MA 374 Financial Engineering Lab Lab 10

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1. Consider an asset which follows a geometric Brownian motion (GBM) with drift $\mu = 10\%$ and volatiltiy $\sigma = 20\%$. Assume that the risk free rate is $r = 5\%$. The initial asset price at time $t = 0$ is $S(0) = 100$.
   Simulate 10 different paths of the asset price making use of the GBM, in both the real and the risk-neutral worlds.
   Now compute the price of a six month fixed-strike Asian option with a strike price of 105 (using arithmetic average). Do the pricing for both call and put options, using Monte Carlo simulation.
   Repeat the above exercise with strike price $K = 110$ and $K = 90$. How do your results compare ?
   Now do a sensitivity analysis of the option prices.

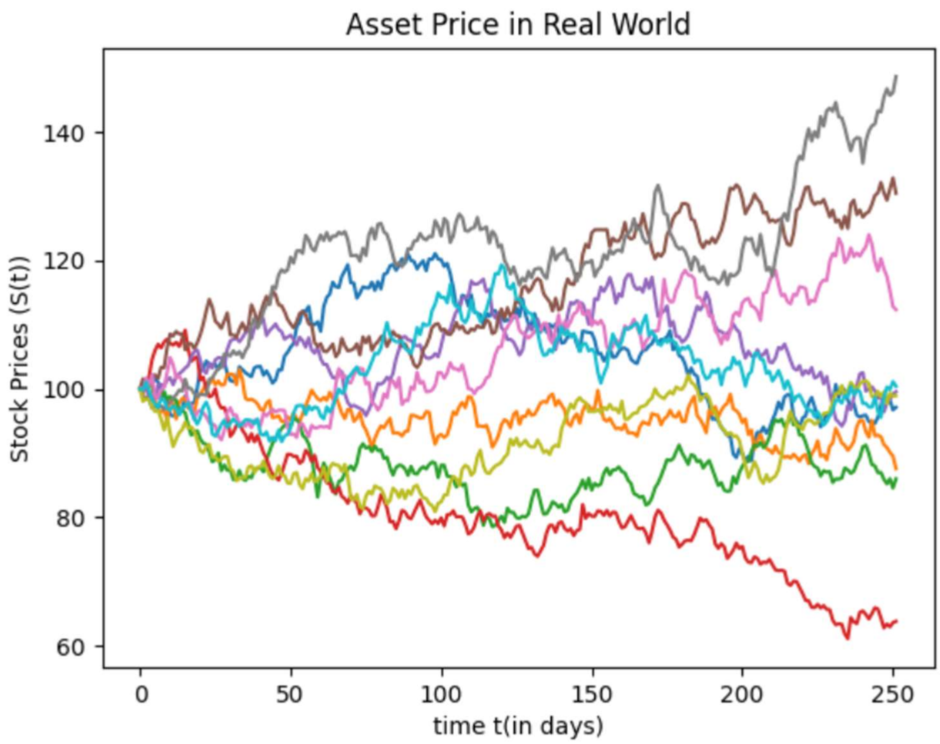
The evolution of the asset price in real world is governed by following differential equation:

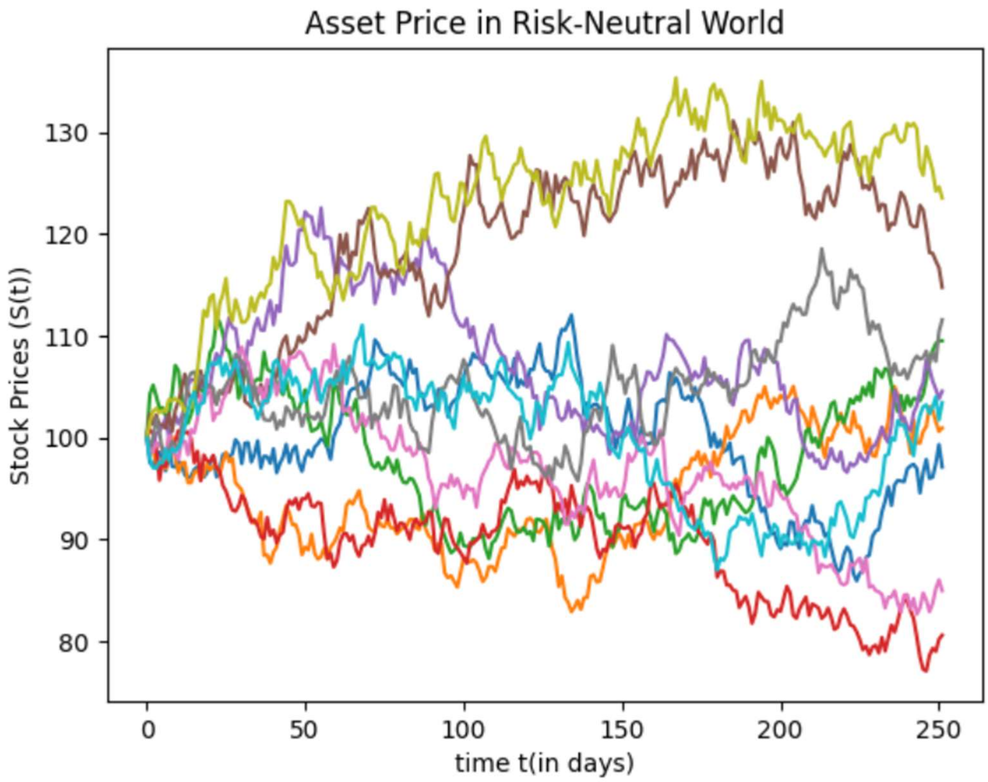
$$dS = \mu S dt + \sigma S dW(t)$$

The evolution of the asset price in the risk-neutral world is governed by following differential equation:

$$dS = rSdt + \sigma S dW^*(t)$$

where,
W* is a Brownian motion under risk-neutral probability





The prices of a six month fixed-strike Asian option with various strike prices are:

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********For K=90********
Asian Call Option Price:                  10.575800153550244
Variance in Asian Call Option Price:      50.644476665116116

Asian Put Option Price:                   0.240433929138292
Variance in Asian Call Option Price:      1.1342227900068602

********For K=105********
Asian Call Option Price:                  1.7273413602424188
Variance in Asian Call Option Price:      13.748667031538643

Asian Put Option Price:                   5.436795983286603
Variance in Asian Call Option Price:      31.744097018467922

********For K=110********
Asian Call Option Price:                  0.722359010489039
Variance in Asian Call Option Price:      5.8894029764017235

Asian Put Option Price:                   8.81327128209042
Variance in Asian Call Option Price:      46.7111282292675
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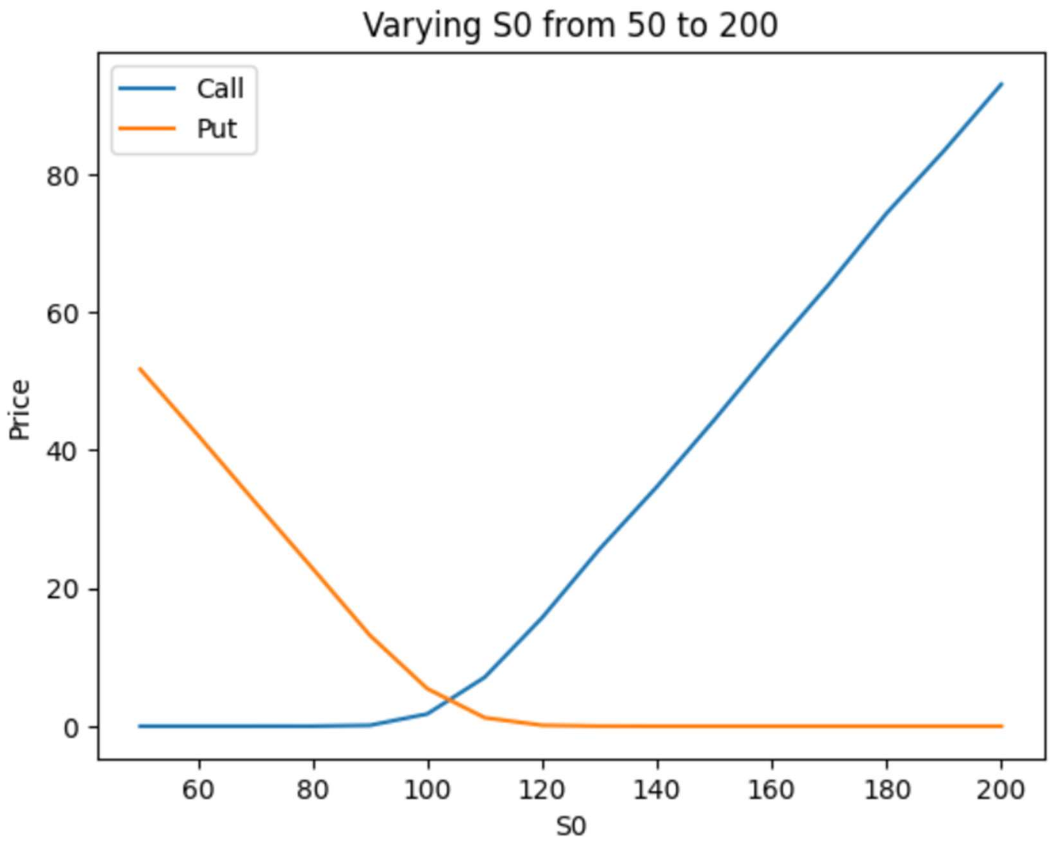
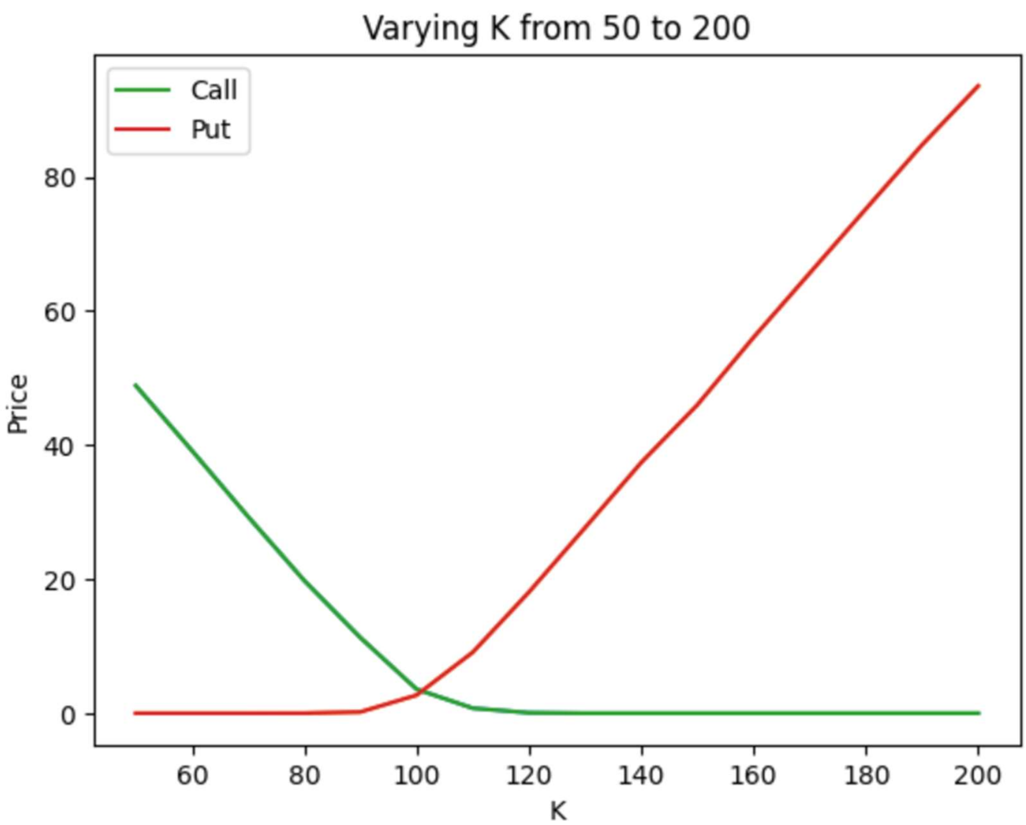
Sensitivity Analysis:

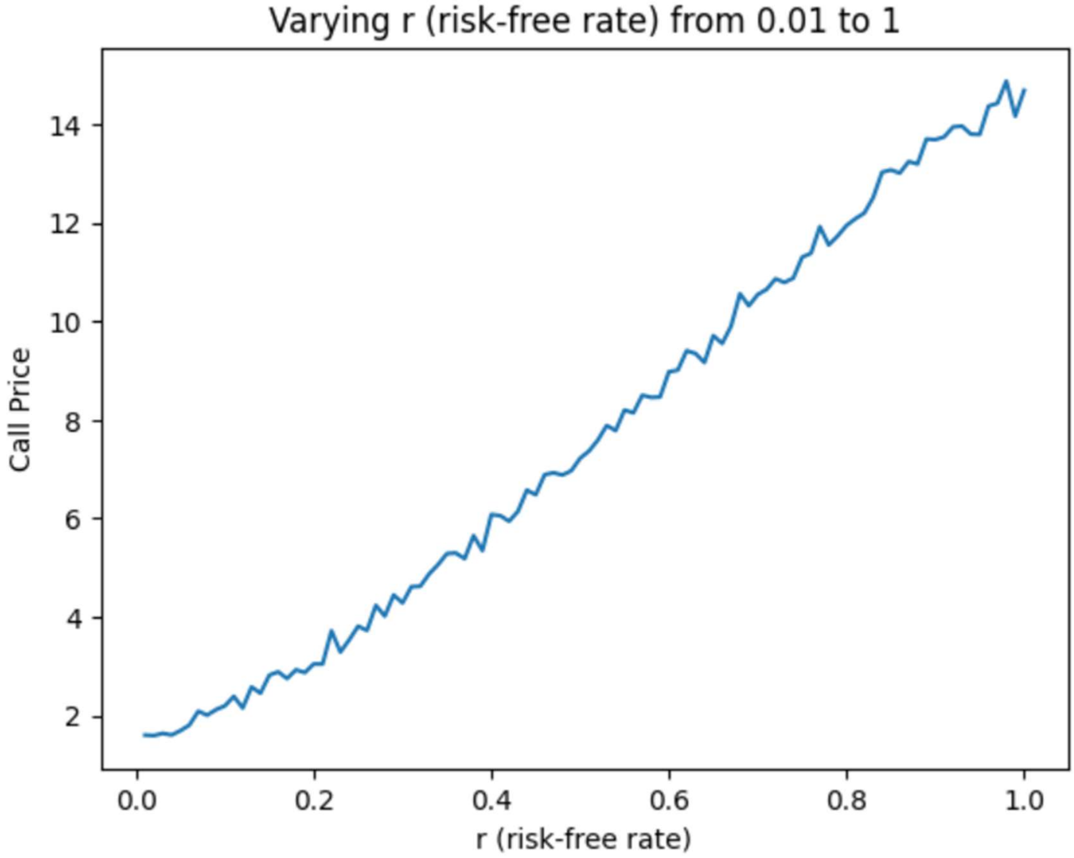
- *Varying* S0 (Initial Asset Price)

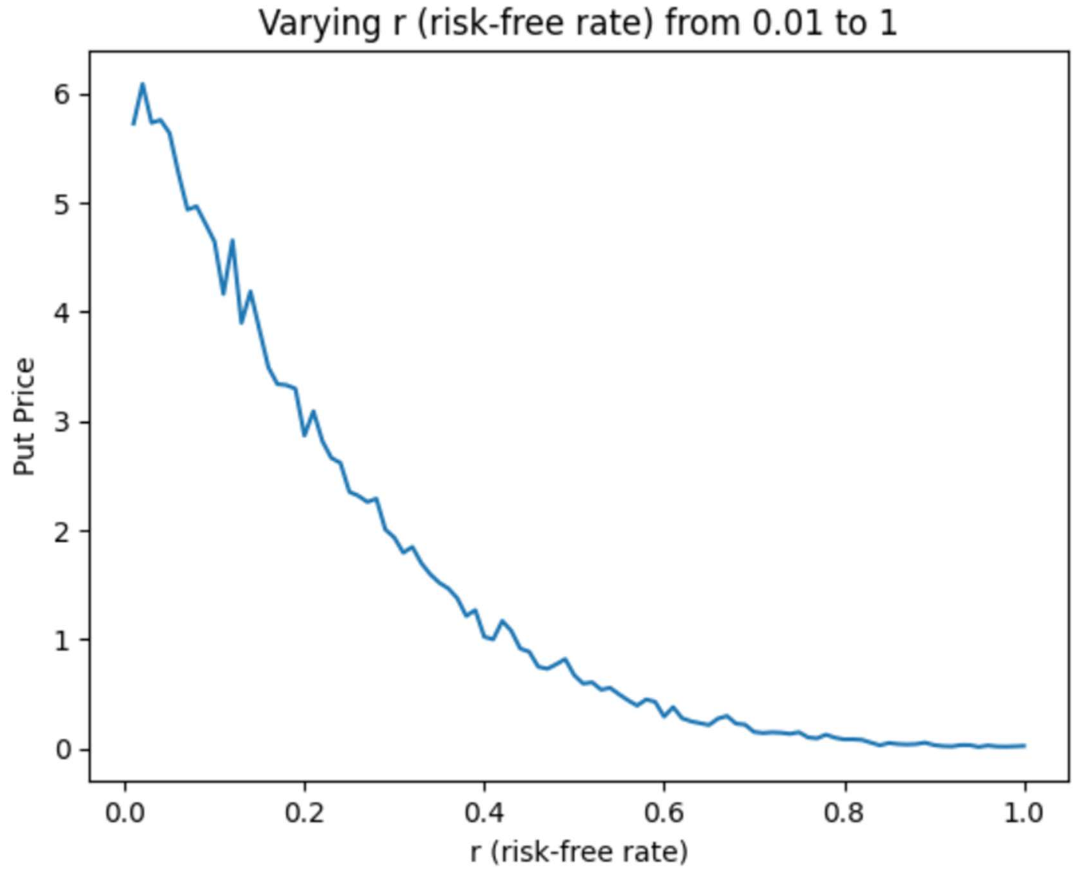


- Varying K (Strike *Price*)

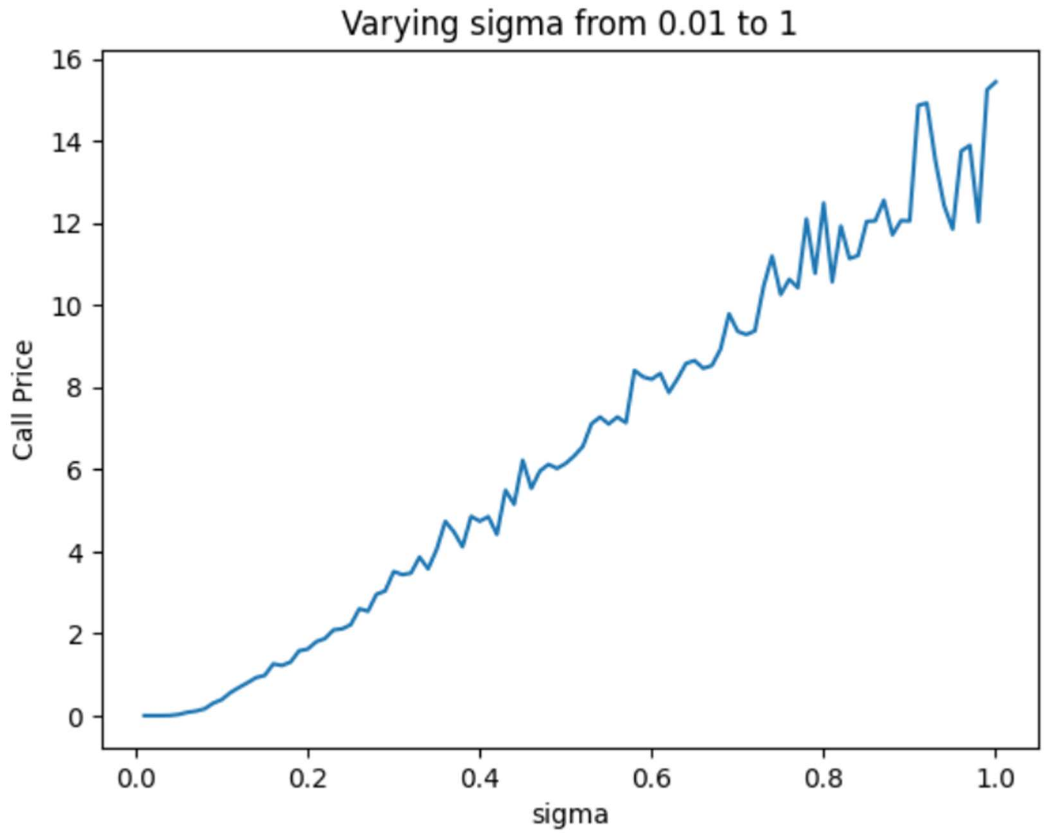


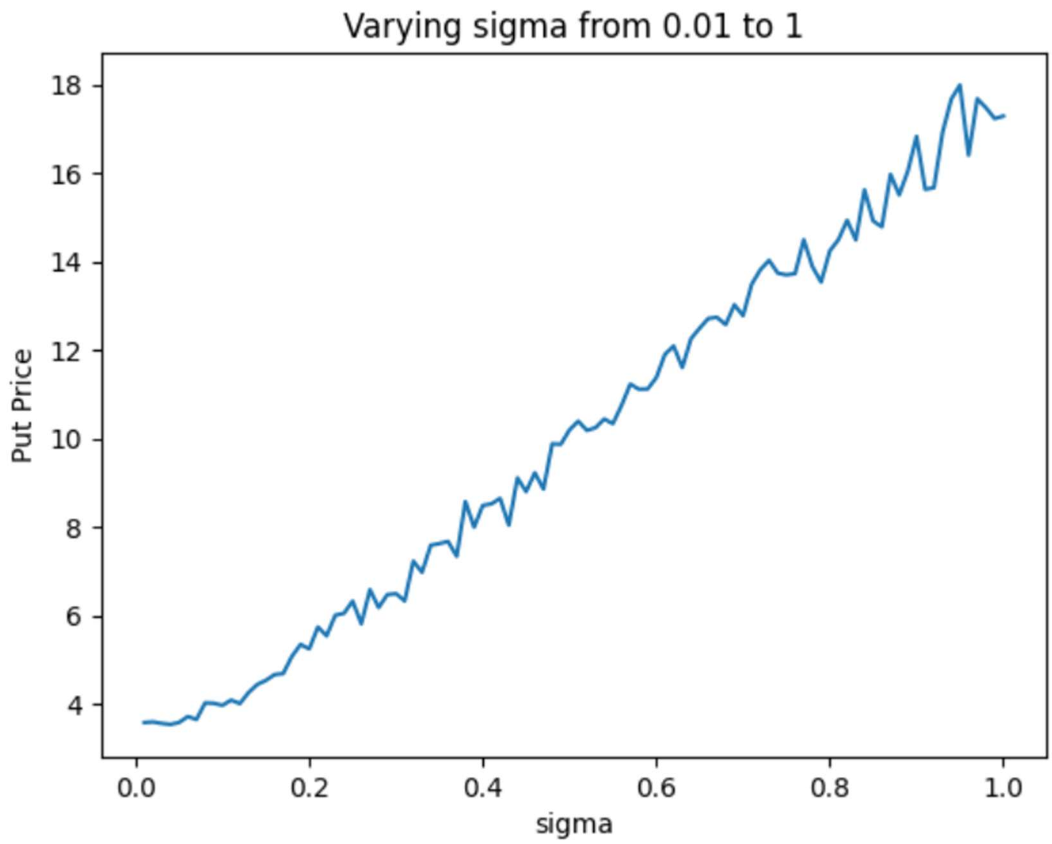
- Varying r (risk-free rate)





- Varying sigma (volatility)





Observations:

1. The price of the call option increases while that of the put option decreases, with an increase in the initial asset price, S_0.

2. The price of the call option decreases while that of the put option increases, with an increase in the strike prices, K.

3. The price of the call option increases while that of the put option decreases, with an increase in the risk free interest, r

4. The price of both call and put option increases with an increase in the volatility.

5. There appears to be some fluctuations in the plots, which we try to minimise using the variance reduction schemes, in the next question.

2. Compute the prices of the Asian options given above by employing variance reduction techniques also and compare your results.

For variance reduction, I use the *Antithetic Technique.*

```
********For K=90********
Asian Call Option Price:                  10.256899989529892
Variance in Asian Call Option Price:      26.127733285020774

Asian Put Option Price:                   0.03515269471140198
Variance in Asian Call Option Price:      0.11176771851549097

********For K=105********
Asian Call Option Price:                  0.7717301582070853
Variance in Asian Call Option Price:      4.346006617245854

Asian Put Option Price:                   4.980082338182267
Variance in Asian Call Option Price:      18.3656354421258

********For K=110********
Asian Call Option Price:                  0.14747994203433265
Variance in Asian Call Option Price:      0.5919201446151733

Asian Put Option Price:                   9.006744012548989
Variance in Asian Call Option Price:      24.689416643638324
```

Observation

Comparing the previous values to the new Monte-Carlo Antithetic Method, the variance has clearly reduced. So, the technique works.

For Call Option:

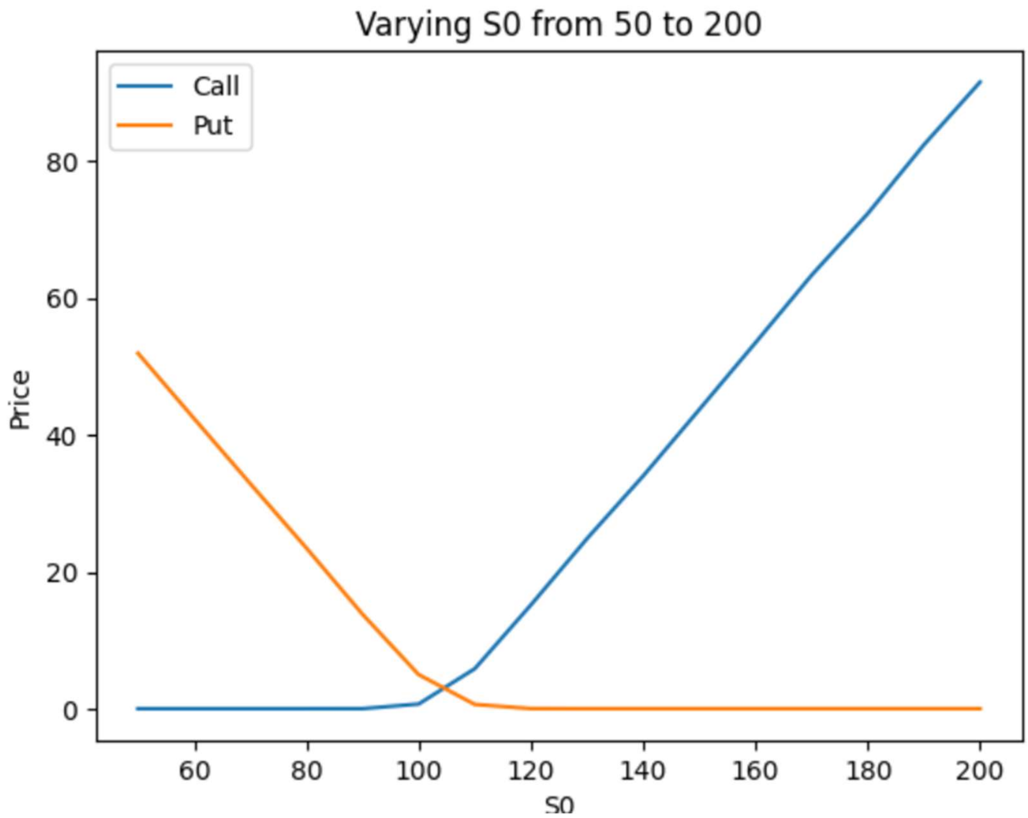
Strike Price	Variance before Reduction	Variance after Reduction
95	50.644476665116116	26.127733285020774
105	13.748667031538643	4.346006617245854
110	5.8894029764017235	0.5919201446151733

For Put Option:

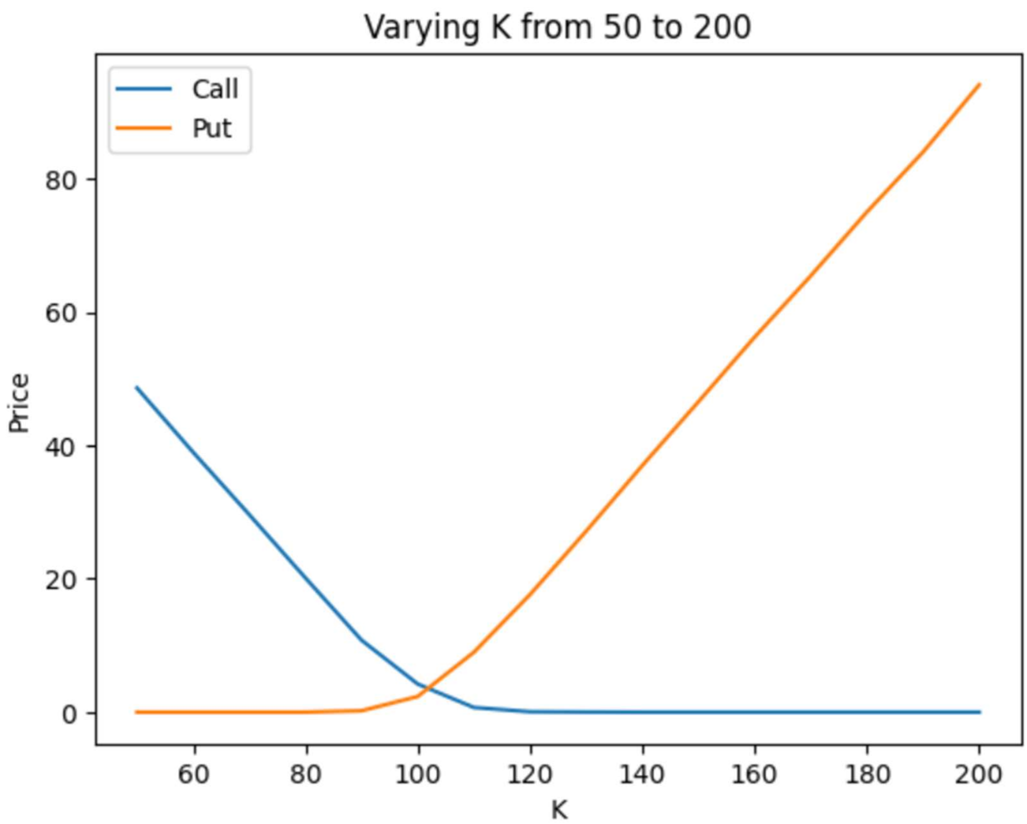
Strike Price	Variance before Reduction	Variance after Reduction
95	1.1342227900068602	0.11176771851549097
105	31.744097018467922	18.3656354421258
110	46.7111282292675	24.689416643638324

Sensitivity Analysis:

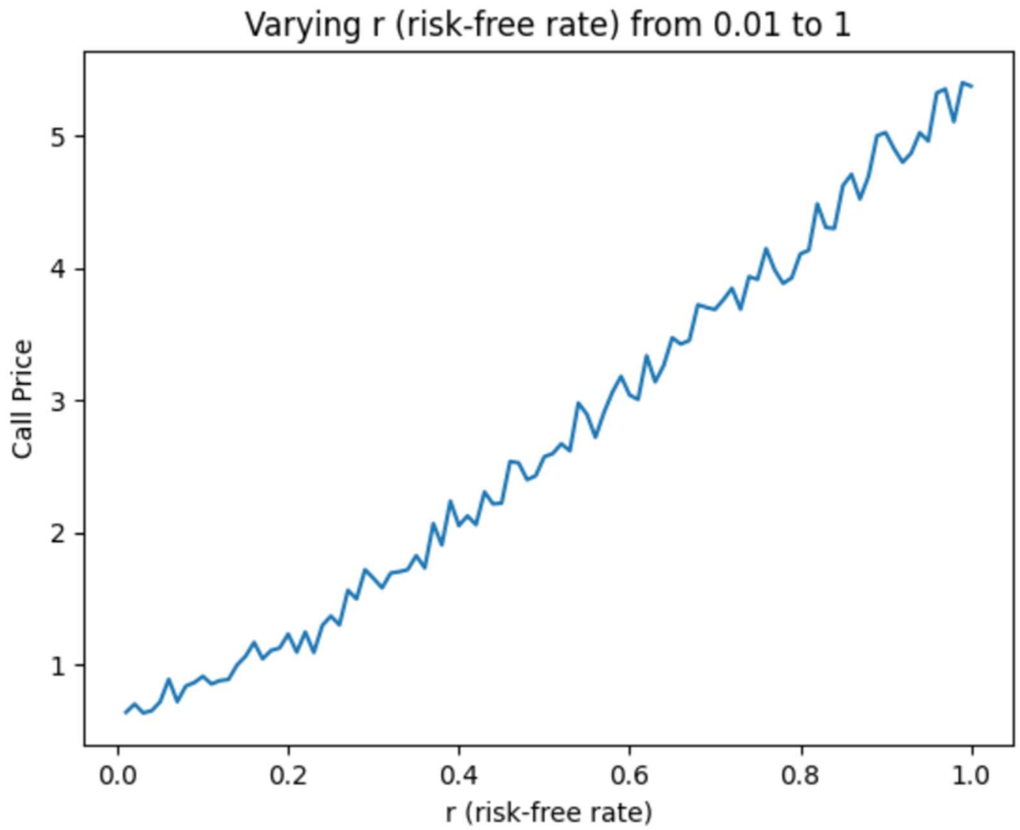
- *Varying* S0 (Initial Asset Price)

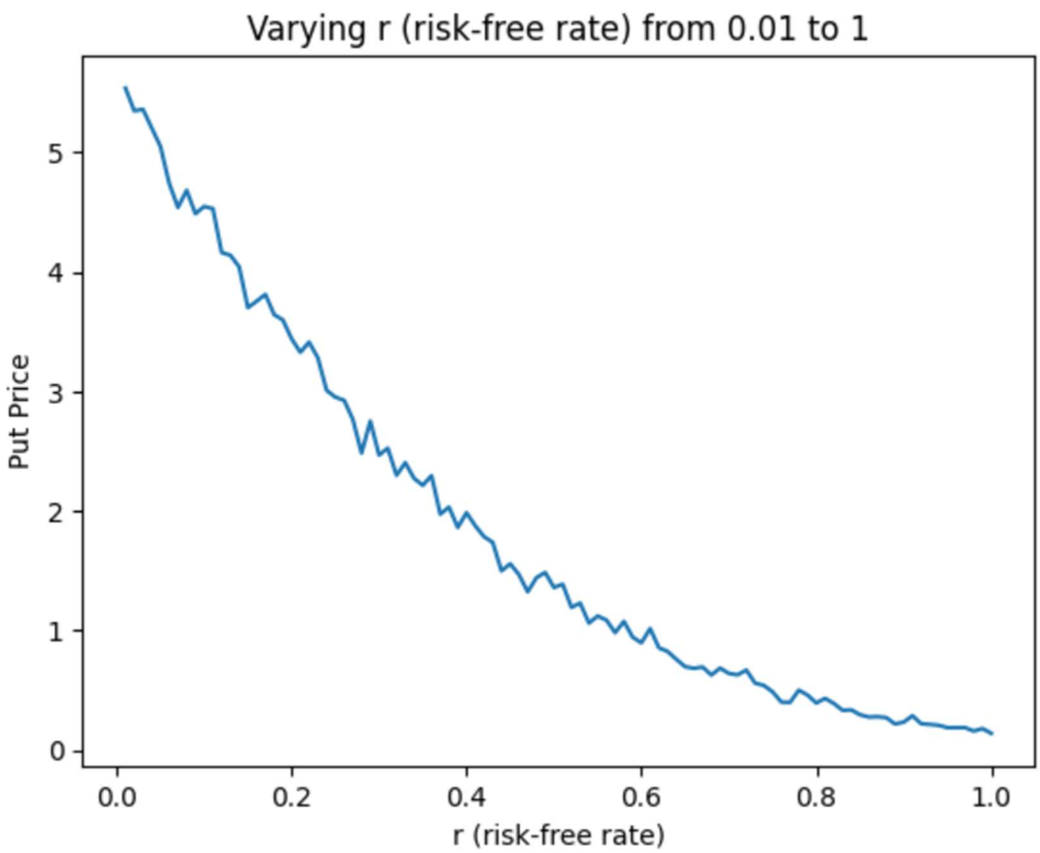


- Varying K (Strike *Price*)

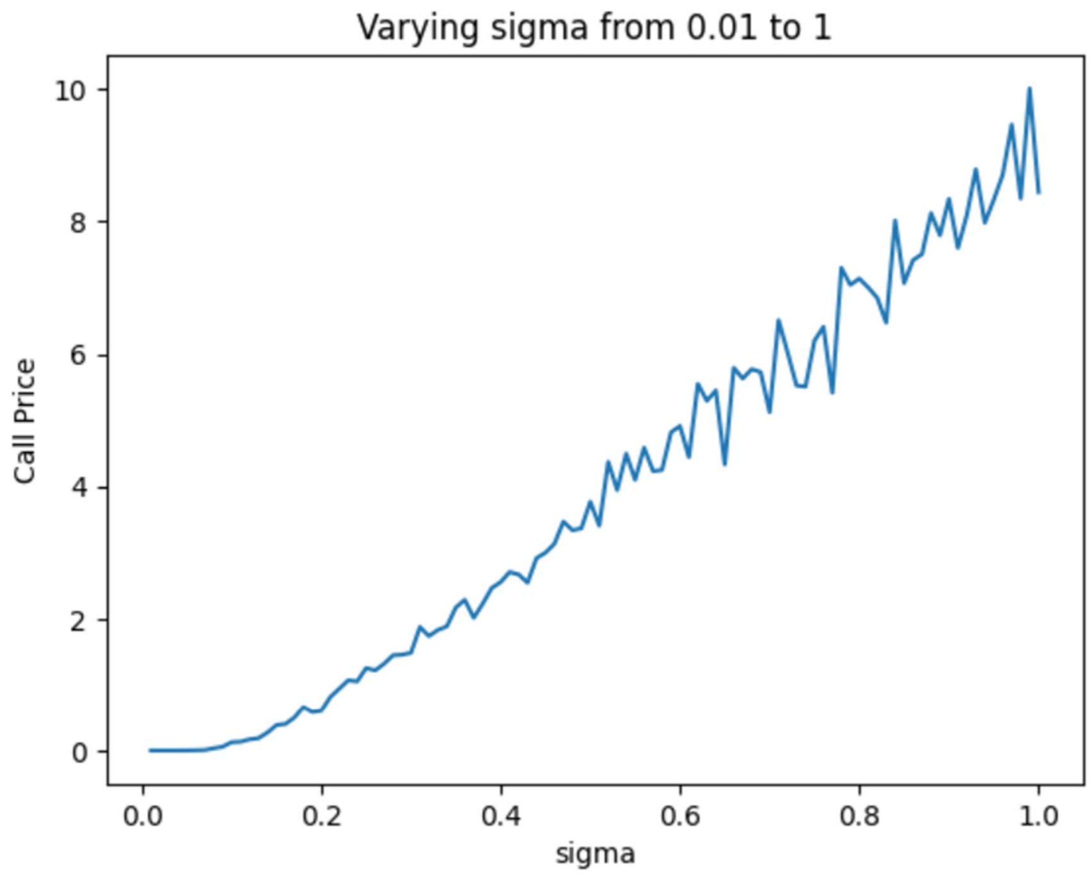


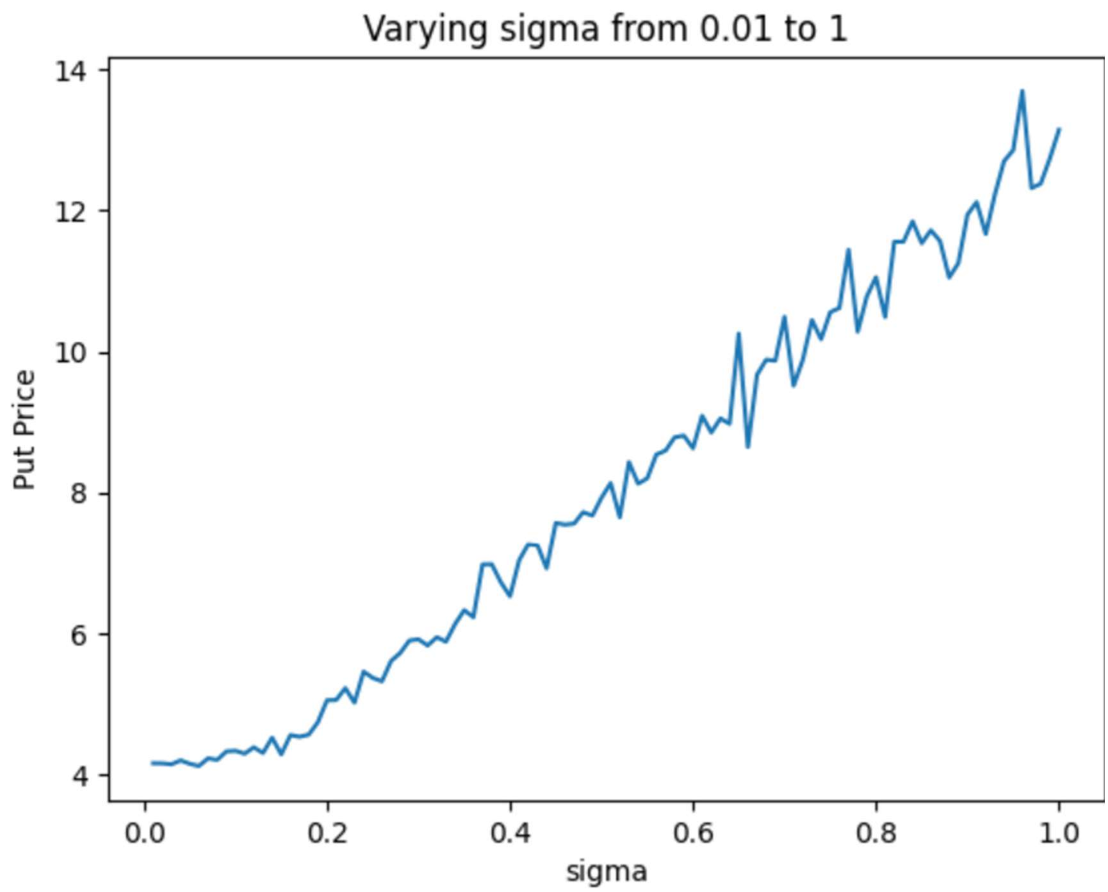
- Varying r (risk-free rate)





- Varying sigma





Variance Reduction Scheme:

This method uses the fact if the two are negatively correlated then the variance of the sum is less than the variance if they are independent.

So we can generate negatively correlated samples using:

$$S_T = S_0 \exp\left[(r - 1/2b^2)T + b \sum_{k \leq n-1} (t_{k+1} - t_k)^{1/2} N_{k+1}\right],$$

and for the second sample we set

$$\tilde{S}_T = S_0 \exp\left[(r - 1/2b^2)T - b \sum_{k \leq n-1} (t_{k+1} - t_k)^{1/2} N_{k+1}\right],$$

Observations:

1. Earlier, we have quantitatively demonstrated that the variance reduction is achieved. This claim is even more supported by the constructed plots.

2. On careful analysis, the fluctuations in the plots seem to be less than the case when variance reduction was not applied. So, the scheme achieves its goal.

3. The nature of the plots is consistent with our expectations, which is explained in the last question.