Analog definitions

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To clearly fix our study, we precisely define the notions of analog signal and systems. The digital counterpart will be treated in a future lecture.

Definition 0.1 (Analog signal)
An **analog signal** is a function from $\mathbb{R}^n$, where $n \in \mathbb{N}^*$, to $\mathbb{K}$, with $\mathbb{K} = \mathbb{R}$ or $\mathbb{C}$. We denote $\mathcal{F}(\mathbb{R}^n, \mathbb{K})$ the vector space of analog signals.

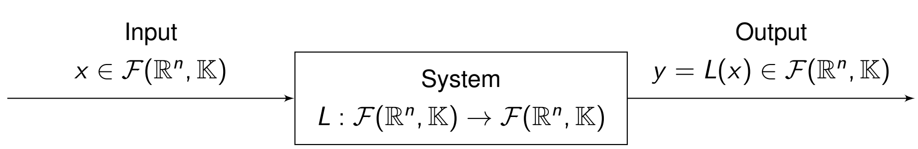
Remarks:

- The vector space structure implies that we can add two signals or multiply a signal by a scalar, which will prove very useful in the following.
- To simplify our study, we restrict the image space to $\mathbb{R}$ or $\mathbb{C}$, although it is possible to give a more general definition dealing with a finite dimensional space. The definitions and properties presented in the following lectures can be easily generalized to this perspective.
- In the following lectures, unless stated otherwise, we are interested in **univariate** analog signals, corresponding to $n = 1$.

Example 0.1
Here is a non-exhaustive list of examples of analog signals met in various fields:

- the classics of signal processing: sound, speech, image, video;
- in many fields of physics, physical quantities can be studied as analog signals: in electronics, the voltage or the intensity of an electrical component, in mechanics, the position, momentum or kinetic energy of a mechanical system, in thermodynamics, the temperature or pressure in a given volume;
- in chemistry and biology, the concentration of a chemical species, the body temperature, arterial pressure, heartbeat;
- in finance and economics: the price of a commodity, a currency exchange rate, unemployment rate, inflation.

Definition 0.2 (Analog system, input, output)
An **analog system** is a mapping from $\mathcal{F}(\mathbb{R}^n, \mathbb{K})$ to $\mathcal{F}(\mathbb{R}^n, \mathbb{K})$. The argument signal of this system is called the **input signal**. The image signal is called the **output signal** or **response**. A system can be represented by the following block diagram:



Example 0.2
We can associate the signals of the previous example with analog systems:

- in signal processing: sensors, converters, filters;
- in physics: electrical circuits, mechanical systems, thermodynamic systems;
- in biology: the human body;
- in economics: a financial market.

Remark: The expression $y = L(x)$ is misleading because it implies that knowing the input x, we can easily deduce the output y, which is generally not true. Indeed, in many occasions, y is the solution of a differential equation governing the system, whose the right member is x or a function of x. Thereby, L rarely provides an explicit definition of y as a function of x. In the following lectures, we study analog system output corresponding to some particular input. Then, we develop a method to explicitly express y as a function of x for a particular class of systems.