The main aim of this project is to write a computer program for simulating the transient response of a half-wave rectifier circuit.

The circuit shown in Figure 1 is called the Half-Wave Rectifier. For this circuit we will compute the transient response of the circuit for all the voltage nodes, namely, $V_1$, $V_2$, and $V_3$.

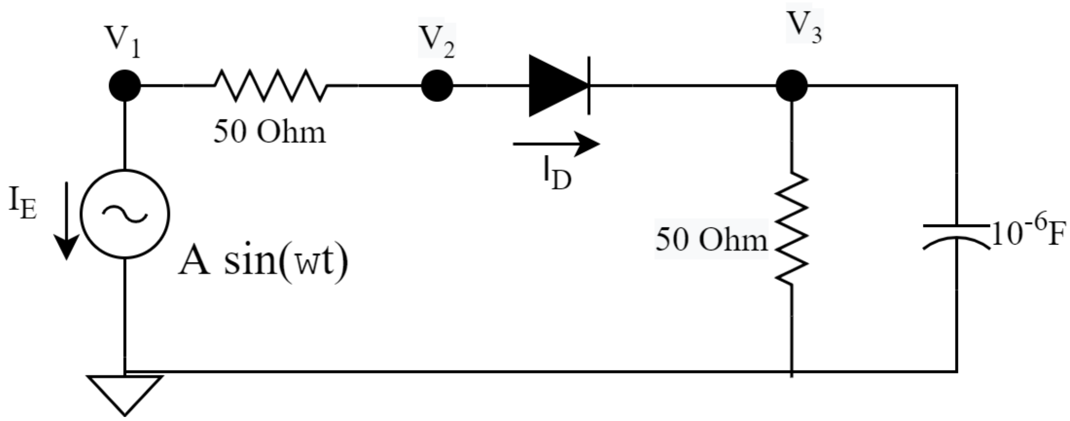


Figure 1: Half-Wave Rectifier.

$I_E$ is the current passing into voltage source. $I_D$ is the diode current shown in (1)

$$I_D = I_S\left(e^{\frac{V_2-V_3}{V_t}} - 1\right) \tag{1}$$

$I_S$ is the saturation current of the diode and $V_t$ is the thermal voltage. As we can see that the diode current is a nonlinear function of the node voltages $V_2$ and $V_3$, this makes the nodal equations of the above circuit (shown in (2)) nonlinear.

$$\underbrace{\begin{bmatrix} \frac{1}{50} & -\frac{1}{50} & 0 & 1 \\ -\frac{1}{50} & \frac{1}{50} & 0 & 0 \\ 0 & 0 & \frac{1}{50} & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}}_{G}\underbrace{\begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ I_E \end{bmatrix}}_{X(t)} + \underbrace{\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 10^{-6} & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}}_{C}\underbrace{\begin{bmatrix} \dot{V}_1 \\ \dot{V}_2 \\ \dot{V}_3 \\ \dot{I}_E \end{bmatrix}}_{\dot{X}(t)} + \underbrace{\begin{bmatrix} 0 \\ I_S\left(e^{\frac{V_2-V_3}{V_t}} - 1\right) \\ -I_S\left(e^{\frac{V_2-V_3}{V_t}} - 1\right) \\ 0 \end{bmatrix}}_{D(X(t))} - \underbrace{\begin{bmatrix} 0 \\ 0 \\ 0 \\ A\sin(\omega t) \end{bmatrix}}_{B(t)} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \tag{2}$$

The above equation can be written in compact form as,

$$\boldsymbol{F}(\boldsymbol{X}) = \boldsymbol{G}\,\boldsymbol{X}(t) + \boldsymbol{C}\,\dot{\boldsymbol{X}}(t) + \boldsymbol{D}(\boldsymbol{X}(t)) - \boldsymbol{B}(t) = \boldsymbol{0} \tag{3}$$

The expression in (3) represents system of Nonlinear Differential Algebraic equations. The solution vector $\boldsymbol{X}(t)$ and the input vector $\boldsymbol{B}(t)$ are time varying. At time, $t = t_n$, $\boldsymbol{X}(t)$ and $\boldsymbol{B}(t)$ can be written as $\boldsymbol{X_n}$ and $\boldsymbol{B}_n$, respectively.

$$\boldsymbol{F}(\boldsymbol{X_n}) = \boldsymbol{G}\,\boldsymbol{X_n} + \boldsymbol{C}\,\dot{\boldsymbol{X}}_n + \boldsymbol{D}(\boldsymbol{X}_n) - \boldsymbol{B}_n = \boldsymbol{0} \tag{4}$$

In this problem, we will use the Backward Euler formula give by

$$\dot{\boldsymbol{X}}_n = \frac{\boldsymbol{X}_n - \boldsymbol{X}_{n-1}}{\Delta t} \tag{5}$$

where $\Delta t$ is the length of the time step. Upon substituting (5) in (4) we get the following difference equation

$$\boldsymbol{F}(\boldsymbol{X_n}) = \left(\boldsymbol{G} + \frac{1}{\Delta t}\boldsymbol{C}\right)\boldsymbol{X}_n + \boldsymbol{D}(\boldsymbol{X}_n) - \boldsymbol{B}_n - \frac{1}{\Delta t}\boldsymbol{C}\boldsymbol{X}_{n-1} = \boldsymbol{0} \tag{6}$$

By using the Backward Euler approximation we have reduced the system of Nonlinear Differential Algebraic Equations in (4) to a system of Nonlinear Difference equations in (6). Using (6) we can compute $\boldsymbol{X}_n$ if we know $\boldsymbol{X}_{n-1}$. Note that in order to solve (6) at each time step we need to use the Newton-Raphson Method.

In order to use Newton Iteration, you need to be able to evaluate $\boldsymbol{F(X_n)}$**.** You also need to be able to evaluate the jacobian of system of nonlinear equations in (6) is given as

$$\frac{\partial \boldsymbol{F}(\boldsymbol{X_n})}{\partial \boldsymbol{X_n}} = \left(\boldsymbol{G} + \frac{1}{\Delta t}\boldsymbol{C}\right) + \frac{\partial \boldsymbol{D}(\boldsymbol{X_n})}{\partial \boldsymbol{X_n}} \tag{8}$$

where $\frac{\partial \boldsymbol{D}(\boldsymbol{X_n})}{\partial \boldsymbol{X_n}}$ is the Jacobian of the diode vector and it given by

$$\frac{\partial \boldsymbol{D}(\boldsymbol{X})}{\partial \boldsymbol{X}} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & \dfrac{\partial I_D}{\partial V_2} & \dfrac{\partial I_D}{\partial V_3} & 0 \\ 0 & \dfrac{-\partial I_D}{\partial V_2} & \dfrac{-\partial I_D}{\partial V_3} & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & \dfrac{I_S}{V_t}\left(e^{\frac{V_2 - V_3}{V_t}}\right) & -\dfrac{I_S}{V_t}\left(e^{\frac{V_2 - V_3}{V_t}}\right) & 0 \\ 0 & \dfrac{-I_S}{V_t}\left(e^{\frac{V_2 - V_3}{V_t}}\right) & \dfrac{I_S}{V_t}\left(e^{\frac{V_2 - V_3}{V_t}}\right) & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \tag{9}$$

Thus, equation (8) becomes,

$$\frac{\partial \boldsymbol{F}(\boldsymbol{X_n})}{\partial \boldsymbol{X_n}} = \begin{bmatrix} \dfrac{1}{50} & -\dfrac{1}{50} & 0 & 1 \\ -\dfrac{1}{50} & \dfrac{1}{50} & 0 & 0 \\ 0 & 0 & \dfrac{1}{50} & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} + \frac{1}{\Delta t}\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 10^{-6} & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & \dfrac{I_S}{V_t}\left(e^{\frac{V_2 - V_3}{V_t}}\right) & -\dfrac{I_S}{V_t}\left(e^{\frac{V_2 - V_3}{V_t}}\right) & 0 \\ 0 & \dfrac{-I_S}{V_t}\left(e^{\frac{V_2 - V_3}{V_t}}\right) & \dfrac{I_S}{V_t}\left(e^{\frac{V_2 - V_3}{V_t}}\right) & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \tag{10}$$

Starting with an initial guess $\boldsymbol{X_n}^{(0)}$ for vector $\boldsymbol{X_n}$, you can obtain an improved guess as shown in (11)

$$\boldsymbol{X_n}^{(1)} = \boldsymbol{X_n}^{(0)} - \left(\left.\frac{\partial \boldsymbol{F}(\boldsymbol{X_n})}{\partial \boldsymbol{X_n}}\right|_{\boldsymbol{X_n} = \boldsymbol{X_n}^{(0)}}\right)^{\boldsymbol{-1}} \boldsymbol{F}\left(\boldsymbol{X_n}^{(0)}\right) \tag{11}$$

Repeating the above process for *k* iteration we get,

$$\boldsymbol{X_n}^{(k+1)} = \boldsymbol{X_n}^{(k)} - \left(\left.\frac{\partial \boldsymbol{F}(\boldsymbol{X_n})}{\partial \boldsymbol{X_n}}\right|_{X_n = X_n^{(k)}}\right)^{-1} \boldsymbol{F}\left(\boldsymbol{X_n}^{(k)}\right) \tag{12}$$

The above procedure is repeated until the following two convergence conditions are simultaneously met,

- The function value at new guess point is sufficiently small, i.e., $\left\|\boldsymbol{F}\left(\boldsymbol{X_n}^{(k+1)}\right)\right\|_2 < \epsilon_{tolerance}$
- The difference between the two consecutive solutions is sufficiently small, i.e., $\left\|\boldsymbol{X_n}^{(k+1)} - \boldsymbol{X_n}^{(k)}\right\|_2 < \epsilon_{tolerance}$

<u>Upon solving the above equations we obtain the solution $\boldsymbol{X}_n$ for time $t = t_n$, we repeat the above procedure for all the time values ranging from $t_1, t_2, \ldots, t_n, \ldots, t_{stop}$.</u>

In other words, start with time $t_1$, solve the nonlinear equations in (6) to obtain $\boldsymbol{X}_1$. Use $\boldsymbol{X}_0 = [0 \quad 0 \quad 0 \quad 0]^T$, at $t_0 = 0$ as an initial condition. The nonlinear equations in (6) are solved using the Newton-Raphson equations described in (12). After obtaining $\boldsymbol{X}_1$, solve the nonlinear equations in (6) at time $t_2$ using the Newton-Raphson to obtain $\boldsymbol{X}_2$. Repeat this process for all time points.

This document showed how to solve the Nonlinear-Differential Equations using the Backward Euler method. In this project your task is to compute find the transient response of the circuit, which means find the solution vector for all values of time. The use transient voltage source with amplitude equal to 5V with the frequency of 60Hz. Use saturation current for the diode, $I_S = 10^{-13}$ and the thermal voltage of the diode,$V_T = 0.025$. Simulate, the circuit for 0.5s with the time step, $\Delta t = 10^{-4}$. Use the tolerance value, $\epsilon_{tolerance} = 10^{-5}$, to check for convergence of the Newton-Raphson. Plot the obtained solution for $V_1$, $V_2$, and $V_3$.

Submit a typed report not more than 2 pages describing your implementation. Discuss in the report, about the initial guess you used for the time points $t_2, \ldots, t_n, \ldots, t_{stop}$. Does the result obtained for node voltage $V_3$ consistent with diode response? How did you check if the simulation results are correct?

Also submit your MATLAB code.