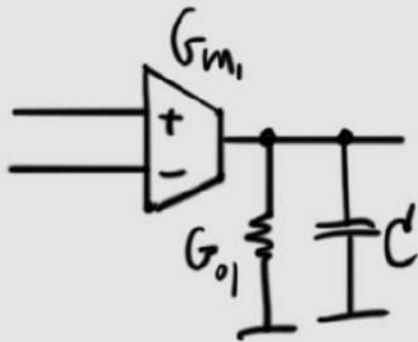


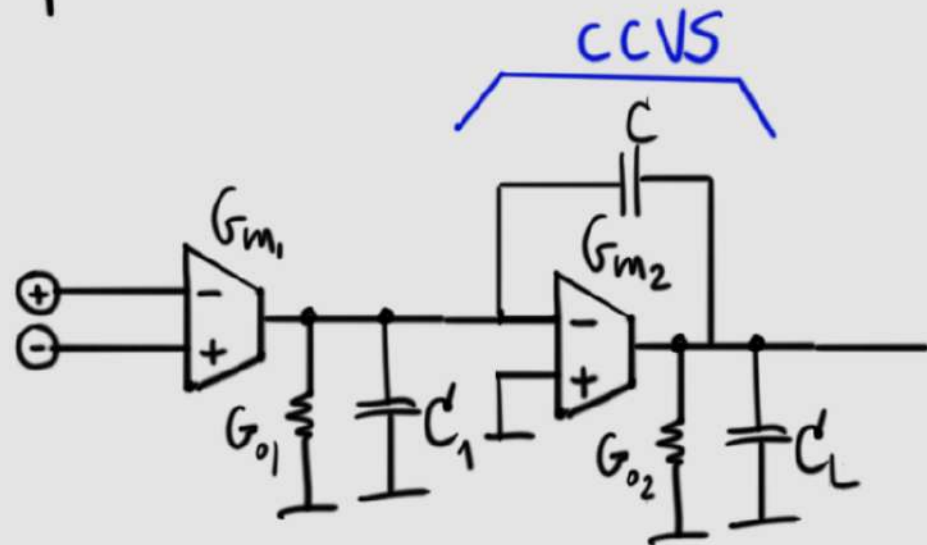
Single stage op amp is a transconductor driving a capacitive load.



$$A_0 = \frac{G_{m1}}{G_{o1}} = g_{m1} (r_{ds1} \parallel r_{ds2})$$

$$\omega_u = \frac{G_{m1}}{C}$$

2-stage op amp is the combination of a transconductor + a "CCVS"



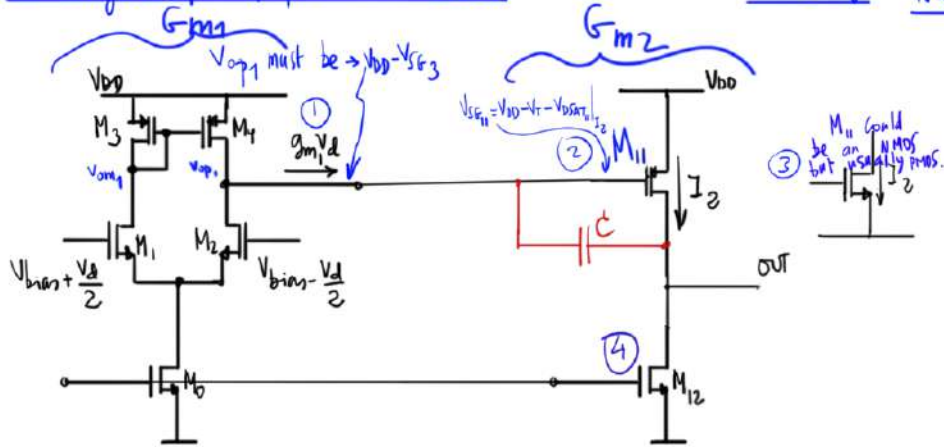
$$A_0 = \frac{G_{m1}}{G_{o1}} \cdot \frac{G_{m2}}{G_{o2}}$$

$$\omega_u = \frac{G_{m1}}{C}, \quad p_2 = -\frac{G_{m2}}{C_L + C_1}$$

↑ Same as the single stage.

Implementation:

1st stage: G_{m1} : Diff pair transconductor



2nd stage: Single transistor = Another transconductor.

- ① From the output of the 1st transconductor a current $g_{m1} v_d$ flows IF WE HAVE THE PROPER TERMINATION ON V_{op1} . Quiescent voltage on V_{op1} must be equal to $-v_{m1}$ which is $V_{DD} - V_{SG3}$.

- ② If we add a PMOS as 2nd stage, this needs to be biased with a specific I_2 that produces its V_{SG11} to be equal to V_{SG3} .

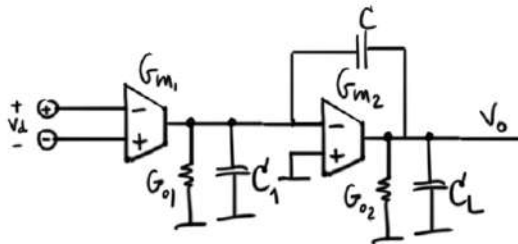
$$V_{op1} = V_{DD} - V_{SG3} = V_{DD} - V_{SG11} = V_{DD} - V_T - V_{DSAT11} |_{I_2}$$

Remember:

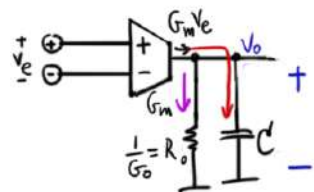
$$V_{DSAT} = V_{OV} = V_{GS} - V_T \quad \text{at limit of saturation}$$

$$V_{DSAT11} = \sqrt{\frac{2 I_2}{\mu C_{ox} \frac{W_{11}}{L_{11}}}}$$

III Miller-compensated op-amp.



- ③ We could use an NMOS 2nd stage but we would need to ensure its $V_{GS} = V_{Tn} + V_{DSAT11}$ was equal to $V_{DD} - V_{SG3}$ which is more inconvenient to match.
- ④ Load of 2nd stage is a current source, to get high DC gain on 2nd stage.

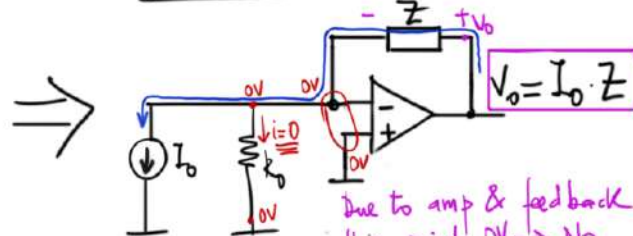


- ① Our intention is to get a voltage amplifier/integrator.
- ② We want to 'convert' the current $G_m V_e$ to a voltage V_o by pushing the current into the cap C .
- ③ * BUT some current flows into R_o hence we don't get all current into cap C . So we don't get full $V_o = \frac{G_m V_e}{sC}$ but a divided version $V_o = \frac{G_m V_e}{sC + G_o}$

It's like this:

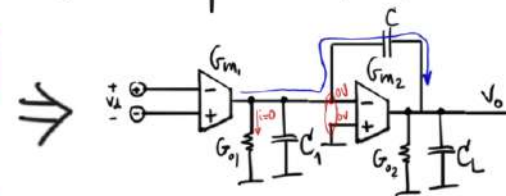
Due to R_o we don't get $V_o = I_o \cdot Z$ but we get $V = I_o \cdot (Z // R_o)$ (which is lower).

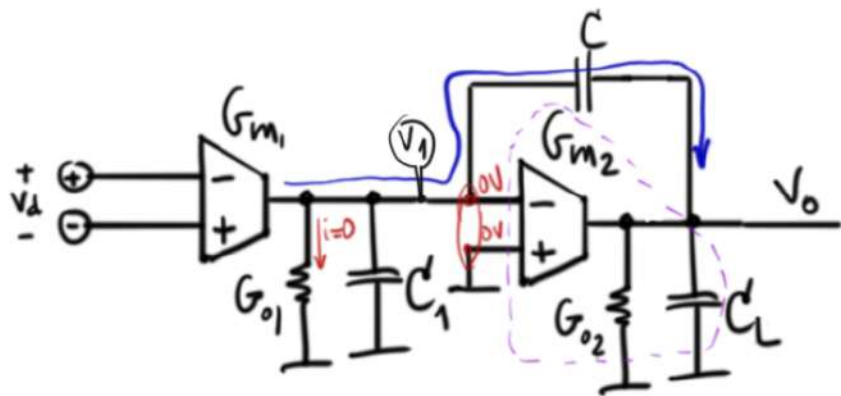
Solution = CCVS



Due to amp & feedback this point 0V \Rightarrow No current flows through $R_o \Rightarrow$ All current flows through target $Z \Rightarrow$ We achieve $V_o = I_o \cdot Z$ as desired. (Z can be a CAP)

This is why we stack the 2 stages like this:
(Miller-compensated op amp)





* It's 2nd order. 2 poles.

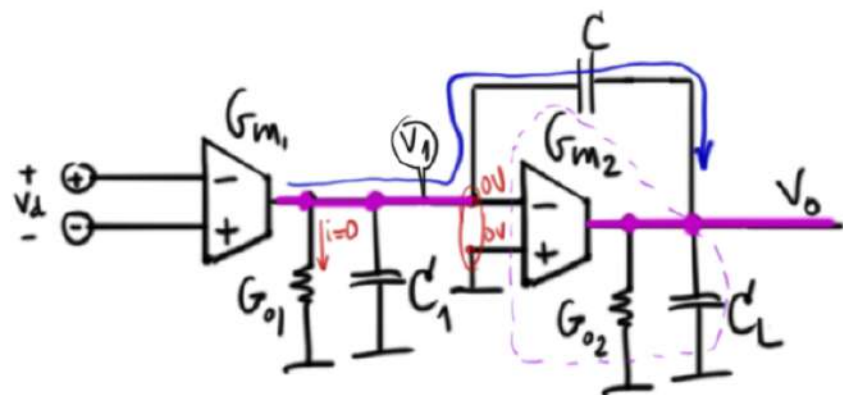
(even though we have 3 caps, but they are in a loop, so only 2 cap voltages are independent).

* DC gain:
at DC, caps are open circuit.

$$I_{out1} = G_{m1} v_d; \quad v_{out1} = \frac{G_{m1} v_d}{G_{01}}$$

$$I_{out2} = G_{m2} v_{out1} = \frac{G_{m1} G_{m2} v_d}{G_{01}}; \quad v_o = \frac{I_{out2}}{G_{02}} = \frac{G_{m1} G_{m2}}{G_{01} G_{02}} v_d; \Rightarrow$$

$$\Rightarrow \boxed{A_o = \left. \frac{v_o}{v_d} \right|_{DC} = \frac{G_{m1}}{G_{01}} \frac{G_{m2}}{G_{02}} = G_{m1} R_{01} \cdot G_{m2} R_{02}}$$



We have 2 nodes (V_1 & V_0). $\triangle \begin{matrix} \Delta \\ 2R \end{matrix}$ ($Y \cdot V = I$)

$$\begin{matrix} \text{Admittance matrix} \\ 2 \times 2 \end{matrix} \times \begin{matrix} \text{Vector of voltages} \\ 2 \times 1 \end{matrix} = \begin{matrix} \text{Vector of currents} \\ 2 \times 1 \end{matrix}$$

Current flowing into V_1 due to V_0

Total admittance hanging off node V_1

$$\begin{bmatrix} s(C_1 + C) + G_{01} & -sC \\ G_{m2} - sC & s(C_L + C) + G_{02} \end{bmatrix} \times \begin{bmatrix} V_1 \\ V_0 \end{bmatrix} = \begin{bmatrix} -G_{m1}V_e \\ 0 \end{bmatrix}$$

Current flowing into node V_0 (from G_{m2} & from C cap) due to V_1

Total admittance hanging off node V_0

We solve V_o
with Cramer's rule:

$$V_o = \frac{\begin{vmatrix} s(C_1+C)+G_{o1} & -G_{m1}V_e \\ G_{m2}-sC & 0 \end{vmatrix}}{\begin{vmatrix} s(C_1+C)+G_{o1} & -sC \\ G_{m2}-sC & s(C_L+C)+G_{o2} \end{vmatrix}} =$$

vector of currents substituted
in place of 2nd column so
we get V_o .

$$\begin{aligned} & C_1 C_L + C^2 + C_1 C + C_L C \\ & s^2 (C_1+C)(C_L+C) + G_{o1} G_{o2} + \\ & s G_{o2} (C_1+C) + s G_{o1} (C_L+C) \\ & + s C G_{m2} - s^2 C^2 \end{aligned}$$

$$= \frac{G_{m1}(G_{m2}-sC)V_e}{s^2(C_1C + CC_L + C_1C_L) + s[C(G_{m2}+G_{o1}+G_{o2}) + C_LG_{o1} + C_1G_{o2}] + G_{o1}G_{o2}}$$



$$\Rightarrow \frac{V_o}{V_e} = \frac{G_{m1}(G_{m2} - sC)}{s^2(C_1C + CC_L + C_1C_L) + s[C(G_{m2} + G_{o1} + G_{o2}) + C_LG_{o1} + C_1G_{o2}] + G_{o1}G_{o2}}$$

DC gain: ($s=0$) 2nd order (as expected)

$$\left. \frac{V_o}{V_e} \right|_{s=0} = \frac{G_{m1} G_{m2}}{G_{o1} G_{o2}}$$

(as expected)

THE FULL TRANSFER FUNCTION OF THE 2-STAGE MILLER-COMPENSATED OP-AMP.

2 poles:

1 zero: $z_1 = + \frac{G_{m2}}{C}$

* To get these values we do an APPROXIMATION.
Too complicated to solve $as^2 + bs + c = 0$
We assume $|s_1| \ll |s_2|$ (the 2 roots)
so $s_1 \approx -\frac{c}{b}$ & $s_2 \approx -\frac{b}{a}$

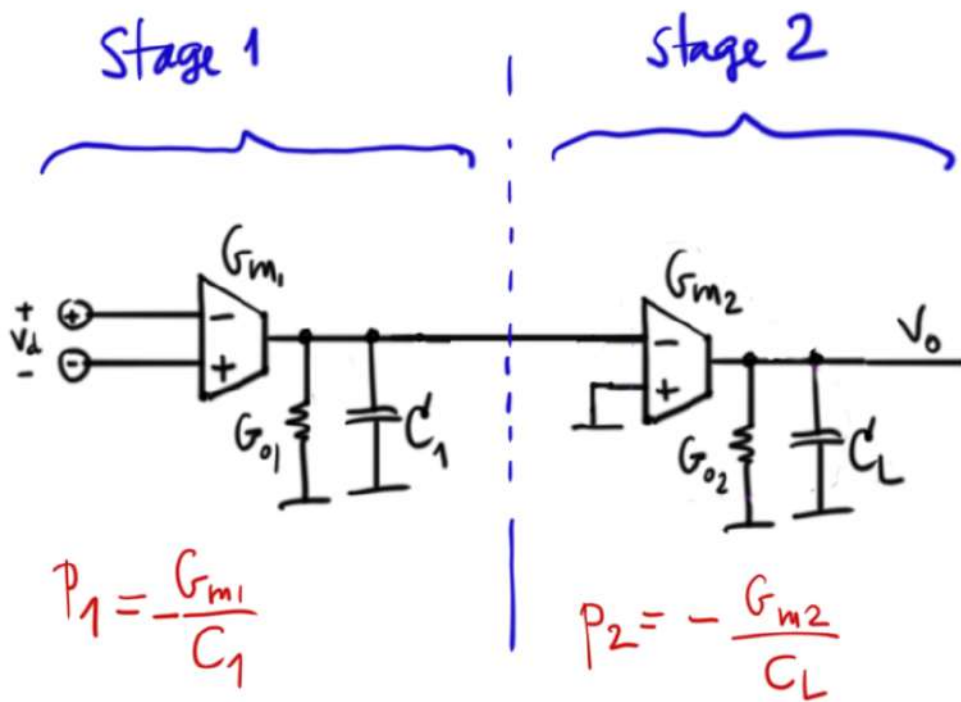
Using the previous approximation, the poles are at:

$$p_1 \approx -\frac{c}{b} = \frac{g_{m1} g_{o2}}{C(g_{m2} + g_{o2} + g_{o1}) + C_1 g_{o2} + C_L g_{o1}} = \boxed{\frac{g_{o1}}{C\left(\frac{g_{m2}}{g_{o2}} + 1 + \frac{g_{o1}}{g_{o2}}\right) + C_1 + C_L \frac{g_{o1}}{g_{o2}}}} \quad p_1$$

$$p_2 \approx -\frac{b}{a} = -\frac{C(g_{m2} + g_{o2} + g_{o1}) + C_1 g_{o2} + C_L g_{o1}}{C C_1 + C_1 C_L + C_L C_1} = \boxed{-\frac{\frac{C}{C+C_1} g_{m2} + g_{o2} + g_{o1} \frac{C+C_L}{C+C_1}}{C_L + \frac{C_1 C}{C+C_1}}} \quad p_2$$

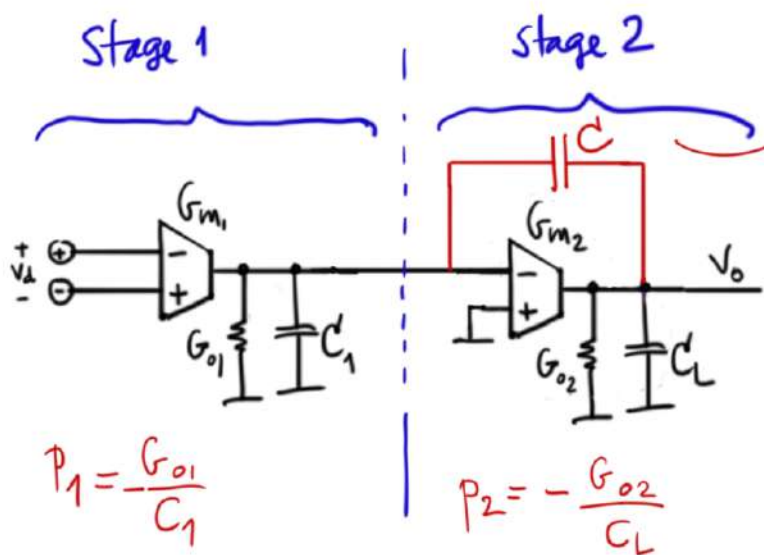
\uparrow
 $\times 8 \div (C+C_1)$

Poles by inspection:



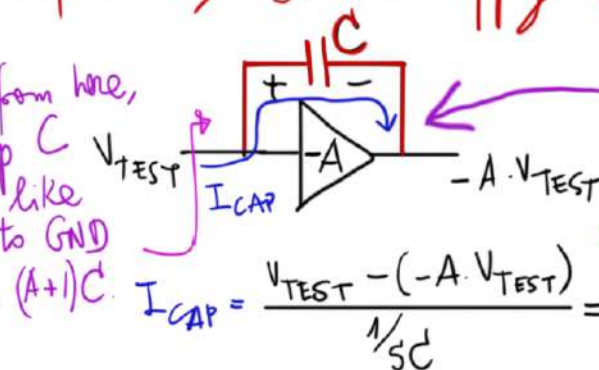
- * If 2 stages concatenated, the 2 transfer functions multiply.
- * So the 2 poles are just the 2 individual poles of the single stages.

Poles by inspection:

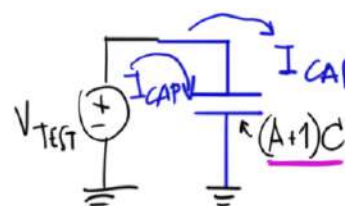


If cap around stage 2, we can no longer look at poles by inspection like before. \Rightarrow So we apply MILLER'S THEOREM.

Seen from here, the cap C looks like a cap to GND of value $(A+1)C$.



MILLER'S THEOREM



$C_{MM} = (A+1)C = \left(\frac{G_{m2}}{G_{o2}} + 1\right)C$

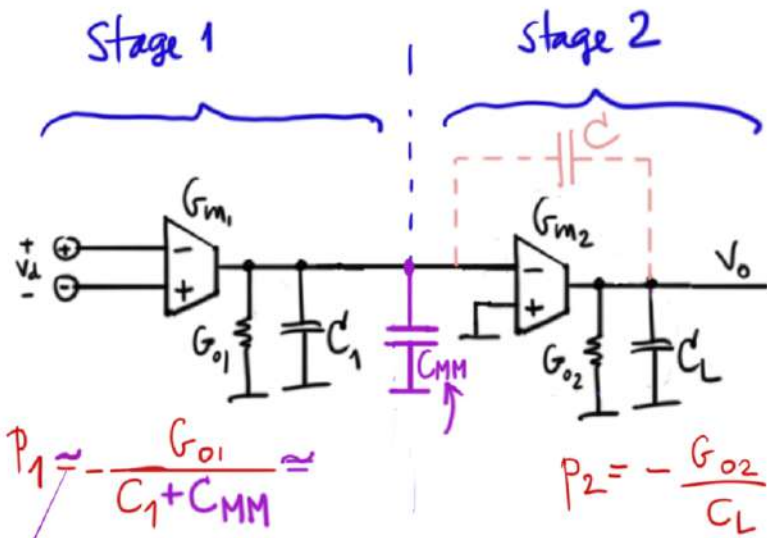
C_{MM} = "Miller-multiplied cap" seen from the INPUT side. Let's call this "CMM"

$C_{MM} = (A+1)C$

Where A is the voltage gain of the 2nd stage $= A_{o2} = G_{m2} \cdot R_{o2} = \frac{G_{m2}}{G_{o2}}$

So $\Rightarrow \boxed{C_{MM} = \left(1 + \frac{G_{m2}}{G_{o2}}\right)C}$

Poles by inspection:



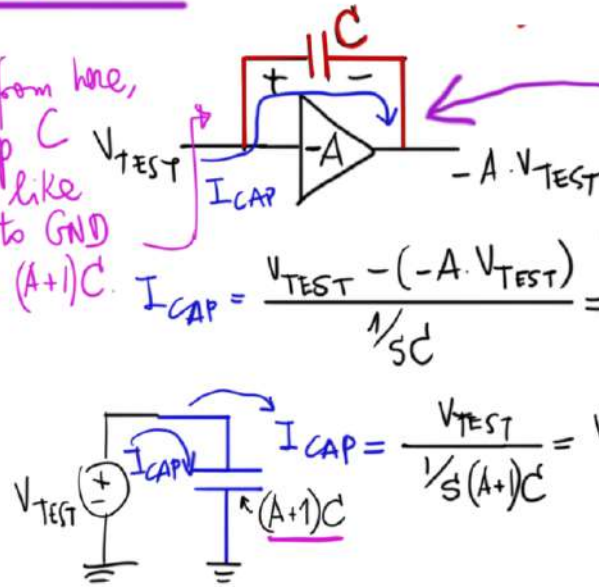
Approximation since 2nd stage is not an ideal opamp.

$$\approx \frac{G_{01}}{C_1 + \left(\frac{G_{m2}}{G_{02}} + 1\right)C}$$

- New expression for the dominant pole p_1 .
- Notice how it has shifted down in frequency (pole splitting)

Miller's Theorem.

Seen from here, the cap C looks like a cap to GND of value $(A+1)C$.



$$I_{CAP} = \frac{V_{TEST} - (-A \cdot V_{TEST})}{1/sC} = (A+1)V_{TEST} \cdot sC$$

$$I_{CAP} = \frac{V_{TEST}}{1/s(A+1)C} = V_{TEST} \cdot s(A+1)C$$

MILLER'S THEOREM

the same

$$C_{MM} = (A+1)C = \left(\frac{G_{m2}}{G_{02}} + 1\right)C$$

C_{MM} = "Miller-multiplied cap" seen from the INPUT side.

without Miller cap C

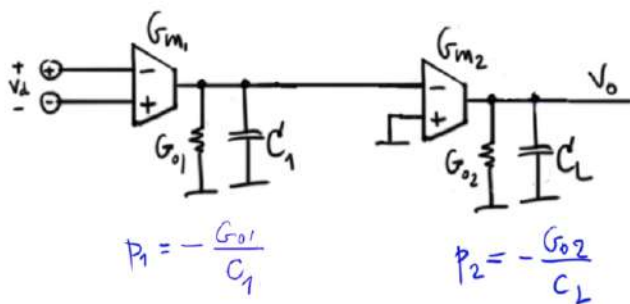
$$p_1 = -\frac{G_{o1}}{C_1}$$

p_1 moved down to lower frequency

$$p_2 = -\frac{G_{o2}}{C_L}$$

"POLE SPLITTING"

p_2 moved to higher frequency



With Miller cap C

$$-\frac{G_{o1}}{C\left(\frac{G_{m2}}{G_{o2}} + 1 + \frac{G_{o1}}{G_{o2}}\right) + C_1 + C_L \frac{G_{o1}}{G_{o2}}} \approx -\frac{G_{o1}}{C_1 + \left(\frac{G_{m2}}{G_{o2}} + 1\right)C}$$

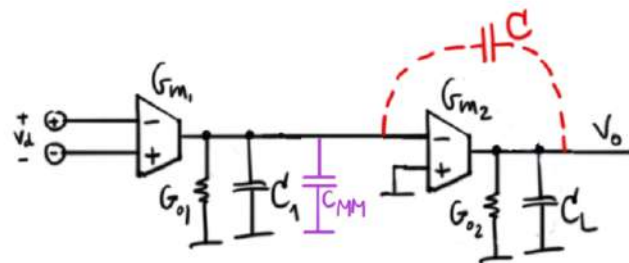
Approximation: this term $C_L \frac{G_{o1}}{G_{o2}}$ happens to be of very low significance when compared to C_1 and to $\left(\frac{G_{m2}}{G_{o2}} + 1\right)C$.

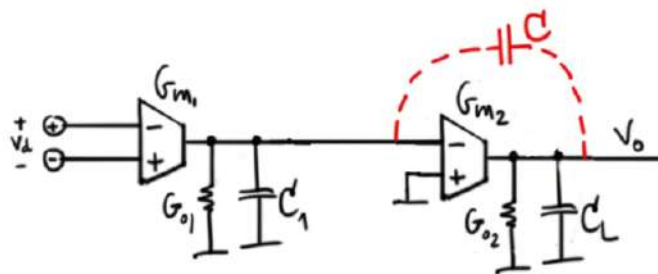
$$-\frac{\frac{C}{C+C_1} G_{m2} + G_{o2} + G_{o1} \frac{C+C_L}{C+C_1}}{C_L + \frac{C \cdot C}{C+C_1}} \approx -\frac{G_{m2} \frac{C}{C+C_1} + G_{o2}}{C_L + \frac{C \cdot C_L}{C+C_1}}$$

Approximation: This term $\frac{G_{o1}}{G_{o2}}$ is small in comparison with $\frac{G_{m2}}{G_{o2}} + 1$ so we can neglect it.

Approximation: this term we also usually neglect it.

This is the expression for p_1 that we will always use





2-STAGE OPAMP (*) At transconductor level

DC gain: $A_D = \frac{G_{m1}}{G_{o1}} \frac{G_{m2}}{G_{o2}}$ Higher DC gain on 2-stage opamp compared to single stage. This is one key advantage.

$$p_1 \approx - \frac{G_{o1}}{C \left(\frac{G_{m2}}{G_{o2}} + 1 \right) + C_1}$$

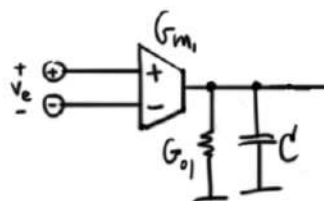
$$p_2 \approx - \frac{G_{m2} \cdot \frac{C}{C+C_1} + G_{o2}}{C_L + \frac{C \cdot C_1}{C+C_1}}$$

$$z = + \frac{G_{m2}}{C}$$

$$\omega_u = A_D \cdot p_1 = \frac{G_{m1} G_{m2}}{G_{o1} G_{o2}} \cdot \frac{G_{o1}}{C \left(\frac{G_{m2}}{G_{o2}} + 1 \right) + C_1} =$$

$$= \frac{G_{m1} G_{m2}}{\underbrace{C \cdot G_{m2}}_{\text{longest term}} + C G_{o2} + C_1} \approx \frac{G_{m1} G_{m2}}{C \cdot G_{m2}} \approx \frac{G_{m1}}{C}$$

Same ω_u as the single stage one.



SINGLE STAGE OPAMP (*)

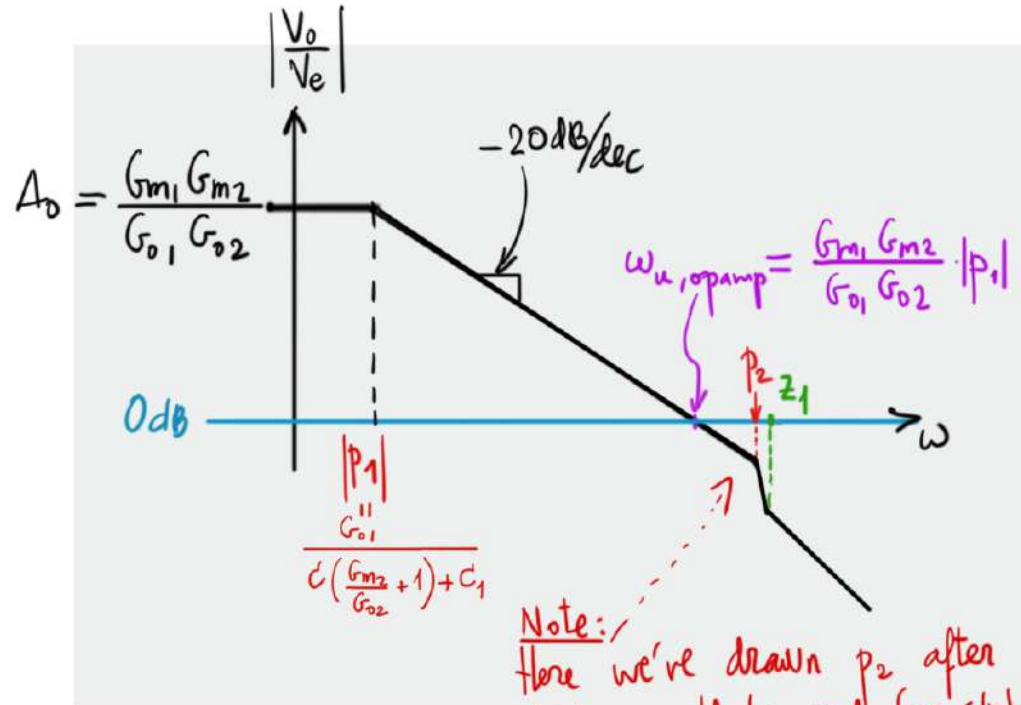
(*) At transconductor level, not at transistor level (no parasitic caps considered, hence just single pole system)

DC gain: $A_D = G_{m1} \cdot R_{out} = \frac{G_{m1}}{G_{o1}}$

$$p_1 = \frac{G_{o1}}{C}$$

$$\omega_u = A_D \cdot p_1 = \frac{G_{m1}}{C}$$

2-STAGE OPAMP (*) At transconductor level



Note:
 Here we've drawn p_2 after w_u which would be good for stability. However it's not like p_2 value will always be higher than w_u , (as was the case in the single stage opamp), so we need to ensure p_2 is at high enough freq.

We call $w_{u, \text{opamp}}$ the "open loop" transfer function of the circuit, WITHOUT FEEDBACK

$$\Rightarrow w_{u, \text{opamp}} = \frac{G_{m1} G_{m2}}{G_{o1} G_{o2}} \cdot \frac{G_{o1}}{C(\frac{G_{m2}}{G_{o2}} + 1) + C_1} \approx \frac{G_{m1}}{C}$$

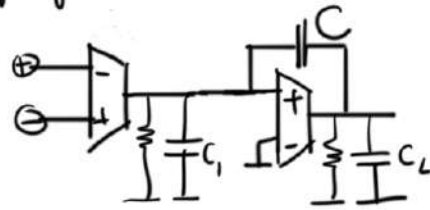
$w_{u, \text{opamp}} = A_0 \cdot p_1$

This matches the w_u of the single stage opamp.

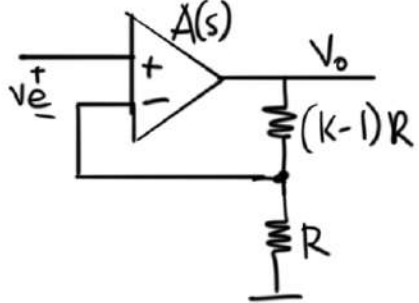
Is the 2-stage op-amp "unconditionally" stable?
No, not unconditionally.

The transfer function of the opamp (WITHOUT FEEDBACK)

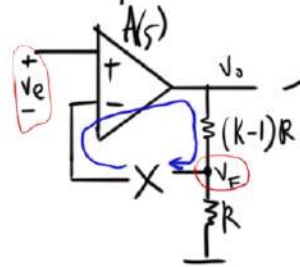
$$A(s) = \frac{V_o}{V_e} = \frac{A_o \left(1 - \frac{s}{z_1}\right)}{\left(1 + \frac{s}{-p_1}\right) \left(1 + \frac{s}{-p_2}\right)}$$



Now if we use this in a feedback configuration:



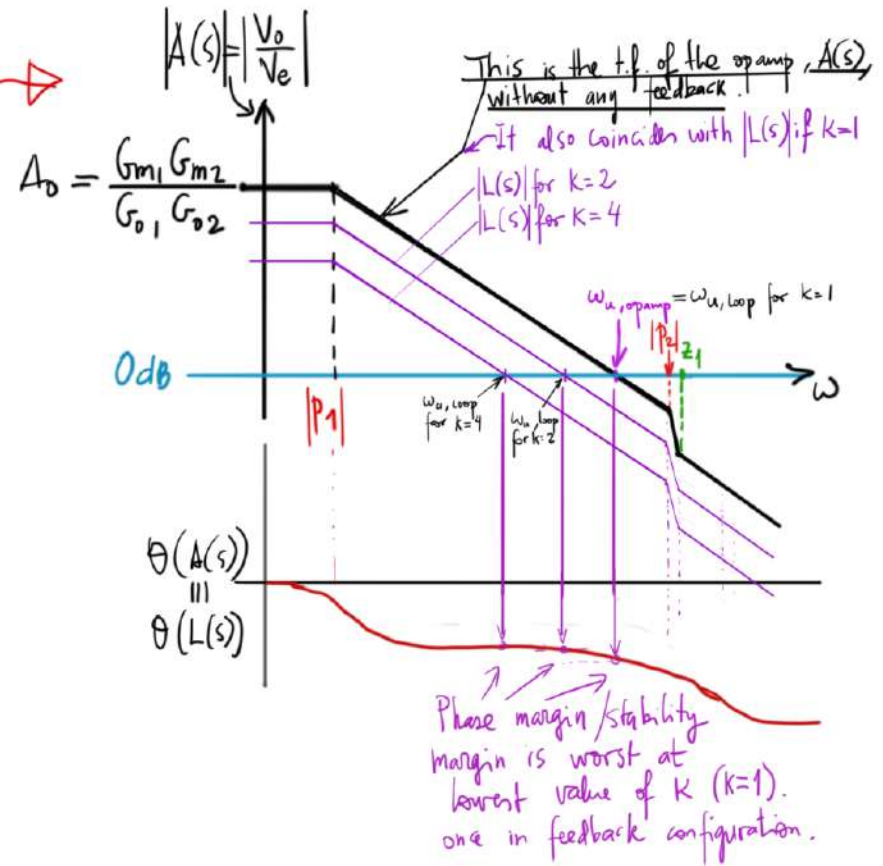
The LOOP GAIN $L(s)$ of this is (breaking the loop & going around it)



$$V_o = A(s) \left(\underbrace{V_p - V_m}_{V_e} \right); V_e = \frac{V_o}{A(s)}$$

$$V_F = \frac{R}{(K-1)R + R} V_o = \frac{R}{KR} V_o = \frac{V_o}{K}$$

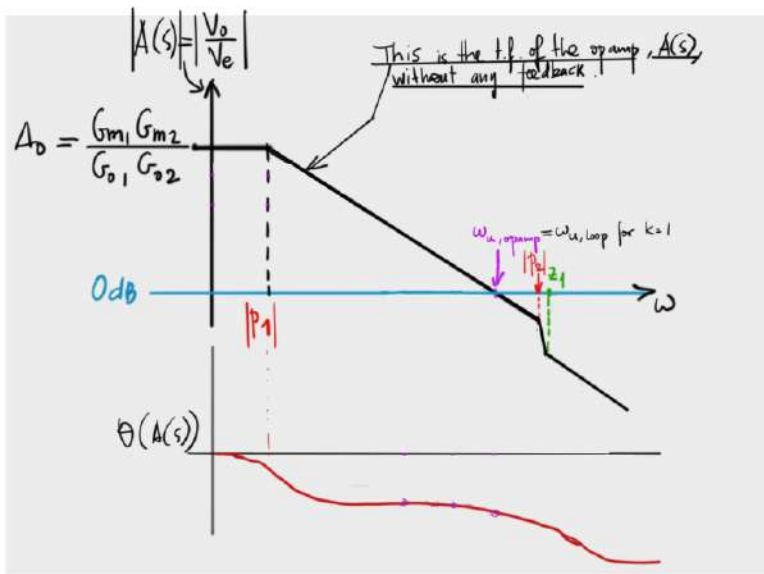
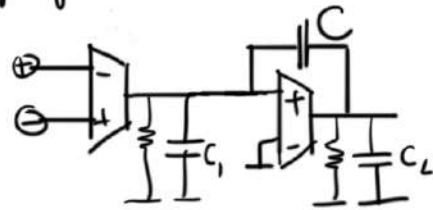
$$L(s) = \frac{V_F}{V_e} = \frac{V_o/K}{V_o/A(s)} = \frac{A(s)}{K}$$



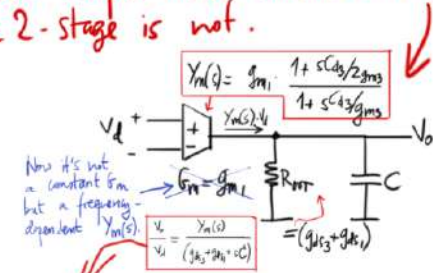
Is the 2-stage op-amp "unconditionally" stable?
No, not unconditionally.

The transfer function of the opamp (WITHOUT FEEDBACK)

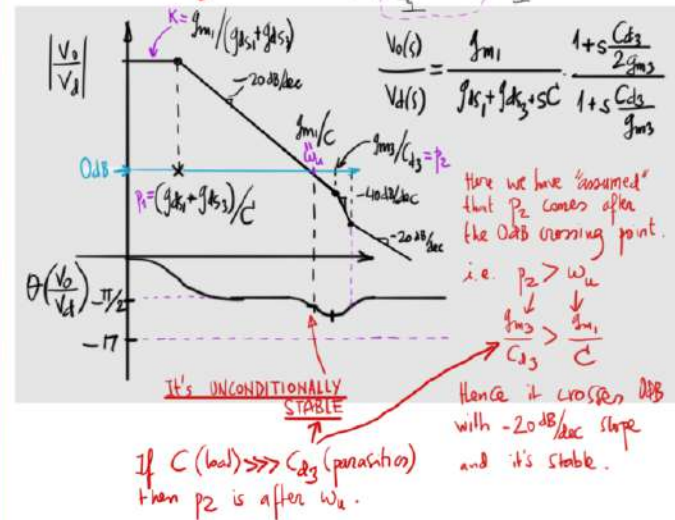
$$A(s) = \frac{V_o}{V_e} = \frac{A_o \left(1 - \frac{s}{z_1}\right)}{\left(1 + \frac{s}{-p_1}\right) \left(1 + \frac{s}{-p_2}\right)}$$



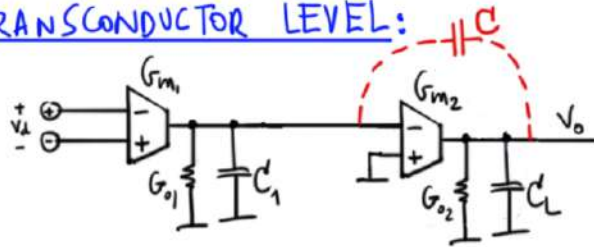
REMINDER, the single stage op amp (5T) with para' caps was UNCONDITIONALLY STABLE but the 2-stage is not.



This is the freq response WITHOUT FEEDBACK



TRANSCONDUCTOR LEVEL:



2-STAGE OPAMP(*) At transconductor level

$$\text{DC gain: } A_0 = \frac{G_{m1}}{G_{o1}} \frac{G_{m2}}{G_{o2}}$$

$$p_1 \approx - \frac{G_{o1}}{C \left(\frac{G_{m2}}{G_{o2}} + 1 \right) + C_1}$$

$$p_2 \approx - \frac{G_{m2} \cdot \frac{C}{C+C_1} + G_{o2}}{C_L + \frac{C \cdot C_1}{C+C_1}}$$

$$z = + \frac{G_{m2}}{C}$$

$$\text{TF: } \frac{V_o}{V_e} = A_0 \cdot \frac{1 - s/z_1}{\left(1 + \frac{s}{-p_1}\right) \left(1 + \frac{s}{-p_2}\right)}$$

Higher DC gain on 2-stage opamp compared to single stage. This is one key advantage.

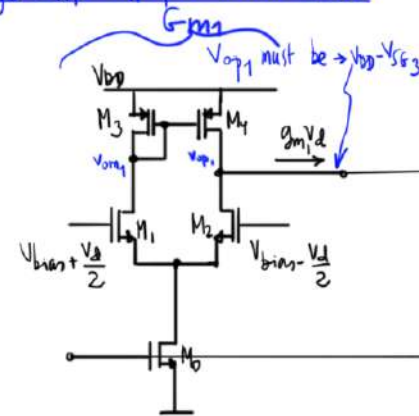
$$\omega_u = A_0 \cdot p_1 = \frac{G_{m1} G_{m2}}{G_{o1} G_{o2}} \cdot \frac{G_{o1}}{C \left(\frac{G_{m2}}{G_{o2}} + 1 \right) + C_1} =$$

$$= \frac{G_{m1} G_{m2}}{C \cdot G_{m2} + C \cdot G_{o2} + C_1} \approx \frac{G_{m1} G_{m2}}{C \cdot G_{m2}} \approx \frac{G_{m1}}{C}$$

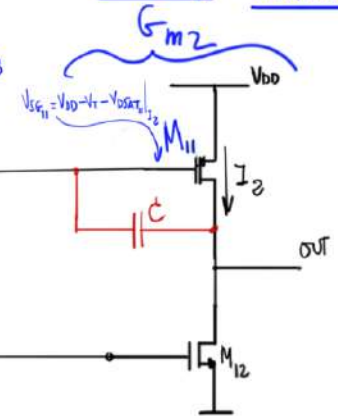
Same ω_u as the single stage one.

TRANSISTOR LEVEL:

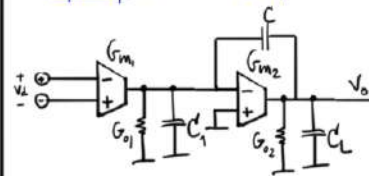
1st stage: G_{m1} : Diff pair transconductor



2nd stage: Single transistor



Miller-compensated op-amp.



$$\text{DC gain: } A_0 = \frac{G_{m1}}{G_{o1}} \frac{G_{m2}}{G_{o2}} \quad \omega_u \approx \frac{G_{m1}}{C}$$

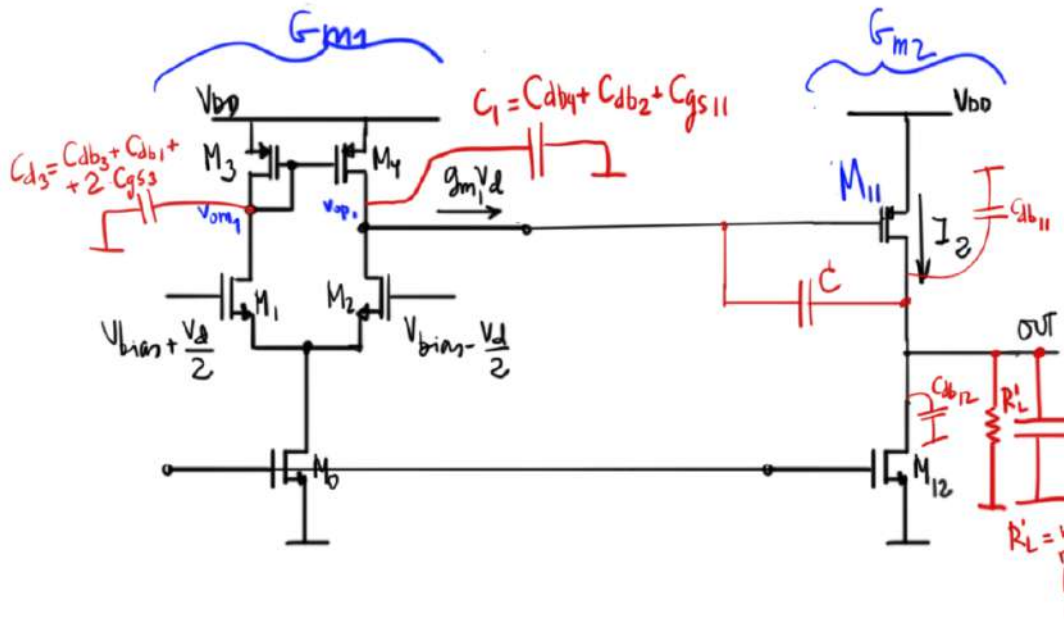
$$p_1 \approx - \frac{G_{o1}}{C \left(\frac{G_{m2}}{G_{o2}} + 1 \right) + C_1}$$

$$p_2 \approx - \frac{G_{m2} \cdot \frac{C}{C+C_1} + G_{o2}}{C_L + \frac{C \cdot C_1}{C+C_1}}$$

$$z = + \frac{G_{m2}}{C}$$

$$\text{TF: } \frac{V_o}{V_e} = A_0 \cdot \frac{1 - s/z_1}{\left(1 + \frac{s}{-p_1}\right) \left(1 + \frac{s}{-p_2}\right)}$$

LET'S IDENTIFY COMPONENT VALUES:



- $C_1 = C_{db4} + C_{db2} + C_{gs11}$

- $C_L = C_L' + C_{db12} + C_{db11}$

Actual values of these we usually just get the values from the simulator.

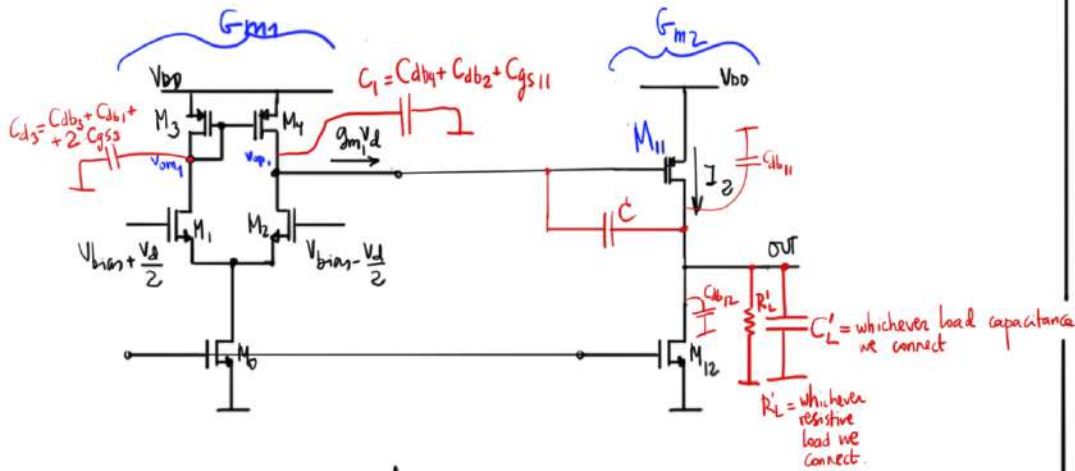
- $G_{o1} = g_{ds1} + g_{ds3} \quad (= \frac{1}{r_{ds1} // r_{ds3}})$

- $G_{o2} = g_{ds11} + g_{ds12} + G_L'$

- $Y_m(s) = g_{m1} \cdot \frac{1 + s \frac{C_{d3}}{2g_{m3}}}{1 + s \frac{C_{d3}}{g_{m3}}} \leftarrow Y_m(s) \text{ is what we use in place of } G_{m1}$

- $G_{m2} = g_{m11}$

AT TRANSISTOR LEVEL:



NOISE: Same as single stage one.

$$S_{V_{in}} = \frac{16}{3} \frac{KT}{g_{m1}} \left(1 + \frac{g_{m3}}{g_{m1}} \right)$$

OFFSET:

Random: Same as single stage one.

$$\sigma_{V_{os}}^2 = \sigma_{V_{T12}}^2 + \sigma_{V_{T34}}^2$$

Systematic: Minimize it by equalizing current densities M_4 & M_{11} .

$$\text{Otherwise } \Delta V = \frac{V_{GS3} - V_{GS11}}{g_{m1}}$$

TF of 2-stage opamp:

$$TF: \frac{V_o}{V_e} = A_0 \cdot \frac{1 + s \frac{C_{d3}}{2g_{m3}}}{1 + s \frac{C_{d3}}{g_{m3}}} \cdot \frac{1 - s/z_1}{(1 + \frac{s}{-p_1})(1 + \frac{s}{-p_2})}$$

Inserting values:

Note: Here it's better to insert values into the "sub-components" of the TF (DC gain, p_1 , p_2 , p_3 ...) rather than trying to get a long expression for TF.

Specific values at transistor level:

$$A_0 = \frac{g_{m1}}{g_{ds1} + g_{ds3}} \cdot \frac{g_{m11}}{g_{ds11} + g_{ds12} + G_L}$$

$$w_u = \frac{g_{m1}}{C}$$

$$p_2 = - \frac{g_{m11} \cdot \frac{C}{C+C_1} + G_{o2}}{\frac{C \cdot C_1}{C+C_1} + C_L}$$

$$z_1 = + \frac{g_{m11}}{C}$$

$g_{m1} \sim g_{m3}$
 $C \gg C_{d3}$ } z_1 lower freq than z_2

$$p_1 = - \frac{g_{ds1} + g_{ds3}}{C \left(\frac{g_{m11}}{G_{o2}} + 1 \right) + C_1}$$

$$p_3 = - \frac{g_{m3}}{C_{d3}}$$

$$z_2 = - \frac{2g_{m3}}{C_{d3}}$$

$$\begin{aligned}
 \text{DC gain: } A_0 &= \frac{g_{m1}}{G_{o1}} \frac{g_{m2}}{G_{o2}} \\
 p_1 &\approx - \frac{G_{o1}}{C \left(\frac{g_{m2}}{G_{o2}} + 1 \right) + C_1} \\
 p_2 &\approx - \frac{g_{m2} \cdot \frac{C}{C+C_1} + G_{o2}}{C_L + \frac{C \cdot C_1}{C+C_1}} \\
 z_1 &= + \frac{g_{m2}}{C} \\
 w_u &\approx \frac{g_{m1}}{C}
 \end{aligned}$$

EXAMPLE:

Let's say:
We want $A_0 = 1000$

with an $R_L = 1\text{K}\Omega$
($G_L = 1\text{mS}$)

With a single stage opamp:

$$A_0 = g_{m1} (\underbrace{r_{ds1} // r_{ds3}}_{\approx R_L}) \approx g_{m1} R_L \Rightarrow$$

$$\Rightarrow g_{m1} = \frac{A_0}{R_L} = \frac{1000}{1000\Omega} = 1\text{S} = \underline{\underline{1000\text{mS}}}$$

To get a g_{m1} of
 1000mS we would
need a huge current
so big power dissipation.

With a 2-stage opamp:

$$A_0 = \frac{g_{m1}}{g_{ds1} + g_{ds3}} \cdot \frac{g_{m11}}{g_{ds11} + g_{ds12} + \underbrace{G_L'}_{1\text{K}\Omega}} \approx \underbrace{\frac{g_{m1}}{g_{ds1} + g_{ds3}}}_{\sim 100 \times \text{from 1st stage}} \cdot \underbrace{\frac{g_{m11}}{G_L'}}_{10 \text{ from 2nd stage}} = 100 \times 10 = 1000$$

From 2nd stage we need to get only a gain of ~ 10 :

$$\frac{g_{m11}}{G_L'} = 10 \Rightarrow \underline{\underline{g_{m11} = 10}} \cdot G_L' = \frac{10}{R_L} = \frac{10}{1\text{K}\Omega} = \frac{1}{100} = \underline{\underline{10\text{mS}}}$$

From 1st stage we need to get a gain of ~ 100 :

$$\frac{g_{m1}}{g_{ds1} + g_{ds3}} = 100 \rightarrow$$

From stability considerations we know that
 G_{m1} has to be significantly less than G_{m2} .
So if $G_{m2} = g_{m11} = 10\text{mS}$, we make $\underline{\underline{g_{m1} = 2\text{mS}}}$
and we get $g_{ds1} + g_{ds3} \approx$

$$\frac{g_{m1}}{g_{ds1} + g_{ds3}} = 100; \quad \underline{\underline{g_{ds1} + g_{ds3} = \frac{1}{(r_{ds1} // r_{ds3})}}} = \frac{g_{m1}}{100} = \frac{2\text{mS}}{100} = \underline{\underline{0.02\text{S}}} \Rightarrow$$
$$\Rightarrow \underline{\underline{r_{ds1} // r_{ds3} = \frac{1}{0.02\text{S}} = 50\text{K}\Omega}}$$

↑
These values
are more
sensible &
easy to
achieve