On Similarities and Differences between Energy Transport Theories in non-sinusoidal Situations

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Abstract: A comparative study of five widely accepted energy-transport theories is presented. The theories studied are multi-vector Poynting Vector theory (PVT), p-q theory (PQT), Current Physical Components theory (CPCT), Conservative (CPT) – time-domain theory, and Conservative (CPT) frequency-domain theory. An analysis of the relations between the sums of physical components in the various theories was performed which yielded a grouping into instantaneous and periodic averaged theories.

The objectives of the study: (i) Similarities among the theories are identified, and significant differences between the theories, which were not emphasized in other works are explained. The laws of each theory are expressed through current/power physical components; (ii) The comparative study combined with the insertion of new rules, gives rise to novel grid applications as demonstrated by four original simulations. The characteristics of the grid technologies required by each theory are presented; (iii) Enable the selection of best theory for a particular purpose. The implications for grid design are outlined for each example; (iv) Examples are developed for grid monitoring, control, and diagnostics usage such as for reverse load identification; (v) Determine suitability for application to renewable energy measurement. The demonstrations display the impact of revisited theories; (vi) Finally, a correction to the continuous time theory reactive energy measurement formula is presented.

1. Introduction

Energy transport theories deals with accurate calculations of the various power components in systems under non-sinusoidal waveforms for the purpose of monitoring and measurement interpretation, power systems optimization, and synthesis of devices such as active filters. Modern concepts of smart grid as described for example in ‎[1], ‎[2] require correct energy computation. Big data analytics require correct load energy profile (quarter hourly to hourly energy consumption) to detect non-technical loss, and forecast consumer load. There are five main energy transport theories: 1) Poynting Vector Theory (PVT) ‎[3], 2) P-Q theory (PQT) also termed as Instantaneous Reactive P-Q theory (IRP) ‎‎[4]-‎[6], 3) Current Physical Components theory (CPCT) ‎‎[7], 4) Conservative time-domain theory ‎[8],‎[9]‎; and 5) Conservative frequency-domain theory ‎[10]. The five theories yield different computational results for various power system scenarios ‎[7],‎[11],‎[12]‎. This may be confusing and a matter of some concern as it can be difficult to Figure out which result is correct. For example in some jurisdictions reactive power tariffs have been proposed. In this case the achievability of universal standardized metering must be considered.

Reactive tariffs are required to quantify the value of increasing reactive power due to renewable energy converters, new lighting methods (LED) etc. Given the complexity of the field there has been much debate about the fundamental correctness of each theory and the effectiveness of the calculation method ‎‎[11]-‎[13] and the number of works on this subject has been increasing in recent. Some of these works focus on one of the theories ‎[4]. Other works view the various theories as simply different descriptions of the same phenomena ‎‎[8],‎[16]. Yet others assume that it is generally understood that there are similarities and dissimilarities between the theories and focus on showing some specific differences ‎[16]. Emanuel ‎‎[8] proposed a method for classifying these power transport theories into two groups. The classification, however, does not apply to instantaneous theories with time-domain parameters but only to *periodic averaged* theories.

In this paper we identify the similarities between clusters (sums) of current/power physical. This equivalence between the sub-theories is not obvious at first glance and not well documented. This paper demonstrates these relations for the five presented theories which are divided into *instantaneous* and *periodic averaged* types as seen in Fig. 1(a). The five selected theories are thus grouped using bridging theoretical computations, based on existing references and demonstrated by simulations.

This paper focuses on demonstrating the mathematical and physical equivalence of the theories. The grouping of the theories is based on whether the method is instantaneous or periodic-averaged and it does not mean that the methods are necessarily equivalent. The equivalence is identified between clusters (sums) of physical components (currents, powers) of the distinct theories, and not necessarily between each specific single physical component in every theory. We also identify several significant differences between the theories, which are not fully emphasized in other works. The paper demonstrates multi-phase equivalence preservation between the theories. This comparative work is demonstrated by simulation of applied practical scenarios generated by Smart Grid issues such as active and reactive power measurement differences due to the various approximate formulas which lead to computational variance. The comparison presents the theories as being more different points of view than fundamentally different theories. We show that the three periodic averaged theories are equivalent, although each theory decomposes the current into different physical components. This paper also shows that some of the apparent power definitions are equivalent and meaningful and the difference between these and other definitions is explained. The above comparison is

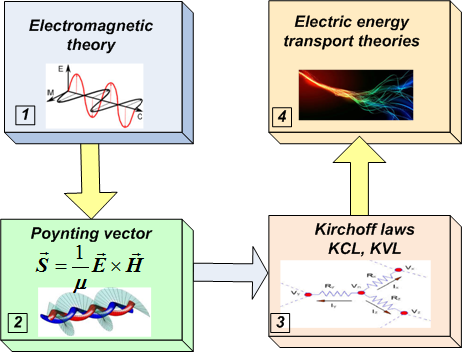
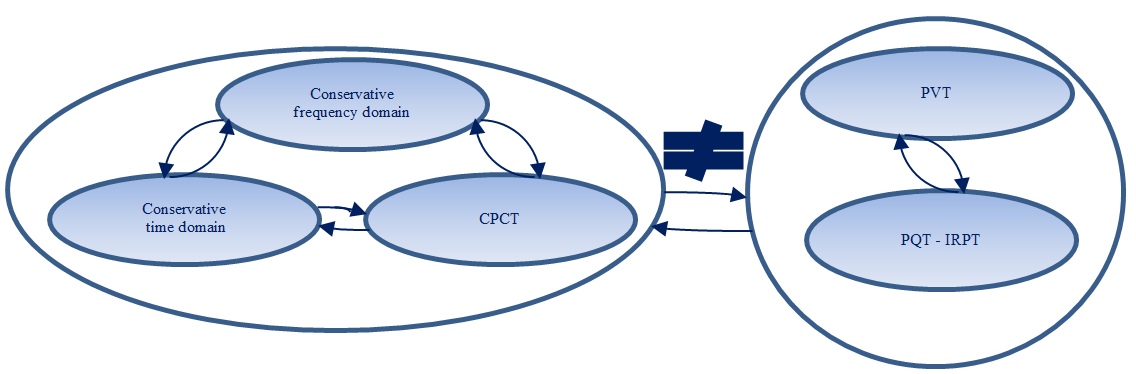


Fig. 1: Two methods for theories re-study applied by the current paper: (a) Unification of the five isolated theories into two major group theories: instantaneous and periodic averaged. (b) Gradual evolution of physical abstraction layers, covering additional rules each, through electric field/current/power decomposition

**(b)**

demonstrated by a simulation example employing waveforms of a simple PWM rectifier which is a non-linear load.

1. **Outline of the five power transport theories**
   1. Common formulation of physical power/current components

As mentioned above, the five theories considered fall into two groups: periodic averaged ‎[7],‎[8] and instantaneous ‎[4],‎[17]. This implies significant physical differences i.e. optimization of electric energy transport instantaneously as opposed to over a period and difference of power quantities which cannot be reconciled by comparing the sums of the physical components. In addition there are areas of strength and areas of weakness for each group of theories ‎[18]. Power theories based on harmonics can be defined using the following equations:



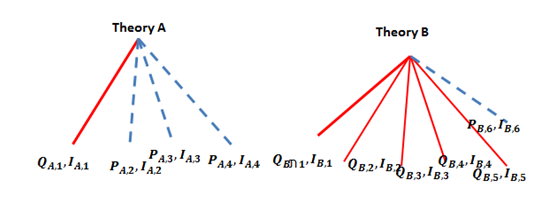
where ***S*** is the apparent power, ***Pj*** is the *jth* component of the active power component with a physical significant implication, ***Qk***, is the *kth* component of reactive power. The current can be expressed as:



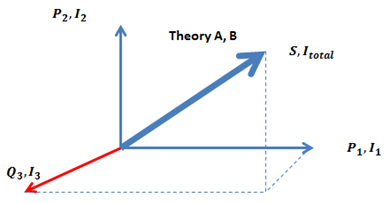
The total current *i*, is decomposed into current components ***ij***and ***ik*** matching the power components ***Pj***and ***Qk*** . Finally, (3) implies orthogonality of the various current components of . This implies that the power components in are also independent.



When presenting the theories we shall show how, for each power theory the relationships expressed in (2) for scalar sum and orthogonality (3),(1) are maintained. The generalization in (1)–(3) is new because so far it has been demonstrated separately for each theory but not generally for all of the theories. This point is important for *unification* or showing the fundamental equivalence of the theories. Fig. 2 demonstrates the main point which will be proved in this paper i.e. the equivalence of clusters (sums) of physical components (current, power). It also demonstrates, in the bottom schematic, the orthogonality of physical components as formulated by (1) and (3). Equivalent clusters are marked in Fig. 2 with similar lines. We will show how these equivalences can be put to use for new practical grid applications.



(a)



(b)

Fig. 2: Equivalence of clusters (sums) of physical components between two power theories. (a) Fig. demonstrates the orthogonality principle stated by (3) and (4). (b) Orthogonality of either theory A or theory B components stand-alone.

A fourth unification equation emerges from and :



where are unit vectors orthogonal to each other and the powers are vector components i.e. summed separately. The reactive ‎[8],‎[10] and apparent power vectors ‎[8] are addressed in some of the theories separately but are not mentioned as a general rule in all the theories. This is because the theories have not been considered as unified. Research experience shows that periodic averaged theories describe well the load active/reactive characteristics, energy measurement and optimal power transfer ‎[16]-‎[18]‎[16]. The instantaneous theories have proved successful for economic power conditioning ‎[4] e.g., for shunt active filter control, as they do not require storage components, but only switching circuits. However they are not suitable for active/reactive characterization ‎[12],‎[13],‎[18].

A brief description of each power transport theory follows.

* 1. Conservative Power Theory (CPT) and FBD method

The terminology of CPT is used in this paper as refined by the FBD (Fryze, Rosenzweig, Depenbrock) method ‎[8], ‎[22] ,‎[23]. We start from the power equation presented by Budeanu and Fryze ‎[24]-‎[26]:



where ***S*** is the apparent power, ***QBudeanu*** is the Budeanu reactive power as in , and ***D*** is the reactive distortion power. In the frequency domain it may be expressed by cross-product of voltage and current harmonics‎[8]. The frequency domain formula reflecting the active power is ‎[8],‎[10]:



The Budeanu distortion power is shown to be ‎[1],‎[8],‎[10]:



where are voltage and current harmonics as areand ***j, k, m*** and ***n*** are the harmonic indices. The result in is also computed in this work in and (48), based not on the macroscopic power equation, but on electromagnetic fields. According to the FBD refinement active current is defined as‎[18]:



where μ is the phase index (1,2,3), ***Gactive*** is the average active inductance and the RMS voltage, ***VΣ ,*** is given by:



where



***vΣ***is the 'instantaneous collective voltage value' and ***vμ***is the phase voltage***.*** The ‘instantaneous active power’ operator ***pΣ (t)*** is given by:



and ***PΣ,*** the ‘periodic averaged active power’ as computed in the time domain is:



The ‘active current’ is the average of the CPT active current component. The current is oscillatory but the power is not. The power current is given by:



The ‘phase power’ current ***ip,μ*** is the total active current consisting of both the average segment named active current, and an oscillating power segment. The next current component is called the ‘variation current’ ***iv,μ***:



This variation current, is comprised of the active oscillatory component around the average inductance current value, named the active current ***ia,μ***. The next current is the non-active current ***in,μ***:



The non-active current is the total phase current minus the active current. It should be kept in mind that FBD active current is an average and not the entire active current. Then the ‘powerless current’ is defined as:



It is the total phase current, ***iμ***, minus the power current, which yields the actual active current.

Finally a rule is maintained for the variation current, ***iv,μ***:



The components in the FBD method are orthogonal as in :



Where ***μ*** is the phase index.

**Example no. 1: Equivalence relations of apparent power definitions at single phase using a PWM rectifier based circuit.** This circuit is used in renewable applications such as MPPT for solar cell arrays. The current example demonstrates, both theoretically and numerically, that implies equality between the Buchholz apparent power and the geometric apparent power for single phase. The relevance of this is that other references emphasize the inequality of apparent power definitions‎[7],‎[8], see Fig. 3. A PWM rectifier MATLAB Simulink model is used applying software for computation of all power parameters. The apparent, active, reactive and distortion powers of periodic averaged theories based on Buchholz definition ‎[29], ‎[30] of and the geometric definition  implied by are shown in table I.



(a) (b)

Fig. 3: PWM rectifier: (a) voltage waveform and harmonic content. (b) current waveform and the associated FFT distribution: amplitude (top), phase (bottom). The circular marker indicates a notch in the voltage waveform. The reactive power is 44% of the apparent power, and current THD is 70%.

Table I: Calculation results for comparing the Buchholz and geometric power

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| ***D* [VA]** | ***QBudeanu***  **[Var]** | ***P*[W]** | ***S*[VA]** | ***VRMS, IRMS*** | **Method** |
| 1116 | 1532 | 2255 | 2947 | 207.67V, 14.22 A | Buchholz |
| 1129 | 1532 | 2255 | 2952 | 207.67V, 14.22 A | Geometric |

The Budeanu distortion power *DB* is calculated using , the reactive power using (14), and the Buchholz using . The Active power *P* is computed in both cases using and yielding identical results. It can be observed from table I that the Buchholz and Geometric definitions yield the same results and the small difference (1.15%) in the apparent and distortion powers is due to the calculation precision. The theoretical demonstration is as follows: From :



is derived from , and and yield :



The 2nd sum is merely a merger of two sums into a single sum. On the other hand, starting from the Buchholz definition:



The last two results are identical. Hence we have the theoretical proof of example no. 1.

* 1. Reactive vector power decomposition method/theory

In ‎[10] the concept of reactive vector power was introduced. In this paper it is the vector of harmonics, similar to PVT as presented above, that is discussed. Identical results such as and are obtained. However this method is based on as the initial condition for developing of the theory. An additional result as in ‎[10] is that there is a reactive power instantaneous operator equivalent to the periodic average reactive power parameter ‎[9]:



Example no. 2 in section 4 will summarize the usability of various CPT theories to active/reactive load characterization.

* 1. Currents’ Physical Components Theory (CPCT)

CPCT was developed by L. S. Czarnecki ‎[7]. Since it was first published there have been developments and adaptations of this power theory ‎[11]-‎[13]. It applies to a linear time invariant load with additional current harmonics generated by the load. In this paper we demonstrate, using example 4, that CPCT is most suitable for grid monitoring and characterization due to its ability to decompose into many physical components. That makes it also suitable for load identification ‎[39]. CPCT potentially isolates weak characteristic effects from averaged strong effects through decomposition. The general way of presenting a harmonic voltage is:



There is an assumption regarding the harmonics not generated by the load, that each harmonic current component is generated only by the matching voltage harmonic ‎[7]:



This leads to a total current expression:



The equivalent active conductance of the system can be defined as:



where ‖U‖ is the RMS value of the voltage. The active current *ia* can now be calculated as:



Subtracting this active current from the general current, yields:



It is evident from that the remaining current can be separated out to give a reactive current *ir* which is π/2 shifted from the same voltage harmonic.



The remaining part is called the scattered current *is*. This current is related to a spectrum of admittances scattered around the equivalent active conductance *Ge*:



Continuing in this manner in the case of a three phase system the total current is decomposed into various components as follows:



where ***ia*** is theactive current, ***ir*** is thereactive current, ***is*** is thescattered (active) current, ***iu*** is theunbalanced current and ***iB*** is thebackward load generated current. ***iB*** is also known as ***iC*** – the customer load generated current. All current physical components have explicit formulas at CPCT. These current components are proven to be orthogonal as shown in ‎[7] and therefore:



This result is identical to (3) as a particular case of CPCT.

Next we will review the instantaneous theories and then a comparison system between the theories will be presented.

1. **Comparison between instantaneous time-domain and frequency domain reactive power theories**
   1. Presentation of instantaneous vs. periodic averaged reactive power definitions

In this section the instantaneous theory is constructed based on the orthogonality of instantaneous voltage and current, through the requirement that the conditions in (1) to (4) are maintained. Theories such as PQT ‎‎[4],‎[17] and its expansion to IRP theory ‎‎[27], and to generalized (N-phase) instantaneous theory ‎[27], ‎[28] define the instantaneous reactive power component as a voltage and current vector product as in ‎[4]:



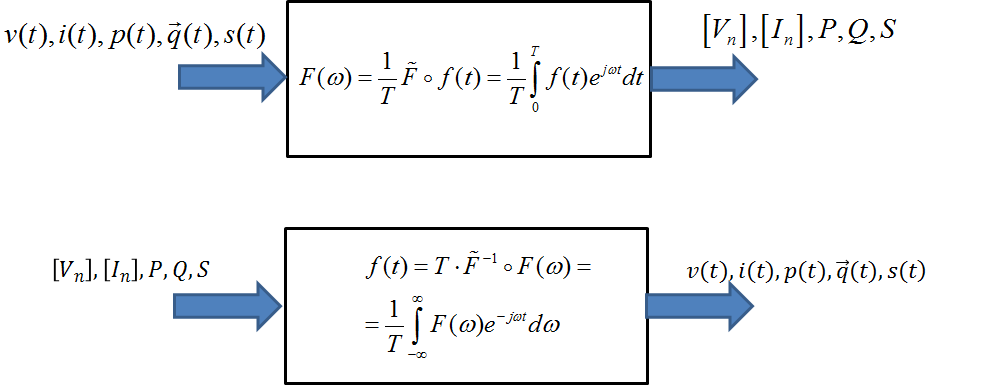
whereand are the voltage and current vectors in an *M* phase system and *[L1,L2,…..,LM]* are the electric phase indices. In ‎[3] multi-vector PVT is presented. Each phase is handled independently and decomposed into harmonic components. This theory is a periodic averaged one. The vector product in (29) is not spatial for lumped circuits, but temporal i.e. the vectors are phasor representations of the voltage phases. The relative phase-angles comprise the difference between the vector components. The instantaneous waveforms are represented by Fourier expansions based on the network frequency ‎[3], similar to periodic averaged theories in ‎[8] and ‎[10]. Time domain voltage and current parameters are instantaneous. The definition in of orthogonality of power components is also correct for an instantaneous theory such as PQT ‎[4]. However the power quantities are the instantaneous rather than the periodic averaged parameters ‎[27],‎[28]. The orthogonality of the instantaneous powers is not explained on a physical basis ‎[5] and ‎[4]. In ‎[10] the definition of *harmonic* vector product  is explained by means of conservative frequency-domain theory but the vector product is periodic averaged and not instantaneous. There is a transformation using the inverse Fourier transform, for transferring the frequency domain voltage, current, active, reactive and distortion powers parameters to instantaneous matching operators as seen in Fig. 4 ‎[3]. The result of this transform, when operating on the periodic-averaged driven, instantaneous reactive power ‎[9],‎[10] is not equal to the instantaneous reactive power operator as described in for the instantaneous theory, but rather to the definition in . In ‎[10] the concept of “reactive power vector space decomposition” is reviewed. The theory developed in ‎[10] is referred to as frequency-domain conservative theory. The reactive power expression is derived in (34), ‎[8], ‎[10], where *P*, *Q* and *S* are the periodic averaged active, reactive, and apparent powers ‎[31] respectively and and are the voltage and current harmonics

Fig. 4: The transformation from ‘instantaneous theory’ parameters to ‘periodic averaged theory’ parameters.



vectors, and are the transposed vectors. In ‎[9] and ‎[10] it is shown that is equivalent to the vector product *.*

* 1. Un-equivalence of the definitions of reactive power vector product in instantaneous and periodic averaged theories

In this section the inequality of the reactive power calculations resulting from periodic averaged theories and instantaneous ones is shown. Next, a modern approach called reactive power vector space decomposition ‎[10] is used as the theoretic tool of c*onservative frequency domain* theory ‎[25],‎[26]. In *periodic averaged* theories, reactive power is defined as:



It is also proved in multi-vector PVT and vector reactive theory (which is a CPT) ‎[10] that instantaneous time-dependent reactive power expression can be written according to , ‎[3]. It can also be proved by reactive power vector decomposition ‎[9],‎[10]. The relationship in can also be derived from multi-vector PVT ‎‎[1] and from conservative time-domain theory ‎[8] converted into the frequency-domain theory ‎[3], as shown in the next section. Orthogonality in Fourier space versus orthogonality in time-domain is the cause for the differences between the reactive power calculations.

1. **Equivalence of reactive energy and power computation in frequency-domain and time-domain**
   1. Comparison of CPT time-domain and frequency domain

In this section we demonstrate the equivalence of power parameters in the periodic averaged time-domain non-active component and in the multi-vector PVT frequency domain non-active physical component. PVT is an instantaneous theory. It has been shown in Fig. 4 that multi-vector PVT is periodic-averaged through Fourier coefficients. The periodic averaged time-domain computation ‎[3] is compared to the periodic-averaged frequency domain computation. Both theories are used as the reactive power and energy calculation by the IEEE 1459-2010 standard ‎[30] and its matching European standard DIN 40110 ‎[31]. The common reactive energy computations in reactive meters are ‎[8] given by:



where *T* is the period. In the presence of harmonics similar formulations ‎[8] use with only the prime harmonic voltage and current wave functions ‎[8]. It was noted that (36) results in high discrepancies (up to 40%!) among energy meters ‎[32]. Now days high end energy meters comply with IEEE 1459 ‎[33]-‎[36], which adopting (6) (and avoiding (36)) settles this discrepancy resolves this issue. As will be shown we introduce a correction to (36), allowing for operation in the time domain.

**Example no. 2:** **numerical differences between frequency-domain and time-domain (periodic-averaged) computations and approximations.** The objectives of the following example are: (i) to demonstrate that there is a large discrepancy between time-domain and frequency-domain periodic averaged computations due to the harmonic inaccuracy of the expression in for both the *QBudeanu*, and the *DBudeanu* computations. (ii) To set the scene for the next sub-section, which is a correction of the time-domain formula. This is of increasing importance for smart grids to (i) increase harmonics, and (ii) inform the development of reactive tariffs as a better reflection than power factor the cost of grid infrastructure. The next calculation results in table II are carried out to compare active vs reactive, and approximate vs accurate formulas, according to standards in ‎[30] and ‎[31] which are based on the equations as marked in brackets beside the results. The waveforms being used for analysis are the single phase PWM rectifier of example no. 1, Fig. 3. PQT as mentioned is totally different and used for active filtering ‎[37]. The computational energy gaps between the active and reactive powers according to the various theories are sumerized in table II (All computations relay on formulas which appear in the IEEE 1459-2010, ‎[34]-‎[36] and DIN 40110 standards). The left column is the frequency domain calculations according to CPT methods, but it may as well be computed by reactive vector power decomposition, or by PVT – all yield same current. The ‘time-domain’ periodic averaged theory is shown in the middle column (CPT- time domain). In the right hand column the time-domain periodic averaged computation for the prime harmonic model is observed (the CPT- time domain prime harmonic is implemented in various power meters).

It will be shown that is inaccurate in the presence of harmonics. As a result an energy gap of 72.36% between non-active power in the time-domain and frequency-domain is calculated in row 11. In the time-domain the net sum of is not maintained (row 10) and so the energy balance between the apparent power S, active power *P*, Budeanu reactive power (*QBudeanu*) and distortion power *DBudeanu* is not maintained accurately. This means that time-domain CPT reactive Budeanu power is approximated, which is sufficiently accurate only for linear loads.

Table II: Computation of predicted deviations of active, reactive, and distortion reactive formulas of energy transport according to FBD method, IEEE 1459-2010 and DIN 40110.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **No.** | **Physical parameter** | **CPT**  **Frequency domain** | **CPT**  **Time domain approximation** | **Time domain approx. on (prime harmonic approx.)** |
| 1 | *VRMS(20)* [V] | 207.67 | 207.67 |  |
| 2 | *IRMS(adaptation to 19)* [A] | 12.0762 (18) | 14.215(18) |  |
| 3 | *Iactive,RMS* [A] | 7.4905 (8) | 12.0762 *()* |  |
| 4 | *Ireactive,RMS* [A] | 14.2107*(15)* | 7.4905 *)* |  |
| 5 |  | 14.2107 | 12.0762 |  |
| 6 | *P*[W] | 2255*(16)* | 2255.93*(21)* | *P1*=2259.7 |
| 7 | *Q*]VAR] | 1555.6*(14)* | 1532.6*(33)* | *Q1*=1555 |
| 8 | *D*[VA] | 1086.1*(17)* | 0 | 0 |
| 9 | *S*[VA] (79) | 2946.9 | 2946.9 | 2946.9 |
| 10 |  | - | 1.48% | 1.44% |
| 11 |  | - | 72.4% |  |
| 12 |  | - | 0.041% | 0.2% |

***(\*) Note: currents computed from CPCT***

The 72.36% difference is due to the harmonic inaccuracy. Note that the distortion power *DBudeanu* is recognized by the majority of researchers as actual (cross-products of voltage and current harmonics are derived from the product of and that is actual power). In row 10 a 1.479% and 1.44% difference is calculated between the Budeanu reactive power, in , and periodic averaged power in the time domain, excluding the distortion power, in . This difference is due to the computation method and the sampling rate error and is within acceptable limits. There are numerical works investigating meters measurement through highly distorted electric environment ‎[32], and numerical works that find discrepancies between the theories at the various current components and at quality of reflection of physical load nature: active, reactive ‎[18]. Table II summarizes the difference between the various power theories. The next section looks further into the theoretical origins. Other works also compare various power computations to IEEE 1459 ‎[19] and suggest accurate time domain computation in accordance with IEEE 1459 for large harmonic disturbances ‎[19] or unbalance conditions ‎[21].

* 1. Harmonic correction of the time-domain computation and exclusion of distortion power

Next we look at a correction of (33) approximation. It is shown that distortion power is included in the correct version of . The relevance is that (i) as stated at section 4, many measurement devices use formula or its prime harmonic version (ii) grids include more harmonics than before. In *reactive vector power decomposition,* the periodic averagedequivalent, instantaneous reactive power is given in (16). The voltage and current in can be presented by their Fourier series. Each harmonic is phase shifted by π/2 according to (16). The entire instantaneous parameter is phase shifted by π/2 while each harmonic component frequency is multiplied by. The relationships in and are equivalent to the relationships in and at the modal level only:



Where *n* is the voltage harmonic index and *m* is the current harmonic index. Then referring to a single element from (34):



where . When ***n≠m***  is notified as ***dn,m***. Another issue is the derivation of the reactive current physical components from . A proof of equivalence of to CPT frequency domain theories and to is shown in the appendix, thus enabling the use of as a correction of electricity meters formula. By substituting the instantaneous voltage and current with their Fourier expansions, two components are obtained, *QBudeanu* and *QDistortion* *(*



where:



Since there are pairs of cross-product harmonics substituted in the entire non-active power can be written as in . This result is identical to the result in .This equality can be shown by integrating over a single period.



By using an orthonormality inner product relation then and represent the total reactive power in Budeanu terminology and not only the Budeanu reactive power. Thus, from the conservative frequency-domain theory, which is a theory of averaged powers, or from PVT, which is an instantaneous theory, identical expressions for power components are obtained. This result will also be demonstrated in the simulation section.

**Example no. 3:** **comparison of precise time-domain CPT (31) and approximate time-domain CPT (33) for a non-linear load: a PWM rectifier.** Next the various components in precise CPT and approximate CPT are compared for the same waveforms shown in Fig. 3, example no. 1. The results are summarized in table III. Note that for convenience the number in the brackets beside each result in table III is the equation according to which the parameter was calculated.

The objectives of the example are: (i) The bifurcation of the approximation — the results demonstrate that the approximation in (33) ceases being equal to the precise solution in (35) due to the insertion of current harmonics, through load non-linearity, which do not exist at the grid’s generator.

Table III: comparison of various parameters between approximate CPT (right) and precise CPT (left)

|  |  |  |  |
| --- | --- | --- | --- |
| **No** | **Physical parameter** | **Accurate CPT***(38)* | **CPT***(33)* |
| 1 | *VRMS))* [V] | 207.67 | 207.67 |
| 2 | *IRMS* ***(adaptation to)*** [A] | 14.215 | 14.215 |
| 3 | *Iactive,RMS [A] (9)* | 10.885 *(23)* | 12.0762 *()* |
| 4 | ]A[ | 14.211A | 14.2107 A |
| 5 | *P*[W] | 2255 *(52)* | 2255.93 *(16,41)* |
| 6 | *Q*[VAR] | 1555.6*(16,35)* | 1532.6 (33) |
| 7 | *Pscatter*[W] | 0 | 1116.313*(17)* |
| 8 | *D* [VA] | 1086.1 | Not separated (\*) |
| 10 | *S*[VA] *(79)* | 2946.9 | 2946.9 |
| 11 |  | 2952 | 2948 |

(\*) “Not separated” in row 8 means that no separation of D exists at the legacy formula, although distortion is there inaccurate.

(ii) Load generated current harmonics reflect as voltage distortion due to voltage drop across transmission lines (and is more pronounced in the case of long lines).



(iii) The ability to measure it all accurately using the methods derived in this paper. (iv)Time-domain formulas turn into quasi frequency-domain formulas where there is sufficient accuracy.

1. **Equating active and reactive powers between conservative time-domain, conservative frequency-domain and PVT**

PVT is the only one of the theories arising from a microscopic physical origin which is the electromagnetic field theory. The other theories deduce the power physical components from macroscopic considerations (voltage, current etc.). It is important to observe the differences between multi-vector PVT ‎[3] and the rest of the theories. The geometric apparent power definition used by multi-vector PVT is:



Calculating using the Poynting vector components yields two non-active power components. A constant conserved component as in and an additional oscillatory non-conserved component , oscillating back and forth at various harmonics as in . PVT is discussed further in ‎[38]. The time dependency is added, in order to emphasize the difference between the *instantaneous* distortion power and the *periodic averaged power*. Integrating over a period in order to obtain the same reference as the periodic averaged theories, yields . This is equivalent to PVT, which is an *instantaneous* theory. However multi-vector PVT ‎[3] is attained through Fourier transform, resulting in a *periodic averaged* theory. The sum over squares is obtained from the orthogonality of the cross-product harmonic waveform components. Distortion power is a non-active power representing an energy flow due to cross-products.

Next, the relation between periodic averaged theories and time-dependent reactive power is shown from microscopic PVT. From ‎[3] it follows for periodic or quasi-periodic that the modal relations between the voltage and the electric field and the current and the magnetic field are obey:



where *p* and *q* are the harmonic indices, is the magnetic field and is the modal electric field. are the characteristic magnetic and electric lengths accordingly**.**  is the modal charge component. It is evident that thecomponent matches the instantaneous active power definition as follows:



Expanding to an M-phase vector, reveals the equivalence between the PVT and IRP theories. The three-phase instantaneous active power can be generalized as follows:



When integrated over a single period, there is equivalence to the periodic-averaged power theories:



From ‎‎[3] an alternative form for the modal reactive power is:



and:



This is the time-dependent equivalent of the reactive component from the conservative time-domain theory. When integrating it over one period the reactive energy is computed as shown at and . The component in is identified as the conservative time-domain reactive energy computation in . An ***m, n(m≠n)*** component is the distortion power of . In fact both components are reactive, the first being inductive while the second is capacitive.

1. **periodic averaged theories** 
   1. Active power and the various currents physical components

In this section the energetic equivalence between CPCT and the rest of periodic averaged theories is presented. The benefit of CPCT is that the current physical components enable isolation of some phenomena into a focused sub-current. CPCT is usually presented from bottom to top ‎‎[7]. Each physical component is presented and added as an orthogonal component for building up the total current. The method of identification of the equivalence here is to collect all *active* physical components and all *reactive* physical components. Therefore it is also possible to use a top-bottom approach and subtract the reactive component from the total current in order to uncover the CPC active component. The CPC formal active component is as previously described in . The instantaneous active power is then:



The CPC defined active power is then:



Note that the orthogonality of the current physical components ‎[7] enables separation of the power components.

The active power as presented in the conservative theory is taken from . We show that the summation of CPCT components (active, scattered, real part of customer, and real part of unbalance) results in the active conservative current. In addition, the sum CPCT components (reactive, imaginary part of costumer, and imaginary part of unbalance) result in conservative reactive currents. As in the FBD method (CPT) the scatter current reflects variations of admittance of active current around an average value. This scattered current is orthogonal to the reactive and active currents. However it is on the active plane and not on the imaginary one. Therefore it is logical to add this current to the active current of . This yields:



Calculating the power resulting from the net sum of the active and scattered currents we get:



The factor 2 is merely the RMS to max ratio and the result is now the same as the active power in the conservative theory in . Next, we address the unbalanced component in the case of a three-phase system:



where, *A* is the unbalanced admittance ‎[7] and *U#* is the negative sequence network. This component may include both active and reactive components. In a linear three-phase case when the unbalanced component exists, it easy to show that the active current of the conservative theory is equal to the net sum of the active current, scattered current and the real part of the unbalanced current of CPCT as follows:



In the general case when a nonlinear load is connected (Harmonics Generating Load) the sum of the active, scattered, the real part of the unbalanced component and the real part of the backward component (customer) is:



Now the net sum of all the imaginary current components is equal to the conservative reactive current and can be written as:



* 1. Comparison of reactive power components

The reactive CPC component is very similar to the conservative theory component ‎[3],‎[10]. An additional reactive component, generated by the load and propagating towards the source ‎[7] exists when a nonlinear load is discussed. In ‎‎[7] the load-generated component is demonstrated and named the Harmonic Generating Load (HGL). This component *iC = icustomer* can be separated from the source-generated component. Then the reactive component of the HGL also contributes to the equivalent conservative reactive current. This can be written as:



The indices *C* and *N* stand for customer (= load) and network respectively. CPCT does not decompose the customer current. Observing the reactive power below, the CPCT reactive power physical component is equivalent to the conservative reactive component as in . The difference is that it is simply computed in terms of admittance.



Fig. 5 demonstrates the active and reactive components of CPC in comparison to the conservative theory. Advanced load identification, using CPCT, is discussed at ‎[39]. Example 4 demonstrates some of of novel applications developed by this research group.

**Example no. 4:** **current, power decomposition according to CPCT for smart grid monitoring and control.**

The objective of this example is to demonstrate the equivalence of the relationships in (72) and (75) between CPCT and CPT. Recent comparison of CPCT, IEEE 1459 standard, and a novel theory by Puche-Nunez is presented at ‎[19]**Error! Reference source not found.**. It is claimed in that paper that, in corner cases, CPCT violates, IEEE 1459. However the comparisons presented in , and results in similar conservative active and reactive components.

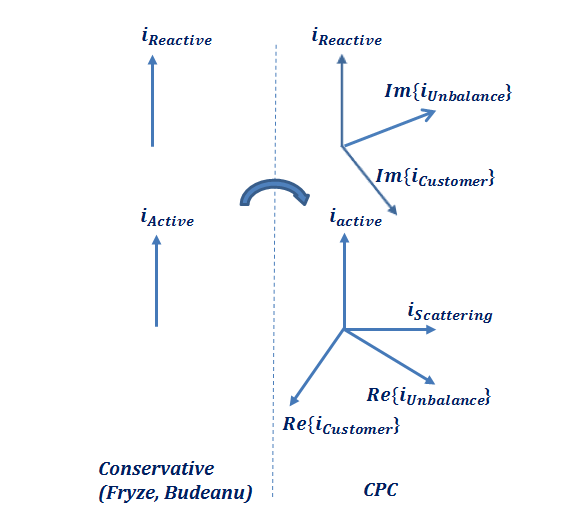
The Implications of this are: (i) CPCT is equivalent to conventional theories and physical components may be used as grid indicators, (ii) CPCT is applicable to grid monitoring 

Fig. 5: A heuristic comparison of active current as defined in conservative theory (Fryze, Budeanu) and in CPCT.

and (iii) CPCT currents can also be used to compute CPT currents. A quick review of the other theories shows that the FBD currents are less useful for reflecting active/reactive load characteristics ‎[18] and that the instantaneous theory currents are different from periodic averaged theories ‎[16]. Thus instantaneous theories are not recommended for active/reactive characterization ‎[12], ‎[18] and CPCT method remains to be one of the most promising candidates for grid monitoring. Two samples were recorded, using a power quality monitor and a class 0.2s meter (American SATECINC EM720). We used two types of load: a PC (with front end rectifier), and an AC motor. CPCT decomposition into five current physical components is advantageous to grid monitoring. It enables the detection of phenomena such as load unbalance; harmonics generated by load and current arriving from load to grid (customer or backward current); reactive current which may indicate transients/faults; or scattered current indicating a highly non-linear load. A software driver from the "monitors management" software database, in Matlab, and a CPCT decomposition module where written in accordance with examples from ‎[7]. CPCT has been shown to be applicable to monitoring grid conditions that are not amenable to direct inspection. In the case of a rectifier: (a) Active current is very small compared to the scattered and reactive currents. (b) Reactive current (Fig. 6-c) is visually similar in voltage waveform (Fig. 6-a) but is 90◦ phase shifted from the voltage. Both voltage and current have a 10th harmonic. For the rectifier: (a) the scattered active component is very small compared to the reactive currents. That is due to being highly non-linear where current harmonics are 90° phase shifted with respect to the voltage (resulting in power oscillations back and forth). (b) Reactive current (Fig. 6-c) is visually similar to the voltage waveform due to the voltage drop across the internal resistive component similar to (Fig. 4-a), but is 90◦ phase shifted from the voltage. Both voltage and current have a 10th harmonic. (c) Scatter current is at 200Hz and high. The voltage also has a harmonic at that frequency. (d) A high customer current i.e. a high load generated current, is identified at the 10th harmonic. Fig. 7 includes further CPCT decomposition for a true recorded AC motor for which the total current/voltage waveforms are shown in Fig. 6-b. Notice that reactive current is similar to voltage.

1. (b)

-

(c) (d)

Fig. 6: Voltage and current recorded waveforms. (a) PC rectifier – highly nonlinear, voltage and current waveforms. (b) An AC motor – partially nonlinear, voltage and current waveforms. (c) Rectifier voltage and current FFT spectra: amplitude (top), phase (bottom). (d) AC motor voltage and current FFT spectra: amplitude and phase.



1. (b)



(c) (d)

Fig. 7: AC motor current physical components decomposed (recorded with Satec INC EM720 power quality monitor): (a) Active current. (b) Scattered current. (c) Reactive current (with 50Hz harmonic). (d) Customer current.

The CPCT acquires now a new interpretation, important for grid monitoring as shown in Table V. Table IV lists the CPCT analysis results for the PC rectifier and AC motor recorded by a meter and a monitor. The relevant equation is shown in brackets. For experimental studies we used the scheme shown in Fig. 8.

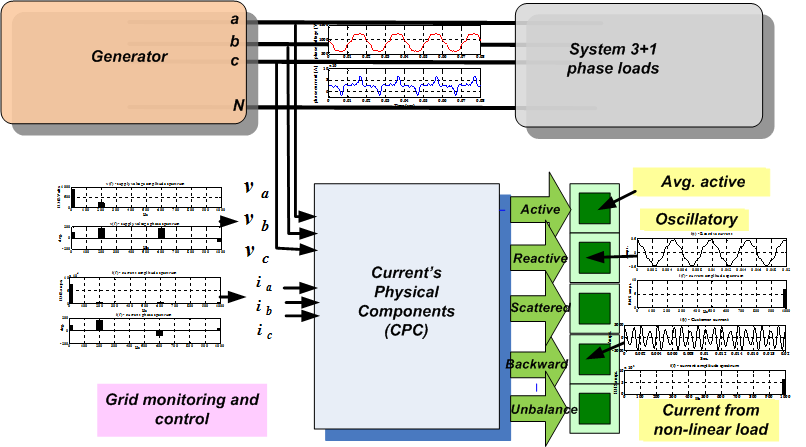


Fig. 8: Schematic of a CPCT based grid monitoring and control software module. The top waveforms are voltage/current of rectifier circuit. The left hand spectra are magnitude/phase spectra of voltage/current. The waveforms on the right hand side are some of the physical components of the current.

Table IV: CPCT analysis results for a rectifier and an AC motor recorded by a meter and a monitor.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  |  |  | **Comparative data** |  | **Comparative data** |
| **No** | **Physical parameter** | **PC rectifier** | **PC rectifier** | **AC motor** | **AC motor** |
| 1 | *VRMS(20)* [V] | 926.66 | *I active, CPT* | 884.74 | *I active, CPT* |
| 2 | *IRMS* *(adaptation to )*  [A] | 73,214 |  | 59,488 | 16,830 |
| 3 | *Iactive, RMS (CPCT)* [A](23) | 71,897 | *I reactive, CPT* | 58,123 | *I reactive, CPT* |
| 4 | *Ireactive,RMS(CPCT)*  [A] (25) | 13,087 |  | 6089.5 | 82,422 |
| 5 | *Iscatter,RMS(CPCT)* [A](25) | 19.714 |  | 6,110.1 |  |
| 6 | *Icustomer(CPCT)* [A](55) | 4408.8 |  | 9254.5 | 1453.4 |
| 7 | [A] | 73,212 | (55) | 59,484 |  |
| 8 | *P*[W](50) | 6.662e7 |  | 5.1423e+7 | 9197.3 |
| 9 | *Q*]VAR](56) | 18,268 |  | 5.3876e+6 |  |
| 10 | *D*S[VA] | 1.2127e7 | (55) | 5.4058e+6 | (74) |
| 11 | *D*Customer[VA] | 4.0855e6 | ***DBudeanu*** | 8.188e+6 | 1.5287e+4 |
| 12 | *SBuchholz*[VA] | 6.7845e7 | (53) | 5.263e+7 |  |
| 13 | [VA](15) | 6.8745e7 |  | 5.263e+7 | 6.5686e+4 |

The software interacted with a portable power quality monitor for diagnostics and leakage detection and at the interface to meter management software through the API. The right hand spectra are the waveforms and amplitude spectra of reactive/backward current physical components. Table V presents a suggested interpretation of CPCT components as load reverse-engineered components. Recent discussions referring to the relevance of CPCT appear in ‎[38]-‎[41] while the relevance of PQT is discussed in ‎[42].

Table V: practical interpretation of grid measurement using CPCT

|  |  |  |  |
| --- | --- | --- | --- |
| **No** | **Interpretation to component** | **Current component taken from CPC** | **Percentile** |
| 1 | Purely resistive load | Active current | 97.7% |
| 2 | Active non-linear load | Scatter current |  |
| 3 | Current returning back from load | Customer current | 9.17% |
| **No** | **Interpretation to component** | **Current component taken from CPC** | **Percentile** |
| 4 | Resistive current back from load | Real{customer current} |  |
| 5 | Reactive (L,C) current returning from load | Imag{customer current} |  |
| 6 | Resistive non-linear load current back from load | “Active”=  {Real{customer}} |  |
| 7 | Percentile of purely capacitive/inductive load | “Reactive”=  {Imag{customer} |  |
| 8 | Unbalance between phases/inter-phase load current | Unbalance |  |
| 9,10 | Unbalance active/reactive | Real, Imag{Unbalance} |  |

1. **Conclusions**

The physical equivalence of clusters of physical components derived through various theories was demonstrated. Although the physical components may not match one another, clusters do match. The five sub-theories reviewed were merged into two groups of theories – periodic-averaged and instantaneous. The groups differ by the optimization criteria ‎[16] and by the construction of the reactive power vector ‎[1],‎[7],‎[10]. The groups may also be equated as this work has shown. Each sub-theory highlights certain points. PVT is a microscopic theory based on electromagnetic field theory. The others are formulated on macroscopic foundations. When there is doubt regarding physical correctness then, for novel physical phenomena, it is useful to use PVT. CPCT is shown to have an advantage in identifying a wide set of current physical components: active, scatter, reactive, unbalance, and load current and harmonics generated in a load. CPCT is especially powerful for grid losses cause identification, be they harmonics or prime harmonic, unbalance, active or reactive; and for either energy flow direction – source or load. Conservative “frequency-domain” theory is the historical root of power transport theory and the foundation of the relevant standards. Conservative “time-domain” theory is the basis of most measurement equipment in use today. In addition it has been shown here to be equivalent to conservative frequency-domain theory. The examples are summarized in the following table.

Table VI: Demonstrated examples, their objectives, and applications

|  |  |  |
| --- | --- | --- |
| **Example** | **Objectives/ implications** | **Implementations** |
| Equality relations of apparent power definitions at single phase, using a Pulse Width Modulation rectifier based circuit. | +Unification of theories  +Grid design | + Reduction of confusion in the choice of formulas for grid design : elec. manufacturing, conduction  + Unification of theories |
| Numerical differences between frequency-domain, and time-domain (periodic-averaged) computations/ approximations. | New relevance to smart grids | + Increase of harmonics  +Thoughts about reactive tariff |
| Comparison of precise time-domain CPT (54), (56) and (57) and approximate time-domain CPT (52) for a non-linear load: a PWM rectifier. | New relevance to smart grids | +Approximation bifurcation  +Grid pollution computation by non-linear load harmonics  +Ability to measure it all accurately through this paper.  +Time-domain formulas turn into quasi frequency-domain formulas |
| Current, power decomposition according to CPCT for smart grid monitoring and control. | Suitability for grid monitoring/control. | +Cannot be observed by inspection without operation of CPC algorithm.  +Grid losses cause identification |

There are numerous studies focused on the reactive tariff cost. It is observed here that various computational models yield significantly different reactive energetic values due to being approximations of an accurate formula. Another aspect of relevance to smart grid research is that renewable energy technology inserts rectifiers. It is shown here that even reactive energy, as computed by various theories, yields different computational results. The equivalence of two apparent power definitions, the geometric\vector and the Buchholz, was shown for the single phase case, both theoretically and through a worked example. The equivalence, to an order of magnitude only, of the three phase case was also demonstrated. This equivalence means that there is a choice of formulas for use in grid design. This also assists further with the unification of the theories.

It was demonstrated that in CPT a frequently used reactive energy computation is merely an approximation. Its correction has been suggested. This enables the development of suitable metering devices for smart grid energy that is not test defined by conventional metrology standards.

**Appendix: Proof of proposed reactive energy correction**

Using and the fact that *T/(4n) ≡ π/2* and the fact that integrals of (*2ωt+φn*) are zero, and using a trigonometric identity to express the product by a sum of two terms:



This is the correct expression known from frequency domain theory. It is easy to prove that is equivalent to . Since it is equivalent for each individual harmonic then it is equivalent for the entire series expansion.

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44. Appendices

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