SOLUTION

a) In magnetic field, the particle will be deflected and follow a helical path.

Lorentz Force in a magnetic field *B,*

$$\frac{mv_\perp^2}{R} = ev_\perp B \tag{1}$$

Where $v_\perp$ is the transverse velocity of the electron, R is the radius of the path.

Since $v_\perp = \omega R$ ($\omega = \dfrac{2\pi}{T}$ is the particle angular velocity and T is the period), then,

$$m\frac{2\pi}{T} = eB \tag{2}$$

To be focused, the period of electron T must be equal to $\dfrac{L}{v_{//}}$, where $v_{//}$ is the parallel component of the velocity.

We also know,

$$eV = \frac{1}{2}m\left(v_\perp^2 + v_{//}^2\right) \approx \frac{1}{2}mv_{//}^2 \tag{3}$$

All the information above leads to

$$B = 2^{3/2}\pi\frac{\left(mV/e\right)^{1/2}}{L} \tag{4}$$

Numerically

$$B = 4.24 \text{ mT}$$

b) The magnetic field of the Solenoid:

$$B = \mu_0 in \tag{5}$$

$$i = \frac{B}{n\mu_0} \tag{6}$$

Numerically

$$i = 6.75 \text{ A.}$$

[*Marking Scheme*] *THEORETICAL Question 2A*

Magnetic Focusing Solenoid

a. (3.0)	0.3	Lorentz force $\frac{mv_{\perp}^2}{R} = ev_{\perp}B$
	0.1	Transverse velocity $v_{\perp} = \omega R$
	0.1	$\omega = \frac{2\pi}{T}$
	0.3	Equation $m\frac{2\pi}{T} = eB$
	0.2	Equation $T = \frac{L}{v_{//}}$
	0.5	Conservation energy $eV = \frac{1}{2}m\left(v_{\perp}^2 + v_{//}^2\right) \approx \frac{1}{2}mv_{//}^2$
	1.0	Formula $B = 2^{3/2}\pi \frac{\left(mV/e\right)^{1/2}}{L}$
	0.5	Numerical value B = 4.23 mT
b. (**1.0**)	0.5	$B=\mu_0 in$
	0.3	$i = \frac{B}{n\mu_0}$
	0.2	i = 6.75 A.

Note: Propagation errors will not be considered.

2B. MAGNETIC FOCUSING (FRINGING FIELD) (6 points)

Two pole magnets positioned on horizontal planes are separated by a certain distance such that the magnetic field between them be B in vertical direction (see Figure 2.2). The poles faces are rectangular with length l and width w. Consider the fringe field near the edges of the poles (fringe field is field particularly associated to the edge effects). Suppose the extent of the fringe field is b (see Fig. 2.3). The fringe field has two components B_x **i** and B_z **k**. For simplicity assume that $|B_x| = B|z|/b$ where z=0 is the mid plane of the gap, explicitly:

- when the particle enters the fringe field $B_x = +B\,z/b$,
- when the particle enters the fringe field after traveling through the magnet, $B_x = -Bz/b$

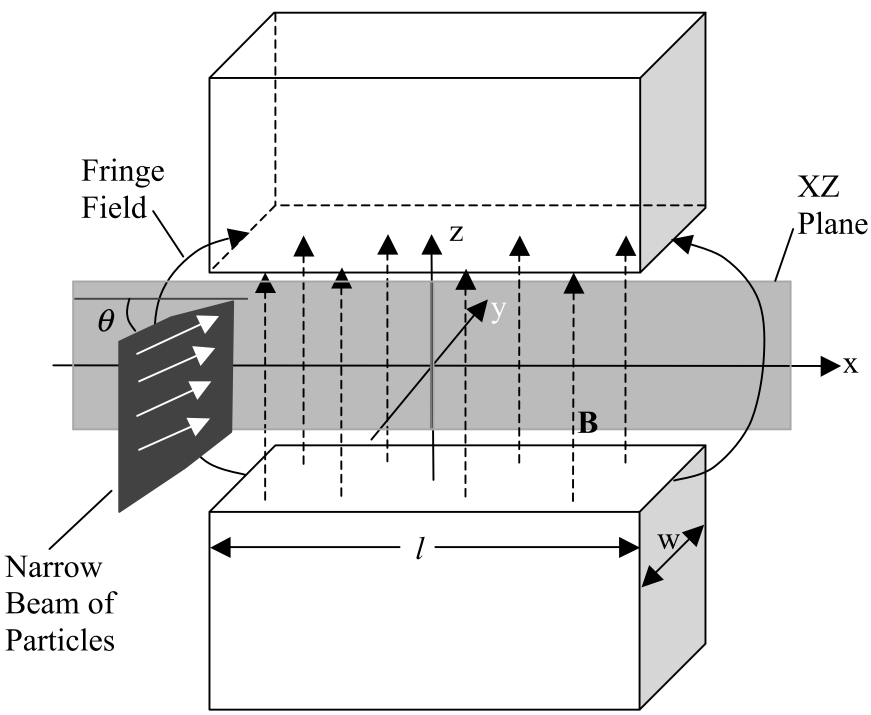

Fig.2.2: Overall view (note that θ is very small).

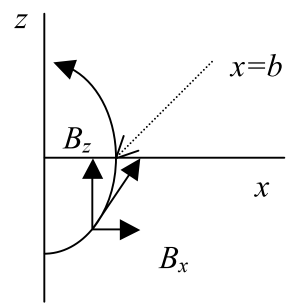

Figure 2.3. Fringe field

A parallel narrow beam of particles, each of mass m and positive charge q enters the magnet (near the center) with a high velocity v parallel to the horizontal plane. The vertical size of the beam is comparable to the distance between the magnet poles. A certain beam enters the magnet at an angle θ from the center line of the magnet and leaves the magnet at an angle $-\theta$ (see Figure 2.4. Assume θ is very small). Assume that the angle θ with which the particle enters the fringe field is the same as the angle θ when it enters the uniform field.

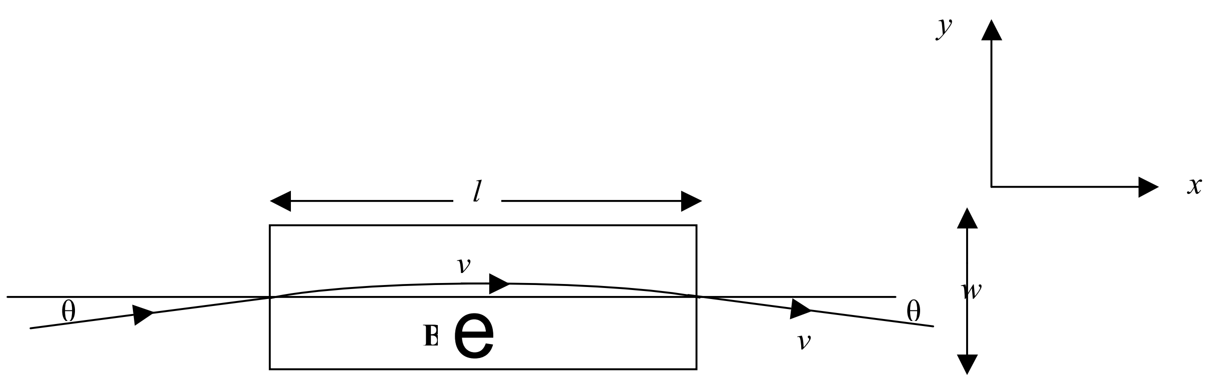

Figure 2.4. Top view

The beam will be focused due to the fringe field. Calculate the approximate focal length if we define the focal length as illustrated in Figure 2.5 (assume $b<<l$ and assume that the z-component of the deflection in the uniform magnetic field B is very small).

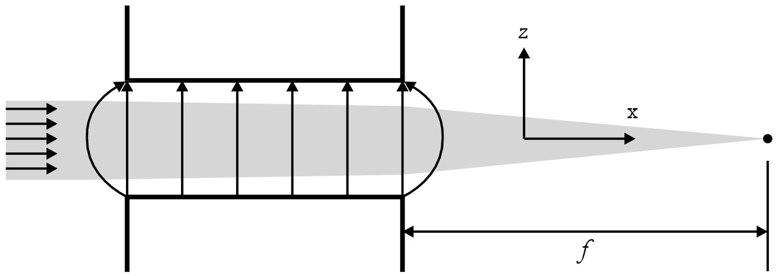

Figure 2.5. Side view

Solution:

The magnetic force due to the fringe field on charge q with velocity v is

$$\vec{F} = q\vec{v}\times\vec{B} \qquad (1)$$

The z-component of the force obtained from the cross product is

$$F_z = q\left(v_x B_y - v_y B_x\right) = -qv_y B_x = -\frac{qv\sin\theta Bz}{b} \qquad (2)$$

The vertical momentum gained by the particle after entering the fringe field

$$\Delta P_z = \int F_z dt = -\frac{qvBz\sin\theta}{b}\Delta t = -\frac{qvBz\sin\theta}{b}\frac{b}{v\cos\theta} = -qBz\tan\theta \qquad (3)$$

The particle undergoes a circular motion in the constant magnetic field B region

$$m\frac{v^2}{R} = qvB \qquad (4)$$

$$v = \frac{qBR}{m} = \frac{qBl}{2m\sin\theta} \qquad (5)$$

Therefore,

$$\sin\theta = \frac{qBl}{2mv} \qquad (6)$$

After the particle exits the fringe field at the other end, it will gain the same momentum.

The total vertical momentum gained by the particle is

$$\left(\Delta P_z\right)_{total} = 2\Delta P_z = -2qBz\tan\theta \approx -2qBz\frac{qBl}{2mv} = -\frac{q^2B^2zl}{mv} \tag{7}$$

Note that for small θ, we can approximate $\tan\theta \approx \sin\theta$

Meanwhile, the momentum along the horizontal plane (xy-plane) is

$$p = mv \tag{8}$$

From the geometry in figure 4, we can get the focal length by the following relation,

$$\frac{\left|\Delta P_z\right|}{p} = \frac{|Z|}{f} \tag{9}$$

$$\boxed{f = \frac{m^2v^2}{q^2B^2l}} \tag{10}$$

[Marking Scheme] **THEORETICAL Question 2B**

Magnetic Focusing (Fringing Field)

(6.0)	0.25	Lorentz force $\vec{F} = q\vec{v} \times \vec{B}$
	0.25	z-component $F_z = q\left(v_x B_y - v_y B_x\right)$
	0.25	z-component $F_z = -qv_y B_x = -\dfrac{qv\sin\theta Bz}{b}$
	0.5	z-component gained momentum $\Delta P_z = \int F_z dt$
	0.75	$\Delta P_z = -qBz\tan\theta$
	0.5	Equation $m\dfrac{v^2}{R} = qvB$
	0.25	$\sin\theta = \dfrac{qBl}{2mv}$
	0.5	$\left(\Delta P_z\right)_{total} = 2\Delta P_z$ (factor of 2)
	0.25	$\left(\Delta P_z\right)_{total} = -\dfrac{q^2B^2zl}{mv}$
	0.5	Horizontal momentum $p = mv$
	1.0	Equation $\dfrac{\left\|\Delta P_z\right\|_{total}}{p} = \dfrac{\left\|Z\right\|}{f}$
	1.0	$f = \dfrac{m^2v^2}{q^2B^2l}$

Note: No propagation error will be considered here.