1

Practice - differentiate $f(x) = (4x^2 - 5x)e^x$

Product rule - $\frac{d}{dx} a(x)g(x) = (a'(x) \cdot g(x)) + (a(x)\, g'(x))$

$a(x) = 4x^2 - 5x$ $g(x) = e^x$

$\hookrightarrow a'(x) = 8x - 5$ $\hookrightarrow g'(x) = e^x$

$f'(x) = e^x(8x - 5) + e^x(4x^2 - 5x)$

Actual - differentiate $f(x) = (9x^2 - 15x)e^x$

$a(x) = 9x^2 - 15x$ $g(x) = e^x$

$\hookrightarrow a'(x) = 18x - 15$ $\hookrightarrow g'(x) = e^x$

$f'(x) = e^x(18x - 15) + e^x(9x^2 - 15x)$

2

Practice - differentiate $y = \frac{8x}{e^x}$

Quotient rule: $\frac{dy}{dx} \frac{f(x)}{g(x)} = \frac{(g(x) \cdot f'(x)) - (f(x) \cdot g'(x))}{g(x)^2}$

$f(x) = 8x$ $g(x) = e^x$

$\hookrightarrow f'(x) = 8$ $\hookrightarrow g'(x) = e^x$

$$y' = \frac{(e^x \cdot 8) - (8x \cdot e^x)}{(e^x)^2}$$

Actual - differentiate $y = \frac{3x}{e^x}$

$f(x) = 3x$ $g(x) = e^x$

$\hookrightarrow f'(x) = 3$ $\hookrightarrow g'(x) = e^x$

$$y' = \frac{(e^x \cdot 3) - (3x \cdot e^x)}{(e^x)^2}$$

3 Practice - Differentiate $y = \frac{7+6x}{5-8x}$

Quotient rule - $\frac{dy}{dx} \frac{f(x)}{g(x)} = \frac{(g(x) \cdot f'(x)) - (f(x) \cdot g'(x))}{g(x)^2}$

$f(x) = 7+6x$ $g(x) = 5-8x$

$\hookrightarrow f'(x) = 6$ $\hookrightarrow g'(x) = -8$

$$y' = \frac{((5-8x) \cdot 6)) - ((7+6x) \cdot -8)}{(5-8x)^2}$$

Actual - Differentiate $y = \frac{1+8x}{3-6x}$

$f(x) = 1+8x$ $g(x) = 3-6x$

$\hookrightarrow f'(x) = 8$ $\hookrightarrow g'(x) = -6$

$$y' = \frac{((3-6x) \cdot 8)) - ((1+8x) \cdot -6)}{(3-6x)^2}$$

4 Practice − Differentiate $y = \frac{\sqrt{x}}{8+x}$

AKA $y = \frac{x^{1/2}}{8+x}$

Quotient rule

$\frac{dy}{dx}\frac{f(x)}{g(x)} = \frac{(g(x)\cdot f'(x)) - (f(x)\cdot g'(x))}{(g(x))^2}$

$f(x) = \sqrt{x} = x^{1/2}$

$\to f'(x) = \frac{1}{2}x^{-1/2} = \frac{1}{2}\cdot\frac{1}{\sqrt{x}} = \frac{1}{2\sqrt{x}}$

$g(x) = 8+x$

$\to g'(x) = 1$

$y' = \frac{((8+x)\cdot\frac{1}{2\sqrt{x}}) - (\sqrt{x}\cdot 1)}{(8+x)^2}$

Actual − Differentiate $y = \frac{\sqrt{x}}{3+x}$

$f(x) = \sqrt{x} = x^{1/2}$

$\to f'(x) = \frac{1}{2}x^{-1/2} = \frac{1}{2\sqrt{x}}$

$g(x) = 3+x$

$\to g'(x) = 1$

$y' = \frac{((3+x)\cdot\frac{1}{2\sqrt{x}}) - (\sqrt{x})}{(3+x)^2}$

5 Practice − $\int x^2\sqrt{x^3+19}\,dx$, $u = x^3+19$

Take der of u ($\frac{du}{dx}$), solve for dx by rearranging, put everything in terms of u, integrate (remember C if indefinite), sub u back

$u = x^3+19 \to \frac{du}{dx} = 3x^2 \to \frac{du}{3x^2} = \frac{3x^2\,dx}{3x^2} \to dx = \frac{du}{3x^2} \to dx = \frac{1}{3x^2}du$

$\int x^2\sqrt{u}\,dx \to \int (u)^{\frac{1}{2}}(x^2\,dx)$

$\to x^2\cdot\frac{1}{3x^2}\cdot du \to \frac{x^2}{3x^2}du \to \frac{1}{3}du$

$\int (u)^{\frac{1}{2}}\cdot\frac{1}{3}du \to \frac{1}{3}\int u^{\frac{1}{2}}du$

$\to \frac{1}{\frac{3}{2}} = \frac{2}{3} \to \frac{2}{3}u^{\frac{3}{2}}$ $\to \frac{2}{3}\cdot\frac{3}{2} = 1$

$\to \frac{1}{3}\cdot\frac{2}{3}u^{\frac{3}{2}} \to \frac{2}{9}u^{3/2} \to \frac{2}{9}(x^3+19)^{\frac{3}{2}}+C$

Actual − $\int x^2\sqrt{x^3+33}\,dx$, $u = x^3+33$

$u = x^3+33 \to \frac{du}{dx} = 3x^2 \to dx = \frac{1}{3x^2}du$

$\int x^2\sqrt{u}\,dx \to \int (u)^{1/2}(x^2\,dx) \to \int (u^{\frac{1}{2}})\cdot\frac{1}{3}du \to \frac{2}{9}u^{\frac{3}{2}}+C$

$\frac{2}{9}(x^3+33)^{3/2}+C$

6 Practice- $\int \frac{x^5}{x^6-8}\,dx$, $u = x^6 - 8$ Do NOT forget C!

$u = x^6 - 8 \to \frac{du}{dx} = 6x^5 \to \frac{du}{6x^5} = \frac{6x^5}{6x^5}\,dx \to \frac{1}{6x^5}\,du = dx$

$\int \frac{x^5}{u}\,dx \to x^5 \cdot \frac{1}{u} \cdot dx \quad | \quad \int \frac{1}{u} \cdot (x^5 \cdot dx) \to x^5 \cdot \frac{1}{6x^5}\,du$

$\frac{x^5}{6x^5}\,du \to \frac{1}{6}\,du$

$\int \frac{1}{u} \cdot \frac{1}{6}\,du \to \frac{1}{6}\int \frac{1}{u}\,du \to \frac{1}{6}\ln|u| + C$

$\frac{1}{6}\ln|x^6-8| + C$

Actual $\to \int \frac{x^3}{x^4-6}\,dx$, $u = x^4 - 6$

$u = x^4 - 6 \to \frac{du}{dx} = 4x^3 \to \frac{du}{4x^3} = \frac{4x^3}{4x^3}\,dx \to \frac{1}{4x^3}\,du = dx \quad | \quad \int \frac{x^3}{u}\,dx = \int x^3 \cdot \frac{1}{u} \cdot dx$

$\int \frac{1}{u}(x^3 \cdot dx) = \int \frac{1}{u}\left(x^3 \cdot \frac{1}{4x^3}\,du\right) = \int \frac{1}{u}\left(\frac{x^3}{4x^3}\,du\right) = \int \frac{1}{u} \cdot \frac{1}{4} \cdot du$

$\frac{1}{4}\int \frac{1}{u}\,du = \frac{1}{4}\ln|u| + C \to \frac{1}{4}\ln|x^4-6| + C$

7 $\int \sin(t)\sqrt{1+\cos(t)}\,dt$ Don't forget C!

$u = 1 + \cos(t) \quad \frac{du}{dt} = -\sin t \quad \frac{du}{-\sin(t)} = \frac{-\sin(t)\,dt}{-\sin(t)}$

$\frac{1}{-\sin(t)}\,du = dt$

$\int \sin(t)\sqrt{u} \cdot \frac{1}{-\sin(t)}\,du$

$\int \sin(t) \cdot \sqrt{u} \cdot -1 \cdot \frac{1}{\sin(t)}\,du = \int \frac{\sin(t)}{\sin(t)} \cdot \sqrt{u} \cdot -1\,du$

$\int \sqrt{u} \cdot -1\,du \to -1\int \sqrt{u}\,du \to -1\int u^{\frac{1}{2}}\,du$

$-1 \cdot \frac{1}{\frac{1}{2}+1}\,u^{\frac{3}{2}} = -1 \cdot \frac{1}{\frac{3}{2}}\,u^{3/2} = -1 \cdot \frac{2}{3}u^{3/2}$

$-\frac{2}{3}u^{3/2} + C \to -\frac{2}{3}(1+\cos(t)) + C$

8

Practice $- \int_7^9 (x^2 + 2x - 7)\,dx$

$= \int_7^9 x^2\,dx + \int_7^9 2x\,dx + \int_7^9 -7\,dx$

$\frac{1}{3}x^3 \Big|_7^9 \qquad x^2 \Big|_7^9 \qquad -7x \Big|_7^9$

$\frac{1}{3}x^3 + x^2 - 7x + C \Big|_7^9$

$\frac{1}{3}(9)^3 + (9)^2 - 7(9) + C - \left(\frac{1}{3}(7)^3 + \underbrace{(7)^2 - 7(7)}_{\times} + C\right)$

$\frac{1}{3}(729) + 81 - 63 + C \quad - \left(\frac{1}{3}(343) + C\right)$

$243 + 81 - 63 + C \quad - \left(\frac{343}{3} + C\right)$

$261 + C$

$261 + C - \left(\frac{343}{3} + C\right) = \frac{3(261)}{3} + C - \frac{343}{3} - C = \frac{783}{3} - \frac{343}{3}$

$= \frac{440}{3}$

Actual $- \int_7^9 (x^2 + 2x - 6)\,dx$

$\int_7^9 x^2\,dx + \int_7^9 2x\,dx + \int_7^9 -6\,dx$

$\frac{1}{3}x^3 \Big|_7^9 \qquad x^2 \Big|_7^9 \qquad -6x \Big|_7^9 \rightarrow \frac{1}{3}x^3 + x^2 - 6x + C \Big|_7^9$

$\frac{1}{3}(9)^3 + (9)^2 - 6(9) + C - \left(\frac{1}{3}(7)^3 + (7)^2 - 6(7) + C\right)$

$243 + 81 - 54 + C - \left(\frac{1}{3}(343) + 49 - 42 + C\right)$

$243 + 27 + C \quad - \left(\frac{1}{3}(343) + 7 + C\right)$

$270 = \frac{3(270)}{3} \quad - \left(\frac{343}{3} + \frac{21}{3} + C\right)$

$\frac{810}{3} + C \quad - \left(\frac{364}{3} + C\right)$

$\frac{810}{3} + C - \frac{364}{3} - C$

$\frac{810}{3} - \frac{364}{3} = \frac{446}{3}$