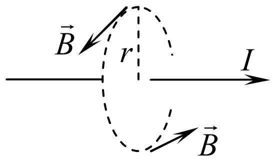
PROBLEM No. 1

a. 1p

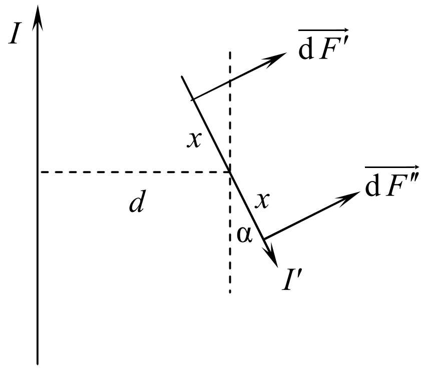
From the symmetry of the situation we take the magnetomotive force along a circular path of radius r, centered on the wire.



$$\oint \vec{B}\,\mathrm{d}\vec{l} = 2\pi r B = \mu_0 I \Rightarrow B = \frac{\mu_0 I}{2\pi r}$$

b. 1.5p

Consider two elements dx of the rod placed symmetrically at distances x from its center. The corresponding forces acting on them are:



$$\mathrm{d}F' = \frac{\mu_0 I I' \,\mathrm{d}x}{2\pi\left(d - x\sin\alpha\right)}$$

$$\mathrm{d}F'' = \frac{\mu_0 I I' \,\mathrm{d}x}{2\pi\left(d + x\sin\alpha\right)}$$

The sum of their torques is:

$$\mathrm{d}M = \left(\mathrm{d}F'' - \mathrm{d}F'\right)x = -\frac{2\mu_0 I I' x^2 \sin\alpha\,\mathrm{d}x}{2\pi\left(d^2 - x^2\sin^2\alpha\right)}$$

For very small angles, the total torque is:

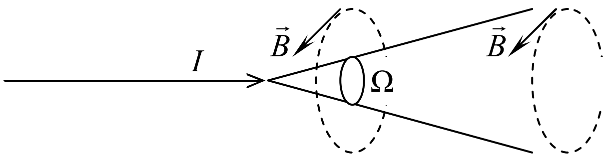
$$M = \int_0^{L/2} -\frac{\mu_0 I I' \alpha x^2 dx}{\pi d^2} = -\frac{\mu_0 I I' \alpha L^3}{24\pi d^2} = \frac{mL^2}{12}\ddot{\alpha} \Rightarrow T_{\text{slant}} = 2\pi d\sqrt{\frac{2\pi m}{\mu_0 I I' L}}$$

c. 1p

Taking path integrals along circular field lines exactly like at the first point, we get:

$$B_{\text{IN}} = 0$$

$$B_{\text{OUT}} = \frac{\mu_0 I}{2\pi r}$$

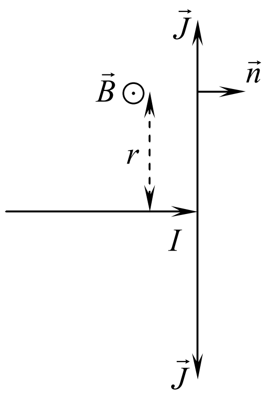


d. 0.5p

The above argument keeps holding, and the results are:

$$B_{\text{WIRE SIDE}} = \frac{\mu_0 I}{2\pi r}$$

$$B_{\text{OTHER SIDE}} = 0$$



e. 1p

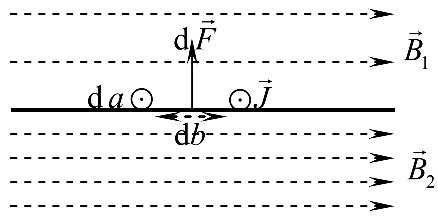
$$J(r) = \frac{I}{2\pi r}$$

$$\Delta B_{\parallel} = B_{\parallel\text{OTHER SIDE}} - B_{\parallel\text{WIRE SIDE}} = 0 - \left(-\frac{\mu_0 I}{2\pi r}\right) = \mu_0 J$$

f. 1.5p

Consider a small region of the plane having dimensions da along J and db across J.

$$J=\frac{B_2-B_1}{\mu_0}$$



Let B_0 be the external magnetic field and B' the field generated by the conducting plane.

$$\left.\begin{aligned}B_1&=B_0-B'\\B_2&=B_0+B'\end{aligned}\right\}\Rightarrow B_0=\frac{B_1+B_2}{2}$$

$$\mathrm{d}F=\mathrm{d}I\cdot\mathrm{d}a\cdot B_0=J\cdot\mathrm{d}b\cdot\mathrm{d}a\frac{B_1+B_2}{2}=\frac{B_2-B_1}{\mu_0}\mathrm{d}S\frac{B_1+B_2}{2}\Rightarrow p=\frac{\mathrm{d}F}{\mathrm{d}S}=\frac{B_2^2-B_1^2}{2\mu_0}$$

g. 0.5p

Just as before,

$$B_{\mathrm{IN}}=0$$

$$B_{\mathrm{OUT}}=\frac{\mu_0 I}{2\pi r}$$

h. 1p

This time the path integrals go the other way around.

$$B_{\mathrm{IN}}=\frac{\mu_0 I}{2\pi r}$$

$$B_{\mathrm{OUT}}=0$$

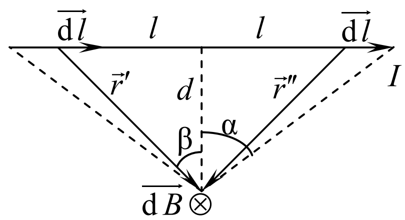
i. 1p

Consider two elements dl of the wire, placed symmetrically at a distance l from the center of the wire. Their contributions to the magnetic field in the mediator plane are equal:

$$\left|\mathrm{d}\vec{B}\right|=\frac{\mu_0 I\,\mathrm{d}l\sin\left(90^\circ-\beta\right)}{4\pi r^2}=\frac{\mu_0 I\,\mathrm{d}l}{4\pi d^2}\cos^3\beta$$

$$l=d\tan\beta\Rightarrow dl=\frac{d}{\cos^2\beta}d\beta\Rightarrow\left|\mathrm{d}\vec{B}\right|=\frac{\mu_0 I}{4\pi d}\cos\beta\,\mathrm{d}\beta$$

$$B=2\int_0^\alpha\frac{\mu_0 I}{4\pi d}\cos\beta d\beta=\frac{\mu_0 I}{2\pi d}\sin\beta\Big|_0^\alpha=\frac{\mu_0 I}{\pi L}\frac{\sin^2\alpha}{\cos\alpha}$$

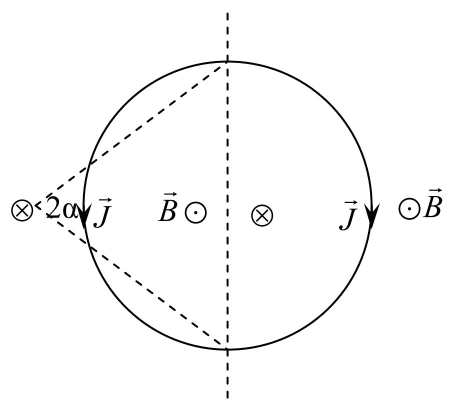


j. 1p

From a point in the equatorial plane, the axis of the poles of the sphere is seen under an angle 2α, with tan α = R/r.

Outside, the sphere behaves similarly to an electric current flowing directly from one pole to the other through a wire connecting the poles directly:

$$B_{\mathrm{OUT}}(r)=\frac{\mu_0 I}{2\pi}\frac{R}{r}\frac{1}{\sqrt{r^2+R^2}}$$



Inside, the sphere behaves similarly to two semi-infinite straight conductors connecting the two poles of the sphere and carrying the current I in the opposite direction:

$$B_{\mathrm{IN}}(r) = \frac{\mu_0 I}{2\pi r}\left(1 - \frac{R}{\sqrt{r^2 + R^2}}\right)$$