

Solution Problem 1

Eclipses of the Jupiter's Satellite

- a. (Total Point : 1) Assume the orbits of the earth and Jupiter are circles, we can write the centripetal force = equal gravitational attraction of the Sun.

$$G \frac{M_E M_s}{R_E^2} = \frac{M_E V_E^2}{R_E}$$

$$G \frac{M_J M_s}{R_J^2} = \frac{M_J V_J^2}{R_J}$$

(0.5 point)

where

G = universal gravitational constant

M_S = mass of the Sun

M_E = mass of the Earth

M_J = mass of the Jupiter

R_E = radius of the orbit of the Earth

V_E = velocity of the Earth

V_J = velocity of Jupiter

Hence

$$\frac{R_J}{R_E} = \left(\frac{v_E}{v_J} \right)^2$$

We know

$$T_E = \frac{2\pi}{\omega_E} = \frac{2\pi R_E}{v_E}, \text{ and}$$

$$T_J = \frac{2\pi}{\omega_J} = \frac{2\pi R_J}{v_J}$$

we get

$$\frac{T_E}{T_J} = \frac{\frac{R_E}{v_E}}{\frac{R_J}{v_J}} = \left(\frac{R_E}{R_J} \right)^{3/2}$$

$$R_J = 779.8 \times 10^6 \text{ km}$$

(0.5 point)

b. (**Total Point: 1**) The relative angular velocity is

$$\omega = \omega_E - \omega_J = 2\pi \left(\frac{1}{365} - \frac{1}{11.9 \times 365} \right)$$

$$= 0.0157 \text{ rad / day}$$

(0.5 point)

and the relative velocity is

$$v = \omega R_E = 2.36 \times 10^6 \text{ km / day}$$

$$= 27.3 \times 10^3 \text{ km}$$

(0.5 point)

c. (**Total Point: 3**) The distance of Jupiter to the Earth can be written as follows

$$d(t) = R_J - R_E$$

$$d(t).d(t) = (R_J - R_E).(R_J - R_E)$$

(1.0 point)

$$d(t) = \left(R_J^2 + R_E^2 - 2R_ER_J \cos \omega t \right)^{1/2}$$

$$\approx R_J \left(1 - 2 \left(\frac{R_E}{R_J} \right) \cos \omega t + \dots \right)^{1/2}$$

$$\approx R_J \left(1 - \frac{R_E}{R_J} \cos \omega t + \dots \right)$$

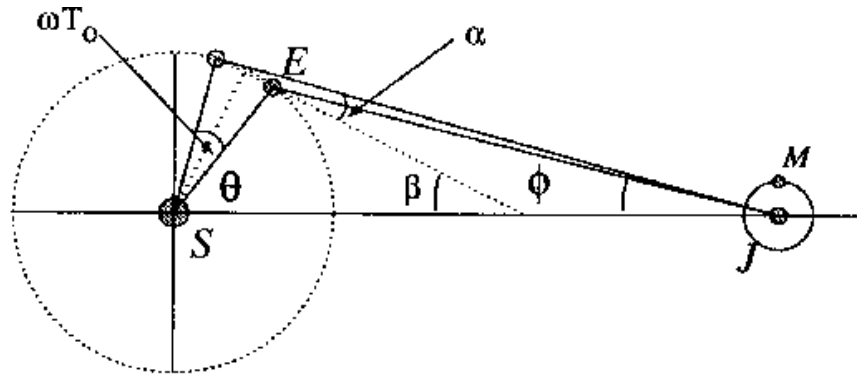


Figure 1: Geometrical relationship to get $\Delta d(t)$

The relative error of the above expression is the order of

$$\left(\frac{R_E}{R_J} \right)^2 \approx 4\%$$

The observer saw M begin to emerge from the shadow when his position was at $d(t)$ and he saw the next emergence when his position was at $d(t + T_0)$. Light need time to travel the distance $\Delta d = d(t + T_0) - d(t)$ so the observer will get apparent period T instead of the true period T_0 .

$$\begin{aligned} \Delta d &= R_E (\cos \omega t - \cos \omega(t + T_0)) \\ &\approx R_E \omega T_0 \sin \omega t \end{aligned}$$

(1.0 point)

because $\omega T_0 \approx 0.03, \sin \omega t + \dots, \cos \omega T_0 \approx 1 - \dots$

We can also get this approximation directly from the geometrical relationship from Figure 1.

(1.0 point)

or we can use another method.

From the figure above we get

$$\beta = (\phi + \alpha)$$

$$\frac{\omega T_0}{2} + \beta + \theta = \frac{\pi}{2}$$

(1.0 point)

$$\Delta d \approx \omega T_0 R_E \cos \alpha$$

$$\approx \omega T_0 R_E \sin \left(\omega t + \frac{\omega T_0}{2} + \phi \right)$$

$$\omega T_0 \approx 0.03 \text{ and } \phi \approx 0.19$$

(1.0 point)

d. (Total Point: 2)

$$T - T_0 \approx \frac{\Delta d(t)}{c}; c = \text{velocity of light}$$

$$T \approx T_0 + \frac{\Delta d(t)}{c} = T_0 + \frac{R_E \omega T_0 \sin \omega t}{c}$$

(1.0 point)

e. Total Point : 2 from

$$T_{\max} = T_0 + \frac{R_E \omega T_0}{c}$$

we get

$$\frac{R_E \omega T_0}{c} = 15$$

Hence

$$C = 2.78 \times 10^5 \text{ km/s}$$

(1.0 point)