.8394 0.0546 0.1825 0.0411 0.2373 0.2021 0.5606
## [1] 0.2160 0.6686 0.8395 0.0547 0.1817 0.0409 0.2377 0.2039 0.5584
## [1] 0.2161 0.6706 0.8397 0.0548 0.1809 0.0406 0.2380 0.2057 0.5563
## [1] 0.2162 0.6725 0.8398 0.0549 0.1801 0.0404 0.2383 0.2073 0.5544
## [1] 0.2163 0.6742 0.8399 0.0549 0.1794 0.0401 0.2386 0.2089 0.5525
## [1] 0.2164 0.6758 0.8400 0.0550 0.1788 0.0399 0.2389 0.2103 0.5508
## [1] 0.2164 0.6773 0.8401 0.0551 0.1781 0.0397 0.2392 0.2117 0.5491
## [1] 0.2165 0.6787 0.8402 0.0552 0.1775 0.0395 0.2394 0.2130 0.5476
## [1] 0.2166 0.6800 0.8403 0.0553 0.1770 0.0394 0.2396 0.2143 0.5461
## [1] 0.2167 0.6812 0.8404 0.0553 0.1765 0.0392 0.2398 0.2155 0.5447
## [1] 0.2167 0.6823 0.8405 0.0554 0.1760 0.0390 0.2400 0.2166 0.5434
## [1] 0.2168 0.6833 0.8405 0.0554 0.1755 0.0389 0.2402 0.2176 0.5422
## [1] 0.2168 0.6843 0.8406 0.0555 0.1750 0.0387 0.2404 0.2186 0.5410
## [1] 0.2169 0.6852 0.8407 0.0556 0.1746 0.0386 0.2406 0.2196 0.5399
## [1] 0.2169 0.6861 0.8407 0.0556 0.1742 0.0385 0.2407 0.2205 0.5388
## [1] 0.2170 0.6868 0.8408 0.0557 0.1739 0.0384 0.2409 0.2213 0.5378
## [1] 0.2170 0.6876 0.8409 0.0557 0.1735 0.0383 0.2410 0.2221 0.5369
## [1] 0.2171 0.6883 0.8409 0.0558 0.1732 0.0382 0.2411 0.2229 0.5360
## [1] 0.2171 0.6889 0.8410 0.0558 0.1729 0.0381 0.2412 0.2236 0.5352
## [1] 0.2172 0.6895 0.8410 0.0558 0.1726 0.0380 0.2414 0.2243 0.5344
## [1] 0.2172 0.6901 0.8411 0.0559 0.1723 0.0379 0.2415 0.2249 0.5336
## [1] 0.2172 0.6907 0.8411 0.0559 0.1720 0.0378 0.2416 0.2255 0.5329
## [1] 0.2173 0.6912 0.8412 0.0559 0.1718 0.0377 0.2417 0.2261 0.5323
## [1] 0.2173 0.6916 0.8412 0.0560 0.1716 0.0376 0.2417 0.2266 0.5316
## [1] 0.2173 0.6921 0.8412 0.0560 0.1713 0.0376 0.2418 0.2271 0.5310
## [1] 0.2174 0.6925 0.8413 0.0560 0.1711 0.0375 0.2419 0.2276 0.5305
## [1] 0.2174 0.6929 0.8413 0.0561 0.1709 0.0375 0.2420 0.2281 0.5299
## [1] 0.2174 0.6933 0.8413 0.0561 0.1708 0.0374 0.2420 0.2285 0.5294
## [1] 0.2174 0.6936 0.8414 0.0561 0.1706 0.0373 0.2421 0.2289 0.5290
## [1] 0.2175 0.6940 0.8414 0.0561 0.1704 0.0373 0.2422 0.2293 0.5285
## [1] 0.2175 0.6943 0.8414 0.0562 0.1703 0.0372 0.2422 0.2297 0.5281
## [1] 0.2175 0.6946 0.8414 0.0562 0.1701 0.0372 0.2423 0.2300 0.5277
## [1] 0.2175 0.6948 0.8415 0.0562 0.1700 0.0371 0.2423 0.2304 0.5273
## [1] 0.2175 0.6951 0.8415 0.0562 0.1699 0.0371 0.2424 0.2307 0.5269
## [1] 0.2176 0.6953 0.8415 0.0562 0.1697 0.0371 0.2424 0.2310 0.5266
## [1] 0.2176 0.6956 0.8415 0.0563 0.1696 0.0370 0.2425 0.2313 0.5263
## [1] 0.2176 0.6958 0.8416 0.0563 0.1695 0.0370 0.2425 0.2315 0.5260
## [1] 0.2176 0.6960 0.8416 0.0563 0.1694 0.0370 0.2426 0.2318 0.5257
## [1] 0.2176 0.6962 0.8416 0.0563 0.1693 0.0369 0.2426 0.2320 0.5254
## [1] 0.2176 0.6964 0.8416 0.0563 0.1692 0.0369 0.2426 0.2322 0.5251
## [1] 0.2176 0.6965 0.8416 0.0563 0.1691 0.0369 0.2427 0.2324 0.5249
## [1] 0.2177 0.6967 0.8416 0.0563 0.1691 0.0368 0.2427 0.2326 0.5247
## [1] 0.2177 0.6968 0.8416 0.0564 0.1690 0.0368 0.2427 0.2328 0.5244
## [1] 0.2177 0.6970 0.8417 0.0564 0.1689 0.0368 0.2427 0.2330 0.5242
## [1] 0.2177 0.6971 0.8417 0.0564 0.1688 0.0368 0.2428 0.2332 0.5240
## [1] 0.2177 0.6973 0.8417 0.0564 0.1688 0.0368 0.2428 0.2333 0.5239
## [1] 0.2177 0.6974 0.8417 0.0564 0.1687 0.0367 0.2428 0.2335 0.5237
## [1] 0.2177 0.6975 0.8417 0.0564 0.1686 0.0367 0.2428 0.2336 0.5235
## [1] 0.2177 0.6976 0.8417 0.0564 0.1686 0.0367 0.2429 0.2338 0.5234
## [1] 0.2177 0.6977 0.8417 0.0564 0.1685 0.0367 0.2429 0.2339 0.5232
## [1] 0.2177 0.6978 0.8417 0.0564 0.1685 0.0367 0.2429 0.2340 0.5231
## [1] 0.2178 0.6979 0.8417 0.0564 0.1684 0.0367 0.2429 0.2341 0.5229
## [1] 0.2178 0.6980 0.8417 0.0564 0.1684 0.0366 0.2429 0.2342 0.5228
## [1] 0.2178 0.6981 0.8418 0.0564 0.1684 0.0366 0.2430 0.2343 0.5227
## [1] 0.2178 0.6981 0.8418 0.0565 0.1683 0.0366 0.2430 0.2344 0.5226
## [1] 0.2178 0.6982 0.8418 0.0565 0.1683 0.0366 0.2430 0.2345 0.5225
## [1] 0.2178 0.6983 0.8418 0.0565 0.1682 0.0366 0.2430 0.2346 0.5224
## [1] 0.2178 0.6983 0.8418 0.0565 0.1682 0.0366 0.2430 0.2347 0.5223
## [1] 0.2178 0.6984 0.8418 0.0565 0.1682 0.0366 0.2430 0.2348 0.5222
## [1] 0.2178 0.6985 0.8418 0.0565 0.1682 0.0366 0.2430 0.2348 0.5221
## [1] 0.2178 0.6985 0.8418 0.0565 0.1681 0.0365 0.2430 0.2349 0.5220
## [1] 0.2178 0.6986 0.8418 0.0565 0.1681 0.0365 0.2431 0.2350 0.5220
## [1] 0.2178 0.6986 0.8418 0.0565 0.1681 0.0365 0.2431 0.2350 0.5219
## [1] 0.2178 0.6987 0.8418 0.0565 0.1680 0.0365 0.2431 0.2351 0.5218
## [1] 0.2178 0.6987 0.8418 0.0565 0.1680 0.0365 0.2431 0.2352 0.5218
## [1] 0.2178 0.6987 0.8418 0.0565 0.1680 0.0365 0.2431 0.2352 0.5217
## [1] 0.2178 0.6988 0.8418 0.0565 0.1680 0.0365 0.2431 0.2353 0.5216
## [1] 0.2178 0.6988 0.8418 0.0565 0.1680 0.0365 0.2431 0.2353 0.5216
## [1] 0.2178 0.6988 0.8418 0.0565 0.1679 0.0365 0.2431 0.2354 0.5215
## [1] 0.2178 0.6989 0.8418 0.0565 0.1679 0.0365 0.2431 0.2354 0.5215
## [1] 0.2178 0.6989 0.8418 0.0565 0.1679 0.0365 0.2431 0.2354 0.5215
## [1] 0.2178 0.6989 0.8418 0.0565 0.1679 0.0365 0.2431 0.2355 0.5214
```

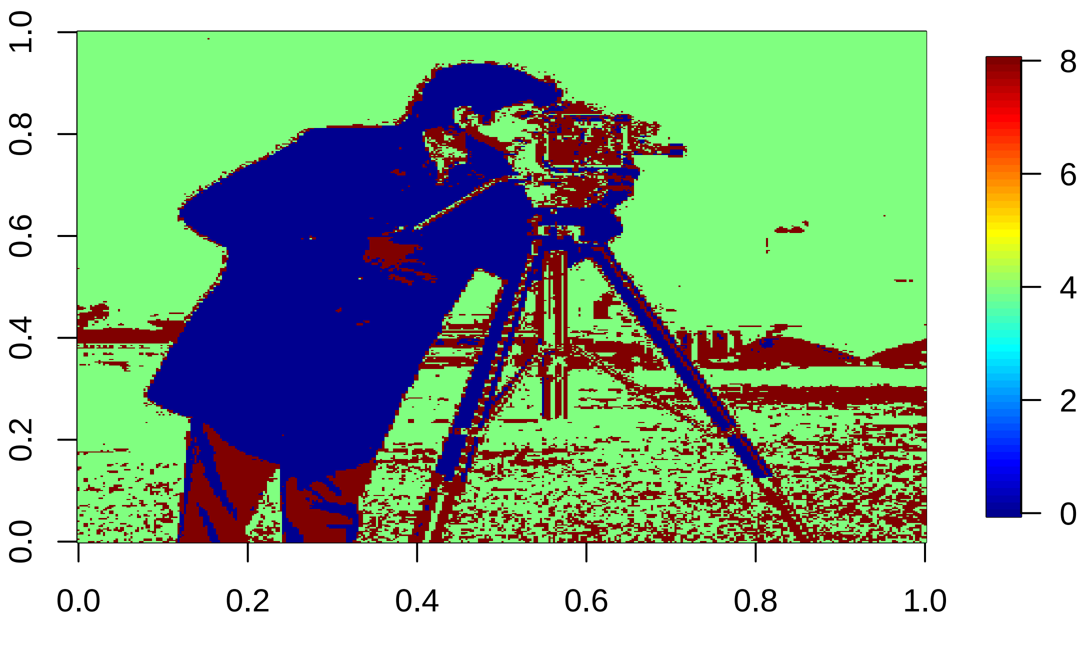
Question 3.4

```r
# Compute the probability of each pixels being in each class
predict_prob <- compute.posterior(as.vector(I), result$p, result$mu, result$sigma)
```

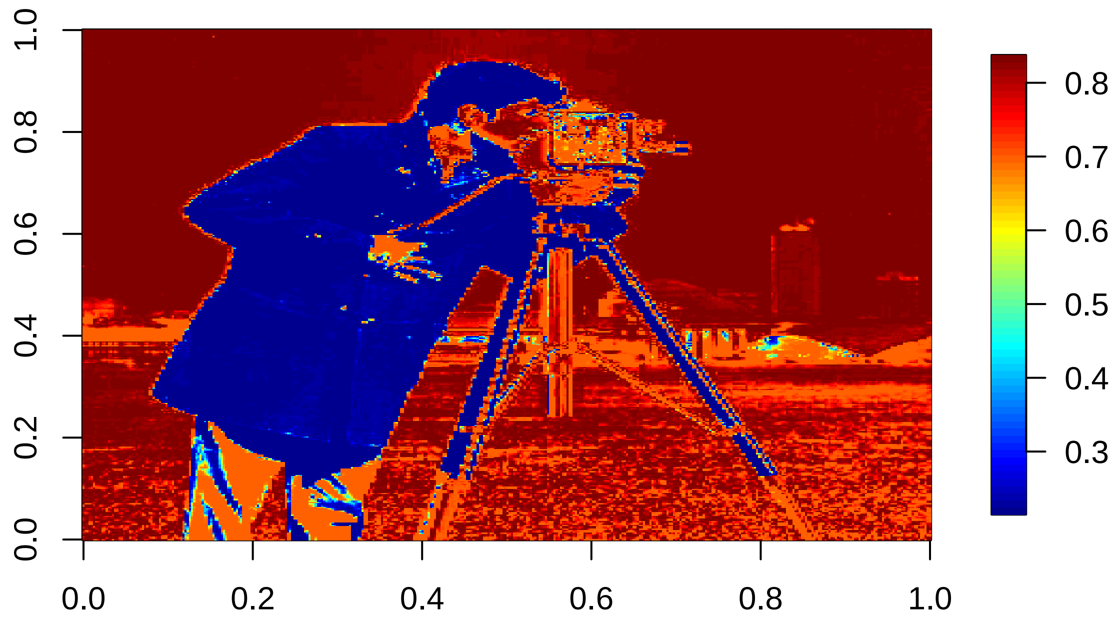
```
# Classify each pixels based on their pdf estimation
label <- rep(0, length(as.vector(I)))
for (i in 1:length(as.vector(I))) {
  label[i] <- which(predict_prob$p[i, ] == max(predict_prob$p[i, ]))
}

# Posterior PDFs multiplied with their mean
posterior <- predict_prob$p / rowSums(predict_prob$p)
mean <- t(replicate(length(as.vector(I)), result$mu))
posterior.mean <- rowSums(posterior * mean)

# Assign pixel values based on figure 3a color bar
label[label == 1] <- 0
label[label == 2] <- 8
label[label == 3] <- 4
```

Question 3.5

```
# Plot figure 3a and 3b
image.plot(matrix(label, 398, 398))
```



```
image.plot(matrix(posterior.mean, 398, 398))
```


0.0
0.2
0.4
0.6
0.8
1.0
0.0
0.2
0.4
0.6
0.8
1.0
0.3
0.4
0.5
0.6
0.7
0.8