MA 374   Financial Engineering Lab   Lab 01

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To ensure no arbitrage during binomial pricing algorithm, the following condition is needs to be satisfied:

$$0 < d < e^r < u$$

Q1)

Result:

M	Call Price	Put Price
1	43.6904	25.4645
5	41.3549	23.129
10	41.5908	23.3648
20	41.4634	23.2375
50	41.2278	23.0019
100	41.1916	22.9656
200	41.2523	23.0263
400	41.2314	23.0055

M can be infinitely large, there is no constraint on its upper bound. This is because the only constraint binding us it of no-arbitrage principle

No Arbitrage Principle

$u > R > d$

$$\implies e^{\sigma\sqrt{\Delta t}+(r-\frac{1}{2}\sigma^2)\Delta t} > e^{r\Delta t} > e^{-\sigma\sqrt{\Delta t}+(r-\frac{1}{2}\sigma^2)\Delta t}$$

$$\implies \sigma\sqrt{\Delta t} - \frac{1}{2}\sigma^2\Delta t > 0 > -\sigma\sqrt{\Delta t} - \frac{1}{2}\sigma^2\Delta t$$

The second inequality always holds true. Thus,

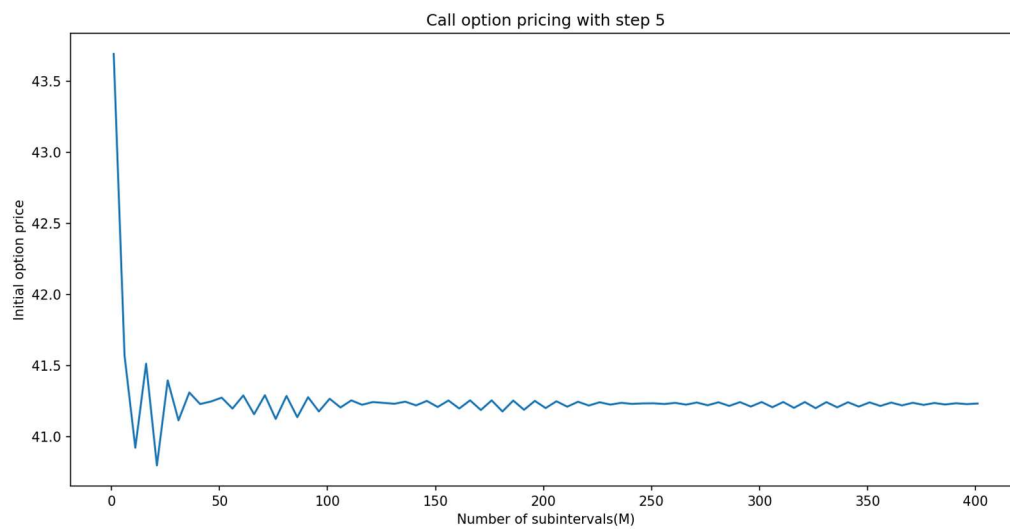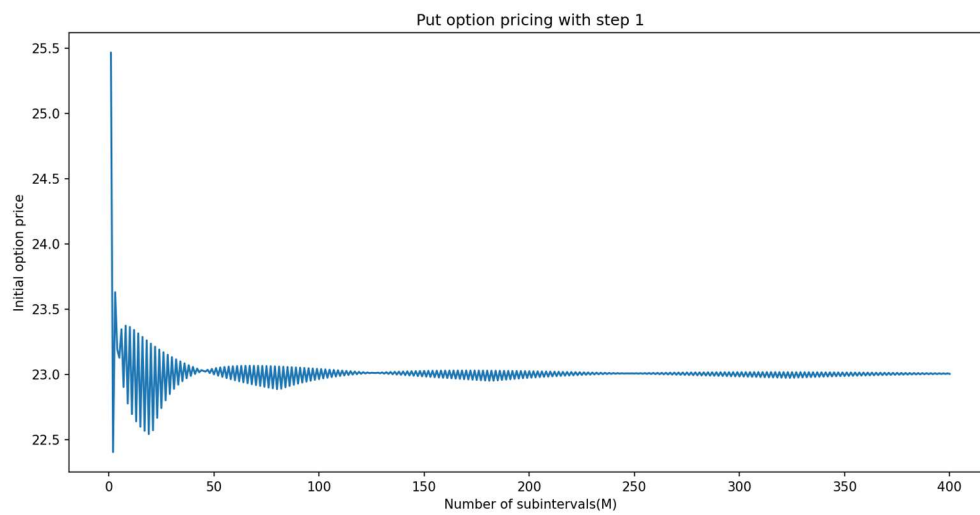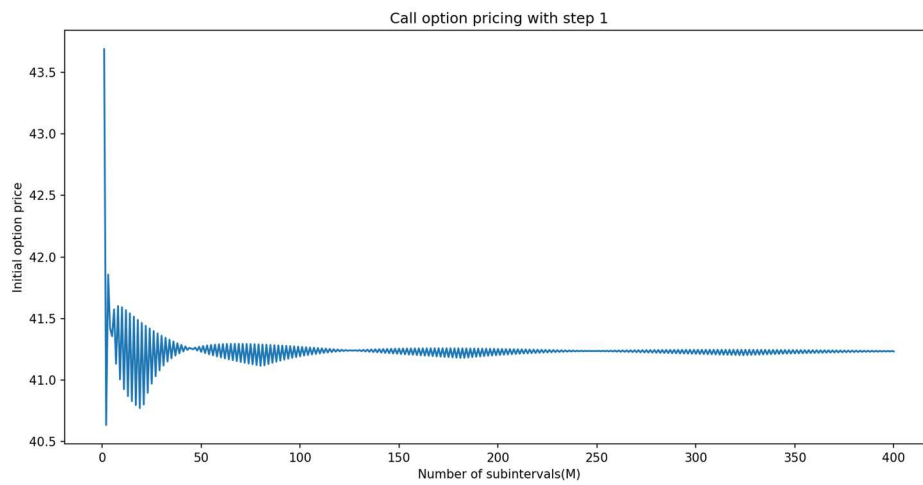$$\sigma\sqrt{\Delta t} > \frac{1}{2}\sigma^2\Delta t$$
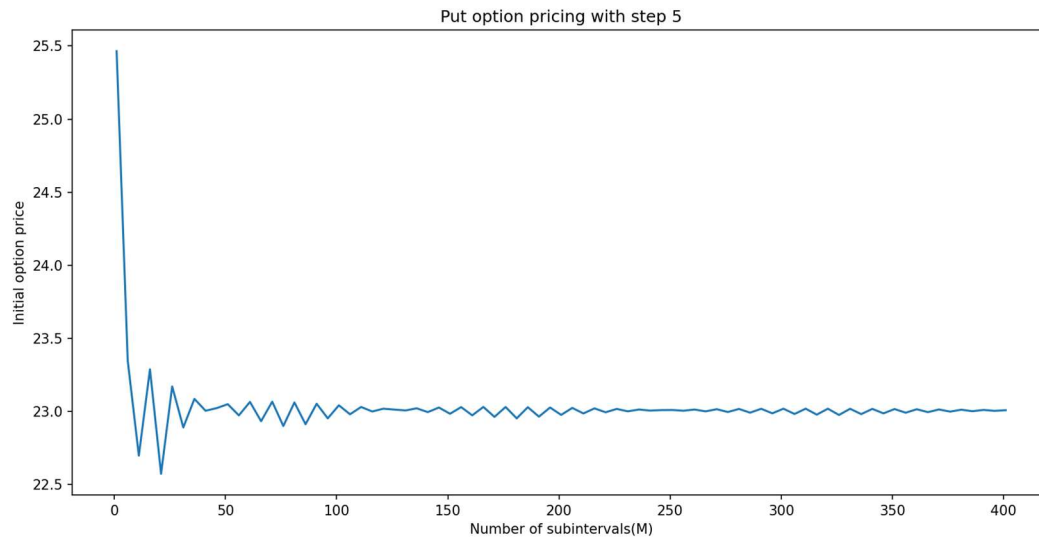
$$\implies \Delta t < 4/\sigma^2$$

$$\implies M > \frac{T\sigma^2}{4} = 0.1125$$

Hence there is no upper bound on M.

Q2)

Graphs:

Call option pricing with step 1

Put option pricing with step 1

Call option pricing with step 5

Put option pricing with step 5

Observations:

In each case, graphs converge to specific value. The converging value is same for step size 1 and 5.

For call option, the converging price is 41.23

For put option, the converging value is 23.00

Q3)

Tabulated values:

Time	Call Price
0	[41.463]
0.5	[77.091 38.064 16.753]
1	[136.669 72.39 34.584 14.474 5.137]
1.5	[231.472 130.635 67.555 31.011 12.189 3.936 0.992]
3	[9.12435e+02 5.80301e+02 3.57732e+02 2.09127e+02 1.11825e+02 5.18910e+01 1.95010e+01 5.42200e+00 9.72000e-01 8.30000e-02 0.00000e+00 0.00000e+00 0.00000e+00]
4.5	[3095.238 2041.039 1334.387 860.705 543.186 330.347 187.676 92.042 32.545 5.714 0. 0. 0. 0. 0. 0. 0. 0. 0.]

Put option:

Time	Put Price
0	[23.237]
0.5	[13.974 23.397 34.563]
1	[6.659 13.583 23.507 35.39 47.499]
1.5	[2.212 6.017 13.08 23.554 36.25 49.123 60.341]
3	[0.0000e+00 0.0000e+00 6.7000e-02 6.9900e-01 3.4340e+00 1.0557e+01 2.3116e+01 3.9167e+01 5.4914e+01 6.7564e+01 7.6556e+01 8.2639e+01 8.6717e+01]
4.5	[0. 0. 0. 0. 0. 0. 0. 0. 4.609 20.75 43.841 63.149 76.092 84.768 90.583 94.481 97.094 98.846 100.02]