1. MAGNETOSTATICS

The magnetomotive force (mmf) along a curve is defined as the path integral of the projection of the magnetic induction B along the curve,

$$\int_{\text{curve}} \vec{B}\vec{dl}\,.$$

Ampere's Circuital Law states that the magnetomotive force along a closed curve (loop) is proportional to the electric current crossing **ANY** surface whose frontier is this loop. The proportionality constant is called *magnetic permeability of the vacuum* (μ_0).

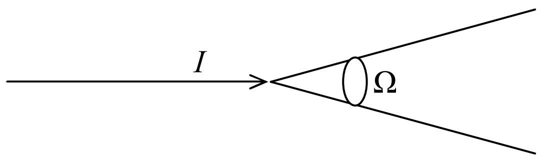
$$\oint_{\text{loop}} \vec{B}\vec{dl} = \mu_0 I_{\text{across}}$$

The positive direction of the current is associated to the path followed on the loop through *the right-handed corkscrew rule*.

a. An infinitely long straight conductor carries a steady current I. Find the magnitude and the orientation of the magnetic induction B generated by this current at a distance r from the wire. Express the result in terms of I, r, and μ_0.

b. A thin uniform rod of mass m and length L is placed parallel to the wire, at a distance d. The rod can only rotate on an axis perpendicular to the plane determined by the wire and the rod, passing through the middle of the rod. The rod carries a steady current I' in the opposite direction of I. The rod is slanted with a small angle from its equilibrium position and let to oscillate freely. Find the period of the small oscillations of the rod in terms of I, I', m, L, d, and μ_0.

c. A semi-infinite straight conductor is continued with an infinite conical conductor surface, whose axis coincides with the wire, as in the diagram alongside. The system carries a steady current I. Find the magnitude and the orientation of the magnetic induction B at a distance r from the axis, both inside and outside the conical conductor. Express the result in terms of I, r, and μ_0.



d. A semi-infinite straight conductor is connected at its end with an infinite conductor plane, placed perpendicular to the wire. The system carries a steady current I. Find the magnitude and the orientation of the magnetic induction B at a distance r from the axis of the wire, on both sides of the plane. Express the result in terms of I, r, and μ_0.

e. Define the linear current density $\vec{J}$ flowing on the plane from the previous point as:

$$J \overset{\text{def}}{=} \frac{dI}{dl},$$

where dl is an elementary length perpendicular to the line carrying an elementary current dI.

Introduce a unit vector $\vec{n}$ perpendicular to the plane, in order to indicate the positive direction of the crossing from one side of the plane to the other. The vectorial product $\vec{J} \times \vec{n}$ determines the positive direction for the component of B parallel to the plane.

Show that when crossing the plane, the difference in magnitude of the component of B parallel to the plane is proportional to the magnitude of J in the crossing point, and find the proportionality constant.

f. An infinite conductor plane is parallel to a uniform magnetic field. The magnetic induction B has the same direction on both sides of the plane, but different values B_1 and B_2. Find the pressure exerted upon the plane. Express the result in terms of B_1, B_2, and μ_0.

g. A conductor hollow sphere is connected at its poles with two semi-infinite straight conductors, oriented on the poles axis. The system carries a steady current I. Find the magnitude and the orientation of the magnetic induction B at a distance r from the axis of the poles, both inside and outside the sphere. Express the result in terms of I, r, and μ_0.

h. A conductor hollow sphere has its poles connected by an interior straight wire. A steady current I flows on the surface of the sphere from one pole to the other, and then back through the wire. Find the magnitude and the orientation of the magnetic induction B at a distance r from the axis of the poles, both inside and outside the sphere. Express the result in terms of I, r, and μ_0.

The Biot-Savart Law gives the expression of the magnetic induction generated in a point in space by an electric current flowing along an elementary path dl:

$$\overrightarrow{dB} = \frac{\mu_0 I\left(\overrightarrow{dl}\times\vec{r}\right)}{4\pi r^3},$$

where r is the position of the point relative to the elementary current.

i. A straight conductor of length L carries a steady current I. The wire is seen from a point in its mediator plane under the angle 2α. Express the magnitude of the magnetic induction in this point in terms of L, I, α, and μ_0.

j. A steady current I flows uniformly on the surface of a conductor sphere of radius R, from one pole to the other. Find the magnitude of the magnetic induction in the equatorial plane of the sphere, in a point at distance r from the axis of the poles, both inside and outside the sphere. Express the result in terms of I, R, r, and μ_0.