The Leidenfrost Phenomenon

The purpose is to estimate the lifetime of a (hemispherical) drop of a liquid sitting on top of a very thin layer of vapour which is thermally insulating the drop from the very hot plate below.

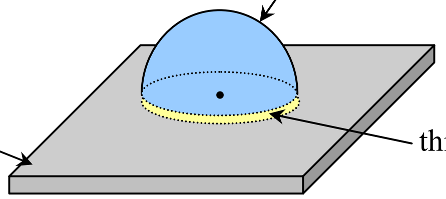

Figure 1

It will be assumed here that the flow of vapour underneath the drop is streamline and behaves as a Newtonian fluid of viscosity coefficient η and of thermal conductivity $\mathcal{K}$. The specific latent heat of vaporization of the liquid is ℓ. And for a Newtonian fluid we have the shear stress $\frac{F}{A} = \eta \times$ the rate of shear $\frac{dv}{dz}$ where v is the flow velocity and z is the perpendicular distance to the direction of flow, and the direction of F is tangential to the surface area A.

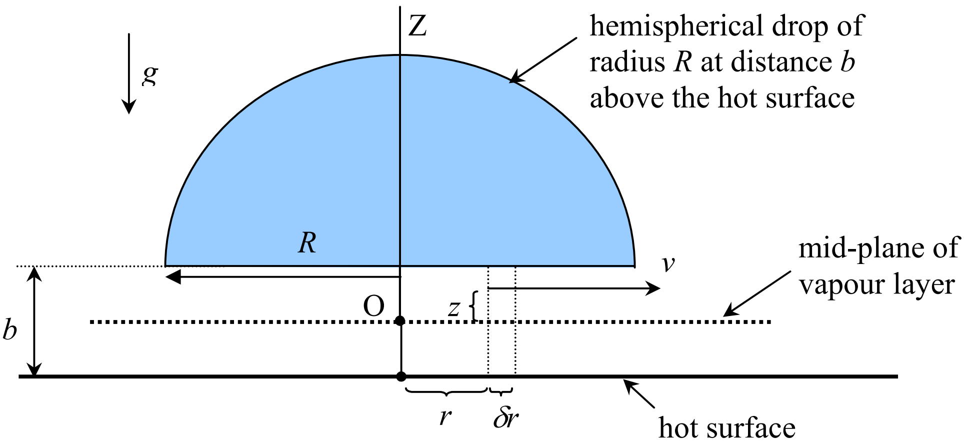

Figure 2

v is the velocity of vapour in the radial direction at the height z above the mid-plane. The pressure P inside the vapour must be higher towards the centre O. This will result in the out-flowing of vapour and in force that holds the drop against the pull of gravity. The thickness of vapour layer under thermal and mechanical equilibria is b.

For a Newtonian flow of vapour we can approximate that

$$\frac{d}{dz}v = \frac{z}{\eta}\frac{d}{dr}P$$

3.1) Show that $$v(z) = \frac{z^2}{2\eta}\frac{d}{dr}P + C$$

where C is an arbitrary constant of integration. (0.5 point)

3.2) Refer to figure 2, find the value of C in terms of $\eta, \frac{d}{dr}P$, and b using the boundary condition $v = 0$ for $z = \pm\frac{b}{2}$. (0.5 point)

3.3) Calculate the volume rate of flow of vapour through the cylindrical surface defined by r. (Hint: the cylinder is of radius r and of height b underneath the drop). (1.0 point)

3.4) By assuming that the rate of production of vapour of density ρ_V is due to heat flow from the hot surface to the drop, find the expression for the pressure $P(r)$. Use P_a to represent the atmospheric pressure, and use ΔT for the temperature difference between those of the hot surface and of the drop. Assume that the system has reached the steady state. (2.0 points)

3.5) Calculate the value of b by equating the weight of the drop to the net force due to pressure difference between the bottom and the top of the drop. The density of the drop is ρ_0. (2.0 points)

3.6) Now, what is the total rate of vaporization? (2.0 points)

3.7) Assume that the drop maintains a hemispherical shape, what is the life-time of the drop? (2.0 points)

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