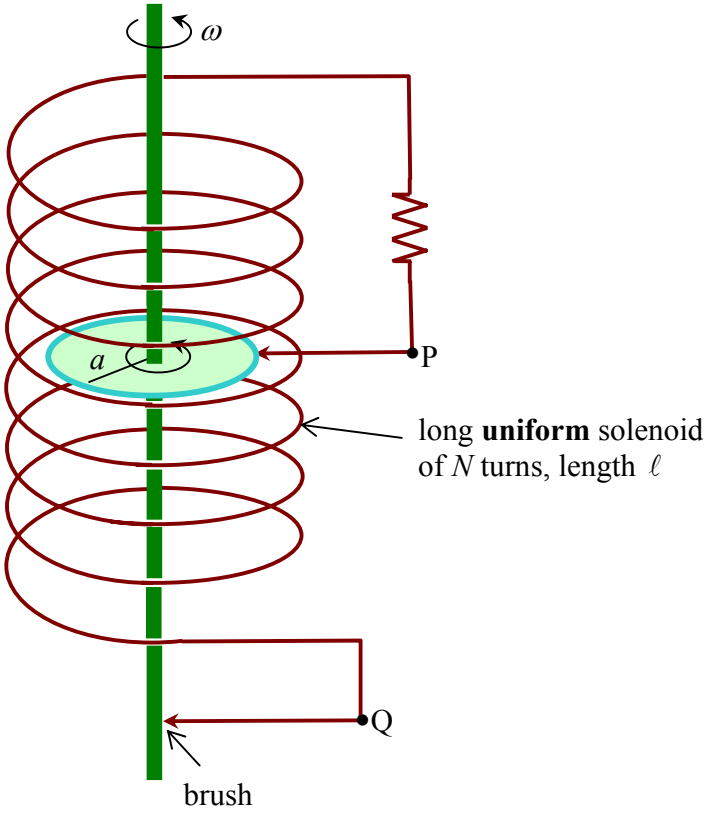
A Self-excited Magnetic Dynamo

A metallic disc of radius a mounted on a slender axle is rotating with a constant angular velocity ω inside a long solenoid of inductance L whose two ends are connected to the rotating disc by two brush contacts as shown. The total resistance of the whole circuit is R. A small magnetic disturbance can initiate the growth of an induced electromotive force across the terminals P, Q.



2.1) Write down the differential equation for $i(t)$, the current through the circuit. Express your answer in terms of $L, R,$ and the induced e.m.f. $(\mathcal{E})$ across the terminals P and Q. (1.0 point)

2.2) What is the value of the magnetic flux density (B) in terms of $i, N, \ell,$ and the permeability of free space μ_0? Ignore the magnetic field generated by the disc and the axle. (1.5 points)

2.3) What is the expression for the induced e.m.f. $(\mathcal{E})$ in terms of $\mu_0, N, a, \ell, i,$ and the angular velocity ω? (2.0 points)

2.4) Solve the equation in question 2.1 for current at any time t in terms of the initial current $i(0)$, and other parameters. (1.5 points)

2.5) What is the minimum value of the angular velocity that will permit the current to grow? Give your answers in terms of $R, \mu_0, N, a,$ and ℓ. (2.0 points)

2.6) In order to maintain a certain steady angular velocity ω, what must be the value of torque applied to the axle at the instant t? (2.0 points)

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