Solution Problem 2

Detection of Alpha Particles

a. From the given range-energy relation and the data supplied we get

$$E = \left(\frac{R\alpha}{0.318}\right)^{\frac{2}{3}} MeV = \left(\frac{5.50}{0.318}\right)^{\frac{2}{3}} = 6.69 MeV$$

(0.5 point)

since $W_{ion\text{-}\rho air}$= 35 eV, then

$$N_{ion-\rho air} = \frac{6.69\times10^{6}}{35} = 1.9\times10^{5}$$

(0.5 point)

Size of voltage pulse:

$$\Delta V = \frac{\Delta Q}{C} = \frac{N_{air-\rho air} e}{C}$$

$$\text{with } C = 45\,pF = 4.5\times10^{-11}$$

(0.5 point)

Hence

$$\Delta V = \frac{1.9\times10^{5}\times1.6\times10^{-19}}{4.5\times10^{-11}} V = 0.68 mV$$

(0.5 point)

b. Electrons from the ions-pairs produced by α particles from a radioactive sources of activity A (=number of α particles emitted by the sources per second) which enter the detector with detection efficiency 0.1, will produce a collected current.

$$I = \frac{Q}{t} = 0.1\times AN_{ion-\rho air}e$$

$$= 0.1\times A\times1.9\times10^{5}\times1.6\times10^{-19} A$$

(1.0 point)

With $I_{min}=10^{-12}$ A, the

$$A_{\min} = \frac{10^{-12}\,\text{dis}\,s^{-1}}{1.6\times1.9\times10^{-15}} = 330\,\text{dis}\,s^{-1}$$

(1.0 point)

Since 1 Ci = 3.7 x 10^{10} dis s^{-1} then

$$A_{\min} = \frac{330}{3.7\times10^{10}} Ci = 8.92\times10^{-9} Ci$$

(1.0 point)

c. With time constant

$$\tau = RC\left(\text{with C=}45\times10^{-12}\text{F}\right) = 10^{-3} s$$

$$R = \left(\frac{1000}{45}\right) M\Omega = 22.22 M\Omega$$

(0.5 point)

For the voltage signal with height Δ V = 0.68 mV generated at the anode of the ionization chamber by 6.69 MeV α particles in problem (a), to achieve q 0.25 V= 250mV voltage signal, the necessary gain of the voltage pulse amplifier should be

$$G = \frac{250}{0.68} = 368$$

(0.5 point)

d. By symmetry , the electric field is directed radially and depends only on distance the axis and can be deducted by using Gauss' theorem.

If we construct a Gaussian surface which is a cylinder of radius τ and length l , the charge contained within it is σl.

The surface integral

$$\int E.dS = 2\pi r l E$$

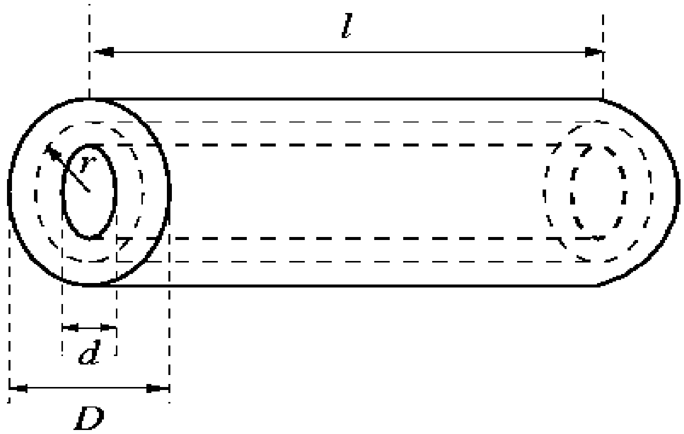

Figure 1 : The Gaussian surface used to calculate the electric field E.

(1.0 point)

Since the field E is everywhere constant and normal to the curved surface. By Gauss's theorem :

$$2\pi r l E = \frac{\lambda l}{\varepsilon_0}$$

so

$$\mathrm{E}(r) = \frac{\lambda}{2\pi\varepsilon_0 r}$$

Since E is radial and varies only with τ, then $E = -\frac{dV}{dr}$ and the potential V can be found by integrating E(τ) with respect to τ, if we call the potential of inner wire V_0, we have

$$V(r) - V_0 = -\frac{\lambda}{2\pi\varepsilon_0}\int_{\frac{d}{2}}^{\tau}\frac{dr}{r}$$

Thus

$$V(r) - V_0 = -\frac{\lambda}{2\pi\varepsilon_0}\ln\left(\frac{2r}{d}\right)$$

(1.0 point)

We can use this expression to evaluate the voltage between the capacitor's conductors by setting $r = \frac{D}{2}$, giving a potential difference of

$$V = \frac{\lambda}{2\pi\varepsilon_0} \ln\left(\frac{D}{d}\right)$$

since the charge Q in the capacitor is σ *l*, and the capacitance C is defined by Q=CV, the capacitance per unit length is

$$\frac{2\pi\varepsilon L_0}{\ln \frac{D}{d}}$$

(1.0 point)

The maximum electric field occurs where r minimum, i.e. at $r = \frac{d}{2}$. if we set the field at $r = \frac{d}{2}$ equal to the breakdown field E_b, our expression for E® shows that the charges per unit length σ in the capacitor must be $E_b\pi_0 d$. Substituting for the potential difference V across the capacitor gives

$$V = \frac{1}{2} E_b d \ln\left(\frac{D}{d}\right)$$

Taking $E_b = 3 \times 10^6$ V , d= 1mm, and D= 1 cm , gives V = 3.453.45 kV.

(1.0 point).