Question 1:

a) For two given matrices $A, B \in \Re^{N\times N}$. Is $e^A . e^B = e^{A+B}$? Explain.

b) Under what condition is $e^A . e^B = e^{A+B}$? Elaborate.

a)

A and B are two nxn matrices

we can write:

$$e^A = I + A + \frac{A^2}{2!} + \frac{A^3}{3!} + \cdots$$

$$e^B = I + B + \frac{B^2}{2!} + \frac{B^3}{3!} + \cdots$$

Multiply them:

$$e^A e^B = I + A + B + AB + \frac{A^2}{2!} + \frac{B^2}{2!} + \frac{A^3}{3!} + \frac{B^3}{3!} + \frac{A^2B}{2!} + \frac{AB^2}{2!} + \cdots$$

$$= I + A + B + \frac{A^2+2AB+B^2}{2!} + \frac{A^3+3A^2B+3AB^2+B^3}{3!} + \cdots$$

Now, if we want to have $(A+B)^2 = A^2+2AB+B^2$

$(A+B)^3 = A^3+3A^2B+3AB^2+B^3$

$\cdots$

We need have the condition $AB = BA$

Since this is not given, the following step:

$$e^A e^B = I + A + B + \frac{(A+B)^2}{2!} + \frac{(A+B)^3}{3!} + \cdots = e^{A+B}.$$

cannot be achieved.

Therefore, given $A, B \in \mathbb{R}^{N\times N}$ does not mean $e^A . e^B = e^{A+B}$.

b) We need to have $AB = BA$

$$e^A = I + A + \frac{A^2}{2!} + \frac{A^3}{3!} + \cdots$$

$$e^B = I + B + \frac{B^2}{2!} + \frac{B^3}{3!} + \cdots$$

Multiply them:

$$e^A e^B = I + A + B + AB + \frac{A^2}{2!} + \frac{B^2}{2!} + \frac{A^3}{3!} + \frac{B^3}{3!} + \frac{A^2B}{2!} + \frac{AB^2}{2!} + \cdots$$

$$= I + A + B + \frac{A^2+2AB+B^2}{2!} + \frac{A^3+3A^2B+3AB^2+B^3}{3!} + \cdots$$

Since $AB = BA$, we have: $(A+B)^2 = A^2+2AB+B^2$

$(A+B)^3 = A^3+3A^2B+3AB^2+B^3$

$\cdots$

So, $$e^A e^B = I + (A+B) + \frac{(A+B)^2}{2!} + \frac{(A+B)^3}{3!} + \cdots$$

$$= e^{A+B}.$$

Q2

We first use [V_real,D_real]=eigs(A) to get the real values that we need to get. Then we implement (a), (b), and (c)and we get the following result, which is correct since they match up with V_real and D_real.

Real values:

```
V_real = 5x5
    0.4478    0.3223   -0.0331   -0.1942    0.8104
    0.4217   -0.5100   -0.6047   -0.4155   -0.1545
    0.3659    0.5409    0.2632   -0.4815   -0.5220
    0.4955   -0.5218    0.6757    0.1601   -0.0003
    0.4921    0.2669   -0.3278    0.7295   -0.2167

D_real = 5x5
   17.7680         0         0         0         0
         0    1.3563         0         0         0
         0         0    0.6116         0         0
         0         0         0    0.3554         0
         0         0         0         0    0.1086
```

(a) power method

```
e = 17.7680
v = 5x1
    0.9038
    0.8511
    0.7385
    1.0000
    0.9932
```

(b) simultaneous orthogonalization

```
V1 = 5x5
   -0.4478    0.3223    0.0331    0.1942   -0.8104
   -0.4217   -0.5100    0.6046    0.4155    0.1545
   -0.3659    0.5409   -0.2632    0.4815    0.5220
   -0.4955   -0.5218   -0.6757   -0.1602    0.0003
   -0.4921    0.2669    0.3278   -0.7295    0.2167

eigen1 = 5x1
   17.7680
    1.3563
    0.6116
    0.3554
    0.1086
```

(c)QR algorithm

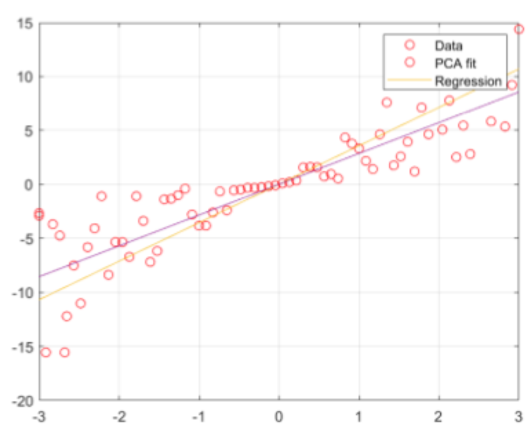
V2 = 5x5

-0.4478	0.3223	0.0331	0.1942	-0.8104
-0.4217	-0.5100	0.6046	0.4155	0.1545
-0.3659	0.5409	-0.2632	0.4815	0.5220
-0.4955	-0.5218	-0.6757	-0.1602	0.0003
-0.4921	0.2669	0.3279	-0.7295	0.2167

eigen2 = 5x1

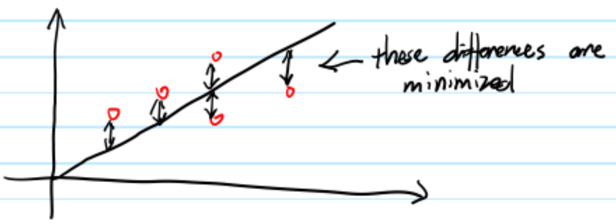
17.7680
1.3563
0.6116
0.3554
0.1086

Q_3



We can see the two lines are slightly different from each other because the error that they are trying to minimize is different.

In linear regression, we use the least square method to find the line to minimize the vertical difference as shown below.



However, in Principle Component Analysis, we are minimizing the orthogonal distance between points and lines as shown below.