| No | Mathematical formulation | Physical interpretation |
|----|---|--|
| | Paper 1 theorems | |
| | Anomaly detection enhancement theorems | |
| 1 | $S_{\omega}^{I_{R}} \triangleq \frac{1}{I_{R}} \left(\frac{\partial I_{R}}{\partial \omega} \right) = \frac{1}{I_{R}} \left(\frac{\partial I_{R,Scattered}}{\partial \omega} \right), \frac{\partial I_{R,fixed}}{\partial \omega} = 0$ | Scattered currents sensitivity theorem: The scattered current may be significantly smaller than total active/reactive current and yet the spectral change information is located there, due to fact that sensitivity measure $S_{\omega}^{I_R}$ of the system's change is there. |
| 2 | $i_R = v / R, i_C(s) = sCv, i_L(s) = v / (sL)$ | Reactive scatter current sensitivity: The reactive→ scatter |
| | $s = \sigma + j\omega : \partial i_R / \partial s = 0,$ | current signature is more likely to catch changes than total |
| | $\partial i_C / \partial s = Cv, \partial i_L / \partial s = v / (-s^2 L)$ | current or active current: Changes over reactive components. Resistive spectral relation is fixed, reactive is pole/zero dependent. |
| 3 | $i_C(j\omega) = j(\omega Cv) = e^{j\pi/2}(\omega Cv), i_L(j\omega) =$ | Reactive current sensitivity: phase may contribute to anomaly |
| | | detection not less than amplitude signature, at reactive |
| | $=e^{-j\pi/2}(v/(\omega L))$ | components and mixed active+ reactive components |
| | Grid/machinery deciphering theorems | $ \begin{array}{c ccccccccccccccccccccccccccccccccccc$ |
| 4 | $Y_{Total}(Z) = Y_{active}(Z) + Y_{Scattered}(Z) + Y_{Reactive}(Z) +$ | Admittance function separability theorem: as observed from |
| | $+Y_{Customer}(Z) + Y_{Unbalanced}(Z) = \sum_{m} I_{m}(Z) / V(Z) \triangleq \sum_{m} Y_{m}$ | above schematics –admittance transfer function may be modeled as composed of several admittance physical components. Published paper. שגיאה! מקור ההפניה לא נמצא |
| 5 | $i_{m,total} = \sum_{n} i_{n,m}$ | Currents physical components separability theorem: |
| | where: $i_{m,total}$ - total current physical component of type m $index \in \{CPC, E - CPC \ current \ types\}$ and n branch index $i_{n,m}$ - current at branch n of CPC type m parallel | Looking at an electric scheme with n parallel branches, the following current relation is maintained at CPC. |
| 6 | $Y_{m,total} = \sum Y_{n,m}$ | "RL serial circuit" reactive admittance theorem: Following |
| | where: | CPC relations then admittance relations are maintained |
| | $Y_{m,total}$ - total current physical component 'admittance' of type $m \in \{CPC \ current \ types\}$ $Y_{n,m}$ - current physical component 'admittance' at branch n of CPC type m including this paper enhanced CPC | |
| 7 | $Y_{total \rightarrow reactive \rightarrow scattered} = Im\{Y_{total \rightarrow active}\} =$ | "RL serial circuit" active admittance theorem: for a linear |
| | $=\sum_{m}-\omega L_{m}/(R_{m}^{2}+\omega^{2}L_{m}^{2})$ | load that includes only resistors and inductors the total— |
| | Reason that is written is because it may mistakenly be considered that following is | reactive→ scatter admittance, equals the sum of all parallel inductors, admittances |
| | correct: $Y_{total \rightarrow reactive} = \sum_{m} 1/(j\omega L_m)$ | |

| 8 | $Y_{total \to active \to scattered} = \text{Re}\{Y_{total \to active}\} =$ | "RLC serial circuit" admittance theorem: For linear loads |
|-----|---|---|
| | $= \sum_{m} R_{m} / (R_{m}^{2} + \omega^{2} L_{m}^{2})$ | with resistive elements at the parallel branches, the |
| | — <i>'''</i> | total→ active admittance, equals to the sum of real |
| | Reason that is written is because it may | segments of admittances (not of the resistors only) and is |
| | mistakenly be considered that following is | not sum only of resistors. |
| | correct: $Y_{total \to active} = \sum_{m} 1/R_m$ | · |
| 9 | (17) | RLC serial total admittance separability: 1) for linear loads |
| | $Y_{total}(s) =$ | with mixture of inductors and capacitors at the load parallel |
| | $= \sum_{m} [R_{m} - j(\omega L_{m} - 1/\omega C_{m})] / [R_{m}^{2} + (\omega L_{m} - 1/\omega C_{m})^{2}]$ | branches, the total admittance, that is computable, equals the |
| | | sum of all parallel active admittances separately+ reactive |
| | The bellow does not show at paper 1. | admittances separately. Example for serial RLC. |
| | ~ | The bellow does not show at paper 1 |
| | Generalization: | 2) generalization: For linear loads with mixture of |
| | $Y_{total \to reactive, m}(\omega) = \text{Im}\{Y_{total, m}\}$ | inductors and capacitors at the load parallel branches, the |
| | $Y_{total \to active, m}(\omega) = \text{Re}\{Y_{total, m}\}$ | total→ reactive admittance, that is computable, equals |
| | $Y_{total \to reactive}(\omega) = \sum_{m} \text{Im}\{Y_{total,m}\}, Y_{total \to active}(\omega) = \sum_{m} \text{Re}\{Y_{total,m}\}$ | the sum of all parallel imaginary segments of branch |
| | | admittances |
| | | $(a) \begin{tabular}{l l l l l l l l l l l l l l l l l l l $ |
| 10- | (18) | Admittance correlation to CPC. Equating between transfer |
| a | $G_e = P/\ u\ ^2 = \sum_{n \in N_D \oplus N_C} G_n u_n^2 / \sum_{n \in N_D} u_n^2$ | measured-computed admittance function for linear load, the total |
| | where: $G_n = R_m / \left[R_m^2 + (\omega_n L_m - 1/(\omega_n C_n))^2 \right]$ | (customer+ distribution)→ reactive→ fixed current is close to |
| | | zero. Although the reactive component of for example parallel |
| | $\begin{bmatrix} \mathbf{p} & \mathbf{Q} & \mathbf{J} \end{bmatrix}^2 \mathbf{\nabla} = \mathbf{p} \cdot \mathbf{Q} / \mathbf{\nabla} \mathbf{Q}$ | branches of serial RLC looks spectral, at CPC spectrally constant |
| | $B_{e} = Q / \ u\ ^{2} = \sum_{n \in N_{D} \oplus N_{C}} B_{n} u_{n}^{2} / \sum_{n \in N_{D}} u_{n}^{2}$ | component is extracted according to equation herein. |
| | where: $B_n = -(\omega_n L_m - 1/(\omega_n C_n)/[R_m^2 + (\omega_n L_m - 1/(\omega_n C_n))^2]$ | |
| | Where: | This theorem relates to important issue: The components based |
| | <i>n</i> - harmonic index | active/reactive admittance transfer functions from eq. are based |
| | m – branch index N_D – set of harmonic indices that are at distribution | on 'total' active and reactive currents. |
| | N_D – set of narmonic indices that are at distribution current. | |
| | N_C - set of harmonic indices that are at customer current | |

| 10- | $i_{reactive,total} = i_{CPC,reactive,distribution} + i_{CPC,reactive,customer}$ | Relation between CPC and classical theory: Eq. is important |
|-----|---|--|
| b | | because "measured-computed admittance" is according to eq. |
| | $i_{active,total} = (i_{CPC,active \to fixed} + i_{CPC,active \to scattered})_{distribution} + (-"-)_{customer}$ | CPC+Z-transform whereas the proposed electric scheme for |
| | | comparison is described by eq. |
| 11 | | Active spectral component: for linear load, the total (customer+ |
| | | distribution)→ active→ scatter current is not zero. The total→ |
| | | reactive→ fixed is not zero. It is according to eq Reason |
| | | theorem is brought is not to suppose by mistake that $Y_{total,real} =$ |
| | | $\sum_{n} 1/R_n$ simply because $i_{total,real} = \sum_{n} i_{n,real}$. The active |
| | | component of each branch is containing spectral reactive |
| | | components. The bellow does not show at paper 1: |
| | | Same, theorem is brought so not to suppose by mistake that |
| | | $Y_{total,imag} = \sum_{n} j\omega C_n - j 1/\omega L_n$ simply because $i_{total,imag} = \sum_{n} j\omega C_n - j 1/\omega L_n$ |
| 12 | | $\sum_{n} i_{n,imag}$. |
| 12 | if rule – is – correct for: $\{all \ n \in N_D\}$ then rule – is – correct – for: $\{all \ n \in N_C\}$ && $\{n \in N_C + N_D\}$ | method enlargement to HGL: theorems 3)-5) are correct also for Harmonic Generating Load (HGL). Practically there might |
| | у тем у тем и | be changes. Mathematically formulating it referring to |
| 13 | | paper.שגיאה! מקור ההפניה לא נמצא. |
| 13 | $(1/Y_m)^2 / \partial_{\omega} (i_m(\omega) / v_m(\omega)) =$ | anomaly enhancement by admittance. Admittance physical |
| | * | component improves the anomaly detection. Proof: the spectral |
| | $= \left[i_{m}(\omega) / i_{m}(\omega) - v_{m}(\omega) / v_{m}(\omega) \right]$ | sensitivity is a measure of 'the change' signature since it's based |
| | Where: m index ∈ | on 'many frequencies change-detection'. |
| | {CPC, enhanced CPC current types} | Eq. is to be correctly interpreted 'not' as admittance dependency |
| | | stand-alone on current i'/i and over voltage v'/v . Correct |
| | | interpretation is 'admittance is independent of voltage |
| | | amplitude'. Since current contains dependency over voltage |
| | | through $i = Yv$ – the voltage dependent component is approx. |
| 14 | | subtracted – leaving only/mainly the 'non-voltage dependent'. |
| 14 | $Y_{customer \to reactive \to scattered}(\omega_i) = Y_{inductive}(\omega_i) + Y_{capacitve}(\omega_i)$ | separability of inductive and capacitive admittances. Inductive and capacitive components may be separated from the total |
| | | (customer+ distribution)→ reactive→ |
| | Theorems outside nones 4 | scatter admittance. Proof: using at least two harmonics $i = 1,2$ |
| 15 | Theorems outside paper 1 | For E CDC (gootton gramouts and for CDC) |
| 15 | All of the above theorems 1-13 are correct both | For E-CPC 6 scatter currents, and for CPC- 1 scatter |
| 1 / | for CPC 5 components,& E-CPC 22 components. All of the above theorems 1-14 are correct for Lap. | current |
| 16 | 1 | |
| 17 | 1)For <u>reactive</u> components – entire reactive current should be taken, and not only customer – | An especially important theorem that is derived from comprehension of definition of customer current as |
| | reactive current. | defined at CPC |
| | 2) For load current – entire current should be taken | Verified experimentally |
| 10 | and not only customer current Signature Conjecture: it is speculated that | Demonstrated ampirically at published paper and to be |
| 18 | originally matching zero-poles are close/ | Demonstrated empirically at published paper, and to be attempted proven theoretically at thesis. Recently an |
| | overlapping for linear loads, deviating apart for | interesting related paper published. |
| | non-linear loads | |

| No | Mathematical formulation | Physical interpretation |
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| | Paper 2 theorems | Load disaggregation |
| 1 | Interpretation: "None" represents not a "vacuum" of | PCA correlation of "None" object instance to device. |
| | devices, but "all the rest". Outside the "kitchen" there | Theorem: the closest the "None" is to a device cluster, the |
| | are devices electrically resembling the kitchen devices. | higher "confusion" scoring between the "None" and the device. |
| | PCA being an orthogonal variables representation, | |
| | when the "None" clusters have a large boundary or | |
| | appear to be embedded with the device cluster – then | |
| | confusion scoring shall be higher. | |
| 2 | | "None" confusion with devices. |
| | | The "None" has larger confusion scoring with devices than a |
| | | device with another device. Explanation: since "None" is a |
| | | collection of devices outside kitchen then it may contain |
| | | signatures of devices that electrically resemble a kitchen device. |
| 3 | This is a weaker more general version of theorem #1. | Correlation between distance at PCA space and confusion. |
| | | Devices that are near each other at PCA space, the "confusion" |
| | | scoring shall be higher between devices. |
| 4 | | |
| | | |
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